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## FORM FACTORS, CHARGE DISTRIBUTION AND THE MEASUREMENT

ASPECT

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## FORM FACTORS, CHARGE DISTRIBUTION AND THE MEASUREMENT ASPECT

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## ABSTRACT

We analyse some aspects of the measurement problem in Relativistic Quantum Field Theory. Although, as suggested in the pioneering work of Bohr and Rosenfeld, what is measurable in Q.F.T. are non local (smeared) operators, no attempt has been made, as yet, in establishing a connection between these objects and quantities which are accessible experimentally, like Form Factors. This paper is an attempt in this direction. I. INTRODUCTION

One of the problems associated to any measurement process is related to the appearence of undesirable effects due to the presence of the measurement apparatus. In order to probe the charge density of a given system, for instance, one has to perform experiments in which momentum and energy are exchanged between the system and an appropriate test body. As the energy increases, the apparatus will disturb the system. Examples of such perturbations are the possibility of exciting the system (innelastic channels) and, within the relativistic context, vacuum fluctuations.

The appearence of such uncontrollable effects of the apparatus is what prevents us from making measurements of densities and Field strenghts at a given point in space. This problem was raised first by Landau and Peierls<sup>(1)</sup>. We intend to make this point clear in section II. Our line of reasoning is different from that of ref.(1) and is based, in the quantum relativistic context, on a subtle consequence of the effect of pair creations which is reflected on the bad properties of the charge operator obtained from integrals of local densities.

As far as the effects of the apparatus are concerned, there is always the possibility of eliminating them or to seek some sort of compensation for them. This program is implementable at the cost of abandoning local measurements. This is the lesson which we have learned from the pioneering work of Bohr and Rosenfeld<sup>(2)</sup> who in a series of papers showed that one can get rid of pair effects if one measures, in contradistinction with measurement of  $J_0(X,t)$  at a given point in space time, averages of the density operator over space-time regions, namely

 $\overline{J}_{o} \equiv \frac{i}{VT} \int d^{3}x \int dt \ J_{o}(x,t)$ 

(I.1)

Although the objects defined in (1.1) are measurable, in principle, the experiments required to achieve this are very difficult to be implemented and, as far as we know, has never been done. This comment applies to others "smeared operators".

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In this paper we will be concerned with a set of nonlocal operators which, in the one-photon exchange approximation, implements the program sought by Bohr and Rosenfeld and whose correspondence with what is accessible experimentally can be implemented. In section III we will present the requirements imposed on the physically relevant set of operators. An explicit example, taken from the Non Relativist Quantum Mechanical context, illustrating the scheme and the ideas is presented in section IV. We end this paper with some conclusions in section V.

II - VACUUM FLUCTUATIONS AND THE CHARGE OPERATOR

In order to understand the difficulties associated with pair creations in implementing local measurements we will analyse the properties of the charge operator obtained by integrating local operators.

Let us define weight functions  $\theta_{1}$ ,  $\theta_{2}$  by

 $\theta_{R}(\mathbf{x}) = e^{-\frac{\mathbf{x}^{2}/\mathbf{R}^{2}}{2}}$ 

 $\Theta_{\tau}(t) = e^{-t \frac{1}{2} \tau t}$ (II.1)

The smeared operator  $\int_{a_{\tau}}^{\mu} (\mathbf{x}, t)$  defined from the local operator  $\int_{\mu} (\mathbf{x}, t)$  as

 $J_{RT}^{\mu}(\mathbf{x},t) = \frac{1}{\left(\sqrt{\pi} R\right)^{3} \left(\sqrt{\pi} T\right)} \int d^{3}\mathbf{x} dt \, \Theta_{T}(t-t) \Theta_{R}(\mathbf{x},\mathbf{x}) \, \tilde{J}^{\mu}(\mathbf{x}',t) \qquad (II.2)$ 

can be interpreted, in close analogy with the operator defined in (I.1), as the averaged value of  $J_{\mu}$  in a 4-sphere centered at P = (X, t) whereas the operator

$$Q_{RT} = (V_{TT} R)^3 J_{RT}^{\circ}(0,0)$$
 (II.3)

