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ON THE NON-LOCAL CHARGE OF THE CP^{n-1} MODEL AND ITS SUPERSYMMETRIC EXTENSION TO ALL ORDERS.

by

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ON THE NON-LOCAL CHARGE OF THE CP^{n-1} MODEL AND ITS SUPERSYMMETRIC EXTENSION TO ALL ORDERS.

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ABSTRACT

We prove that the conservation of quantum non-local charge of the CP^{n-1} model is spoiled by an anomaly calculable to all orders in the 1/n expansion while for its supersymmetric extension it is restored.

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ON THE NON-LOCAL CHARGE FOR THE CP^{n-1}. MODEL AND ITS SUPERSYMMETRIC. EXTENSION TO ALL ORDERS.

I - INTRODUCTION

Two dimensional non-linear sigma-models (chiral models, for short), defined on symmetric spaces, have in recent years aroused considerable interest among field theoretical physicists. This interest is partly justified by the numerous analogies of the chiral models with four-dimensional Yang-Mills theories(1), analogies which include "gauge" content, non-trivial topological structure, instantons and so on. They become even more striking if we go to loop space: there is impressive evidence that Wilson loops can be interpreted as chiral fields in loop space(2).

One of the most important properties of these models is their classical integrability, which leads to an infinite number of non local conservation laws(3) (there are also local conservation laws, but they will not concern us in this paper). These conservation laws were first discovered in the O(n) non-linear σ-model(4), and subsequently generalized to various other models(5). They can also be described as Noether currents associated with a non-local field transformation leaving the action unchanged(6).

At the quantum level, the conservation of the non-local charges also imposes severe restrictions on the dynamics of the models. This is exemplified by the O(n) σ-model, for which it has been shown that they imply the absence of pair production and the factorization equation(7). As is well known, these are fundamental blocks for the construction of the exact S-matrix.

For all these reasons we think it to be very important to study "in extenso" the properties of two-dimensional σ-models. In this sense, we have in two recent papers discussed the construction of the quantum non-local charges in the CP^{n-1} model[8] and its supersymmetric extention[9].

The results obtained can be summarized as follows:

a) The would be quantum non-local charge of the CP^{n-1} model is not conserved[8]. Therefore, the exact S-matrix program can not be completed following this tread. Of course, the absence of an S-matrix, for the quanta of the basic CP^{n-1} field, is intuitively expected from the confining properties of the model[10]. An examination of the local charges shows a similar result[11].

b) In contrast to (a), quantum non-local conserved charges seem to exist if fermions are coupled to the CP^{n-1} field in a minimal or supersymmetric way[9]. The mechanism by which this happens is the same as the one which is responsible for mass generation in the Schwinger model[12]: vacuum polarization from the coupling to fermions. This gives mass to the topological gauge field (thus liberating the basic CP^{n-1} quanta) and also provides an additional, Adler type anomaly[13] which cancels the one coming from the pure CP^{n-1} model. Similary to the O(n) σ-model, the existence of the quantum conservation laws justifies the construction of exact S-matrices for these models[14].

So far, the above results were obtained only in the dominant order of the 1/n expansion whereas we would expect them to hold in all orders. In this communication we will show this to be indeed the case. Our result follows from a conjuction of general arguments with a detailed graphical analysis.

The content of the paper is organized as follows: in section II we show the absence of radiative corrections to the anomaly of the pure CP^{n-1} model. In section III we prove the conservation of the quantum non-local charge in the supersymmetric case to all orders. In section IV we present some conclusions. Various technical details are delegated to two Appendices.

II - ABSENCE OF RADIATIVE CORRECTIONS TO THE ANOMALY OF THE PURE CP^{n-1} MODEL

We begin by listing some basic properties of the CP^{n-1} model (all calculations will be done in Euclidean space). This is the theory of an n-component complex field z_i, described by the Lagrangian density:

$$\mathcal{L} = \overline{D_\mu z}\, D_\mu z_i \tag{II-1}$$

where

$$D_\mu z = \partial_\mu z - A_\mu z_i \tag{II-1a}$$

with the constraint

$$\bar{z}z = \sum_i \bar{z}_i z_i = \frac{n}{2f} \tag{II-1b}$$

The Feynman rules for the 1/n expansion are found in ref. (15). They are:

$$A_\mu\text{-propagator} \leftrightarrow \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \left[(p^2+4m^2)A(p) - \frac{1}{\pi}\right]^{-1} \tag{II-2a}$$

$$Z - \text{propagator} \leftrightarrow (p^2+m^2)^{-1} \tag{II-2b}$$

$$\alpha - \text{propagator} \leftrightarrow [A(p)]^{-1} \tag{II-2c}$$

with

$$A(p) = \frac{1}{2\pi}\left[p^2(p^2+4m^2)\right]^{-1/2} \ln \frac{\sqrt{p^2+4m^2}+\sqrt{p^2}}{\sqrt{p^2+4m^2}-\sqrt{p^2}} \tag{II-2d}$$

where α is the Lagrange multiplier field which enforces the constraint $\bar{z}z = \frac{n}{2f}$. The mass m is dynamically generated and is given by $m^2=\mu^2 e^{-n/2f}$, where μ is the renormalization point. Remember that on the quantum level the topological gauge field has acquired the status of an independent field.

The simplest non-local charge is classically given by:

$$Q^{ij} = \int dy_1\, dy_2\, \varepsilon(y_1-y_2) J_0^{ik}(t,y_1) J_0^{kj}(t,y_2) - \frac{n}{2f}\int J_1^{ij}(t,y)\,dy \qquad \text{(II-3)}$$

where

$$J_\mu^{ij} = z^i \overleftrightarrow{\partial_\mu}\, \bar{z}^j + 2A_\mu z^i \bar{z}^j \qquad \text{(II-3a)}$$

is the classical (traceless) Noether current generating isospin rotations in the plane ij.

In the quantum case, to give a proper definition of (II-3) we need to consider the singular short distance behavior of the product of the traceless part of the two currents. To leading order of 1/n we showed in ref.8 that the following expansion holds:

$$[J_\mu(x+\varepsilon), J_\nu(x)]^{ij} = C^\rho_{\mu\nu}(\varepsilon) J_\rho^{ij}(x) + D^{\rho\sigma}_{\mu\nu}(\varepsilon)\partial_\sigma J_\rho^{ij}(x) + E^{\rho\sigma}_{\mu\nu}(\varepsilon) z_i \bar{z}_j F_{\rho\sigma}(x) \qquad \text{(II-4a)}$$

where

$$C^\rho_{\mu\nu} = \frac{n}{2\pi}\left[-\frac{\delta_{\mu\nu}\varepsilon^\rho}{\varepsilon^2} + \frac{\delta^\rho_\mu \varepsilon_\nu}{\varepsilon^2} + \frac{\delta^\rho_\nu \varepsilon_\mu}{\varepsilon^2} + 2\,\frac{\varepsilon_\mu\varepsilon_\nu\varepsilon^\rho}{\varepsilon^2}\right] \qquad \text{(II-4b)}$$

$$D^{\rho\sigma}_{\mu\nu} = \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \frac{1}{4}\ln\frac{m^2\varepsilon^2}{4}\right)(\delta^\sigma_\mu\delta^\rho_\nu - \delta^\sigma_\nu\delta^\rho_\mu) + \frac{\delta^\sigma_\nu\varepsilon_\mu\varepsilon^\rho}{2\varepsilon^2} - \frac{\delta^\sigma_\mu\varepsilon_\nu\varepsilon^\rho}{2\varepsilon^2} - \frac{\delta_{\mu\nu}\varepsilon^\rho\varepsilon^\sigma}{2\varepsilon^2} + \frac{\delta^\rho_\mu\varepsilon_\nu\varepsilon^\sigma}{2\varepsilon^2} - \frac{\delta^\rho_\nu\varepsilon_\mu\varepsilon^\sigma}{2\varepsilon^2} + \frac{\varepsilon_\mu\varepsilon_\nu\varepsilon^\sigma\varepsilon^\rho}{(\varepsilon^2)^2}\right] \qquad \text{(II-4c)}$$

$$E^{\rho\sigma}_{\mu\nu} = \frac{n}{2\pi}\left[2\delta^\rho_\mu\frac{\varepsilon_\nu\varepsilon^\sigma}{\varepsilon^2} - 2\delta^\sigma_\nu\frac{\varepsilon_\mu\varepsilon^\rho}{\varepsilon^2}\right] \qquad \text{(II-4d)}$$

This result can be used to verify that the quantum charge

$$Q^{ij} = \lim_{\delta\to 0} Q^{ij}_\delta \qquad \text{(II-5a)}$$

where

$$Q^{ij}_\delta = \frac{1}{n}\left\{\int_{|y_1-y_2|\geq\delta} dy_1\, dy_2\, \varepsilon(y_1-y_2) J_0^{ik}(t,y_1) J_0^{kj}(t,y_2) - Z_\delta \int dy\, J_1^{ij}(t,y)\right\} \qquad \text{(II-5b)}$$

$$Z_\delta = \frac{n}{2\pi}\ln\frac{e^{\gamma-1}\mu\delta}{2} \qquad \text{(II-5c)}$$

is well-defined. However, instead of being conserved it satisfies:

$$\frac{dQ^{ij}}{dt} = -\frac{2}{\pi}\int_{-\infty}^{\infty} z_i\bar{z}_j F_{10}\,dy \qquad \text{(II-6)}$$

Therefore Lüscher's construction[7] can not be applied.

