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ON THE PEIERLS-BRAYSHAW RESONANCES

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[Nuclear reactions, one dimensional Faddeev-Lovelace equations, scattering theory, Peierls-Brayshaw mechanism].

The connection between bound states and/or resonances of a three-particle system and bound states and/or resonances of a two-particle subsystem is investigated for three identical bosons in one-dimension. It is found that, in general, if the two-particle subsystem has a bound state of energy ν_o then the three-particle system will have a bound state at the energy $4\nu_o$.

I. INTRODUCTION

In a previous work D.D. Brayshaw and R.F. Peierls[1] considered the connection between bound states and/or resonances of a three-particle system and the bound states and/or resonances of a two-particle system.

This was done in the three-particle case by considering the one-dimensional Faddeev equation for the scattering of one of the particles by a bound state of the two remaining particles. The Lovelace[2] separable approximation was used. Their result was: "if the scattering of two identical spinless particles moving in one dimension can be described by a single separable term corresponding to a resonance of energy ν_o , with a slowly varying form factor, then the corresponding three-particle scattering amplitude will develop a denominator pole at an energy $4\nu_o$ on the second physical sheet. For a narrow two-body resonance this suggests the occurence of a three-body resonance near $4\nu_o$. The corresponding statement in the case of a two-particle bond state is not true".

We shall refer to this result as the Peierls-Brayshaw mechanism and the pole at $4\nu_o$ as the Peierls singularity.

This result was considered by Simonov and Badalyan[3] and Simonov[4] using a different approach. They considered the three-body scattering by defining a "physical" on-shell particle-resonance amplitude and described its analytical properties. By solving exactly the resulting N/D equations they found a result at variance with the above mentioned Peierls-Brayshaw mechanism, namely, they found no Peierl's singularity.

The "existence" of the Peierls singularity has not been settled however. A number of papers by Brayshaw[5,6] and Brayshaw and collaborators[7], related experimental findings with the Peierls-Brayshaw mechanism or a variant of it, called Brayshaw Mechanism[5]. Also some authors[8] refer to the Simonov work as

"numerical investigation" which apparently is unable to disprove the validity of the Peierls-Brayshaw mechanism.

Part of the difficulty in settling the issue comes from the fact that the original Brayshaw-Peierls paper[1] uses a method invented by Brayshaw[9], to handle the Faddeev-Lovelace equations which is little known. Also their paper in its crucial part is difficult to follow.

The purpose of this paper is to present the results of a reinvestigation of the Peierls-Brayshaw mechanism.

We have used Brayshaw's method of handling integral equations for the same one-dimensional problem. We found an error in one analytical continuation so that our results differ from theirs. Our results show that in general, if a system of three-identical spinless particles moving in one dimension possesses a bound-state of energy ν_o in the two-particle subsystem, then there is a bound-state in the three-particle system with energy $4\nu_o$. This result is in agreement with a number of investigations[10,11,12] carried out for particular one-dimensional systems using different methods. On the other hand if there is a resonance in the two-particle subsystem we show that it is not possible to guarantee that a resonance is present in the three-particle system. This is in agreement with Simonov and Badalyan[3] and Simonov[4].

In the next section we outline the Brayshaw[9] method of handling integral equations. We have presented it in a way that facilitates the reproduction of all the steps involved. In section III we show that a bound-state in the two body system with energy ν_o strongly suggests a three-particle bound state at $4\nu_o$. In section IV we show that a two-particle resonance does not necessarily produce a resonance in the three-particle subsystem and certainly not at the value $4\nu_o$. Finally we show in section V, for completness, that the Brayshaw method can be used to reduce the resolution of a integral equation to a much more rapidily converging integral equation.

II. THE BRAYSHAW METHOD

We consider a system of three-identical spinless particles of unit mass moving in one dimension. The Faddeev equation for the amplitude $\tau(x,x';W)$ of scattering of a particle by a bound state (binding energy ν_o) of the remaining two-particles is[1]

$$\tau(x,x';W) = Z(x,x';W) - \int_{-\infty}^{+\infty} dx'' \frac{Z(x,x'';W)\,\tau(x'',x';W)}{D_o(W - \frac{3}{4}x''^2)} \qquad (1)$$

where

$$Z(x,x';W) = \frac{-2\, g(-1/2x-x')\, g^*(x+1/2x')}{x^2 + x'^2 + xx' - W - i\varepsilon}$$

and $D_o(\nu)$ vanishes linearly at $\nu = \nu_o$ that is, $D_o(\nu) \approx (\nu-\nu_o)D_o'(\nu_o)$ near ν_o. W is the total energy of the three-particle system. The on-the-energy shell condition for this amplitude is obtained by putting $x = x' = \sqrt{E}$ where $E = \frac{4}{3}(W-\nu_o)$.

