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by

B.V. Carlson and M.S. Hussein

Instituto de Física, Universidade de São Paulo,
C.P. 20.516, São Paulo, SP, Brasil

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UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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C.P. 20.516, São Paulo, S.P., Brasil

ABSTRACT

We develop a simple model for the average S-matrix that describes heavy ion direct processes in the presence of absorption due to compound nucleus formation. The fluctuation cross section and the fusion cross section are then calculated for deformed heavy ion systems where multiple Coulomb excitation is important. A simple expression for the fusion cross section valid for above-barrier energies is then obtained. The formula clearly displays the modification, due to Coulomb excitation, in the usual geometrical expression.

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I. INTRODUCTION

Heavy ion compound and fusion cross sections have usually been calculated assuming the complete separation from the fast, direct transitions. Of course the latter are partially accounted for through the use of appropriate absorptive potentials. It is by now, however, well established that the presence of directly coupled channels affects not only the values of the transmission coefficients needed in the calculation of the compound nucleus (fluctuation) and fusion cross sections, but, more importantly, the structure of the statistical theory (i.e. the Hauser-Feshbach theory). This is borne out by several investigations[1].

An interesting example of directly coupled-channels effects on the compound nucleus and fusion cross sections of heavy ions is that of multiple Coulomb excitation. These effects have been very nicely demonstrated for the system $^{16}O + {}^{A}Sm$ (A=148, 150,152,154) at sub-barrier energies by Stokstad et al.[2]. Earlier discussion of these effects in α -induced reactions on deformed targets was given in [3].

For the purpose of simple analyses, several concepts have been introduced, e.g., static deformation[4], dynamic deformation[5], zero-point vibration[6], etc.. Clearly these concepts are physically motivated and represent to a large extent a simulation of the overall physics involved in a more complete, coupled channel description of the fusion process. In such a calculation one identifies the fusion cross section σ_F, with the difference $\sigma_F \equiv \sigma_R - \sum_{I\neq 0} \sigma_I^D$, where σ_R is the total reaction cross section in the entrance channel, and $\sum_{I\neq 0} \sigma_I^D$ represents the total <u>direct</u> reaction cross section.

In the present paper, we develop a theory for σ_F as well as the different components of σ_F, the fluctuation cross sections, which takes explicitly into account multiple Coulomb excitation. We predict, in the sharp cut-off limit, appropriate for above-barrier energies, the following general two-parameter expression for the fusion cross-section involving quadrupole deformed target nuclei

$$\sigma_F = \pi R_c^2 \left[1 - \frac{E_c}{E} - \frac{16\pi}{225} \frac{2\mu}{\hbar^2} \varepsilon_{2^+} \frac{B(E2)\uparrow}{e^2} \frac{R_2^2(\bar{\theta},\xi) E^2}{Z_p^2 Z_T^4 e^4} \right] \quad (1)$$

where E_c and R_c represent the critical energy (fusion barrier) and critical fusion radius respectively, ε_{2^+} the excitation energy of the 2^+ state in the target, Z_p, Z_T are the projectile and target charge numbers respectively, and finally $R_2^2(\bar{\theta},\xi)$ is a well known function in the theory of Coulomb excitation[7]. Note that E_c and R_c are quite close to the corresponding quantities of the Coulomb barrier for CM energies not too high above the barrier.

II. SALIENT RESULTS OF THE STATISTICAL THEORY

The general formula for the fluctuation cross section describing the transition $\alpha \to \beta$ for a given partial wave-J is[1]

$$\sigma^{fl}_{\alpha\beta}(J) = \frac{T^J_{\alpha\alpha} T^J_{\beta\beta} + T^J_{\alpha\beta} T^J_{\beta\alpha}}{\mathrm{Tr}\, \underset{\sim}{T}} \quad (2)$$

where $\underset{\sim}{T}$ is Satchler's transmission matrix, given in terms of the average S-matrix, $\underset{\sim}{\bar{S}}$, that describes the coupled direct channels part of the problem.

$$\underset{\sim}{T} = 1 - \underset{\sim}{\bar{S}}^{\dagger} \underset{\sim}{\bar{S}} \quad (3)$$