Although derived in the lowest order of the 1/n expansion, (II-6) is nevertheless valid to all orders. This result can be stated more precisely as follows:

Let $J_\mu^{ij}(x)$ be the current generating isospin rotations in the plane ij, so that the following Ward identity holds:

$$\partial^x_\mu \langle 0|T[J_\mu(x),J_\nu(y)]^{ij} X|0\rangle = -2n\delta(x-y)\langle 0|T\, J_\nu^{ij}(y) X|0\rangle - \sum_\ell \delta(x-x_\ell)\langle 0|T(J_\nu^{\alpha_\ell j}(y) z^i(x) - J_\nu^{\alpha_\ell k}(y) z^k(x) X_{\hat{\alpha}_\ell}|0\rangle +$$

$$+ \sum_{m,k} \delta(x-y_m) <0|T(\delta^{i\beta_m} \bar{z}^k(x) J_\nu^{kj}(y) - \delta^{k\beta_m} J_\nu^{ik}(y)\bar{z}^j(x)\ X_{\widehat{\beta_m}}\ |0> \qquad \text{(II-7)}$$

Here $X = \prod_{\ell,m} z_{\alpha_\ell}(x_\ell)\bar{z}_{\beta_m}(y_m)$ and

$X_{\widehat{\alpha_\ell}\,\widehat{\beta_m}}$ means that $z_{\alpha_\ell}(x_\ell)\,(\bar{z}_{\beta_m}(y_m))$ is to be deleted. Furthermore, the current normalization is given by:

$$<0|T\ J_\mu^{ij}(x) z^\ell(y)\ X|0> \;=\; \frac{1}{2\pi}\,\frac{(x-y)_\mu}{(x-y)^2+i0}\ <0|(\delta^{j\ell}z^i(y) - \frac{1}{n}\ \delta^{ij}z^\ell(y))X|0>$$

$$+\ O(\ell n\ x-y) \qquad \text{(II-8)}$$

in accordance with (II-7).

With these assumptions, it follows that (II-4a-d) hold in a weak sense (i.e., for time-ordered products and discarding convergent surface terms) in all orders of the 1/n expansion.

We would like to remark that the Green-functions of the basic CP^{n-1} field are infrared divergent. Thus in (II-8) and in equations containing non-gauge-invariant operators, we implicitly assume that an infrared regulator for the propagator of the A_μ field is used.

The above result is proved in two steps. First, we employ methods completely analogous to those of ref.(7), namely, we use arguments such as covariance (under Lorenz transformation, charge conjugation, parity, time reversal) and current conservation, to determine the coefficients $C^\rho_{\mu\nu}$ and $D^{\rho\sigma}_{\mu\nu}$. This is done in Appendix A. Next, to find the remaining coefficients $E^{\rho\sigma}_{\mu\nu}$, we argue with more detailed graphical methods:

From (II-4a) and using that $C^\rho_{\mu\nu}=C^\rho_{\nu\mu}$ (see Appendix A) we have:

$$\int d^2p\, e^{-i\varepsilon p} \left\{ \frac{\partial}{\partial k_\alpha} <0|T[\tilde{J}_\mu(p), J_\nu(0)]\tilde{z}_k(q)\tilde{\bar{z}}_\ell(r)\tilde{A}_\lambda(k)|0>^{\text{Prop}}\Big|_{q=r=k=0} - (\mu\leftrightarrow\nu) \right\} =$$

$$= (D^{\rho\sigma}_{\mu\nu}(\varepsilon) - D^{\rho\sigma}_{\nu\mu}(\varepsilon))\frac{\partial}{\partial k_\alpha} <0|T\ \partial_\rho J_\sigma(0)\tilde{z}_k(q)\tilde{\bar{z}}_\ell(r)\tilde{A}_\lambda(k)|0>^{\text{Prop}}\Big|_{q=r=k=0} -$$

$$-4A(\varepsilon^2)\frac{\partial}{\partial k_\alpha} <0|T[z_i\bar{z}_j\ F_{\mu\nu}](0)\tilde{z}_k\tilde{\bar{z}}_\ell A_\lambda(k)|0>^{\text{Prop}}\Big|_{q=r=k=0} \qquad \text{(II-9)}$$

where the tildes indicate Fourier transform and Prop means proper. Assuming that our normal products are normalized at zero external momenta, the right hand side (r.h.s.) of (II-9) turns into

$$-\ 2\ (D^{\alpha\lambda}_{\mu\nu} - D^{\alpha\lambda}_{\nu\mu})\ -\ 4iA(\varepsilon^2)(\delta^\alpha_\mu\delta^\lambda_\nu - \delta^\alpha_\nu\delta^\lambda_\mu) \qquad \text{(II-10)}$$

Note that $\delta^\alpha_\mu\delta^\lambda_\nu - \delta^\alpha_\nu\delta^\lambda_\mu = -\varepsilon_{\mu\nu}\varepsilon^{\alpha\lambda}$ and therefore

$$E^{\rho\sigma}_{\mu\nu} = A(x^2)\ \varepsilon_{\mu\nu}\varepsilon^{\rho\sigma}$$

Because of current conservation, $A(x^2)$ is a constant, which remains to be determined:

Only the graphs in fig.2 contribute to the left hand side (l.h.s.) of (II-4a). To verify this notice that the graphs contributing to $<0|T[J_\mu(\varepsilon), J_\nu(0)]z_\ell\bar{z}_k A_\lambda|0>$ have the general structure shown in fig.1. Now if the derivative $\frac{\partial}{\partial k_\alpha}$ does not act directly on the momentum factors associated with the current vertex, we obtain a result symmetric under the exchange $\mu\leftrightarrow\nu$. Therefore this type of term (fig. 1a) will not contribute to the l.h.s. of (II-9).

On the other hand, if the derivative acts on the momentum factors at the current vertex, the only graphs which contribute are those of fig.2. For the graphs of fig.1b-c this happens because the insertion of a zero momentum external wavy line will produce a result proportional to the derivative with respect to the loop momentum p. (The graph of fig.1d is trivially zero). Thus after integration we will get zero except for the graphs of fig.2, in which case there will be non-vanishing surface terms. But these terms have already been computed in ref. (8), so that the result (II-4a) holds in every finite order of the 1/n expansion.

III - CONSERVATION OF THE SUPERSYMMETRIC QUANTUM NON-LOCAL CHARGE TO ALL ORDERS

The formal Lagrangian density that couples the CP^{n-1} - model supersymmetrically to fermions is

$$\mathcal{L} = \overline{D_\mu z} D_\mu z + \bar{\psi}\ (\not\partial - \gamma_\mu \bar{z}\partial_\mu z)\psi + \frac{f}{2n}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 - (\bar{\psi}\gamma_\mu\psi)^2\right] \qquad \text{(III-1)}$$

where

$$D_\mu z = \partial_\mu z - \frac{2f}{n}(\bar{z}\partial_\mu z)z \qquad \text{(III-1a)}$$
$$\bar{z}z = \frac{n}{2f} \qquad \text{(III-1b)}$$
$$\bar{\psi}z = \bar{z}\psi = 0 \qquad \text{(III-1c)}$$

The 1/n expansion as well as the Feynman rules were derived in ref. (10). We follow the graphical notation of ref.9 to which the reader is also refered for details. To enforce the constraints (III-1b-c), one introduceds the α and c fields,

respectivelly. The quadrilinear interactions are reduced to bilinear ones by introducing the auxiliary fields π, ϕ, λ_μ.

Our strategy will be the same as for the pure CP^{n-1} case but the technical details are more complicated.

The model has a classical non-local charge specified by:

$$Q^{ij} = \int dy_1 dy_2\, \varepsilon(y_1 - y_2) J_0^{ik}(t,y_1) J_0^{kj}(t,y_2) - \int dy (j_1 + 2i_1)^{ij}(t,y) + $$
$$+ i\int dy \left[z_i \bar{z}_j (\bar{\psi}\gamma_1\psi)\right](t,y) \qquad \text{(III-2)}$$

where

$$J_\mu^{ij} = z^i \overleftrightarrow{D}_\mu z^j + \bar{\psi}^j \gamma_\mu \psi^i = (j_\mu + i_\mu)^{ij} \qquad \text{(III-2a)}$$

$$D_\mu z = \partial_\mu z - A_\mu z \qquad \text{(III-2b)}$$

To give a correct quantum definition of (III-2) we have, as before, to examine the short-distance behaviour of the product of two currents.

A priori there will be a huge number of local operators, of dimension equal or less than two*, which appear in this Wilson expansion. However, as shown in Appendix B, P.T., charge conjugation, charge conservation and general graphical arguments strongly restrict the number of allowed candidates and we are left with:

$$\left[J_\mu(x+\varepsilon),\ J_\nu(x)\right] = C_{\mu\nu}^{\rho} j_\rho + 2C'^{\rho}_{\mu\nu}\, i_\rho + D_{\mu\nu}^{\sigma\rho}\partial_\sigma j_\rho + 2D'^{\sigma\rho}_{\mu\nu}\partial_\sigma i_\rho$$
$$+ E_{\mu\nu}^{\rho\sigma} z_i \bar{z}_j F_{\rho\sigma} + N\left[J_\mu, J_\nu\right] \qquad \text{(III-3)}$$

---

* As in Luscher's case we argue that assymptotic freedom, restricts the dimension of the local operators to be ≤ 2.

where j_ρ is the pure CP^{n-1} current and $i_\rho = \bar{\psi}_j \gamma_\rho \psi_i$ is the fermionic current as specified in (III-2a).