In obtaining equation (1) the two-particle off-the-energy shell t-matrix

$$<-\frac{1}{2}x-x'|t(\nu)|x+\frac{1}{2}x'> = \sum_n \frac{g_n(-\frac{1}{2}x'-x)\,g_n^*(x+\frac{1}{2}x')}{D_n(\nu)}$$

$$+ <-\frac{1}{2}x-x'|\tilde{t}(\nu)|x+\frac{1}{2}x'> \qquad (2)$$

was approximated by $\frac{g(-\frac{1}{2}x-x')\,g^*(x+\frac{1}{2}x')}{D_o(\nu)}$. This approximation is of course very poor but is acceptable if one wants to investigate the connection between bound states and/or resonances in the three-particle system with bound states and/or resonances in the two-particle subsystem. This will be demonstrated in section III by showing that the neglected terms of equation (2) do not alter our results. We also assume for the purpose of the subsequent discussions that $g(-\frac{1}{2}x-x')$ is a constant. Again it will be shown in section III that this assumption can be removed without altering our results and is done only for expediency. Defining

$$\tau^{\pm}(x,x';W) = \frac{1}{2}\left[\tau(x,x';W) \pm \tau(-x,x';W)\right]$$

and

$$Z^{\pm}(x,x';W) = \frac{1}{2}\left[Z(x,x';W) \pm Z(-x,x';W)\right]$$

we find

$$\tau^{\pm}(x,x';W) = Z^{\pm}(x,x';W) - \int_{-\infty}^{+\infty} \frac{Z(x,x'';W)\,\tau^{\pm}(x'',x';W)}{D_o(W-\frac{3}{4}x''^2)}\,dx'' \qquad (3)$$

We assume to begin with, that ν_o is real and negative (that is, a bound state) and that the real and imaginary parts of $(W)^{1/2}$ are >0. Equation (3) defines $\tau\pm(x,x';W)$ for all real values of x. We now extend it for complex values of x and find that this can be done until x reaches the lines $\phi_{\pm}(x') = -\frac{1}{2}x' \pm |W-\frac{3}{2}x'^2|^{1/2}$ at which point the kernel blows up. This is so because

$$\frac{1}{x^2 + x'^2 + xx' - W} = \frac{1}{(x-\phi_+(x'))(x-\phi_-(x'))}$$

However, by allowing x to approach each of the lines ϕ^+ and ϕ^- from above and below, one obtains the difference of these limits for each line and thus may account for the singularities and continue, $\tau^{\pm}(x,x';W)$.

The result is

$$\tau^{\pm}(z,x';W) = Z^{\pm}(z,x';W) - \int_{\infty}^{\infty} dx'' \frac{Z(x,x'',W)\,\tau^{\pm}(x'',x';W)}{D_o(W - \frac{3}{4}x''^2)} -$$

$$- (2\pi i)(2gg^*) \left[\frac{\theta_+(z)}{2(W - \frac{3}{4}z^2)^{1/2}} \frac{\tau^{\pm}(\phi_-(z),x';W)}{D_o(W - \frac{3}{4}\phi_-^2(z))} + \right.$$

$$\left. + \frac{\theta_-(z)}{2(W - \frac{3}{2}z^2)^{1/2}} \frac{\tau^{\pm}(\phi_+(z),x';W)}{D_o(W - \frac{3}{4}\phi_-^2(z))} \right] \qquad (4)$$

where

$$\theta_+(z) = 1 \quad \text{above } \phi_+(x)$$
$$= 0 \quad \text{otherwise}$$

and

$$\theta_-(z) = 1 \quad \text{below } \phi_-(x)$$
$$= 0 \quad \text{otherwise} \quad .$$

Now that $\tau^{\pm}(z,x';W)$ has been extended to the whole complex z plane we can write a new representation for it. Observe that $\tau^{\pm}(z,x';W) - Z(z,x';W)$ has two branch point at $\pm(4/3\ W)^{1/2}$ and two poles at $\phi_+(\pm E^{1/2})$.