Eq. (2) is valid when the number of open channels, N, is large. Notice that unitarity is approximately satisfied in the sense:

$$\sum_\beta \sigma^{fl}_{\alpha\beta}(J) = T^J_{\alpha\alpha} + \frac{(T^{J^2})_{\alpha\alpha}}{\mathrm{Tr}\, \underset{\sim}{T}^J}$$
$$= T^J_{\alpha\alpha} + \mathcal{O}\left(\frac{1}{N}\right) \quad (4)$$

where the second term on the right hand side, of order 1/N, measures the violation of unitarity in Eq. (2). However Eq. (2) was originally obtained[1], by neglecting the same type of term as the one above. Therefore to be consistent, we shall neglect this term when calculating the total compound (i.e. fusion) cross section, σ_F, and write for channel α

$$\sigma_F^{(\alpha)} = \frac{\pi}{k^2} \sum_{J=0} (2J+1)\, T^J_{\alpha\alpha} \quad (5)$$

where J is the compound nucleus angular momentum (which is the same as the incident orbital angular momentum for the case of spinless projectile and target considered here).

Although we agree fully with Mahaux and Weidenmüller's[8] warning that the Hauser-Feshbach formula lacks a foundation in the case of heavy ion systems, we shall however, use it as the basis of our theory. We remind the reader that the statistical, Hauser-Feschbach formula Eq. (2) (without the second term) has been widely used, and with success, in heavy ion compound reactions[9].

It is clear from our Eqs. (2) and (3) that the basic quantity in our theory is the transmission matrix, $\underset{\sim}{T}$. In order to construct this matrix one has to solve for the average S-matrix, $\bar{S}$, which requires a solution of the full coupled channels problem describing multiple Coulomb excitation in the presence of compound nucleus absorption. Although exact numerical solution of this problem is now feasible for heavy systems[10], what we seek here is an approximate analytical solution for $\underset{\sim}{\overline{S}}$ that would lead to a transparent expression for $\underset{\sim}{T}$ and accordingly σ_F, valid in the presence of multiple Coulomb excitation.

III. SIMPLE MODEL FOR THE AVERAGE S-MATRIX

A very powerful method for solving iteratively the many-coupled-channels equations of heavy ion Coulomb excitation problem is the inward-outward integration scheme developed at Copenhagen[11]. We shall use this formulation to obtain an approximate expression for the matrix $\underset{\sim}{\overline{S}}$. In the inward-outward method, the solutions, $\psi_\alpha(r)$, to the coupled radial Schroedinger equations

$$\left[\frac{d^2}{dr^2} + k_\alpha^2 - \frac{\ell_\alpha(\ell_\alpha+1)}{r^2} - \frac{2\mu}{\hbar^2}\,U_\alpha(r)\right]\psi_\alpha(r)$$

$$= \sum_\beta V_{\alpha\beta}(r)\;\psi_\beta(r) \tag{6}$$

which are regular at the origin $(\psi_\alpha(0) = 0)$, are written in terms of r-dependent coefficients $a(r)$, $a^{(+)}(r)$ of the regular and outgoing, $\varphi(r)$, $h^{(+)}(r)$, parts of the homogeneous (uncoupled) optical wave function

$$\psi_\alpha(r)\frac{1}{k_\alpha^{1/2}} = a_\alpha(r)\;\varphi_\alpha(r)\frac{1}{k_\alpha^{1/2}} - a_\alpha^{(+)}(r)\;h_\alpha^{(+)}(r)\;\frac{1}{k_\alpha^{1/2}} \tag{7}$$

Inserting Eq. (7) into Eq. (6) we then obtain the following two sets of coupled linear equations[11]

$$\frac{d}{dr}a_\alpha(r) = \frac{1}{k_\alpha^{1/2}}\Big[h_\alpha^{(+)}(r)\sum_\beta V_{\alpha\beta}(r)\,\varphi_\beta(r)\frac{1}{k_\beta^{1/2}}a_\beta(r) - h_\alpha^{(+)}(r)\sum_\beta V_{\alpha\beta}(r)\,h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta^{(+)}(r)\Big] \tag{8}$$

$$\frac{d}{dr}a_\alpha^{(+)}(r) = \frac{1}{k_\alpha^{1/2}}\Big[\varphi_\alpha(r)\sum_\beta V_{\alpha\beta}(r)\,h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta(r) - \varphi_\alpha(r)\sum_\beta V_{\alpha\beta}(r)\,h_\beta^{(+)}(r)\frac{1}{k_\beta^{1/2}}a_\beta^{(+)}(r)\Big] \tag{9}$$

It should be clear that we have the freedom to consider a part of the channel-channel coupling in the generalized potential U. This is precisely what we shall do. The interaction V contain only the long-range Coulomb coupling, whereas U contains all short range (nuclear) couplings as well as the average effect of the coupling between the open-coupled-channel space and the compound nucleus (absorption due to fusion). We shall see that such a decomposition would be quite convenient for our purposes. The above implies that the S-matrix resulting from solving the homogeneous equation (6), $\underset{\sim}{\overline{S}}^{(0)}$, would be generally non-diagonal.