Now using the same arguments as in the pure CP^{n-1} case we see that only the lowest order graphs contribute. The coefficients $C^\rho_{\mu\nu}$ and $D^{\sigma\rho}_{\mu\nu}$ are the same as those of section II, while for $C'^\rho_{\mu\nu}$ and $D'^{\sigma\rho}_{\mu\nu}$ we have the result in ref.9, $E^{\rho\sigma}_{\mu\nu}$ being zero in all orders. Explicitely,

$$C^\rho_{\mu\nu}(\varepsilon) = \frac{n}{2\pi}\left[-\frac{\delta_{\mu\nu}\varepsilon^\rho}{\varepsilon^2} + \frac{\delta^\rho_\mu \varepsilon_\nu}{\varepsilon^2} + \frac{\delta^\rho_\nu \varepsilon_\mu}{\varepsilon^2} + \frac{2\varepsilon_\mu \varepsilon_\nu \varepsilon^\rho}{\varepsilon^2}\right] \tag{III-4a}$$

$$D^{\rho\sigma}_{\mu\nu}(\varepsilon) = \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \frac{1}{4}\ \ln\frac{m^2\varepsilon^2}{4}\right)(\delta^\sigma_\mu\delta^\rho_\nu - \delta^\sigma_\nu\delta^\rho_\mu) + \frac{\delta^\sigma_\nu \varepsilon_\mu \varepsilon^\rho}{2\varepsilon^2} - \frac{\delta^\sigma_\mu \varepsilon_\nu \varepsilon^\rho}{2\varepsilon^2} - \right.$$
$$\left. - \frac{\delta_{\mu\nu}\varepsilon^\rho\varepsilon^\sigma}{2\varepsilon^2} + \frac{\delta^\rho_\mu \varepsilon_\nu \varepsilon^\sigma}{2\varepsilon^2} - \frac{\delta^\rho_\nu \varepsilon_\mu \varepsilon^\sigma}{2\varepsilon^2} + \frac{\varepsilon_\mu\varepsilon_\nu\varepsilon^\sigma\varepsilon^\rho}{(\varepsilon^2)^2}\right] \tag{III-4b}$$

$$C'^\rho_{\mu\nu}(\varepsilon) = \frac{n}{2\pi}\left[-\frac{\delta_{\mu\nu}\varepsilon^\rho}{\varepsilon^2} + \frac{\delta^\rho_\mu \varepsilon_\nu}{\varepsilon^2} + \frac{\delta^\rho_\nu \varepsilon_\mu}{\varepsilon^2}\right] \tag{III-4c}$$

$$D'^{\rho\sigma}_{\mu\nu}(\varepsilon) = \frac{1}{2}\, C'^\rho_{\mu\nu}\varepsilon^\sigma + D'_1\varepsilon^\rho\ (\varepsilon_\mu\delta^\sigma_\nu - \varepsilon_\nu\delta^\sigma_\mu) + D'_2\varepsilon^\sigma\ (\varepsilon_\mu\delta^\sigma_\nu + \varepsilon_\nu\delta^\sigma_\mu) \tag{III-4d}$$

where

$$-x^2 D'_1 + D'_3 = \frac{1}{4\pi}\ \ln\,\mu^2 x^2 \tag{III-4e}$$

It follows that the quantum-non-local charge.

$$Q^{ij}_\delta = \frac{1}{n}\int_{|y_1-y_2|\geq\delta} dy_1 dy_2\, \varepsilon(y_1-y_2)\, J_0^{ik}(t,y_1)\, J_0^{kj}(t,y_2) - \frac{z_\delta}{n}\int dy\ [j_1(t,y) +$$
$$+ 2i_1(t,y)]^{ij} + \frac{i}{n}\int dy\ (z_i\bar{z}_j\bar{\psi}\gamma_1\psi)(y,t) \tag{III-5}$$

is conserved to all orders of 1/n.

IV - CONCLUSIONS

We have proved that to all orders in the 1/n expansion, conservation of the non-local charge in the pure CP^{n-1} model is spoiled by an anomaly with a calculable coefficient while for its supersymmetric extension it is restored. In this last case, this means that the quantum S-matrix can be calculated, using standard procedures such as factorization equations, justified by the non-local charge conservation.

The case of the pure CP^{n-1} model is more involved. Here, the existence of the anomaly is in accord with the confining[15] properties of the model. The next question concerns the scattering of bound states. We conjecture that the anomaly will not contribute (with the same implications as above) if the relevant asymptotic states are constructed from the vacuum by application of operators which commute with the anomaly.

We remark also that our results are valid to all orders of the 1/n expansion but neglect non-perturbative aspects such as θ-vacua and pseudo-particles. The existence of an anomaly in the pure CP^{n-1} model and its absence in the $O(n)$ σ-model puts forward the following question: what are the fundamental properties determining the possibilities of anomalies in quantum non-local charges?

As will be shown elsewhere[16], the absence of an anomaly in the purely bosonic non-linear σ-models, defined on symmetric spaces G/H, can be traced back to the fact that H is simple.

Finally, we would like to remark that CP^1 is anomaly free: the would be anomaly is a total derivative which can be absorbed into a redefinition of the charge.

APPENDIX A

In this Appendix we determine the coefficients $C^{\rho}_{\mu\nu}$ and $D^{\rho\sigma}_{\mu\nu}$ of equation (II-4a) using general properties such as parity, time reversal and current conservation. This will give us non-perturbative information also about the coefficient $E^{\rho\sigma}_{\mu\nu}$ of the Wilson expansion.

The results obtained in this way are listed below.

i) P.T.:

$$C^{\rho\sigma}_{\mu\nu}(-\varepsilon) = -C^{\rho}_{\mu\nu}(\varepsilon) \qquad \text{(A-1a)}$$

$$D^{\rho\sigma}_{\mu\nu}(-\varepsilon) = D^{\rho\sigma}_{\mu\nu}(\varepsilon) \qquad \text{(A-1b)}$$

$$E^{\rho\sigma}_{\mu\nu}(-\varepsilon) = E^{\rho\sigma}_{\mu\nu}(\varepsilon) \qquad \text{(A-1c)}$$

ii) Charge conjugation:

$$C^{\rho}_{\mu\nu}(\varepsilon) = -C^{\rho}_{\nu\mu}(-\varepsilon) \qquad \text{(A-2a)}$$

$$D^{\rho\sigma}_{\mu\nu}(\varepsilon) = -D^{\rho\sigma}_{\nu\mu}(-\varepsilon) - \left[C^{\sigma}_{\mu\nu}(-\varepsilon)\varepsilon^{\rho} - \frac{1}{2}\delta^{\rho\sigma}C^{\lambda}_{\nu\mu}(-\varepsilon)\varepsilon_{\lambda}\right] \qquad \text{(A-2b)}$$

$$E^{\rho\sigma}_{\mu\nu}(\varepsilon) = -E^{\rho\sigma}_{\nu\mu}(-\varepsilon) \qquad \text{(A-2c)}$$

iii) Current conservation:

Using (II-4a) we have

$$\partial^{\mu}_{\varepsilon}\left[\langle 0|T[J_{\mu}(x+\varepsilon),J_{\nu}(x)]^{ij}\, X|0\rangle\right] = \partial^{\mu}_{\varepsilon}\, C^{\rho}_{\mu\nu}\langle 0|TJ^{ij}_{\rho}(x)\;X\;|0\rangle + $$
$$+\,\partial^{\mu}D^{\rho\sigma}_{\mu\nu}(\varepsilon\;)\;\langle 0|T\partial_{\sigma}J^{ij}_{\rho}(x)\;X\;|0\rangle + \partial^{\mu}E^{\rho\sigma}_{\mu\nu}(\varepsilon)\langle 0|Tz^{i}\bar{z}^{j}F_{\rho\sigma}(x)X\;|0\rangle \qquad \text{(A-3)}$$

As a consequence of (II-7), the term containing $\delta(\varepsilon)$ on the l.h.s. is given by

$$-2n\delta(\varepsilon)\langle 0|J^{\nu}_{ij}(x)\;X\;|0\rangle \qquad \text{(A-4)}$$

Thus we require that

$$\partial^{\mu}C^{\rho}_{\mu\nu}(x) = 2n\delta(x)\delta^{\rho}_{\nu} \qquad \text{(A-5a)}$$

$$\partial^{\mu}D^{\rho\sigma}_{\mu\nu}(x) = \partial^{\mu}E^{\rho\sigma}_{\mu\nu} = 0 \qquad \text{(A-5b)}$$

Now, we use that the above coefficients have the follwing tensorial decomposition.

$$C^{\rho}_{\mu\nu}(x) = C_1(x^2)g_{\mu\nu}x^{\rho} + C_2(x^2)(x_{\mu}\delta^{\rho}_{\nu} + x_{\nu}\delta^{\rho}_{\mu}) + C_3(x^2)x_{\mu}x_{\nu}x^{\rho} \qquad \text{(A-6a)}$$

$$D^{\rho\sigma}_{\mu\nu}(x) = D_1x^{\sigma}(x_{\mu}\delta^{\rho}_{\nu} - x_{\nu}\delta^{\rho}_{\mu}) + D_2x^{\rho}(x_{\mu}\delta^{\sigma}_{\nu} - x_{\nu}\delta^{\sigma}_{\mu}) + \frac{1}{2}(C^{\sigma}_{\mu\nu}x^{\rho} - \frac{1}{2}g^{\rho\sigma}C^{\lambda}_{\mu\nu}x_{\lambda}) \qquad \text{(A-6b)}$$

$$E^{\rho\sigma}_{\mu\nu}(x) = A(x^2)\varepsilon_{\mu\nu}\varepsilon^{\rho\sigma} \qquad \text{(A-6c)}$$