can be interpreted as the charge measured within a sphere of radius R centered at the origin averaged over the time interval T. We have defined the operators in (II.2) and (II.3) in such a way that the naive charge operator and the local density operator are obtained as limits of (II.2) and (II.3), namely

$$\begin{aligned} \mathbf{k} &= \lim_{R \to \infty} \lim_{T \to 0} \mathbb{Q}_{RT} &= \int d^{3}\mathbf{x}^{i} \, \mathbf{J}^{0}(\mathbf{x}^{i}, t^{i}) \\ \mathbf{J}^{A}(\mathbf{x}, t) &= \lim_{R \to 0} \lim_{T \to 0} \int \mathbf{J}^{A}(\mathbf{x}, t) \\ \mathbf{R} &= 0 \text{ T} \to 0 \end{aligned}$$
(II.4)

In the relativistic case, one of the most important effects is the appearance of vacuum polarization induced by the apparatus. In order to understand this effect let us study the vacuum polarization charge density induced by an external charge distribution. The induced charge polarization density will be infered from the matrix element

$$\langle \psi(t) | J_{(x,t)}^{\circ} | \psi(t) \rangle = \langle 0 | U_{(t,0)}^{\circ} | J_{0}(x,t) | U_{(t,0)} | 0 \rangle$$
 (II.5)

where U(t,0) is the time evolution operator

$$\mathbf{U}(\mathbf{t},\mathbf{o}) \equiv \mathbf{T} \exp - \mathbf{i} \int_{-\infty}^{\infty} d\mathbf{k}^{*} \int d^{2}\mathbf{x}^{*} \mathbf{J}^{*}(\mathbf{x}^{*},\mathbf{t}^{*}) \mathbf{A}_{\mu}(\mathbf{x}^{*},\mathbf{t}^{*}) \qquad (\mathbf{II}.6)$$

In lowest order perturbation theory one can write

$$\langle \psi_{lb} | J_{0}(\mathbf{x}',t') | \psi_{lb} \rangle = -i \int d^{3}x'' \int dt' A_{\mathbf{x}}^{act} (\mathbf{x}'',t'') \theta(t-t') \langle 0 | L J''_{\mathbf{x}}',t'' \rangle J_{0}(\mathbf{x}',t') | h \rangle (II.7)$$

For t' $\langle$  t the result obtained from (II.7) can be interpreted as the effect of the measurement on the state at later times, and for t'>t as the effect of the developing state on a measurement in the future<sup>(3)</sup>.

Following ref.(3), let us consider a very simple, and important for what follows, example in order to illustrate the situation in the relativistic case. Suppose  $A_{jk}^{vit} \approx (A_{\sigma}^{vit}, \mathfrak{O})$  and that Ab (4.1) is given by

$$A_{o}^{\text{sub}} = \frac{\Theta_{T_{s}}(t)}{\sqrt{\pi} T_{s}} \qquad (II.8)$$

The charge density associated to (II.8) can be easily infered. The most important feature is that for  $|\mathbf{x}| \leq R_5$  the charge density is approximately constant and given by  $\mathcal{P}(\mathbf{x}) \sim \frac{1}{R_5^2}$ For the charge density given by (II.8) it can be shown<sup>(3)</sup>, by using the Jost-Lehman representation that

$$\langle \psi(t) | J^{\circ}(x_{1},t') | \psi(t) \rangle \sim \frac{1}{R_{s}^{2}}$$
 (II.9)

which means that the polarization charge follows the external charge distribution for  $|\kappa|<R_{s}$  .

The asymptotic behavior of  $Q_{\rm RT}^{\rm pol}$ , the total polarization charge within the volume R, is  $^{(3)}$ 

$$\langle \psi_{(k)} | Q_{RT} | \psi_{(k)} \rangle \sim \frac{R^3 R^3}{(R^2 + R^4_5)^{5/2}}$$
 (II.10)

It can be readily seen that if one makes R=R<sub>s</sub> then the

total polarization within the volume R behaves like

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In view of (II.9) this result is easily understood. Since, within the volume R, the induced polarization charge density is approximately constant the total polarization charge within the volume is just given by the product of the charge density times the volume. An interesting lesson to be learned from this example is that as the volume increases the total charge increases leading to a divergent result in the infinite volume limit.

