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by

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ABSTRACT

We suggest that spin- and charge-exchange resonances could be associated with a R(4) symmetry that should emerge from the consideration of these resonance states as those of many particle-hole phonons with various spin-isospin modes, taking a self-conjugate nucleus as our vaccum. Experimental results are compatible with the corresponding theoretical estimates.

Since the discovery of giant resonances involving spin- and charge- exchange excitations[1], the spin-isospin modes of the nuclear excitation have aroused much interest. Recently, the systematic study of these excitation modes has been advanced through investigations with (p,n) experiments carried out by several research groups[2,3]. These studies[1,2] and others[4] have demonstrated that a Gamow-Teller (GT) resonance state of a N>Z nucleus is observed with an energy somewhat above the isobaric analog state (IAS). The observations of other isovector resonances of spin-flip type and spin-nonflip type have also been reported[1-3]. Some of these experimental data have been currently confronted with the theoretical estimates[5] which are based on the Bohr-Mottelson model of spin- and charge-exchange resonances[6]. Slightly earlier than this, Krewald, Speth, Osterfeld and Brown[7] have made the first systematic attempt to gain an undestanding of the nuclear structure effects connected with these resonances.

In these theoretical treatments as well as that of many others, the structure of an observed resonance state, say, resulting from a (p,n) reaction, is believed to be characterized by a coherent sum of proton-particle-neutron-hole states, or in the spirit of TDA or RPA, with a one ph-phonon over a vaccum, say, the ground state of the target nucleus. It is the purpose of this note to suggest that by considering the resonance state of interest as that of *many* particle-hole phonons now built on the the lowest 0^+ of the self-conjugate nucleus with the mass number equal to that of the target nucleus. This increase in the complexity of the problem is rewarded later on. We then allow an interplay among the various spin-iso-

spin modes of excitation taking place in the self-conjugate nuclei. We propose that each of these phonons can be considered as a system of double pseudo-spin, 1/2, and double pseudo-isospin 1/2 similar to the well known case of the Hydrogen atom. Using these elementary excitations as bulding blocks, we reproduce a Hydrogen-like spectrum for our nuclei. The resonance states of interest are thus eventually associated with a rotational symmetry, R(4), in four dimensions.

To spell out our idea in detail, we begin with a consideration of the modes excited by the fields

$$F(\kappa,\sigma,\tau,\lambda\mu) = r^{\kappa}Y^{\kappa}(\theta,\phi)\ x \begin{cases} 1 & \sigma = 0 \quad \tau = 0 \\ \vec{\sigma} & \sigma = 1 \quad \tau = 0 \\ \vec{\tau} & \sigma = 0 \quad \tau = 1 \\ \vec{\sigma}\vec{\tau} & \sigma = 1 \quad \tau = 1 \end{cases} \tag{1}$$

where $\vec{\sigma}$ and $\vec{\tau}$ are the spin and isospin variables. The quantum numbers κ, σ, τ and λ stand for the orbital angular momentum, the spin, the isospin and the total angular momentum carried by the excitations. Such a field can result as the average effect of a two-body interaction of the type

$$V_0 + V_\sigma\ \vec{\sigma}_1\cdot\vec{\sigma}_2 + V_\tau\ \vec{\tau}_1\cdot\vec{\tau}_2 + V_{\sigma\tau}\ \vec{\sigma}_1\cdot\vec{\sigma}_2\ \vec{\tau}_1\cdot\vec{\tau}_2 \tag{2}$$

For the sake of illustration, we limit our discussion for the moment to the case of Gamow-Teller resonances and set κ=0 and λ=σ. We now allow four spin-isospin modes of excitation in (1) to interplay in a self-conjugate nucleus. For a certain manner of coupling one of the many phonon states built on the

lowest 0^+ state could then be identified with the GT resonance. For example, in a (p,n) reaction using $^{90}_{40}Zr_{50}$ a target, one could consider the T=4 GT resonance state in $^{90}_{41}Nb_{49}$ as that of six phonons built on the lowest 0^+ state of $^{90}_{45}Rh_{45}$. The many-phonon states here are then described in terms of representations of a $U_\lambda(4)xU_\tau(4)$ group. Now the experimental fact that, for heavy nuclei, T-1 GT resonances are close to the IAS seems to be in favor of the group chain

$$U_\lambda(4)xU_\tau(4) \supset R_\lambda(4)xR_\tau(4) \supset R_\lambda(3)xR_\tau(3) \tag{3}$$