We obtain, for $x \neq 0$

$$C_1 + 2x^2C'_2 + 3C_2 = 0 \qquad \text{(A-7a)}$$
$$C'_1 + C'_2 + x^2C'_3 + 2C_3 = 0 \qquad \text{(A-7b)}$$
$$2x^2D'_1 + 2D_1 + \frac{1}{2}C_1 = 0 \qquad \text{(A-7c)}$$
$$-2D'_1 - 2D'_2 + \frac{1}{2}C_3 = 0 \qquad \text{(A-7d)}$$
$$-D_1 - D_2 - \frac{C_2}{4} - \frac{1}{4}C_1 - \frac{1}{4}C_3\,x^2 = 0 \qquad \text{(A-7e)}$$
$$2x^2D'_2 + 2D_2 + \frac{1}{2}C_2 = 0 \qquad \text{(A-7f)}$$
$$A' = 0 \qquad \text{(A-7g)}$$

The above equations are not enough to fix the C's and D's. To get more information we proceed as follows:

From (II-4a) we have:

$$\partial_x^\nu <0|T[J_\mu(y),J_\nu(x)]^{ij} z^\ell(x') \ X \ |0> = \partial_x^\nu C^\rho_{\mu\nu}(x-y)<0|TJ_\rho^{ij}(x) z^\ell(x') X \ |0> +$$

$$+ C^\rho_{\mu\nu}(y-x)\partial_x^\nu <0|TJ_\rho^{ij}(x) z^\ell(x') \ X \ |0> + \partial_x^\nu D^{\rho\sigma}_{\mu\nu}(y-x)<0|T\partial_\sigma J_\rho^{ij} z^\ell(x') X \ |0>$$

$$+ \partial_x^\nu E^{\rho\sigma}_{\mu\nu}(y-x)<0[T|z_i \bar{z}_j F_{\rho\sigma}](x) z^\ell(x') \ X \ |0> + 0(\ln(x-y)^2) \qquad \text{(A-8)}$$

In computing the l.h.s. of (A-8) we will retain only terms involving delta functions of (y-x) or (x-x').

Using (A-5a-b) we get (at $x^0 = y^0$):

$$-in\delta(x-x')<0|T[J_1^{ik}(y)\delta^{\ell j} z^k(x^1) - \delta^{\ell k} z^i(x^1) J_1^{kj}(y)]|0> =$$

$$= \{C^\rho_{1\nu}\partial^\nu <0|TJ_\rho^{ij}(x) z^\ell(x^1) \ X \ |0> + \partial^\nu D^{\rho\sigma}_{\mu\nu}\partial_\sigma <0|TJ_\rho^{ij}(x) z^\ell(x) X \ |0>\}\Big|_{\substack{y^0=x^0 \\ y'-x'=\varepsilon'}}$$

$$= C_2(x^2)\varepsilon_1 \partial^\nu <0|TJ_\nu^{ij}(x) z^\ell(x^1) \ X \ |0> \qquad \text{(A-9)}$$

So that

$$<0|T[J_1^{ik}(y) z^k(x)\delta^{\ell j} - J_1^{\ell j}(y) z^i(x)]X \ |0> =$$

$$= C_2(x^2)\varepsilon_1 \delta^{j\ell} <0|T(\delta^{j\ell} z^i(x) - \frac{1}{n}\delta^{ij} z^\ell(x))X \ |0> \qquad \text{(A-10)}$$

From (A-10) and (II-8) we obtain, finally

$$C_2(x^2) = \frac{n}{2\pi}\,\frac{1}{x^2} \qquad \text{(A-11)}$$

Having found C_2, C_1 is evaluated with the help of (A-7a)

$$C_1(x^2) = -\frac{n}{2\pi}\,\frac{1}{x^2} \qquad \text{(A-12)}$$

and from (A-7b) we find

$$C_3(x^2) = \frac{\lambda}{(x^2)^2} \qquad \text{(A-13)}$$

The value of λ above is fixed by imposing (A-5a), which gives

$$C_3(x^2) = \frac{n}{\pi}\,\frac{1}{(x^2)^2} \qquad \text{(A-14)}$$

Using the equations (A-7c,d,f) we find D_1 and D_2 .

$$D_1 = \frac{n}{8\pi x^2} \ln \mu^2 x^2 \qquad \text{(A-15)}$$

$$D_2 = \frac{-n}{8\pi \ x^2} \ln\mu^2 x^2 - \frac{n}{4\pi}\,\frac{1}{x^2} \qquad \text{(A-16)}$$

APPENDIX B

In this Appendix we show that the short distance expansion for the product of two currents of the CP^{n-1} supersymmetric model is given by (III-3). To prove this we note that the local operators contributing to the commutator $[J_\mu(x+\varepsilon),J_\nu(x)]^{ij}$ have at most dimension two. Therefore the allowed candidates are those listed in table I which shows also the behavior of the coefficients under P.T. and charge conjugation.

Using table I we analyse whether each of the coefficients contribute to the Wilson expansion or not. Since the first five will survive we begin the discussion with the coefficient number 6.

1 - Coefficient number 6 ($E'_{\mu\nu}(\varepsilon)$).

By P.T. and charge conjugation (C.C.) we conclude that

$$E'_{\mu\nu}(\varepsilon) = -E'_{\nu\mu}(\varepsilon) \qquad \text{(B-1)}$$

so that

$$E'_{\mu\nu}(\varepsilon) = \varepsilon_{\mu\nu} f(\varepsilon^2) \qquad \text{(B-2)}$$

Because of current conservation $f(\varepsilon^2)$ is a constant f. The normalization condition gives (our normal products are always normalized at zero external momenta).

$$f\varepsilon_{\mu\nu} = \langle 0|Tz_i(p)\bar{z}_j(q)\pi(r)\ [J_\mu(\varepsilon),J_\nu(0)]\ |0\rangle\Big|_{p=q=r=0} \qquad \text{(B-3)}$$

The graphs contributing to the r.h.s. of (B-3) have the structure shown in fig. 3. Thus we have

$$\text{fig 3} = \int \frac{d^2k}{(2\pi)^2}\ e^{ik\varepsilon}\ (k+2p)_\mu\ \ \Delta(k,p,r)(k+p+r+q)_\nu\Big|_{p=q=r=0} \qquad \text{(B-4)}$$

where the $\Delta(k,p,r)$ factor can be explicited using the Feynman rules. The equation (B-4) is symmetric under the interchange $\mu \leftrightarrow \nu$, and we conclude, therefore, that $E'_{\mu\nu}(\varepsilon)=0$. The same arguments can be applied to show that the coefficients number 8,12,14,20,44,47,48, 52,55,56,59,62 also vanish.

2 - Coefficient number 7 ($F_{\mu\nu}(\varepsilon)$).

Taking the adjoint of the Wilson expansion one readily sees that $F_{\mu\nu}(\varepsilon)$ must be purely imaginary. On the other hand

$$F_{\mu\nu}(\varepsilon) = \langle 0|Tz_i\bar{z}_j\phi\ [J_\mu(\varepsilon),J_\nu(0)]\ |0\rangle = \int \frac{d^2k}{(2\pi)^2} e^{ik\varepsilon}\ g_{\mu\nu}(k), \qquad \text{(B-5)}$$

where by inspection $g_{\mu\nu}(k)$ is real. (Although the fermion propagators have both imaginary and real parts, only products of an even number of imaginary parts contribute). It follows that:

$$F_{\mu\nu}(-\varepsilon) = F^*_{\mu\nu}(\varepsilon) = -F_{\mu\nu}(\varepsilon) \qquad \text{(B-6)}$$

By P.T.

$$F_{\mu\nu}(\varepsilon) = F_{\mu\nu}(-\varepsilon), \qquad \text{(B-7)}$$

and we conclude that $F_{\mu\nu}(\varepsilon) = 0$. An identical argument can be used for the coefficients numbers 10,11.

3 - Coefficients number 9,18 ($G^{\rho}_{\mu\nu}(\varepsilon)$, $K'^{\rho\sigma}_{\mu\nu}(\varepsilon)$).

First of all, due to P.T., c.c. and

$$G^{\rho}_{\mu\nu}(\varepsilon) = g_1 \frac{g_{\mu\nu}\varepsilon^{\rho}}{\varepsilon^2} + g_2 \frac{\varepsilon_{\mu} g^{\rho}_{\nu} + \varepsilon_{\nu} g^{\rho}_{\mu}}{\varepsilon^2} + g_3 \frac{\varepsilon_{\mu}\varepsilon_{\nu}\varepsilon^{\rho}}{(\varepsilon^2)^2} \tag{B-8}$$

where g_1, g_2, g_3 are constants.

Using that (up to logarithmically divergent terms)

$$\partial^{\nu}_{x} <0|\, T\, [J_{\mu}(y), J_{\nu}(x)]_{ij}\ \psi^{k}_{\alpha}(x')\ X\ |0> =$$

$$= \partial^{\nu}_{x} G^{\rho}_{\mu\nu}(y-x) <0|T(z_i \bar{z}_j \bar{\psi}\gamma_{\rho}\psi)(x)\,\psi^{k}_{\alpha}(x')\ X\ |0> +$$

$$+ G^{\rho}_{\mu\nu}(y-x)\, \partial^{\nu}_{x} <0|T(z_i \bar{z}_j \bar{\psi}\gamma_{\rho}\psi)(x)\,\psi^{k}_{\alpha}(x')\, X\ |0> +$$

$$+ \partial^{\nu}_{x} K'^{\rho\sigma}_{\mu\nu}(y-x) <0|T\partial_{\sigma}(z_i \bar{z}_j \bar{\psi}\gamma_{\rho}\psi)(x)\,\psi^{k}_{\alpha}(x')\ X\ |0> +$$

$$+ \text{other terms} \tag{B-9}$$

we see that the l.h.s. provides terms linear in the J_{μ}, ψ and X fields (X denotes a products of the z's, ψ's and their adjoints), but no term proportional to $z_i \bar{z}_j \bar{\psi}\gamma_{\rho}\psi$. These linear contributions cancel those coming from $C'^{\rho}_{\mu\nu}$ on the r.h.s., but no contributions arise which are able to cancel $G^{\rho}_{\mu\nu}(\varepsilon)$. It follows that g_1=0, implying by current conservation that g_2=0. To see that g_3 is also zero, we reconsider equation (A-3), with the r.h.s. supplemented by the term:

$$\partial^{\mu} G^{\rho}_{\mu\nu}(\varepsilon)\ <0|T(z_i \bar{z}_j \bar{\psi}\gamma_{\rho}\psi)\ (x)\ X\ |0> \tag{B-10}$$

and the analogue to (A-5a) is

$$\partial^{\mu} G^{\rho}_{\mu\nu}(\varepsilon) = 0 \tag{B-11}$$

Note that no $\delta(\varepsilon)$ term appears, and as a consequence, g_3=0. It is straightforward to see that this implies $K'^{\rho\sigma}_{\mu\nu}(\varepsilon)$=0.