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These vacuum fluctuations are yet responsable for another peculiarity of the relativistic theory: the difficulty related to Noether's theorem<sup>(4,5)</sup> (for a pedagogical review see ref.5). From Noether's theorem one would expect that the naive charge operator (II.4) would be the generator of the symmetry transformation. That will require, for instance, that Q annihilates the vacuum. That is not so. In fact the norm of the operator  $Q_{\rm RT}$  applied to the vacuum has the following asymptotic behavior:

$$\| Q_{RT} |0\rangle \|^{2} \sim R \qquad (II.12)$$

Expression (II.12) indicates that the norm of the state resulting from the application of Q to the vacuum is a divergent one. The reason for this peculiar behavior of the naive charge operator is known<sup>(4,5)</sup>. The operator  $\tilde{J}_{\nu}(\mathbf{x},t)$  when applied to the vacuum creates pairs. These vacuum fluctuations are evenly distributed over the space in such a way that when summed over the whole space one gets a divergent result. The situation is very similar to that described in the previous example.

Integrals of local densities do not have the expected properties of the generator of symmetry transformations. The physical meaning for the divergent result (II.12) is that local measurements of densities are not possible. The formal object obtained by integrating a local density is meaningless as a result of the vacuum fluctuations which are bound to be induced by any attempt of measuring  $\vec{J}_0(\mathbf{X},t)$  at each point in space.

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The program of constructing the generator associated to a symmetry transformation was implemented in ref.(4). (For a review see ref.(5)). In scattering theory, where the kinematics is described by the asymptotic configurations, a Generator G can be written in terms of the asymptotic "in" and "out" creation and annihilation operators  $^{(5)}$ . The relevant properties of the Generator are that it satisfies the commutation relations

$$[G_{r}, Q_{in}^{\dagger}, Q_{2}] = f(\mathbf{w}) Q_{in}^{\dagger}, \text{ out } (\mathbf{i}\mathbf{k})$$

$$[G_{r}, Q_{in}, g_{in}, g_{i$$

and that

$$\langle 0|G|0\rangle = 0 \tag{II.14}$$

where  $\int (ik)$  can be inferred, by using (II.13) and (II.14), from the matrix element of the generator between one particle states.

III. NON-LOCAL DENSITY OPERATORS

In the last section we have seen that, as a result of the vacuum fluctuations induced by the apparatus, local measurements of the charge density operators cannot be implemented. At the formal level one would say that the charge, associated to a local operator like (II.4), is not a hermitian operator. Since within the Quantum Field Theoretical context smeared fields yields to well defined operators one expects that the analogue of a classical observable density, which one requires to be an hermitian operator, to be a set of such operators. The motivation, from the point of view of measurement, for introducing the smearing is to eliminate the uncontrolable effects of the apparatus.

A class of smeared densities relevant in the construction of the Generator is given by

$$J_{RT}^{o}(\mathbf{x},t) = \int dt' \int d^{3}\mathbf{x}' \eta_{T}(t'-t) \int_{R} (\mathbf{x}'-\mathbf{x}) J^{o}(\mathbf{x}',t') \qquad (III.1)$$

where  $\int R(\mathbf{x}) (\eta_{\tau}(t))$  is a sequence of  $\int (\mathbf{x}^3) (S(\mathbf{x}))$  real functions which converges, when  $R \rightarrow 0$  ( $\tau \rightarrow 0$ ), to  $\delta^3(\mathbf{x}) (\delta(t))^{(5)}$ .

In close analogy with (I.1) and (II.2),  $J_{RT(k,t)}^{0}$  will represent the average value of  $J_0$  in a 4-sphere with volume  $TR^3$ centered at P=(X,t).