To simplify the discussion we shall use in what follows matrix notation. Recognizing the fact that outside the range of the short-range nuclear coupling the wave functions $\varphi(r)$ and $h^{(+)}(r)$ behave like

$$\varphi(r)\,\frac{1}{k^{1/2}} = \frac{i}{2}\left[H^{(-)}(r)\,\frac{1}{k^{1/2}} - \underset{\sim}{S}^{(0)}\,\frac{1}{k^{1/2}}\,H^{(+)}(r)\right]e^{i\sigma} \tag{10}$$

$$h^{(+)}(r) = e^{i\sigma} H^{(+)}(r) \frac{1}{k^{1/2}} \tag{11}$$

where $H^{+(-)}$ is the outgoing (incoming) Coulomb wave function and σ is the Coulomb phase shift. Inserting Eqs. (10) and (11) into Eqs. (8) and (9) and dropping rapidly oscillating terms involving the products $H^{(+)}(r)\, H^{(+)}(r)$ and $H^{(-)}(r)\, H^{(-)}(r)$, we obtain[10b)]

$$\frac{d}{dr} e^{i\sigma} a(r) = \frac{i}{2} \frac{1}{k^{1/2}} H^{(+)}(r) V(r) H^{(-)}(r) \frac{1}{k^{1/2}} e^{i\sigma} a(r) \tag{12}$$

$$\begin{aligned} \frac{d}{dr} e^{-i\sigma} a^{(+)}(r) = \frac{1}{2} \Big[& \frac{1}{2} \frac{1}{k^{1/2}} H^{(+)}(r) V(r) H^{(-)}(r) \underset{\sim}{\bar{S}}^{(0)} \\ & + \frac{1}{2} \frac{1}{k^{1/2}} \underset{\sim}{\bar{S}}^{(0)} H^{(+)}(r) V(r) H^{(-)}(r) \frac{1}{k^{1/2}} \Big] e^{i\sigma} a(r) \\ & - \frac{i}{2} \frac{1}{k^{1/2}} H^{(-)}(r) V(r) H^{(+)}(r)\, e^{-i\sigma} a^{(+)}(r) \end{aligned} \tag{13}$$

Equations (12) and (13) may be further simplified by recognizing the fact that because of strong absorption, the long-range coupling potential, V(r), modifies the wave function of the system at radial distances larger than the classical turning points. At these separation distances the following approximation is quite good.

$$H^{(+)}(r) V(r) H^{(-)}(r) \simeq 2\, F(r) V(r) F(r) \tag{14}$$

where F represents a vector whose components are the regular Coulomb functions in the different channels. Corrections to Eq. (14), involve rapidly oscillating terms that would contribute very little when integrated. In what follows we use Eq. (14) for all values of r.

We introduce the following matrix

$$\underset{\sim}{C}(r) \equiv \frac{1}{k^{1/2}} \int_0^r F(r') V(r') F(r') dr' \frac{1}{k^{1/2}} \tag{15}$$

We then have

$$\frac{d}{dr} e^{i\sigma} a(r) = i \left[\frac{d}{dr} \underset{\sim}{C}(r)\right] e^{i\sigma} a(r) \tag{16}$$

$$\begin{aligned} \frac{d}{dr} e^{-i\sigma} a^{(+)}(r) = & \frac{1}{2}\left[\underset{\sim}{\bar{S}}^{(0)} \frac{d}{dr} \underset{\sim}{C}(r) + \frac{d}{dr} \underset{\sim}{C}(r)\, \underset{\sim}{\bar{S}}^{(0)} \right] e^{i\sigma} a(r) \\ & - \frac{i}{2} \left[\frac{d}{dr} \underset{\sim}{C}(r)\right] e^{-i\sigma} a^{(+)}(r) \end{aligned} \tag{17}$$

Eqs. (16) and (17) have to be solved in conjunction with the boundary conditions.

$$a(\infty) = 1 \tag{18}$$

$$a^{(+)}(0) = 0 \quad , \quad a^{(+)}(\infty) = \pi\left(\underset{\sim}{t} - \underset{\sim}{t}^{(0)}\right) \tag{19}$$

where $\underset{\sim}{t}$ is the total t-matrix and $\underset{\sim}{t}^{(0)}$ is the corresponding one for the homogeneous equation (V=0), i.e. $\underset{\sim}{t}^{(0)} = \frac{1}{2i}\left(1 - e^{i\sigma} \underset{\sim}{\bar{S}}^{(0)} e^{i\sigma}\right)$. Of course the original equations (8) and (9) also satisfy the above boundary conditions.