4 - Coefficients number 13,19 ($I_{\mu\nu}(\varepsilon)$, $L_{\mu\nu}(\varepsilon)$).

Taking the symmetric and antisymmetric parts of the coefficients (which have definite transformation properties under charge conjugation), we see that both parts are zero, using the arguments of the cases number 1 and 2.

5 - Coefficient number 15 ($J^{\rho}_{\mu\nu}(\varepsilon)$).

By P.T., c.c. and P. we get for this coefficient

$$J_{\mu\nu}(\varepsilon) = j_1(\varepsilon^2)\varepsilon_{\mu\nu}\varepsilon^{\rho} + j_2(\varepsilon^2)(\varepsilon^{\rho}_{\mu}\varepsilon_{\nu} - \varepsilon^{\rho}_{\nu}\varepsilon_{\mu}) \tag{B-12}$$

However current conservation implies:

$$J^{\rho}_{\mu\nu}(\varepsilon) = j\,(\varepsilon_{\mu\nu}\varepsilon^{\rho} - \varepsilon^{\rho}_{\mu}\varepsilon_{\nu} + \varepsilon^{\rho}_{\nu}\varepsilon_{\mu}) \tag{B-13}$$

which tends to zero as $\varepsilon \to 0$. Analogously for the coefficients number 16, 22, 25, 26, 43, 46, 51, 60.

6 - Coefficient number 17 ($K^{\rho\sigma}_{\mu\nu}(\varepsilon)$).

P.T. and c.c. enforce:

$$K^{\rho}_{\mu\nu}(\varepsilon) = k(\varepsilon^2)(\varepsilon_\mu g^\rho_\nu - \varepsilon_\nu g^\delta_\mu) \qquad \text{(B-14)}$$

and this is consistent with current conservation only if $K^{\rho}_{\mu\nu}=0$. The same argument holds for coefficients number 21, 29, 36, 40.

7 - Coefficients number 23, 24 ($N^{\rho}_{\mu\nu}(\varepsilon)$, $N'^{\rho}_{\mu\nu}(\varepsilon)$).

First we take the symmetric and antisymmetric parts which have definite transformation properties under charge conjugation. Combining the arguments used in the last two cases we get that these parts are zero. For the pair (27,28) one can use the same argument.

8 - Coefficients number 30, 31 ($Q'^{\rho}_{\mu\nu}(\varepsilon)$, $R^{\rho}_{\mu\nu}(\varepsilon)$).

For this pair we use a combination of the arguments of the second ($F_{\mu\nu}(\varepsilon)$) and the sixth case ($K^{\rho}_{\mu\nu}(\varepsilon)$).

9 - Coefficient number 32 ($R'^{\rho\sigma}_{\mu\nu}(\varepsilon)$).

The graphical contribution is shown in fig. 4. We have

$$R'^{\rho\sigma}_{\mu\nu}(\varepsilon) = \gamma^\rho_{\alpha\beta}\left(\frac{\partial}{\partial r_\sigma} - \frac{\partial}{\partial s_\sigma}\right)\langle 0|T\bar z_i(p) z_j(q)\bar\psi_{k_\alpha}(r)\psi_{k_\beta}(s)\left[J_\mu(\varepsilon),J_\nu(0)\right]|\,\rangle\Big|_{p=q=r=s=0}$$

$$= \int\frac{d^2k}{(2\pi)^2}\, k_\mu(k+p+q+r+s)_\nu\, \gamma^\rho_{\alpha\beta}\left(\frac{\partial}{\partial r_\sigma} - \frac{\partial}{\partial s_\sigma}\right) f_{\alpha\beta}(p,k,s,r)\, e^{ik\varepsilon}\Big|_{p=q=r=s=0} - \binom{\mu\leftrightarrow\nu}{\varepsilon\leftrightarrow\bar\varepsilon}$$

$$= \int\frac{d^2k}{(2\pi)^2}\, k_\mu k_\nu \gamma^\rho\left(\frac{\partial}{\partial r_\sigma} - \frac{\partial}{\partial s_\sigma}\right) f_{\alpha\beta}(p,k,r,s)\,(e^{ik\varepsilon} - e^{-ik\varepsilon}) \qquad \text{(B-15)}$$

which is antisymmetric under the intercharge $\varepsilon \leftrightarrow \varepsilon-$. But this is forbidden by P.T. The same holds for the coefficient number 37.

10 - Coefficient number 33 ($S^{\rho\sigma}_{\mu\nu}(\varepsilon)$).

By P.T., c.c. and current conservation

$$S^{\rho\sigma}_{\mu\nu}(\varepsilon) = s\varepsilon_{\mu\nu}\,\varepsilon^{\rho\sigma} \qquad \text{(B-16)}$$

on the other hand

$$S^{\rho\sigma}_{\mu\nu}(\varepsilon) = \mathrm{tr}\gamma^\sigma\left(\frac{\partial}{\partial r_\rho} - \frac{\partial}{\partial s_\rho}\right)\int\frac{d^2k}{(2\pi)^2}\, k_\mu(k+r+s+p+q)_\nu\, f(p,k,r,s)\, e^{ik\varepsilon} - \binom{\mu\leftrightarrow\nu}{\varepsilon\leftrightarrow-\varepsilon}$$

In view of the equation (B-16), only the antisymmetric part survives and the derivatives act only on the $k_\mu(k+r+s+p+q)_\nu$ factor. After multiplying by $\varepsilon^{\mu\nu}\varepsilon_{\rho\sigma}$, we get

$$s = \mathrm{tr}\int\frac{d^2k}{(2\pi)^2}\,(k_\mu g^\rho_\nu\, e^{ik\varepsilon} - k_\nu g^\rho_\mu\, e^{-ik\varepsilon})\,\varepsilon^{\mu\nu}\gamma_\rho\gamma_5 f(k) \qquad \text{(B-17)}$$

which is antisymmetric under $\varepsilon\leftrightarrow\varepsilon-$, violating P.T.

11 - Coefficients number 34, 35 ($S'^{\rho\sigma}_{\mu\nu}(\varepsilon)$, $T^{\rho\sigma}_{\mu\nu}(\varepsilon)$).

For the symmetric part of this pair we use the same argument as in the case of $R'^{\rho\sigma}_{\mu\nu}(\varepsilon)$ (number 32). For the antisymmetric part the argument is the same as in the case of $S^{\rho\sigma}_{\mu\nu}(\varepsilon)$ (number 33).

12 - Coefficients number 38, 39 ($U'^{\rho}_{\mu\nu}(\varepsilon)$, $V^{\rho}_{\mu\nu}(\varepsilon)$).

Taking the antisymmetric part of this pair we use the sixth argument. For the symmetric part we see that:

$$U'^{\rho}_{\{\mu\nu\}}(\varepsilon) = \mathrm{tr}\left(\frac{\partial}{\partial q_\rho} - \frac{\partial}{\partial p_\rho}\right)\int \frac{d^2k}{(2\pi)^2}(k+2p)_\mu (k+p+r+s+q)_\nu \; f\, e^{ik\varepsilon}\Big|_{p=q=r=s=0} - \left(\begin{smallmatrix}\mu\leftrightarrow\nu\\ \varepsilon\leftrightarrow-\varepsilon\end{smallmatrix}\right)$$

$$= \mathrm{tr}\int \frac{d^2k}{(2\pi)^2} k_\mu k_\nu \left(\frac{\partial}{\partial q_\rho} - \frac{\partial}{\partial p_\rho}\right) f(e^{ik\varepsilon} - e^{-ik\varepsilon}) + \int \frac{d^2k}{(2\pi)^2}\, 2(g^\rho_\mu k_\nu e^{ik\varepsilon} - g^\rho_\nu k_\mu e^{-ik\varepsilon}) f \qquad \text{(B-18)}$$

The first integral is convergent, whereas the second one Furnishes:

$$U'^{\rho}_{\{\mu\nu\}}(\varepsilon) = u(\varepsilon^2)(g^\rho_\mu \varepsilon_\nu + g^\rho_\nu \varepsilon_\mu) \qquad \text{(B-19)}$$

At this point current conservation is enough to imply $U'^{\rho}_{\{\mu\nu\}}(\varepsilon)=0$.

13 - Coefficient number 41 ($X^{\rho}_{\mu\nu}(\varepsilon)$).