Another class of smeared operators of interest, denoted by  $J_0(X,\eta_M)$ , is the one defined in a manner similar to (III.1), namely<sup>(6)</sup>

$$J^{\circ}(\mathbf{x}, \eta_{ad}) = \int dt' \eta_{ad}(t') J^{\circ}(\mathbf{x}, t') \qquad (III.2)$$

where an adiabatic time smearing,  $\eta_{\rm ad}\left(t\right)$  , is defined as the one satisfying

$$\widetilde{\eta}_{\text{od}}(\dot{p}_{0}) = 0$$
 for  $\dot{p}_{0} < \lambda$  (III.3)

where  $\eta_{\alpha\beta}(b_0)$  is the Fourier transform of  $\eta(t)$  and  $\lambda$  is a parameter characteristic of the system and which will be denoted by the "cut off parameter".

The choice of an adiabatic  $\eta_{\mu}$  is associated to a very careful process of measuring the charge density where by an appropriate adjustment of the cut off parameter we will be able to eliminate the undesirable effects of the apparatus<sup>(6)</sup>. For instance if one wants to avoid that  $\tilde{J}(\mathbf{x},\eta_{\mathrm{M}})$  creates pairs and consequently, that it annihilates the vacuum

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(III.4)

we have just to require that  $\lambda = \mu$ , where  $\mu$  is the smallest mass in the theory (for a theory with no massless particles).

An explicit example of  $\eta_{Ad}(4)$  is given by

$$\eta_{ul(t)} = \int dq_0 \Theta\left(1 - \frac{1}{2} \log t\right) e^{iq_0 t}$$

where  $\lambda$  is the cut off parameter and  $\Theta$  is the usual step functions. Let us consider the smearing functions  $\bigcap_{R} (t, x'-x)$ 

defined by

where  $\bigcup_{R(X)}$  is a sequence of real functions which converges to  $\delta^3(x)$  when  $R \rightarrow 0$  and  $\eta(q_0,q_1^2)$  is, as yet, not specified.

By using these smearing functions one can construct a fairly broad class of smeared operators, denoted by  $\overline{J}^b_R$  , where

$$\overline{J}_{\mathbf{x}}^{\circ}(\mathbf{x},0) \equiv \int dt \int d^{3}\mathbf{x}' \ \overline{J}^{\circ}(\mathbf{x}',t) \ \eta_{\mathbf{x}}(t,\mathbf{x}'-\mathbf{x}) \qquad (\text{III.6})$$

The operator  $\overline{J}^{p}(\mathbf{X}_{,0})$  will be defined as the limit

$$\overline{\mathbf{J}}^{\circ}(\mathbf{x},0) \equiv \lim_{n \to 0} \overline{\mathbf{J}}^{\circ}_{\mathbf{x}}(\mathbf{x},0) \qquad (\mathbf{III}.7)$$

By choosing appropriate 1 functions in (III.5) one can get the smeared operators (III.1) or (III.2) as particular cases of (III.6). A specific example of operators of the form (III.6) will be studied in chapter IV. Before proceeding we will recall some basic facts on the measurement problem. In order to probe the charge distribution of a given system one has to make use of a Test Body (The electron, usually ). The effect of the Test Body on the system to be probed can be represented (within the one photon exchange approximation) by the action of the operator  $\tilde{J}_0(x,t)$  on the state |B> which describes the state of the system, namely

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As we know, the application of  $J_0(X_1t)$  to  $|B\rangle$  leads to the possibility of pair creations as well as to the possibility of exciting the system. From the point of view of the measurement of the charge density this effects should be avoided. This requirement follows from the need of making ourselves sure that all the momentum transferred to the Test Body be solely due to the system one wants to probe.

Any experimental set up, or any mathematical construct like the smeared operators (III.1) which eliminates the undesirable effects of the apparatus will be said to implement the Bohr-Rosenfeld program.

We are now ready to state some requirements which should be met by the non-local operator which we would select as being more appealing from the physical point of view.

A) From any such an operator one should infer the Generator of symmetry transformation.

In the case of an operator of the form (III.2) or (III.7) this is equivalent to requiring that the integrals of these operators over the whole space leads to well defined operators satisfying (II.13)-(II.14).

B) This operator should implement the Bohr-Rosenfeld program.

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C) This set of operators should allows us to establish some analogy with classical observable densities.