Equations (16) and (17) with the conditions (18) and (19) can be solved analytically, if we ignore ordering effects, namely set the commutator $\left[\left[\frac{d}{dr}\underset{\sim}{C}\right]_r, \left[\frac{d}{dr}\underset{\sim}{C}\right]_{r'}\right] = 0$, which we believe to be related to the sudden approximation[7)]. We thus find

$$a(r) = e^{-i\sigma} \exp\left[i\left[\underset{\sim}{C}(r) - \underset{\sim}{C}\right]\right] e^{-i\sigma} \tag{20}$$

$$a^{(+)}(r) = \frac{e^{i\sigma}}{2i}\left[\bar{S}^{(0)} e^{iC(r)} - e^{-iC(r)}\bar{S}^{(0)}\right] e^{-iC} e^{i\sigma} \tag{21}$$

$$, \quad C \equiv C(r=\infty)$$

Thus

$$a^{(+)}(\infty) = \pi\left[t - t^{(0)}\right] = \frac{e^{i\sigma}}{2i}\left[\bar{S}^{(0)} - e^{-iC}\bar{S}^{(0)} e^{-iC}\right] e^{i\sigma} \tag{22}$$

and finally

$$\pi t = \frac{1 - e^{i\sigma} e^{-iC} \bar{S}^{(0)} e^{-iC} e^{i\sigma}}{2i} \tag{23}$$

Therefore the total average S-matrix can be identified through

$$\pi t = \frac{1-\bar{S}}{2i} \quad ,$$

$$\bar{S} = e^{i\sigma} e^{-iC} \bar{S}^{(0)} e^{-iC} e^{i\sigma} \tag{24}$$

Equation (24) is our principal result of this section. It is interesting to observe that in the limit of pure Coulomb excitation , $\bar{S}^{(0)} = 1$, the $\bar{S}$-matrix becomes exactly the one obtained by Alder and Winther in the sudden limit[7], namely

$$\bar{S}^{AW} = e^{i\sigma} e^{-2iC} e^{i\sigma} \tag{25}$$

The approximation (14) have been used previously in a more restricted sense and was found to give results quite close to the coupled channel calculations[12]. We call the above approximation the on-shell plus off-shell corrections method[13].

We shall discuss the compound and fusion cross sections at energies higher than but close to the barrier. We

thus feel confortable in ignoring the short-range nuclear channel-channel coupling and take $\bar{S}^{(0)}$ to be diagonal.

With $\bar{S}^{(0)}$ diagonal, Eq. (24) written as

$$\bar{S}^{(0)} = e^{iC} e^{-i\sigma} \bar{S} e^{-i\sigma} e^{iC} \tag{26}$$

supplies a nice example of the Engelbrecht-Weidenmüller transformation[14]. The matrix $U \equiv e^{iC}$ diagonalizes $e^{-i\sigma}\bar{S}e^{-i\sigma}$, in the sense $U^T e^{-i\sigma}\bar{S}e^{-i\sigma} U \equiv \bar{S}^{(0)}$ and also diagonalizes the transmission matrix given in Eq. (3) in the sense $U^\dagger T U = 1-\bar{S}^{(0)\dagger}\bar{S}^{(0)} =$ diagonal. However, since the matrix e^{-iC} corresponds to a physical process, namely the transition operator for pure Coulomb excitation at half the strength (see Eq. (25), we do not need to deal explicitly with the EW transformation in our analysis, as we show below.

IV. THE TRANSMISSION MATRIX AND THE FUSION CROSS SECTION

In applying our results of the previous section, we shall assume that several collective channels (members of a rotational band of the deformed nucleus) are strongly coupled, thus giving rise to non-diagonal elements of the transmission matrix. We also assume the presence of many more weakly coupled channels. The totality of all the channels is assumed to be very large such that $\mathrm{Tr}\, T \gg 1$.