We have

$$X^{\rho}_{\mu\nu}(\varepsilon) = \mathrm{tr}\left(\frac{\partial}{\partial p_\rho} - \frac{\partial}{\partial q_\rho}\right) \frac{d^2k}{(2\pi)^2}\gamma_\mu \Gamma(p,q)\gamma_\nu \; e^{ik\varepsilon}\Big|_{p=q=0} - \left(\begin{smallmatrix}\mu\leftrightarrow\nu\\ \varepsilon\leftrightarrow-\varepsilon\end{smallmatrix}\right), \qquad \text{(B-20)}$$

where in general.

$$\Gamma(p,q) = b(p,q) + c\,(p,q)\gamma_5 + a_n(p,q)\gamma^n, \qquad \text{(B-21)}$$

giving us 3 terms:

1) $\mathrm{tr}\left[\gamma_\mu\gamma^n\gamma_\nu e^{ik\varepsilon} - \gamma_\nu\gamma^n\gamma_\mu e^{-ik\varepsilon}\right] f(k)$

This is zero because $\mathrm{tr}\,\gamma_\mu\gamma^n\gamma_\nu = 0$.

2) $\mathrm{tr}\,\gamma_5\left[\gamma_\mu\gamma_\nu e^{ik\varepsilon} - \gamma_\nu\gamma_\mu e^{-ik\varepsilon}\right] f(k)$

This is proportional to $\varepsilon_{\mu\nu}$, which is forbidden by P.

3) $\mathrm{tr}\left[\gamma_\mu\gamma_\nu e^{ik\varepsilon} - \gamma_\nu\gamma_\mu e^{-ik\varepsilon}\right] f(k)$

This has the tensor structure.

$$X^{\rho}_{\mu\nu}(\varepsilon) = x(\varepsilon^2)\, g_{\mu\nu}\varepsilon^\rho \qquad \text{(B-22)}$$

and again current conservation requires $X^{\rho}_{\mu\nu}(\varepsilon) = 0$.

This argument can be used for the coefficient number 54, 57, 58.

14 - Coefficient number 42 ($X'^{\rho}_{\mu\nu}(\varepsilon)$).

By P.T., P. and charge conjugation.

$$X'^{\rho}_{\mu\nu}(\varepsilon) = x(\varepsilon^2)(\varepsilon^\rho_\mu \varepsilon_\nu + \varepsilon^\rho_\nu \varepsilon_\mu) \qquad \text{(B-23)}$$

and current conservation implies $X'^{\rho}_{\mu\nu}(\varepsilon)=0$.

15 - Coefficient number 45 $(W_{\mu\nu}(\varepsilon))$.

P.T. and c.c. imply

$$W_{\mu\nu}(\varepsilon) = \omega_1(\varepsilon^2)\, g_{\mu\nu} + \omega_2(\varepsilon^2)\varepsilon_\mu\varepsilon_\nu \tag{B-24}$$

On the other hand,

$$W_{\mu\nu}(\varepsilon) = \int \frac{d^2k}{(2\pi)^2}\, k_\mu k_\nu f(k) e^{ik\varepsilon} - \left({\mu\leftrightarrow\nu \atop \varepsilon\leftrightarrow-\varepsilon}\right) \tag{B-25}$$

which is antisymmetric under $\varepsilon\leftrightarrow\varepsilon-$, violating P. The same argument holds for coefficients number 50, 64, 65, 66, 68.

16 - Coefficient number 49 $(a^\rho_{\mu\nu}(\varepsilon))$.

We have

$$a^\rho_{\mu\nu}(\varepsilon) = \int \frac{d^2k}{(2\pi)^2}\, k_\mu k_\nu f(k)\left(e^{ik\varepsilon} - e^{-ik\varepsilon}\right)$$

Each of the terms above is logarithmically divergent so that their dif erence is finite. This argument can also be applied to coefficient number 53.

17 - Coefficient number 61 $(h^\rho_{\mu\nu}(\varepsilon))$.

We have

$$h^\rho_{\mu\nu}(\varepsilon) = \mathrm{tr} \int \frac{d^2k}{(2\pi)^2} \left[\gamma_\mu f(k)\gamma_\nu e^{ik\varepsilon} - \gamma_\nu f(k)\gamma_\mu\, e^{-ik\varepsilon}\right] \tag{B-27}$$

and using the cyclicity of the trace:

$$h^\rho_{\mu\nu}(\varepsilon) = \int \frac{d^2k}{(2\pi)^2} \left[\gamma_\nu\gamma_\mu\, e^{ik\varepsilon} - \gamma_\mu\gamma_\nu\, e^{-ik\varepsilon}\right] f(k) \tag{B-28}$$

But $\gamma_\mu\gamma_\nu = g_{\mu\nu} + \varepsilon_{\mu\nu}\gamma_5$ (B-29)

where the last term does not contribute due to P., and for the first term we use the same argument as for coefficient number 49 $(a^\rho_{\mu\nu}(\varepsilon))$. The same holds for coefficient number 63.

18 - Coefficient number 67 $(k^{\rho\sigma}_{\mu\nu}(\varepsilon))$.

Here we have a product of two traces but the procedure is similar to the preceding cases.

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FIGURE CAPTIONS

FIG. 1 - Graphs contributing to the l.h.s. of equation II-9. Dot means derivative with respect to the momentum. In case (a) the derivative acts on all places except in the upper two vertices.

FIG. 2 - Non-vanishing lowest order graphs of Fig. 1.

FIG. 3 - Graph corresponding to the coefficient number 6 (equation B-4).

FIG. 4 - Graph corresponding to the coefficient number 32 (equation B-15).

TABLE I - Allowed operators in the Wilson expansion for the commutator of two currents. Listed are the behavior of the coefficients under P.T. and charge conjugation.

TABLE- 1

	OPERATOR	COEFFICIENT	P.T.	CHARGE CONJUGATION
1	j_ρ	$C^{\rho}_{\mu\nu}(\varepsilon)$	$-C^{\rho}_{\mu\nu}(-\varepsilon)$	$-C^{\rho}_{\nu\mu}(-\varepsilon)$
2	i_ρ	$C'^{\rho}_{\mu\nu}(\varepsilon)$	$-C'^{\rho}_{\mu\nu}(-\varepsilon)$	$-C'^{\rho}_{\nu\mu}(-\varepsilon)$
3	$\partial_\sigma j_\rho$	$D^{\sigma\rho}_{\mu\nu}(\varepsilon)$	$+D^{\sigma\rho}_{\mu\nu}(-\varepsilon)$	$-D^{\sigma\rho}_{\nu\mu}(-\varepsilon)-C^{\rho}_{\nu\mu}(-\varepsilon)\varepsilon^{\sigma}$
4	$\partial_\sigma i_\rho$	$D'^{\sigma\rho}_{\mu\nu}(\varepsilon)$	$+D'^{\sigma\rho}_{\mu\nu}(-\varepsilon)$	$-D'^{\sigma\rho}_{\nu\mu}(-\varepsilon)-C'^{\rho}_{\nu\mu}(-\varepsilon)\varepsilon^{\sigma}$
5	$z_i\bar{z}_j F_{\sigma\rho}$	$E^{\sigma\rho}_{\mu\nu}(\varepsilon)$	$+E^{\sigma\rho}_{\mu\nu}(-\varepsilon)$	$-E^{\sigma\rho}_{\nu\mu}(-\varepsilon)$
6	$z_i\bar{z}_j\pi$	$E'_{\mu\nu}(\varepsilon)$	$+E'_{\mu\nu}(-\varepsilon)$	$-E'_{\nu\mu}(-\varepsilon)$
7	$z_i\bar{z}_j\phi$	$F_{\mu\nu}(\varepsilon)$	$+F_{\mu\nu}(-\varepsilon)$	$+F_{\nu\mu}(-\varepsilon)$
8	$z_i\bar{z}_j\bar{\psi}\gamma_5\psi$	$F'_{\mu\nu}(\varepsilon)$	$+F'_{\mu\nu}(-\varepsilon)$	$-F'_{\nu\mu}(-\varepsilon)$
9	$z_i\bar{z}_j\bar{\psi}\gamma_\rho\psi$	$G^{\rho}_{\mu\nu}(\varepsilon)$	$-G^{\rho}_{\mu\nu}(-\varepsilon)$	$-G^{\rho}_{\nu\mu}(-\varepsilon)$
10	$z_i\bar{z}_j\bar{\psi}\psi$	$G'_{\mu\nu}(\varepsilon)$	$+G'_{\mu\nu}(-\varepsilon)$	$+G'_{\nu\mu}(-\varepsilon)$
11	$\bar{\psi}_j\psi_i$	$H_{\mu\nu}(\varepsilon)$	$+H_{\mu\nu}(-\varepsilon)$	$+H_{\nu\mu}(-\varepsilon)$
12	$\bar{\psi}_j\gamma_5\psi_i$	$H'_{\mu\nu}(\varepsilon)$	$+H'_{\mu\nu}(-\varepsilon)$	$-H'_{\nu\mu}(-\varepsilon)$
13	$z_i\bar{\psi}_j c$	$I_{\mu\nu}(\varepsilon)$	$+I_{\mu\nu}(-\varepsilon)$	$I_{\nu\mu}(-\varepsilon)=L_{\mu\nu}(\varepsilon)$
14	$z_i\bar{\psi}_j\gamma_5 c$	$I'_{\mu\nu}(\varepsilon)$	$+I'_{\mu\nu}(-\varepsilon)$	$I'_{\nu\mu}(-\varepsilon)=-L'_{\mu\nu}(\varepsilon)$
15	$\partial_\rho(z_i\bar{z}_j\bar{\psi}\gamma_5\psi)$	$J^{\rho}_{\mu\nu}(\varepsilon)$	$-J^{\rho}_{\mu\nu}(-\varepsilon)$	$-J^{\rho}_{\nu\mu}(-\varepsilon)$
16	$\partial_\rho(z_i\bar{z}_j\pi)$	$J'^{\rho}_{\mu\nu}(\varepsilon)$	$-J'^{\rho}_{\mu\nu}(-\varepsilon)$	$-J'^{\rho}_{\nu\mu}(-\varepsilon)$
17	$\partial_\rho(z_i\bar{z}_j\phi)$	$K^{\rho}_{\mu\nu}(\varepsilon)$	$-K^{\rho}_{\mu\nu}(-\varepsilon)$	$K^{\rho}_{\nu\mu}(-\varepsilon)$
18	$\partial_\rho(z_i\bar{z}_j\bar{\psi}\gamma_\sigma\psi)$	$K'^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+K'^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$-K'^{\rho\sigma}_{\nu\mu}(-\varepsilon)$
19	$\bar{z}_j\bar{c}\psi_i$	$L_{\mu\nu}(\varepsilon)$	$+L_{\mu\nu}(-\varepsilon)$	$L_{\nu\mu}(-\varepsilon)=I_{\mu\nu}(\varepsilon)$
20	$\bar{z}_j\bar{c}\gamma_5\psi_i$	$L'_{\mu\nu}(\varepsilon)$	$+L'_{\mu\nu}(-\varepsilon)$	$L'_{\nu\mu}(-\varepsilon)=I'_{\mu\nu}(\varepsilon)$

