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ABSTRACT

A detailed discussion of the optical theorem for heavy-ion scattering is given. It is pointed out that a careful application of this theorem to light heavy-ion systems may yield information about the nuclear interaction at distances corresponding to forward glory trajectories. Applications to several cases are presented.

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The total reaction cross section σ_R is the most inclusive of all cross sections, containing the least explicit information about the colliding system. It is, however, of paramount importance as a measure of its "dynamical geometry"[1].

There exist several methods for extracting σ_R from elastic scattering, the most widely used one being through optical model analysis of the data. A variant of this method, to which it bears a close relation, is the quarter-point recipe[2]. Recently, the sum-of-differences method has been proposed as a means of extracting σ_R for heavy ions in a less model-dependent way[3]. This method is based on an approximate version of the optical theorem, adapted to the physical conditions prevalent in heavy-ion elastic scattering, i.e., strong Coulomb interaction and strong absorption[4].

In this letter, we carefully examine the optical theorem for heavy-ion scattering. We point out that the usual identification of σ_R with $2\pi\int_0^\pi [\sigma_{Ruth}(\theta)-\sigma(\theta)]\sin\theta d\theta$, where $\sigma(\theta)$ and $\sigma_{Ruth}(\theta)$ are the elastic and Rutherford differential cross sections, respectively, is not quite valid for light heavy ions, e.g., $^{16}O+^{12}C$. We interpret the difference as arising mainly from forward glory scattering. This effect is well known in atomic collisions[5]. Several numerical examples are given to support our claim.

According to this interpretation, glory undulations should be present in the difference $\Delta\sigma_R=\sigma_R-2\pi\int_0^\pi [\sigma_{Ruth}(\theta)-\sigma(\theta)]\sin\theta d\theta$, i.e., $\Delta\sigma_R$ should oscillate as a function of the center-of-mass energy E, with a characteristic period directly related to the nuclear passage time for the forward glory trajectory. Observation of this quantity would supply useful information about the heavy-ion interaction potential at the short distances involved.

We start with the usual partial-wave decomposition of the elastic scattering amplitude.

$$f(\theta) = \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell+1)(1-S_\ell)P_\ell(\cos\theta) \quad (1)$$

where k denotes the asymptotic wave number in the elastic channel, $k=(2\mu E/\hbar^2)^{1/2}$, μ being the reduced mass. In (1), S_ℓ is the partial wave S-function, given by

$$S_\ell = S_\ell^C S_\ell^n = |S_\ell^n| \exp[2i(\sigma_\ell+\delta_\ell)], \quad (2)$$

where S_ℓ^C is the point-Coulomb S-function and S_ℓ^n represents the effect of the short-range nuclear interaction; σ_ℓ and δ_ℓ are the phase shifts associated with S_ℓ^C and S_ℓ^n, respectively.

As a consequence of absorption, $|S_\ell^n| \leq 1$, with a significant deviation from unity holding usually only for low partial waves, except in cases where strong multiple Coulomb excitations are present. Such excitations, usually found in deformed systems, give rise to long-range absorption in the elastic channel, rendering $|S_\ell|$ significantly less than unity in a wider ℓ-range[6]. For a discussion of the effects of long-range absorption on the extraction of σ_R see, e.g., Ref. [7]. In the present paper, we deal mainly with light heavy-ion systems, so that we can justifiably ignore long-range absorption effects.