The transmission matrix, T , is obtained directly from the average S-matrix through Eq. (3). Using our expression for $\bar{S}$ of Eq. (24) we obtain the Hermitian matrix

$$T = e^{-i\sigma} e^{+iC} T^{(0)} e^{-iC} e^{i\sigma} \tag{27}$$

with

$$\underset{\sim}{T}^{(0)} = 1 - \bar{\underset{\sim}{S}}^{(0)\dagger} \bar{\underset{\sim}{S}}^{(0)} \tag{28}$$

If we further assume $\bar{\underset{\sim}{S}}^{(0)}$ to be diagonal we obtain for the element $T_{\beta\alpha}$

$$T_{\beta\alpha} = \sum_{\gamma} \left(e^{+i\underset{\sim}{C}}\right)_{\beta\gamma} T_{\gamma}^{(0)} \left(e^{-i\underset{\sim}{C}}\right)_{\gamma\alpha} e^{i(\sigma_\alpha - \sigma_\beta)} \tag{29}$$

The channel label γ implies $\{\ell I, J\}$ with ℓ being the orbital angular momentum, I the intrinsic spin of the excited state, and J is total angular momentum of the channel which is conserved and also represents the angular momentum of the compound nucleus.

The diagonal elements, $T_{\alpha\alpha}$, have a very simple physical interpretation

$$T_{\alpha\alpha} = \sum_{\gamma} \left|\left(e^{-i\underset{\sim}{C}}\right)_{\alpha\gamma}\right|^2 T_{\gamma}^{(0)} \tag{30}$$

Eq. (3) demonstrates the fact that the flux in the entrance channel is distributed among the strongly coupled channels before fusion takes place. Since the intermediate channels are not observed, the transitions from the entrance channel to these channels is described by the factors $\left|\left(e^{-i\underset{\sim}{C}}\right)_{\alpha\gamma}\right|^2$ which are the usual inelastic probabilities calculated at <u>half</u> the value of the coupling strength.

Using the fact that the operator $e^{-i\underset{\sim}{C}}$ is unitary we can rewrite Eq. (30) as

$$T_{\alpha\alpha} = T_{\alpha}^{(0)} + \sum_{\gamma} \left|\left(e^{-i\underset{\sim}{C}}\right)_{\alpha\gamma}\right|^2 \left[T_{\gamma}^{(0)} - T_{\alpha}^{(0)}\right] \tag{31}$$

At above-barrier energies, the second term on the right hand side of Eq. (31), which represents the coupled-channels effects on the compound nucleus transmission , is always negative since the critical angular momentum associated with the bare entrance channel transmission coefficient $T_{\alpha}^{(0)}$ is larger than that of the inelastic channels $T_{\gamma}^{(0)}$. Therefore one may conclude that $T_{\alpha\alpha} < T_{\alpha}^{(0)}$. The reason is leakage of some of the flux to other channels. This is clearly seen in the particular case of two channels labeled 1 and 2,

$$T_{11} = T_{1}^{(0)} - \left|\left(e^{-i\underset{\sim}{C}}\right)_{12}\right|^2 \left[T_{1}^{(0)} - T_{2}^{(0)}\right] \tag{32}$$

$$T_{22} = T_{2}^{(0)} + \left|\left(e^{-i\underset{\sim}{C}}\right)_{12}\right|^2 \left[T_{1}^{(0)} - T_{2}^{(0)}\right] \tag{33}$$

and the non-diagonal elements

$$T_{12} = e^{-i(\sigma_1 - \sigma_2)} \left[\left(e^{+i\underset{\sim}{C}}\right)_{11} \left(e^{-i\underset{\sim}{C}}\right)_{12} \left[T_{1}^{(0)} - T_{2}^{(0)}\right]\right] \tag{34}$$

$$T_{21} = e^{i(\sigma_1 - \sigma_2)} \left[\left(e^{+i\underset{\sim}{C}}\right)_{21} \left(e^{-i\underset{\sim}{C}}\right)_{11} \left[T_{1}^{(0)} - T_{2}^{(0)}\right]\right] \tag{35}$$

Therefore the corrections, to the compound nucleus transmission coefficients, due to channel coupling is proportional <u>to the differences between bare transmission coeffcients pertaining to different channels</u>. As functions of the angular momentum (J) , these differences correspond to narrow windows centered close to the critical angular momentum.