The third requirement is a very relevant requirement and a more specific way of formulating it, is the following: let us consider a classical density,  $\rho^{B}(x)$ associated to a system B. From such a density one can construct a classical two-point correlation W(x) defined by

$$W(\mathbf{x}) = \int d^{3}\mathbf{y} \ \rho^{\theta}(\mathbf{x},\mathbf{y}) \ \rho^{\theta}(\mathbf{x},\mathbf{y}) \qquad (\text{III.8})$$

Since after the integration of W(x) over the  $\chi$  -variable one gets the total charge of the systems B squared, one can interpret W(x) as the probability of finding charge within the system B at a distance  $\chi$ <sup>(8)</sup>.

The requirement of analogy with classical observable densities will be met if correlation functions involving these operators are Identical (or have Identical behavior) to classical correlation functions like (III.8). For the operator defined in (III.7) all one has to do is to require that

$$\langle B|\overline{J}_{0}(\mathbf{x},0)|\overline{J}_{0}(0,0)|B\rangle = \int \partial^{2}\mathbf{y} \, \rho^{2}(\mathbf{x},\mathbf{y}) \, \rho^{2}(\mathbf{x},\mathbf{y}) \qquad (III.9)$$

Having stated the basic requirement, the next step will be to show that there are, in fact, operators obeying them.

Operator defined in (III.1)do obey condition A. It is possible to infer the Generator of symmetry transformation from such operator. The equivalence of such operator to that satisfying (II.13) was shown by Orzalesi Sucher and Woo in ref.(7).

The operators defined in (III.2) satisfies requirement A and B. In order to see this let us stick to the particular example of smearing functions given by (III.6). In this case one can see that it is possible to ajust the parameter  $\lambda$  in order to implement the Bohr and Rosenfeld program. For keeping  $\lambda$  small means to keep the energy transferred to the system small and consequently one can allways prevent the application of  $J_0(X, \eta_{ad})$ to a given state (B) from making transition to another state (excited state)or from pair creation. That integrals of  $J_0(X, \eta_{ad})$ over the whole space leads to well defined operators having properties similar to those of the Generator of symmetry was shown in Ref.(3).

In this example properties A and B are intimately connected. In fact this was one of the motivations of Horowitz and Raby in the paper of Ref.(3), namely, to show that the construction of a well behaved charge operator is intimately associated to Bohr-Rosenfeld's prescription.

Within the relativistic context one doesn't know of operator satisfying conditions (A-C). However, we shall see, in the next section, that by an appropriate choice of the smearing function  $\eta_{(x,t)}$  we will find operators having these desirable properties within the Non-Relativistic context.

#### IV. A SIMPLE EXAMPLE

Let us consider a Non-Relativistic system of N particles whose dynamics is described by the Hamiltonian H. In this case one can give an expression for the charge density operator in terms of the position operator of the i-th constituent at the time  $t - \hat{\Pi}_{i}(t)$  This expression is

$$\int_{O}(\mathbf{x}_{i}t) = \sum_{i=1}^{M} \int d^{2}q + i (q) e^{i q (\mathbf{x}_{i} - \mathbf{i}_{i}(t))}$$
(IV.1)

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Where  $t_{ivg}$  is the Form Factor of the i-th constituent.

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By using the Heisenberg representation one can write

$$\hat{J}_{0}(\mathbf{x},t) = \mathbf{e} \quad \hat{J}_{0}(\mathbf{x},0) =$$
(IV.2)

Within Born's approximation, the action of the apparatus on a system whose state is  $|\psi_b\rangle$  (which will be taken as the ground state of the system) is described by the application of the charge operator density to the state  $|\psi_b\rangle$ , that is

$$\hat{J}_{0}(\mathbf{x},t)|\Psi_{0}\rangle = \sum_{i \neq i} \langle \Psi_{i}|\hat{J}_{0}(\mathbf{x},t)|\Psi_{0}\rangle |\Psi_{i}\rangle \quad (IV.3)$$

By making use of (IV.2) (and making  $f_{i}(q) = q'_{i}$ i.e. the charge of the i-th constituent) one can write, for the matrix element in (IV.3), the following expression

$$\{\psi_{f}|J^{\circ}(x,t)|\psi_{o}\rangle = e e J_{of}(q)$$
 (IV.4)