	OPERATOR	COEFFICIENT	P.T.	CHARGE CONJUGATION
21	$\partial_\rho(z_i\bar{z}_j\bar{\psi}\psi)$	$M^{\rho}_{\mu\nu}(\varepsilon)$	$-M^{\rho}_{\mu\nu}(-\varepsilon)$	$+M^{\rho}_{\nu\mu}(-\varepsilon)$
22	$z_i\bar{z}_j\partial_\rho\pi$	$M'^{\rho}_{\mu\nu}(\varepsilon)$	$-M'^{\rho}_{\mu\nu}(-\varepsilon)$	$-M'^{\rho}_{\nu\mu}(-\varepsilon)$
23	$z_i\partial_\rho\bar{z}_j\pi$	$N^{\rho}_{\mu\nu}(\varepsilon)$	$-N^{\rho}_{\mu\nu}(-\varepsilon)$	$N^{\rho}_{\nu\mu}(-\varepsilon)=-N'^{\rho}_{\mu\nu}(\varepsilon)$
24	$\partial_\rho z_i\bar{z}_j\pi$	$N'^{\rho}_{\mu\nu}(\varepsilon)$	$-N'^{\rho}_{\mu\nu}(-\varepsilon)$	$N'^{\rho}_{\nu\mu}(-\varepsilon)=-N^{\rho}_{\mu\nu}(\varepsilon)$
25	$z_i\bar{z}_j\partial_\rho(\bar{\psi}\gamma_5\psi)$	$O^{\rho}_{\mu\nu}(\varepsilon)$	$+O^{\rho}_{\mu\nu}(-\varepsilon)$	$-O^{\rho}_{\nu\mu}(-\varepsilon)$
26	$z_i\bar{z}_j\bar{\psi}\gamma_5\overleftrightarrow{\partial}_\rho\psi$	$O'^{\rho}_{\mu\nu}(\varepsilon)$	$-O'^{\rho}_{\mu\nu}(-\varepsilon)$	$+O'^{\rho}_{\nu\mu}(-\varepsilon)$
27	$\partial_\rho z_i\bar{z}_j\bar{\psi}\gamma_5\psi$	$P^{\rho}_{\mu\nu}(\varepsilon)$	$-P^{\rho}_{\mu\nu}(-\varepsilon)$	$P^{\rho}_{\nu\mu}(-\varepsilon)=-P'^{\rho}_{\mu\nu}(\varepsilon)$
28	$z_i\partial_\rho\bar{z}_j\bar{\psi}\gamma_5\psi$	$P'^{\rho}_{\mu\nu}(\varepsilon)$	$-P'^{\rho}_{\mu\nu}(-\varepsilon)$	$P'^{\rho}_{\nu\mu}(-\varepsilon)=-P^{\rho}_{\mu\nu}(\varepsilon)$
29	$z_i\bar{z}_j\partial_\rho\phi$	$Q^{\rho}_{\mu\nu}(\varepsilon)$	$-Q^{\rho}_{\mu\nu}(-\varepsilon)$	$+Q^{\rho}_{\nu\mu}(-\varepsilon)$
30	$z_i\partial_\rho\bar{z}_j\phi$	$Q'^{\rho}_{\mu\nu}(\varepsilon)$	$-Q'^{\rho}_{\mu\nu}(-\varepsilon)$	$Q'^{\rho}_{\nu\mu}(-\varepsilon)=R^{\rho}_{\mu\nu}(\varepsilon)$
31	$\partial_\rho z_i\bar{z}_j\phi$	$R^{\rho}_{\mu\nu}(\varepsilon)$	$-R^{\rho}_{\mu\nu}(-\varepsilon)$	$R^{\rho}_{\nu\mu}(-\varepsilon)=Q'^{\rho}_{\mu\nu}(\varepsilon)$
32	$z_i\bar{z}_j\bar{\psi}\gamma_\rho\overleftrightarrow{\partial}_\sigma\psi$	$R'^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+R'^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$+R'^{\rho\sigma}_{\nu\mu}(-\varepsilon)$
33	$z_i\bar{z}_j\partial_\rho(\bar{\psi}\gamma_\sigma\psi)$	$S^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+S^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$-S^{\rho\sigma}_{\nu\mu}(-\varepsilon)$
34	$\partial_\rho z_i\bar{z}_j\bar{\psi}\gamma_\sigma\psi$	$S'^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+S'^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$S'^{\rho\sigma}_{\nu\mu}(-\varepsilon)=-T^{\rho\sigma}_{\mu\nu}(\varepsilon)$
35	$z_i\partial_\rho\bar{z}_j\bar{\psi}\gamma_\sigma\psi$	$T^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+T^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$T^{\rho\sigma}_{\nu\mu}(-\varepsilon)=-S'^{\rho\sigma}_{\mu\nu}(\varepsilon)$
36	$z_i\bar{z}_j\partial_\rho(\bar{\psi}\psi)$	$T'^{\rho}_{\mu\nu}(\varepsilon)$	$-T'^{\rho}_{\mu\nu}(-\varepsilon)$	$+T'^{\rho}_{\nu\mu}(-\varepsilon)$
37	$z_i\bar{z}_j\bar{\psi}\overleftrightarrow{\partial}_\rho\psi$	$U^{\rho}_{\mu\nu}(\varepsilon)$	$-U^{\rho}_{\mu\nu}(-\varepsilon)$	$-U^{\rho}_{\nu\mu}(-\varepsilon)$
38	$z_i\partial_\rho\bar{z}_j\bar{\psi}\psi$	$U'^{\rho}_{\mu\nu}(\varepsilon)$	$-U'^{\rho}_{\mu\nu}(-\varepsilon)$	$U'^{\rho}_{\nu\mu}(-\varepsilon)=V^{\rho}_{\mu\nu}(\varepsilon)$
39	$\partial_\rho z_i\bar{z}_j\bar{\psi}\psi$	$V^{\rho}_{\mu\nu}(\varepsilon)$	$-V^{\rho}_{\mu\nu}(-\varepsilon)$	$V^{\rho}_{\nu\mu}(-\varepsilon)=U'^{\rho}_{\mu\nu}(\varepsilon)$
40	$\partial_\rho(\bar{\psi}_j\psi_i)$	$V'^{\rho}_{\mu\nu}(\varepsilon)$	$-V'^{\rho}_{\mu\nu}(-\varepsilon)$	$+V'^{\rho}_{\nu\mu}(-\varepsilon)$