For heavy-ion systems, one usually considers the ratio $\sigma(\theta)/\sigma_{Ruth}(\theta)$, where $\sigma(\theta) \equiv d\sigma_{el}/d\Omega = |f(\theta)|^2$ is the differential elastic cross section and $\sigma_{Ruth}(\theta) = |f_{Ruth}(\theta)|^2$, where $f_{Ruth}(\theta)$ is the Rutherford scattering amplitude. We compute instead the difference $\sigma(\theta)-\sigma_{Ruth}(\theta)$, which comes out to be

$$\sigma(\theta)-\sigma_{Ruth}(\theta) = -(1/4\pi) \sum_{\ell=0}^{\infty} (2\ell+1)\sigma_{R,\ell}\, P_\ell^2(\cos\theta) -$$

$$- (2k)^{-2} \sum_{\ell \neq \ell'} (2\ell+1)(2\ell'+1)S_\ell^C S_{\ell'}^{C*}(1-S_\ell^n S_{\ell'}^{n*}).$$

$$\times P_\ell(\cos\theta)P_{\ell'}(\cos\theta) + (2/k)\mathrm{Im} f_n(\theta)\delta(1-\cos\theta), \quad (3)$$

where $\sigma_{R,\ell} = \pi k^{-2}(2\ell+1)(1-|S_\ell^n|^2)$ are the partial reaction cross sections ($\sigma_R = \sum_{\ell=0}^{\infty} \sigma_{R,\ell}$) and $f_n(\theta) = f(\theta) - f_{Ruth}(\theta)$.

Equation (3) has been derived, albeit in a slightly different form, by several authors[4]. A more realistic discussion, taking many-body aspects into account, should also include the compound elastic contribution. This entails using for the transmission coefficient, $T_\ell = 1-|S_\ell^n|^2$, the modified form (ignoring spin effects) $T'_\ell = 1-|S_\ell^n|^2 - 2(T_\ell^C)^2/\Sigma_\beta T_\beta^C$, where T_ℓ^C is the compound nucleus transmission coefficient in the elastic channel and the sum Σ_β is over all possible decay channels of the compound nucleus. The inclusion of the compound elastic contribution results[8] in a small "window" in ℓ-space centered around the grazing angular momentum ℓ_g. However, in heavy-ion scattering, the sum $\Sigma_\beta T_\beta^C$ tends to be very large. This renders the contribution of the above window quite insignificant, justifying the neglect of the compound elastic contribution.

In order to emphasize the physical content of each one of the three terms on the RHS of Eq.(3), we rewrite (3) as

$$\sigma(\theta)-\sigma_{Ruth}(\theta) = -\sigma_{inc}(\theta) + \sigma_{coh}(\theta) + \sigma_{fgl}(\theta) \quad (4)$$

where the subscripts stand for "incoherent", "coherent" and "forward glory" contributions to the elastic "nuclear differential cross section" $\sigma(\theta)-\sigma_{Ruth}(\theta)$. Note that $\sigma_{coh}(\theta)$ and $\sigma_{fgl}(\theta)$, in contrast with the other three quantities appearing in (4), are not positive definite.

Equations (3) and (4) clearly exhibit several of the most significant features of heavy-ion scattering. The quantity $\sigma_{inc}(\theta)$

in the first term on the RHS represents the incoherent contribution to $\sigma(\theta)$ due to absorption. The integral $\int\sigma_{inc}(\theta)d\Omega$ is precisely the total reaction cross section σ_R.

The angular dependence of $\sigma_{inc}(\theta)$ is symmetric about $\theta=\pi/2$. If the main contribution arises from values of ℓ such that $(\ell+\frac{1}{2})\theta>>1$, $(\ell+\frac{1}{2})(\pi-\theta)>>1$, we may substitute $P_\ell^2(\cos\theta)$ by its asymptotic expansion in the first term of (3), leading to

$$<\sigma_{inc}(\theta)> \simeq \sigma_R/(2\pi^2\sin\theta), \qquad (5)$$

where the angle brackets denote an average over an interval in θ sufficient to smooth out angular oscillations. The denominator in (5) is just the expected phase space factor. For large values of the grazing angular momentum ℓ_g, the angular dependence of $\sigma_{inc}(\theta)$ is very well approximated by (5) for θ not too close to 0 or π, as Fig. 1 clearly demonstrates. A reasonable estimate of $\sigma_{inc}(0)=\sigma_{inc}(\pi)$ may be obtained by computing the anisotropy $R=[\sigma_{inc}(0)/\sigma_{inc}(\pi/2)]-1$ in the sharp-cutoff model. The result is

$$R = \frac{2\pi}{3}\left[(\ell_g+1) - \frac{1}{4(\ell_g+1)}\right] - 1 . \qquad (6)$$