Where  $\widehat{\mathcal{J}}_{o_{1}}(\mathbf{q}_{1})$  is the transition form factor defined in terms of the intrinsic wave functions  $\mathcal{Y}_{o}$  and  $\mathcal{Y}_{f}$  of the ground state and excited state respectively as

$$\overline{F}_{o_{\mathcal{F}}}(q_{\mathcal{I}}) = \sum_{j=1}^{n} q_{j} \int \prod_{k=1}^{n} d^{3} w_{k} \ \overline{e}^{(q_{\mathcal{F}})} \delta(\Sigma m_{\mathcal{F}} \overline{v}_{\mathcal{I}}) \Psi_{o}(\overline{v}_{\mathcal{I}} \cdots \overline{v}_{n}) \Psi_{f}(\overline{v}_{\mathcal{I}} \cdots \overline{v}_{n})$$
(IV.5)

and Q is the momentum transferred by the Test Body.

By looking at (IV.3) and (IV.4) one can readily see that in this example the apparatus will induce transitions from the original state  $|\Psi_i\rangle$  to an excited state  $|\Psi_i\rangle$ . This is, from the point of view of measurement of the density, an undesirable effect of the apparatus since the final state is not the same we have started with. As a consequence of the different objects we have at the initial and final stages of the experiment one cannot say that the momentum transfered to the Test Body is solely due to the system one wants to probe.

Suppose now that one eliminates all the unwanted effects due to the Test Body. It is possible to construct an operator associated to such a careful measurement. Let us consider the test function  $\Im R(t, X'-X)$  given by

$$\eta_{R(t_1,x'_1,x)} \equiv \int d^{3}\eta' \int dq'_{0} 2\theta \left(1 - \frac{q_{0}'}{q_{1}'^{3} f_{2M}}\right) \frac{e}{s\pi} \frac{e}{(3\pi)^{3}} \frac{G}{(3\pi)^{3}} (q_{1}'), \quad (IV.6)$$

By using this explicit  $\eta_R$  in (III.6) and from (IV.3) and (IV.4) it follows that the action of  $\overline{J}^o_R(\mathfrak{X}, \mathfrak{O})$ on  $|\psi_0\rangle$  will be expressed as

$$\overline{J}_{R}^{o}(\mathfrak{X}_{10})|\mathfrak{W},\mathfrak{P}_{=0}\rangle = \frac{1}{2} \int_{\mathcal{H}} \left[ d^{3}\mathfrak{P} 2\theta \left( 1 - \underline{\mathbf{e}}_{\underline{\mathbf{f}}} - \underline{\mathbf{e}}_{\underline{\mathbf{f}}} \right) \mathcal{E} \right] \widetilde{\mathcal{J}}_{0}^{\mathbf{F}}(\mathfrak{P}) \widetilde{\mathcal{G}}_{R}(\mathfrak{P}) |\mathfrak{M}_{\underline{\mathbf{f}}},\mathfrak{P}\rangle$$

$$(\mathrm{IV.7})$$

IP in (IV.7) is the total momentum of the system. Due to the recoil of the target it can be inferred that  $\tilde{E}_{f}$  in (IV.7) will be expressed as  $E_{f} = \frac{\tilde{E}}{f_{f}} + \frac{R^{2}}{2M}$  ( $E_{0} = \tilde{E}_{0}$ ) and consequently one can write for  $\tilde{J}_{0}(\mathbf{x}_{0})|\psi_{0}\rangle$  defined in (III.7), the following expression

$$\overline{J}_{0}(\mathbf{x},0) | \psi_{0}, P=0 \rangle = \sum_{44} \int d^{3}P \, \mathcal{L} \Theta(-\overline{e}_{f} + \overline{e}_{n}) e^{iP \mathbf{x}} \overline{\mathcal{L}}_{f}(P) | \psi_{1}, P \rangle \quad (IV.8)$$