We turn now to the fusion cross section in channel α (entrance channel with the two nuclei in their ground states) defined in Eq. (5). Using our result for $T_{\alpha\alpha}$ given in Eq. (31) we obtain

$$\sigma_F^{(\alpha)} = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\,T^{(0)\,\ell}_{\ell,0}$$

$$-\frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\sum_{\ell' I\neq 0}\left|\left(e^{-i\underset{\sim}{\varsigma}}\right)^{\ell}_{\ell 0,\ell' I}\right|^2\left[T^{(0)\ell}_{\ell,0}-T^{(0)\ell}_{\ell',I}\right] \tag{36}$$

where the total angular momentum of the compound nucleus, J, is set equal to the orbital angular momentum in the entrance channel, ℓ, since the ground state spins of the two nuclei are assumed to be zero. Notice that ℓ' could only have the values permitted by the selection rule[7)]

$$\ell' + \lambda + \ell = \text{even} , \tag{37}$$

where λ denotes the multipolarity of the transition.

At sub-barrier energies, the sum over ℓ' in the second term on the right-hand-side of Eq. (36) has to be evaluated very casefully since the difference $\left(T^{(0)\ell}_{\ell,0} - T^{(0)\ell}_{\ell' I}\right)$ is non-zero even for very small value of ℓ. At higher energies, the contribution of this difference is centered about the critical angular momentum for fusion in the entrance channel, i.e. the angular momentum that specifies the value of $\frac{\pi}{k^2}\sum_{\ell}(2\ell+1)T^{(0)\ell}_{\ell,0}$, which in the sharp cut off limit becomes

$$\frac{\pi}{k^2}\sum_{\ell}(2\ell+1)\,T^{(0)\ell}_{\ell,0} \cong \frac{\pi}{k^2}\left(\ell^{\alpha}_{cr}+1\right)^2 \tag{38}$$

Since $\ell^{\alpha}_{cr} \gg 1$ for heavy systems, even at energies slightly higher than the barrier (e.g. for $^{16}O + {}^{152}Sm$ at $\frac{E_{cm}}{E_B} \simeq 1.1$ MeV, $\ell^{(\alpha)}_{cr} \sim 25$), we expect that an approximate evaluation of the ℓ'-sum involving consideration of the average value of $T^{(0)\ell}_{\ell,0} - T^{(0)\ell}_{\ell' I}$, to be adequate. The correction to this approximation would be proportional to $\frac{q_{0\to 2}}{\ell^{\alpha}_{cr}}$, (where $q_{0\to 2}$ is the quadrupole strength parameter[7)]), which is a small quantity for the systems studied so far (for $^{16}O + {}^{152}Sm$ considered above, $\frac{q_{0\to 2}}{\ell^{\alpha}_{cr}} \simeq \frac{3}{25} \sim 0.1$).

Therefore we set

$$\sum_{\ell' I\neq 0}\left|\left(e^{-i\underset{\sim}{\varsigma}}\right)^{\ell}_{\ell 0,\ell' I}\right|^2\left(T^{(0)\ell}_{\ell,0} - T^{(0)\ell}_{\ell' I}\right)$$

$$\simeq \sum_{I\ell'}\left|\left(e^{-i\underset{\sim}{\varsigma}}\right)^{\bar{\ell}}_{\bar{\ell}0,\ell' I}\right|^2\left[\Theta\left(\ell^{(\alpha)}_{cr}-\ell\right) - \Theta\left(\ell^{(\gamma)}_{cr}-\ell\right)\right] \tag{39}$$

with

$$\ell^{(\gamma)}_{cr} \simeq \sqrt{\frac{2\mu}{\hbar^2}R_c^2\left(E-\varepsilon_I-E_c\right)}$$
$$\ell^{(\alpha)}_{cr} = \sqrt{\frac{2\mu}{\hbar^2}R_c^2\left(E-E_c\right)} \tag{40}$$

and ε_I is the excitation energy, and Θ is the step function.

In Eq. (39) $\bar{\ell} \equiv \frac{\ell_{\alpha\gamma}}{\eta}$ represents an average value of angular momentum which corresponds roughly to the mean of $\frac{\ell^{(\alpha)}_{cr}}{\eta_\alpha}$ and $\frac{\ell^{(\gamma)}_{cr}}{\eta_\gamma}$, with η representing the Sommerfeld parameter.