The second term, $\sigma_{coh}(\theta)$, represents the genuine coherence present in the system. This manifests itself through, e.g., the usual oscillations in $\sigma(\theta)/\sigma_{Ruth}(\theta)$ at forward angles. The integral $2\pi\int_0^\pi \sigma_{coh}(\theta)\sin\theta\, d\theta$ vanishes identically.

Finally, the third term, $\sigma_{fg\ell}(\theta)$, being zero everywhere except at $\theta = 0$, is usually set equal to zero at all angles in heavy-ion scattering, since it is assumed that $f_n(0) = 0$. However, an appreciable value of $\sigma_{fgl} = \int\sigma_{fgl}(\theta)d\Omega$ may result if the classical deflection function goes through zero at a nonzero

value of ℓ, i.e., if there is a forward glory. For a recent discussion of forward glory effects in the scattering of light, see Ref. [9].

Although previously not realized, we believe that the decomposition (4) of $\sigma(\theta)$ may supply a powerful alternative method for analysing heavy-ion elastic scattering. Through (4), $\sigma(\theta)$, for $\theta \neq 0$, is uniquely split into three well-defined pieces: $\sigma_{Ruth}(\theta)$, $\sigma_{inc}(\theta)$ and $\sigma_{coh}(\theta)$. Since σ_{Ruth} is given and $\sigma_{inc}(\theta)$ is well determined for large values of ℓ_g, as shown above, one is therefore able to extract from the data $\sigma_{coh}(\theta)$, the piece that contains the useful physics. Further elaboration of these ideas is being pursued.

Integrating (4) over all solid angles, we obtain the optical theorem[4)]

$$\Delta\sigma_R \equiv \sigma_R - 2\pi\int_0^\pi [\sigma_{Ruth}(\theta) - \sigma(\theta)] \sin\theta\, d\theta$$

$$= \frac{4\pi}{k} \operatorname{Im} f_n(0) \quad . \qquad (7)$$

Special care should be taken when evaluating the integral in (7). Usually the lower limit is replaced by a small angle θ_o, and the contribution from the region $0 \leq \theta \leq \theta_o$ is found to be oscillatory and small as long as the angle θ_o is chosen small enough[4,7,10)]

In the presence of a forward glory, the amplitude $f_n(0)$ may be approximately evaluated by the method of stationary phase, as was done originally by Ford and Wheeler[11)]. A more accurate treatment by Berry[12)] yields a uniform approximation for $f_n(\theta)$, which reduces to Ford and Wheeler's result for $\theta = 0$. The result is

$$f_n(0) \approx \frac{1}{k}\left(\ell_{gl} + \frac{1}{2}\right)\left(2\pi \Big/ \left|\frac{d\Theta}{d\ell}\right|_{\ell_{gl}}\right)^{1/2} \left|S^n_{\ell_{gl}}\right| \quad .$$

$$\times \exp\left[2i(\sigma_{\ell_{gl}} + \delta_{\ell_{gl}}) - i\,\frac{\pi}{4}\right] \quad , \tag{8}$$

where ℓ_{gl} is the glory angular momentum, $\Theta \equiv 2\,\frac{d}{d\ell}\,(\sigma_\ell + \delta_\ell)$ is the total deflection function and σ_ℓ and δ_ℓ were defined in (2). It is assumed that $|S^n_\ell|$ is slowly-varying (and therefore, in our case, close to unity) near $\ell = \ell_{gl}$.