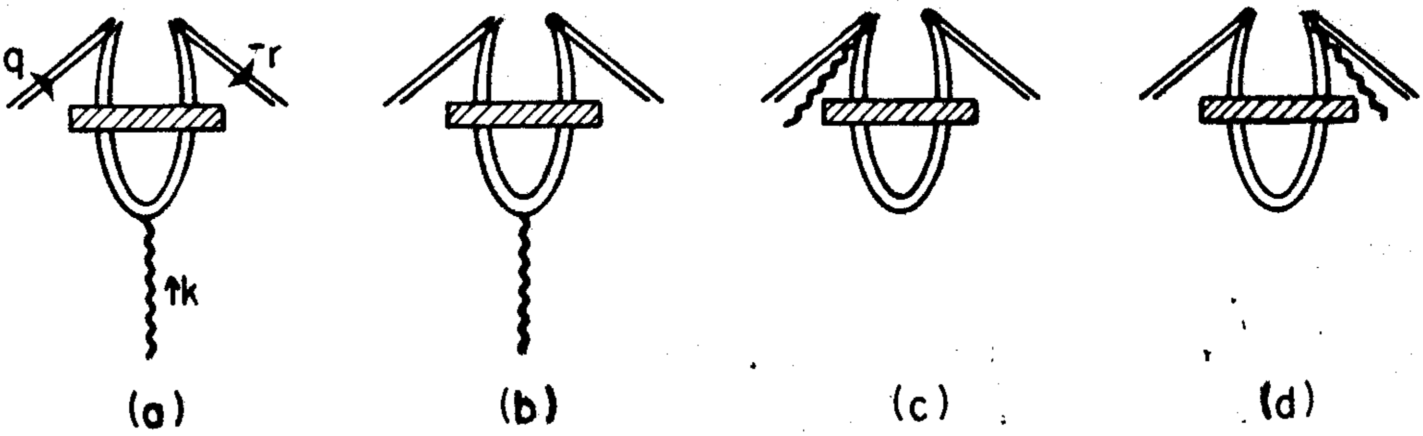

FIG. 1

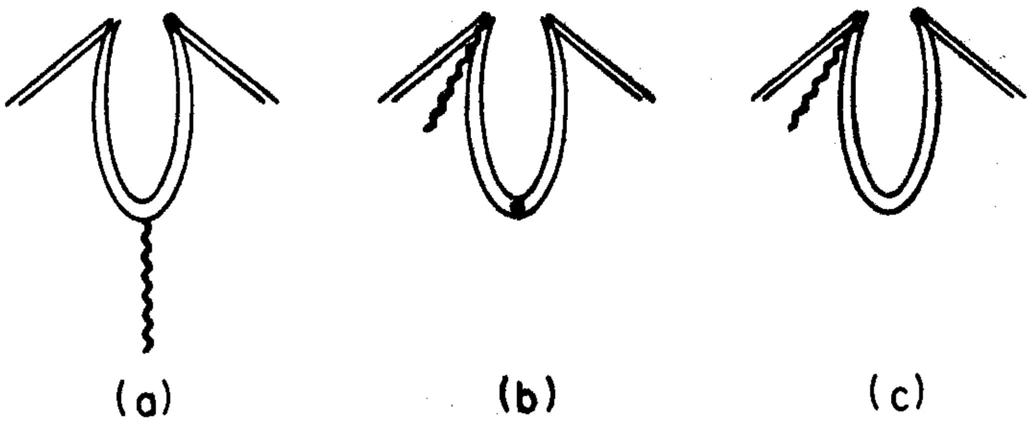

FIG. 2

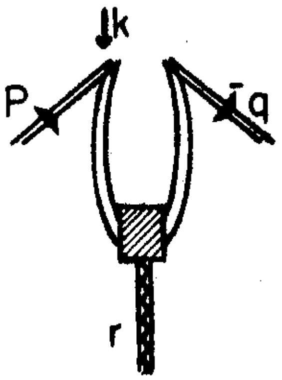

FIG. 3

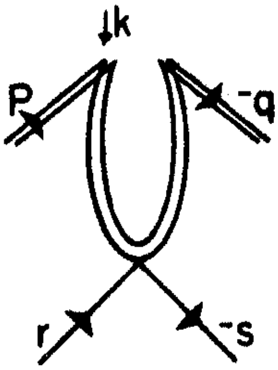

FIG. 4

	OPERATOR	COEFFICIENT	P.T.	CHARGE CONJUGATION
41	$\bar{\psi}_j \overleftrightarrow{\partial}_\rho \psi_i$	$X^\rho_{\mu\nu}(\varepsilon)$	$-X^\rho_{\mu\nu}(-\varepsilon)$	$-X^\rho_{\nu\mu}(-\varepsilon)$
42	$\partial_\rho(\bar{\psi}_j \gamma_5 \psi_i)$	$X'^\rho_{\mu\nu}(\varepsilon)$	$-X'^\rho_{\mu\nu}(-\varepsilon)$	$-X'^\rho_{\nu\mu}(-\varepsilon)$
43	$\bar{\psi}_j \gamma_s \overleftrightarrow{\partial}_\rho \psi_i$	$Z^\rho_{\mu\nu}(\varepsilon)$	$+Z^\rho_{\mu\nu}(-\varepsilon)$	$+Z^\rho_{\nu\mu}(-\varepsilon)$
44	$z_i \bar{z}_j \pi\phi$	$Z'_{\mu\nu}(\varepsilon)$	$+Z'_{\mu\nu}(-\varepsilon)$	$-Z'_{\nu\mu}(-\varepsilon)$
45	$z_i \bar{z}_j \pi\bar{\psi}\gamma_5\psi$	$W_{\mu\nu}(\varepsilon)$	$+W_{\mu\nu}(-\varepsilon)$	$+W_{\nu\mu}(-\varepsilon)$
46	$z_i \bar{z}_j \pi\bar{\psi}\gamma_\rho\psi$	$W'^\rho_{\mu\nu}(\varepsilon)$	$-W'^\rho_{\mu\nu}(-\varepsilon)$	$+W'^\rho_{\nu\mu}(-\varepsilon)$
47	$z_i \bar{z}_j \pi\bar{\psi}\psi$	$Y_{\mu\nu}(\varepsilon)$	$+Y_{\mu\nu}(-\varepsilon)$	$-Y_{\nu\mu}(-\varepsilon)$
48	$z_i \bar{z}_j \phi\bar{\psi}\gamma_5\psi$	$Y'_{\mu\nu}(\varepsilon)$	$+Y'_{\mu\nu}(-\varepsilon)$	$-Y'_{\nu\mu}(-\varepsilon)$
49	$z_i \bar{z}_j \phi\bar{\psi}\gamma_\rho\psi$	$a^\rho_{\mu\nu}(\varepsilon)$	$-a^\rho_{\mu\nu}(-\varepsilon)$	$-a^\rho_{\nu\mu}(-\varepsilon)$
50	$z_i \bar{z}_j \phi\bar{\psi}\psi$	$a'_{\mu\nu}(\varepsilon)$	$+a'_{\mu\nu}(-\varepsilon)$	$+a'_{\nu\mu}(-\varepsilon)$
51	$z_i \bar{z}_j \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_\rho\psi$	$b^\rho_{\mu\nu}(\varepsilon)$	$-b^\rho_{\mu\nu}(-\varepsilon)$	$+b^\rho_{\nu\mu}(-\varepsilon)$
52	$z_i \bar{z}_j \bar{\psi}\gamma_5\psi\bar{\psi}\psi$	$b'_{\mu\nu}(\varepsilon)$	$+b'_{\mu\nu}(-\varepsilon)$	$-b'_{\nu\mu}(-\varepsilon)$
53	$z_i \bar{z}_j \bar{\psi}\gamma_\rho\psi\bar{\psi}\psi$	$c^\rho_{\mu\nu}(\varepsilon)$	$-c^\rho_{\mu\nu}(-\varepsilon)$	$-c^\rho_{\nu\mu}(-\varepsilon)$
54	$\pi\bar{\psi}_j\gamma_5\psi_i$	$c'_{\mu\nu}(\varepsilon)$	$+c'_{\mu\nu}(-\varepsilon)$	$+c'_{\nu\mu}(-\varepsilon)$
55	$\pi\bar{\psi}_j\gamma_i\psi$	$d_{\mu\nu}(\varepsilon)$	$+d_{\mu\nu}(-\varepsilon)$	$-d_{\nu\mu}(-\varepsilon)$
56	$\phi\bar{\psi}_j\gamma_5\psi_i$	$d'_{\mu\nu}(\varepsilon)$	$+d'_{\mu\nu}(-\varepsilon)$	$-d'_{\nu\mu}(-\varepsilon)$
57	$\phi\bar{\psi}_j\psi_i$	$f_{\mu\nu}(\varepsilon)$	$+f_{\mu\nu}(-\varepsilon)$	$+f_{\nu\mu}(-\varepsilon)$
58	$\bar{\psi}\gamma_5\psi\bar{\psi}_j\gamma_5\psi_i$	$f'_{\mu\nu}(\varepsilon)$	$+f'_{\mu\nu}(-\varepsilon)$	$+f'_{\nu\mu}(-\varepsilon)$
59	$\bar{\psi}\gamma_5\psi\bar{\psi}_j\psi_i$	$g_{\mu\nu}(\varepsilon)$	$+g_{\mu\nu}(-\varepsilon)$	$-g_{\nu\mu}(-\varepsilon)$
60	$\bar{\psi}\gamma_\rho\psi\bar{\psi}_j\gamma_5\psi_i$	$g'_{\mu\nu}(\varepsilon)$	$-g'_{\mu\nu}(-\varepsilon)$	$+g'_{\nu\mu}(-\varepsilon)$

	OPERATOR	COEFFICIENT	P.T.	CHARGE CONJUGATION
61	$\bar{\psi}\gamma_\rho\psi\bar{\psi}_j\psi_i$	$h_{\mu\nu}(\varepsilon)$	$-h_{\mu\nu}(-\varepsilon)$	$-h_{\nu\mu}(-\varepsilon)$
62	$\bar{\psi}\psi\bar{\psi}_j\gamma_5\psi_i$	$h'_{\mu\nu}(\varepsilon)$	$+h'_{\mu\nu}(-\varepsilon)$	$-h'_{\nu\mu}(-\varepsilon)$
63	$\bar{\psi}\psi\bar{\psi}_j\psi_i$	$i_{\mu\nu}(\varepsilon)$	$+i_{\mu\nu}(-\varepsilon)$	$+i_{\nu\mu}(-\varepsilon)$
64	$z_i \bar{z}_j \pi\pi$	$i'_{\mu\nu}(\varepsilon)$	$+i'_{\mu\nu}(-\varepsilon)$	$+i'_{\nu\mu}(-\varepsilon)$
65	$z_i \bar{z}_j \phi\phi$	$j_{\mu\nu}(\varepsilon)$	$+j_{\mu\nu}(-\varepsilon)$	$+j_{\nu\mu}(-\varepsilon)$
66	$z_i \bar{z}_j \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$	$j'_{\mu\nu}(\varepsilon)$	$+j'_{\mu\nu}(-\varepsilon)$	$+j'_{\nu\mu}(-\varepsilon)$
67	$\bar{\psi}\gamma_\rho\psi\bar{\psi}\gamma_\sigma\psi$	$k^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+k^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$+k^{\rho\sigma}_{\nu\mu}(-\varepsilon)$
68	$z_i \bar{z}_j \bar{\psi}\psi\bar{\psi}\psi$	$k'^{\rho\sigma}_{\mu\nu}(\varepsilon)$	$+k'^{\rho\sigma}_{\mu\nu}(-\varepsilon)$	$+k'^{\rho\sigma}_{\nu\mu}(-\varepsilon)$