The sum over ℓ can then be performed immediately, yielding

$$\sigma_F^{(\alpha)} \cong \frac{\pi}{k^2}\left(\ell_{cr}^{(\alpha)}+1\right)^2 - \frac{\pi}{k^2}\left(\frac{2\mu R_c^2}{\hbar^2}\right)\sum_{I\ell'}\left|\left(e^{-i\frac{C}{\sim}}\right)^{\bar{\ell}}_{I_0,\ell' I}\right|^2 \mathcal{E}_I \quad (41)$$

Since $\sum_{\ell'}\left|\left(e^{-i\frac{C}{\sim}}\right)^{\bar{\ell}}_{\ell_0,\ell' I}\right|^2$ represents the Coulomb excitation probability for the transition $0 \to I$ at an angle given by $\frac{\bar{\theta}}{2}=\arctan\left(\frac{1}{\bar{\ell}}\right)$, evaluated at half the value of the coupling strength, we identify the I-sum in Eq. (41) with the average collective energy transferred in the excitation process, corresponding to half the value of the coupling strength. This quantity is evaluated by Alder and Winther[7]. Thus for a quadrupole rotational coupling

$$\sum_{I\ell'}\left|\left(e^{-i\frac{C}{\sim}}\right)^{\bar{\ell}}_{I_0,\ell' I}\right|^2 \mathcal{E}_I \cong \frac{16}{45}\,\mathcal{E}_{2^+}\left(\frac{q_{0\to 2}}{2}\right)^2 R_2^2(\bar{\theta})\, g_2(\xi) \quad (42)$$

Where, $q_{0\to 2}$ is the quadrupole coupling strength given by

$$q_{0\to 2} = \sqrt{\frac{\pi}{5}}\,\frac{\sqrt{\eta_0\,\eta_2}}{a_0\,a_2}\,\frac{\langle 0\|M(E2)\|2\rangle}{Z_T\, e} \quad (43)$$

$R_2^2(\bar{\theta})$ is given by $\left(\frac{\bar{\theta}}{2} = \tan^{-1}\frac{1}{\bar{\ell}}\right)$

$$R_2^2(\bar{\ell}) = \frac{9}{4}\,\frac{1}{\bar{\ell}^4}\left(1-\frac{\tan^{-1}\bar{\ell}}{\bar{\ell}}\right)^2 + \frac{3}{4}\left(\frac{1}{1+\bar{\ell}^2}\right)^2 \quad (44)$$

with $R_2^2(\bar{\ell}=0) = 1$,

and $g_2(\xi)$ is the semiclassical energy loss factor tabulated in Ref. 7) and ξ is the adiabaticity parameter for the $0 \to 2$ transition

$$\xi = \eta_2 - \eta_0 \simeq \frac{1}{2}\,\eta_0\,\frac{\mathcal{E}_{2^+}}{E_0} \quad (45)$$

with $\mathcal{E}_{2^+}$ being the excitation energy of the 2^+ state. In Eq. (43) η_I and a_I represent, respectively, the Sommerfeld parameter and half the distance of closest approach for head-on collision, in channel I.

Ignoring the difference between η_2 and η_0 and that between a_2 and a_0, and using Eq. (40b) for $\ell_{cr}^{(\alpha)}$, we finally obtain Eq. (1)*

$$\sigma_F^{(\alpha)} = \pi R_c^2\left[1 - \frac{E_c}{E} - \frac{16\pi}{225}\,\frac{2\mu}{\hbar^2}\,\mathcal{E}_{2^+}\,\frac{B(E2)\uparrow}{e^2}\,\frac{R_2^2(\bar{\theta})\,g_2(\xi)\,E^2}{Z_P^2 Z_T^4 e^4}\right] \quad (1')$$

The correction to the usual geometrical formula of σ_F, arising from multiple Coulomb excitation, depends on the center of mass energy as depicted in Fig. (1), where the function $R_2^2(\bar{\theta})\,E^2$ is plotted vs. $\left(\frac{E_c}{E}\right)^2$. It is clear therefore that the correction term is important at energies close to the Coulomb barrier. At much higher energies the function $R_2^2(\bar{\theta})\,E^2$ attains the following asymptotic form

$$R_2^2(\bar{\theta})\,E^2 \simeq \frac{3}{16}\,\frac{E_c^4}{E^2} \quad (46)$$

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* The total reaction cross section in channel α, $\sigma_R^{(\alpha)}$, is of course obtained from the elastic element of the average $\bar{S}$-matrix, namely $\sigma_R^{(\alpha)} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[1-|\bar{S}_{00}|^2\right] = \sigma_F^{(\alpha)} + \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sum_{\gamma\neq 0}|\bar{S}_{0\gamma}|^2$

We have estimated the above correction factor for the system $^{16}O + {}^{152}Sm$ at $E = 70$ MeV assuming a value of $E_c \sim 63$ MeV. The effect is about 5%. Although this is not a very large correction, we expect equation (1) to give a more precise fitted values of the parameters R_c and E_c. We remind the reader that our finding that at above-barrier energies, Coulomb excitation results in a reduction in σ_F is substantiated by the results of Ref. 5).