If we cut the plane from $\pm(4/3\ W)^{1/2}$ to $\pm i\infty$ as shown in figure 1 we can write using Cauchy theorem

$$\tau^{\pm}(t,x';W) = Z^{\pm}(t,x';W) - \int_{\Gamma_1(W)} \frac{dz}{t-z} \frac{2gg^*\,\tau^{\pm}(\phi_-(z),x';W)}{2(W - \frac{3}{4}z^2)^{1/2}\, D_o(W - \frac{3}{4}\phi_-^2(z))}$$

$$+ \frac{(2\pi i)(2gg^*)\,\tau^{\pm}(\phi_-(E^{1/2}),x';W)}{\left(t - \phi_+(E^{1/2})\right) 2\left(W - \frac{3}{4}\phi_+^2(E^{1/2})\right)^{1/2} D_o\left(W - \frac{3}{4}\phi_+^2(E^{1/2})\right) E^{1/2} D_o'(\nu_o)} -$$

$$- \int_{\Gamma_2(W)} \frac{dz}{(t-z)} \frac{(2gg^*)\ \tau^{\pm}(\phi_+(z),x';W)}{2(W - \frac{3}{4}z^2)^{1/2}\, D_o(W - \frac{3}{4}\phi_+^2(z))} -$$

$$- \frac{(2\pi i)(2gg^*)\ \tau^{\pm}(\phi_+(-E^{1/2}),x';W)}{(t - \phi_-(E^{1/2})\ 2(W - \frac{3}{4}\phi_-^2(E^{1/2}))\ E^{1/2}\ D_o'(\nu_o)} \qquad (5)$$

where the contours of integration $\Gamma_1(W)$ and $\Gamma_2(W)$ are as show in figure 1. For example Γ_1 , runs from $+i\infty$ to $(4/3\ W)^{1/2}$ on the right hand side of the upper cut and from $(4/3\ W)^{1/2}$ back to $+i\infty$ on the left hand side of the upper cut.

Using $\phi_{\pm}(-z) = -\phi_{\mp}(z)$ and $\tau^{\pm}(-x,x';W) = \pm\tau^{\pm}(x,x';W)$ equation (5) can be rewritten as

$$\tau^{\pm}(t,x';W) = Z^{\pm}(t,x';W) \quad +$$

$$+ \int_{\Gamma_1(W)} dz \left(\frac{1}{t-z} \pm \frac{1}{t+z}\right) \frac{(2gg^*)\ \tau^{\pm}(\phi_-(z),x';W)}{2(W - \frac{3}{4}z^2)^{1/2}\ D_o(W - \frac{3}{4}\phi_-^2(z))} -$$

$$- \frac{(2gg^*)(2\pi i)\ \tau^{\pm}(\phi_-(E^{1/2}),x';W)}{2\left\{W - \frac{3}{4}\phi_+^2(E^{1/2})\right\}^{1/2} E^{1/2}\ D_o'(\nu_o)} \left[\frac{1}{\phi_+(E^{1/2})-t} \pm \frac{1}{\phi_+(E^{1/2})+t}\right] \qquad (6)$$

The result (6) is valid if both the real and imaginary part of $W^{1/2}$ is positive and ν_o is real and negative. If we try to extend this result to an arbitrary value of W we find that when we cross the curve (Im means imaginary part)

$$\mathrm{Im}(3/2\, E^{1/2} - \nu_o^{1/2}) = 0 \tag{7}$$

where $E = \frac{4}{3}(W-\nu_o)$, the pole of $D_o(W - \frac{3}{4}\phi_-^2(z))$ reaches the branch point $(4/3\, W)^{1/2}$. Thus the integral in equation (6) develops an end point singularity. The discontinuity across the curve (7) can be calculated as sketched in the book by Eden et al.[13]. The discontinuity is

$$\pm \frac{(2gg^*)(2\pi i)\, \tau^{\pm}(\phi_-(E^{1/2}),x';W)}{2(W - \frac{3}{4}\phi_+^2(E^{1/2}))\, E^{1/2}\, D_o'(\nu_o)} \left[\frac{1}{\phi_+(E^{1/2})-t} \pm \frac{1}{\phi_+(E^{1/2})+t}\right] \tag{8}$$