Furthermore, each of particle-hole phonons can be considered as a system of double spins (1/2, 1/2) and double isospins (1/2, 1/2), one spin or isospin is for the particle and the other is for the hole. This is reminiscent of the fact that the Hydrogen atom could be also considered as a system of double pseudo-spins in association with a R(4) symmetry. If our nuclear system does possess a R(4) symmetry, then the pseudo-spins that are used to classify our states are no longer mere mathematical symmetry labels but have now indeed some definite physical meanings in connection with spins and isospins for the particle world as well as the hole world. Using the group theoretical treatment of H-atom as a guide, the model Hamiltonian for our system of many-phonons that preserve the R(4) symmetry and gives a possible description to GT resonances is found to be of the following form:

$$H = \frac{a}{2}\left[(4\hat{s}_1+1)^{1/2} + (4\hat{s}_2+1)^{1/2}\right] + b\hat{S}^2 + 2c\ \vec{t}_1\cdot\vec{t}_2 \tag{4}$$

where $\hat{s}_1$, $\hat{s}_2$ are the pseudo-spin operators and $\hat{t}_1$, $\hat{t}_2$, the pseudo-isospin operators. $\hat{S}$ is the total spin. The constants a,b, and c are the free parameters. For completeness, let us now consider all charge and spin exchange resonances with positive parities by also taking the $\kappa = 2$ phonons into consideration and assuming the group chain

$$U_\kappa(6) \times U_\sigma(4) \times U_\tau(4) \supset SU_\kappa(3) \times R_\sigma(4) \times R_\tau(4) \supset R_\kappa(3) \times R_\sigma(3) \times R_\tau(3) \qquad (5)$$

We now have an energy spectrum as given by

$$E = a(s_1+s_2+1)+bS(S+1)+c\left[T(T+1)- t_1(t_1+1)-t_2(t_2+1)\right]+dL(L+1) \qquad (6)$$

where L and T are total orbital angular momentum and total isospin. In (6) we did not write down the terms with the SU(3) symmetry labels since we consider only the resonances with the same SU(3) representation as the target state. Eq. (6) is expected also to be valid for the resonances with negative parities in which case only κ=1 phonons or a mixture of κ=1 and κ=3 phonons are involved. We illustrate in Fig. 1 how to assign the R(4) symmetries to some states of the target nucleus (T_z=T) and their analogs (isospin=T) and antianalogs (isospin= = T-1) in the T_z=T-1 nucleus resulting from a (p,n) reaction.

To determine the free parameters in (6), we demand that, in the cases of spin-nonflip resonances, our formula should reproduce well the energy systematics of the giant multipole resonances (GMR) in the target nuclei. We borrow immediately the results from the interacting sp-boson model[8] for GMR. Consequently we are left with only two

parameters to be fixed by comparing the data of giant GT resonances with some of our states with half-integer pseudo-spins. We finally obtain

$$a = 39\ A^{-1/3}, \quad b = -6.5\ A^{-1/3}, \quad c = 18/A, \quad d = -2\ A^{-1/3}$$

With these parameters, we now proceed to extract estimates of the energy differences between a T-1 dipole spin-flip resonance and the IAS and of the energy splitting between the T and T-1 components of the vector-dipole resonance. The results are shown in Table I along with the data and they seem to reproduce the experimental values reasonably well.

It still remains to be seen, as more data is accumulated, whether Eq. (6) can predict the energy systematics of giant resonances in nuclei other than GMR and those of GT, vector-dipole and dipole spin-flip. If these predictions continue to prove to be encouraging, then the pain which we have taken from the beginning to increase the complexity of the problem is justified and compensated by the simplicity which eventually emerges from the resulting symmetries in nuclei. This is in a way similar to the shell model calculations when we reach the middle of the shell, the complexity of the problem increases with the number of nucleons. However, a simple rotational spectrum emerges in the midst of despair. Now if the present model proves not to be too far fetched, we may always think of a hydrogen-like energy spectrum in connection with giant resonances in nuclei. Furthermore, the present encouraging results seem to indicate that the theory which we have outlined is a promising candidate for furnishing a foundation for the interacting sp boson model for resonances[8].

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FIGURE CAPTIONS

Fig. 1 - Hidrogen-like spectrum for resonances. On the left are shown some states of the target nucleus (T_z=T) and on the right their analogs (isospin=T) and antianalogs (isospin=T-1) in the T_z=T-1 nucleus resulting from a (p,n) reaction. The numbers in the two parentheses beside each level line refer to the double pseudo-spins (left) and the double pseudo-isospins (right).