Substituting (8) in (7), we finally obtain

$$\Delta\sigma_R \approx \frac{4\pi}{k^2}\left(\ell_{gl} + \frac{1}{2}\right)\left(2\pi \Big/ \left|\frac{d\Theta}{d\ell}\right|_{\ell_{gl}}\right)^{1/2} \left|S^n_{\ell_{gl}}\right| \quad .$$

$$\times \sin\left[2(\sigma_{\ell_{gl}} + \delta_{\ell_{gl}}) - \frac{\pi}{4}\right] \quad . \tag{9}$$

According to (9), $\Delta\sigma_R$ should oscillate as a function of energy (glory oscillations), with a local period given approximately by

$$\Delta E \approx \frac{\pi}{\frac{\partial}{\partial E}\,(\sigma_{\ell_{gl}} + \delta_{\ell_{gl}})} \approx \frac{2\pi\,\hbar}{\tau_{\ell_{gl}}} \quad , \tag{10}$$

where $\tau_{\ell_{gl}}$ is the collision time (Wigner time delay[13]) associated with the forward glory trajectory. Under semiclassical conditions, this time may be related to the interaction potential through the usual classical relation. It should be of the order of a typical heavy-ion direct reaction characteristic time ($\sim 10^{-22}$ sec).

We further note that $\Delta\sigma_R$ should become zero (or very small) at center of mass energies E_n satisfying the condition

$$2\left[\sigma_{\ell_{gl}}(E_n) + \delta_{\ell_{gl}}(E_n)\right] = \left(n + \frac{1}{4}\right)\pi \quad .$$

$$n = 0,1,2,\ldots \tag{11}$$

Therefore, only at these energies E_n would the usual identification[3] of $2\pi \int_0^\pi \left[\sigma_{Ruth}(\theta) - \sigma(\theta)\right] \times \sin\theta\, d\theta$ with σ_R be strictly valid.

In Fig. 2 we exhibit our calculation of $\Delta\sigma_R$ for the $^{16}O + {}^{12}C$ system*, using two different optical model potentials that seem to account well for the forward elastic scattering data[14]. Though these potentials were adjusted to fit the data at one particular energy, we have nevertheless employed them to calculate $\Delta\sigma_R$ over the whole CM energy range 8-55 MeV. As is apparent in Fig. 2, the forward glory contribution to $^{16}O + {}^{12}C$ scattering oscillates as a function of energy, with a local period of about 20 MeV. This corresponds to a passage time of about 2.0×10^{-22} sec. This value is consistent with our interpretation of the origin of $\Delta\sigma_R$. The energies at which $\Delta\sigma_R = 0$ come out to be $E_0 \approx 11$ MeV, $E_1 \approx 21.5$ MeV, etc., for the full curve in Fig. 2. Also clear from Fig. 2 is the fact that the local period increases with energy, a clear indication of the shortening of $\tau_{\ell_{gl}}$, as Eq. (10) demonstrates.

The amplitude of oscillation, which, according to (9), is determined by $\ell_{gl}(E)$, $\left|d\Theta/d\ell\right|_{\ell_{gl}}(E)$ and $|S^n_{\ell_{gl}}(E)|$, increases with energy in Fig. 2, suggesting that $\left|d\Theta/d\ell\right|_{\ell_{gl}}$ is decreasing, i.e., that $\Theta(\ell)$ near ℓ_{gl} tends to become flatter as the energy increases. It must be emphasized, however, that a more realistic calculation of $\Delta\sigma_R$ should take into account the energy dependence of the optical potential, specially the effect on its imaginary part of the gradual opening of new reaction channels. This will damp out the oscillations (as a result of the decrease of $|S^n_{\ell_{gl}}(E)|$ with increasing energy) and change the period as well.

* The well-known oscillations in $\sigma_R(E)$, commonly attributed to quasi-molecular resonances, will not be discussed here.

However, the general qualitative features of $\Delta\sigma_R$ shown in Fig. 2, namely, its oscillatory behavior with a period directly related with the nuclear passage time for the glory trajectory, should remain.