 $\label{eq:From (IV.8)} \mbox{ it can be seen that the role of the test} function $$\eta_R$ is just to select the elastic contribution, since all $$$ 

other states have energies higher than the fundamental state. More explicitly

$$\overline{J}_{\alpha}(x,0)|\psi_{0},P_{=0}\rangle = \int d^{3}P e^{iPx} \overline{J}_{00}(e)|\psi_{0},P\rangle \qquad (IV.9)$$

 $\mathbf{J}_{00}$  (**IP**) in (IV.7) is the elastic Form Factor of the system whose definition in terms of the wave function is

$$\widetilde{f}_{\infty(\mathfrak{p})} = \sum_{j=1}^{N} q_{j} \int \left( \prod_{k=1}^{N} d^{j} \overline{u_{k}} \right) e^{i \left[ p \cdot \overline{u_{j}} \right]} \delta(\overline{z} m_{i} \overline{u_{i}}) \left[ U_{0}(\overline{u_{i}} \dots \overline{u_{w}}) \right]^{2}$$
(IV.10)

We will check now that the operator  $J_0(x,0)$ whose action on the state  $|\Psi_0\rangle$  is given by (IV.9), satisfies properties A-C of section III.

First of all one notes that

$$\hat{Q} | \psi_0 \rangle = \int \hat{J}_0(x_1 + j) d^3x | \psi_0 \rangle = \left( \sum_{j=1}^{N} q_j \right) | \psi_0 \rangle \qquad (IV.11)$$

On the other hand, from (IV.9) and (IV.10), one can

see that

$$\int d^3x \, \overline{J}_0(x,0) \, |\Psi_0\rangle = \left(\sum_{j=1}^{M} q_j\right) \, |\Psi_0\rangle \qquad (IV.12)$$

Expressions (IV.11) and (IV.12) shows the equivalence of the charge operator and the operator obtained by integrating  $\overline{J}_{\sigma}(\mathbf{X}_{,0})$  over the whole space.

As explained before, the operator  $\vec{J}_0(X,0)$  was deviced in such a way as to get rid of all the excited states. These are precisely the ones which should be eliminated in order to implement, in this case, the Bohr-Rosenfeld program.

Finally, property C emerges when we consider the

two-point correlation function defined in (III.9). By using (IV.9) one can see that

$$(\psi_0 | \overline{J}_0(x, v) | \overline{J}_0(0, 0) | \psi_0) = \int d^{3|p|} \overline{J}_{00}(p) \overline{J}_{00}(p) = (1P, x)$$
(IV.13)

It is easy to see that the right hand side of (IV.13) can indeed be written under the form proposed in (III.8) and (III.9). The associated charge distribution will be written as

$$\rho(\mathbf{x}) = \begin{bmatrix} d^{3} \mathbf{P} \ e^{i\mathbf{P}\cdot\mathbf{x}} \ \tilde{\mathbf{J}}_{00}(\mathbf{P}) \end{bmatrix}$$
(IV.14)

and consequently we have been led, ultimately, to the well know relation between Form Factors and charge distribution. The procedure for getting the charge density in terms of Form Factors is, in this context, somewhat related to that one proposed by us in Ref. (9).

### V. CONCLUSIONS

We have reviewed the problems associated with the measurement of some local operators within the context of Relativistic Quantum Theory. In the case of the charge density we have emphasized that the restriction in implementing local measurements can be infered from the bad behavior of the naive charge operator. From this it follows that only smeared (or non local) operators are measurable.

In order to select some smeared operators we have required that the physically relevant non-local operators exhibts three properties: that the Generator of symmetry transformation be infered from them, that they implement the program of eliminating

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all the undesirable effects due to the apparatus and that correlation functions constructed with these operators assume the form of classical correlation functions.

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Within the relativistic context we have pointed that there are operators satisfying some of the properties required but not all of them. Consequently, the construction of operators similar to the one described in section IV deserves more investigation.

In the non-relativistic case we have constructed operators exhibiting the desired properties. Furthermore correlation functions constructed with them assume the form of correlation functions involving classical densities whose Fourier transform are the Form Factors. This latest property is relevant because it sheds some light on the connection between what one measures experimentally (Form Factors) and some formal constructs of Quantum Field Theory (smeared operators).

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