TABLE CAPTIONS

TABLE I - Comparison between theory and experiment. Theoretical estimates according to Eq. (6), of energy differences E between the T-1 giant GT resonance and the IAS appear in column (1) along with the experimental results[2]. In column 2 the energy differences between the dipole spin-flip resonance and the IAS are also compared with experiment[2]. Column 3 contains the isospin splitting D(T-1) obtained from Eq. (6) compared to the experimental findings[3].

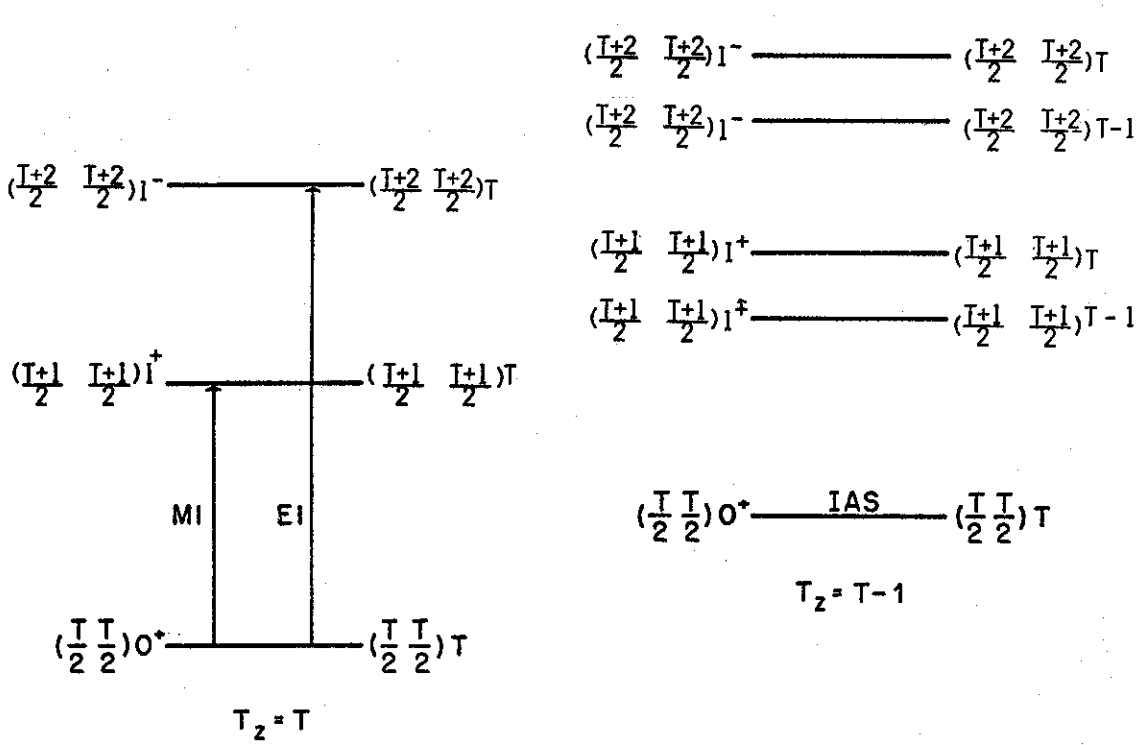

FIG. 1

TABLE I

Nucleus	$\sigma=1,\ \kappa=0$ E_{th} (MeV)	$\sigma=1,\ \kappa=0$ E_{exp} (MeV)	$\sigma=1,\ \kappa=1$ E_{th} (MeV)	$\sigma=1,\ \kappa=1$ E_{exp} (MeV)	$\sigma=0,\ \kappa=1$ D_{th} (MeV)	$\sigma=0,\ \kappa=1$ D_{exp} (MeV)
^{112}Sn	3.5	2.9	10.7	10.0	2.7	2.7
^{116}Sn	2.9	2.0	10.0	9.3	3.3	2.5
^{124}Sn	1.7	1.0	8.8	7.2	4.3	4.6
^{90}Zr	3.8	3.5	11.6	12.8	2.9	2.2
^{92}Zr	3.4	3.0	11.2	12.5	3.2	1.7
^{94}Zr	3.1	2.1	10.7	12.0	3.6	2.0
^{169}Tm	1.4	1.3	7.7	7.6	4.0	---
^{208}Pb	0.6	0.4	6.5	6.6	4.5	4.5