The sensitivity of $\Delta\sigma_R(E)$ to the optical potential employed, which is apparent in Fig. 2, may be further explored to remove the well-known ambiguities[15] present in optical model analyses of heavy-ion elastic scattering. As seen in Fig. 2, two different optical model potentials that both fit rather well[14] the forward elastic scattering data of $^{16}O + ^{12}C$ generate quite different $\Delta\sigma_R(E)$.

It should be emphasized that the glory oscillations may come out to be appreciably damped even in the case of weak absorption at ℓ_{gl} . This would indicate that $\theta = 0$ is in a classically forbidden region. The contribution to $f_n(0)$ would then arise from quantum-mechanical tunnelling (surface wave contribution). Such situations have been successfully treated by the complex angular momentum method in the case of light scattering[16]. We believe that the method would also be successful for heavy-ion scattering. However, we will not present a detailed discussion here. We mention only that the damping factor in $\Delta\sigma_R$ due to the complex nature of the glory trajectory should be of the form $\exp(-\mathrm{Im}\, \ell_{gl}\, \theta_{min})$, where ℓ_{gl} is a complex parameter associated with the glory trajectory and θ_{min} is the height of the angle barrier, i.e., the angular distance separating the minimum deflection from the forward direction.

Of course, if there is actually a real minimum in the deflection function very close to $\theta = 0$, one would encounter the situation of forward glory scattering forming a part of a nuclear rainbow. Such a situation has been discussed in a different context

by Pechukas[17]. In this case, the rainbow enhancement in $\Delta\sigma_R(E)$ would partly compensate for the surface wave damping factor referred to above[9], rendering the oscillations more conspicuous. This may well be a more realistic situation in heavy-ion scattering.

The angular distribution of glory scattering may also be obtained[12]; note that, for $\theta \neq 0$, it is described by the term $\sigma_{coh}(\theta)$ in (3) and (4), which contains the interference between nuclear and Rutherford scattering. As was already mentioned, however, this term does not contribute to $\Delta\sigma_R$.

We have calculated $\Delta\sigma_R$ for several light heavy ion systems and we have found in many of these cases a damped glory contribution amounting to as much as 20%. We call these damped because the corresponding deflection functions do not exhibit a negative branch.

In Table I we present the results for six light heavy-ion systems and one intermediate system, chosen arbitrarily[14]. As is clearly seen, $\Delta\sigma_R$ can be positive or negative, depending on the system and the energy (cf. Fig. 2). The case of $^{14}N + ^{12}C$ at $E_{\ell ab} = 78$ MeV is rather anomalous, with $\Delta\sigma_R$ being almost equal to σ_R . It would seem worth while to perform measurements on these and other systems at several energies in order to verify the existence of glory oscillations. These oscillations would yield important information about the nuclear interaction at distances shorter than the Coulomb rainbow[18] or strong absorption[15] radii.

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TABLE CAPTIONS

TABLE 1 - The contribution $\Delta\sigma_R$ calculated for several systems. The value of θ_o used in the calculation was set equal to 1.0×10^{-5} degrees (see text).

FIGURE CAPTIONS

FIG. 1 - The incoherent contribution, $\sigma_{inc}(\theta)$, plotted vs. the center of mass angle, for three values of the grazing angular momentum, ℓ_g . A sharp cut-off approximation was used for the transmission coefficients $1-|S_\ell^n|^2$. For comparison the function, $\sigma_R/(2\pi^2 \sin\theta)$, for $\ell_g = 25$, is also shown. All results are given in arbitrary units.

FIG. 2 - The function $\Delta\sigma_R(E)$ (in units of fm^2) plotted vs. the center of mass energy for the system ${}^{16}O + {}^{12}C$. The full curve was obtained with the optical model potential of Ref. 14a, and the dashed curve with OM potential of Refs. 14b (see Table 1). The classical deflection function of this system seems to exhibit a forward glory at a real value of ℓ_{g1} (see text).