As we have mentioned earlier, at sub-barrier energies, the dispersion in angular momentum exemplified by the ℓ'-sum in Eq. (36), must be treated very carefully, as in this case small ℓ-values give the dominant contribution to the difference $\left[T^{(0)\ell}_{\ell,0} - T^{(0)\ell}_{\ell' I}\right]$. It is expected that the correction factor would become positive at these lower energies[2]. What is interesting about our formula Eq. (36) is the clear separation between what is called static deformation effect, having to do with ℓ'-sum i.e the dispersion in angular momentum, and the dynamic deformation effects related primarily to the presence of the "reduced" transition probabilities. A fuller accounts of the results presented in this paper, together with further developments, will be published later.

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FIGURE CAPTION

FIG. 1 - The function $R_2^2(\bar{\theta})\, E^2$ (see Eq. (44)) plotted vs. $\left(\frac{E_c}{E}\right)^2$

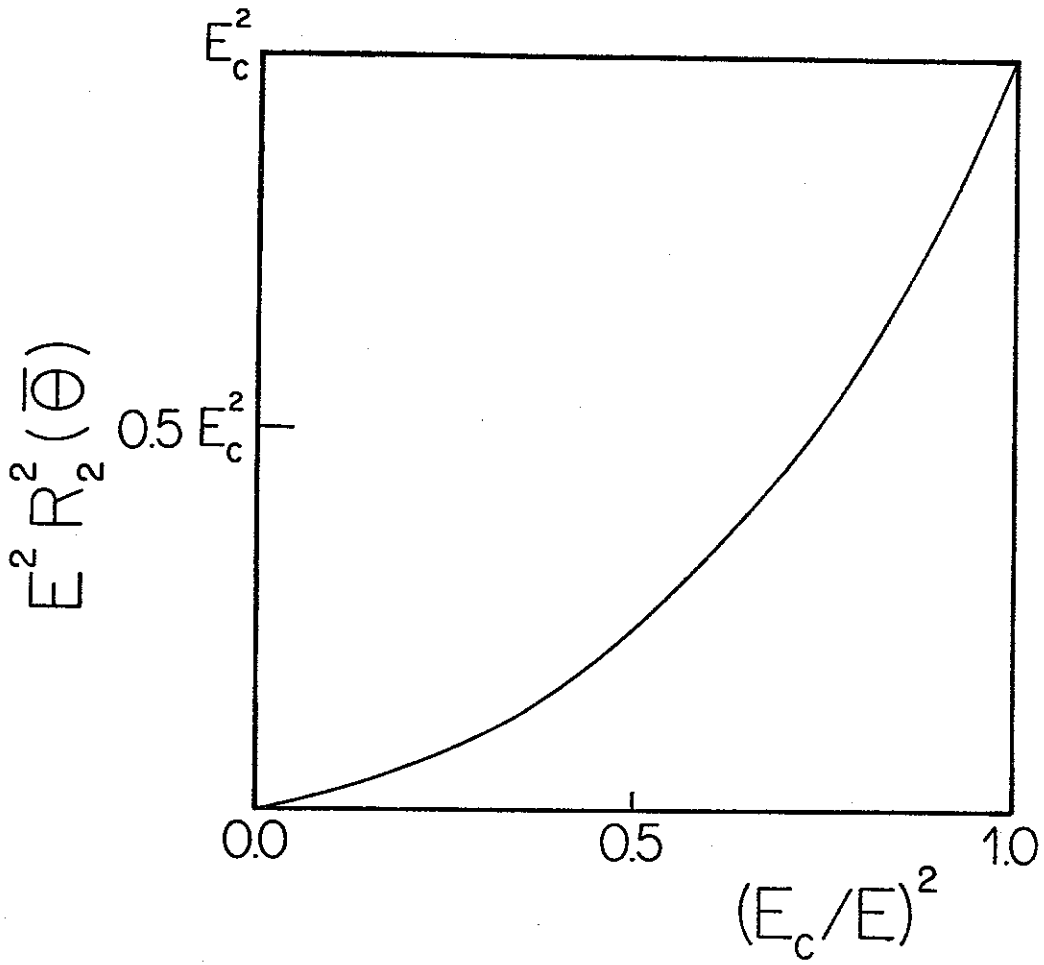

Figure 1