TABLE 1

SYSTEM	E_B (MeV)	E_{CM} (MeV)	σ_R (Barns)	$\Delta\sigma_R$ (Barns)	$\int_{\theta_o}^{\pi}[\sigma_{Ruth}(\theta)-\sigma(\theta)]d\Omega$ (Barns)	OM potential parameters [V(MeV), R_V(fm), a_V(fm), W(MeV), R_W(fm), a_W(fm)].
${}^{14}N + {}^{12}C$ [14c]	7.8	36.0	1.25	1.18	0.36	30.0, 4.79, 0.57, 7.1, 5.64, 0.79.
${}^{16}O + {}^{12}C$ [14a]	8.5	8.6	0.10	0.0	0.10	100.0, 9.19, 0.48, 10.0, 6.06, 0.26.
${}^{16}O + {}^{12}C$ [14b]	8.5	15.4	1.03	-0.25	0.98	51.3, 6.24, 0.533, 2.45, 6.24, 0.533.
${}^{16}O + {}^{27}Al$ [14b]	18.2	29.5	1.30	0.11	1.28	44.9, 7.17, 0.522, 24.74, 7.17, 0.522.
${}^{12}C + {}^{28}Si$ [14d]	14.4	17.5	0.56	-0.05	0.55	10.0, 7.31, 0.458, 11.35, 5.20, 0.786.
${}^{16}O + {}^{56}Fe$ [14e]	31.5	42.0	0.94	0.0	0.94	28.0, 7.93, 0.6, 11.0, 7.93, 0.6.

FIGURE 1

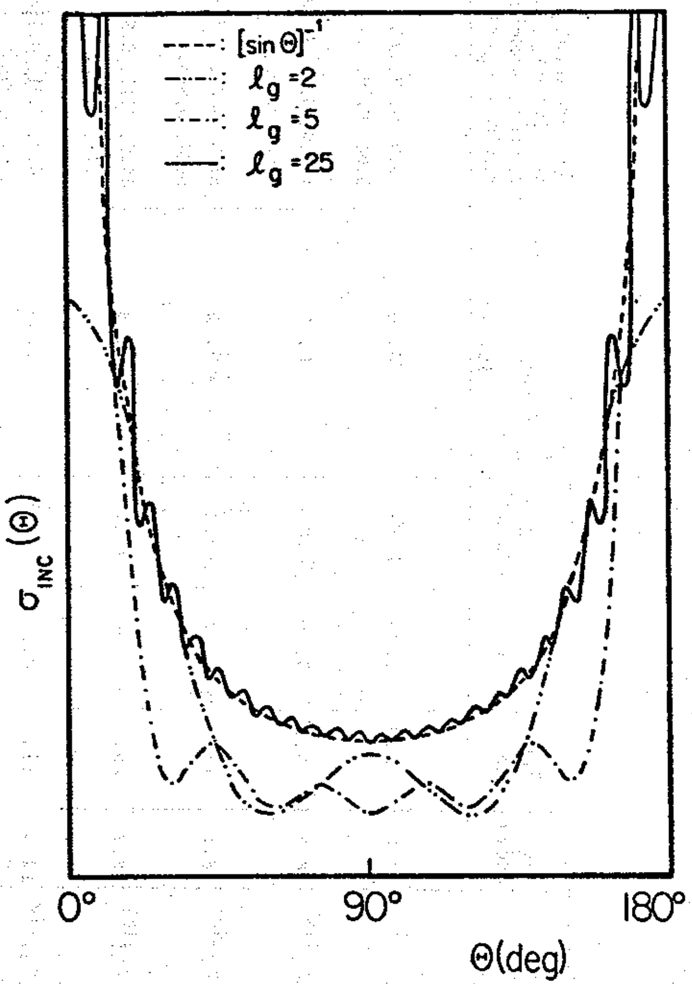

FIGURE 2