To continue $\tau^{\pm}(x,x';W)$ one must be very careful about the choice of the correct sign. We find, using the book by Goursat[14] that the sign should be such that the correct analytic continuation is

$$\tau^{\pm}(t,x';W) = Z\,(t,x';W) + \int_{\Gamma_1(W)} dz \left(\frac{1}{t-z} \pm \frac{1}{t+z}\right) \frac{(2gg^*)\ \tau^{\pm}(\phi_-(z),x';W)}{2(W-\frac{3}{4}z^2)^{1/2}\, D_o(W-\frac{3}{4}\phi_-^2(z))}$$
$$- (1+\theta(E,\nu_o)) \frac{(2gg^*)(2\pi i)\ \tau^{\pm}(\phi_-(E^{1/2}),x';W)}{2(W-\frac{3}{4}\phi_+(E^{1/2}))^{1/2}\, E^{1/2}\, D_o'(\nu_o)} \left(\frac{1}{\phi_+(E^{1/2})-t} \pm \frac{1}{\phi_+(E^{1/2})+t}\right) \tag{9}$$

where $\theta(E,\nu_o)$ is 0 for W inside the curve defined by equation (7) and 1 for W outside it. Note that this continuation differs

from the one given by Peierls-Brayshaw in that they chose the opposite sign.

III. CONNECTION BETWEEN BOUND STATES

Equation (9) is a new representation for equation (3). The advantage of this new representation is that we can infer some results for the half-on-shell amplitude without having to solve the integral equation.

In fact if we set $t = E^{1/2}$ in equation (9) we get

$$\tau^{\pm}(E^{1/2},x';W) = Z^{\pm}(E^{1/2},x';W) + \int_{\Gamma_1(W)} dz \left(\frac{1}{E^{1/2}-z} \pm \frac{1}{E^{1/2}+z}\right) \frac{(2gg^*)\ \tau^{\pm}(\phi_-(z),x';W)}{2(W-\frac{3}{4}z^2)^{1/2}\, D_o(W-\frac{3}{4}\phi_-^2(z))}$$
$$+ \frac{(2gg^*)(2\pi i)\ \tau^{\pm}(\phi_-(E^{1/2}),x';W)}{(W-\frac{3}{4}\phi_-^2(E^{1/2}))E^{1/2}\, D_o'(\nu_o)} \left(\frac{1}{\phi_+(E^{1/2})-E^{1/2}} \pm \frac{1}{\phi_+(E^{1/2})+E^{1/2}}\right) \tag{10}$$

This can now be written, algebraically for $\tau^{\pm}(E^{1/2},x';W)$. The result is

$$\tau^{\pm}(E^{1/2},x';W) = N + D \tag{11}$$

where

$$D = \frac{\tau^{\pm}(\phi_-(E^{1/2}),x';W)\ (2gg^*)(2\pi i)}{(W-\frac{3}{4}\phi_+^2(E^{1/2}))\, E^{1/2}\, D_o'(\nu_o)} \left(\frac{1}{\phi_+(E^{1/2})-E^{1/2}} \pm \frac{1}{\phi_+(E^{1/2})+E^{1/2}}\right) \tag{12}$$

$\tau^{\pm}(\phi_+(E^{1/2}),x',W)$ can be obtained by setting $t = \phi_-(E^{1/2})$ in equation (9).

Note that when $W = 4\nu_o$ D is infinite independently of the value of x'. In this case if $\tau_\pm(\phi_-(E^{1/2}),x';W)$ does not vanish at $4\nu_o$ this corresponds to a bound state of the three-particle system.

Note that this result is independent of the remaining terms of the expansion (2) and also of possible contributions of singularities from $g(-1/2x-x')$ and $g^*(x+1/2x')$. This is so because further terms and singularities of the form factors g will just add further terms to N. They can therefore alter this conclusion only if they make $\tau^\pm_\pm(\phi_-(E^{1/2}),x';W)$ vanishes at $4\nu_o$.

It is of course difficult to ascertain if $\tau^\pm(\phi_-(E^{1/2}),x';W)$ vanishes or not for $W = 4\nu_o$. It is however possible to test this by doing a calculation with a specific model. Fortunately at least three(10,11,12) such calculations have already been made and although the two-particle potentials were different each presented a two-particle bound state and exhibited a pole at $W = 4\nu_o$. This result combined with those in equation (11) and (12), suggests that the pole at $W = 4\nu_o$ is a general feature.

IV. CONNECTION BETWEEN RESONANCES

Up to this point we have assumed ν_o real and negative associating it with a two-particle bound state. We would like to investigate the case when the two-particle amplitude has a pole for a complex ν_o whose real part is >0 and imaginary part <0, i.e., a resonance. We might think that this result could be obtained from equation (10) by analytically continuing ν_o from the negative real value down to the second sheet. This however would violate two-body unitarity and consequentely three-body unitarity as well. In fact, as shown very carefully in the classical paper by

Lovelace(2), if the two-body subsystem has a resonance, the off-shell two body t-matrix should be approximated by

$$t(x,x';W) = \frac{2gg^*}{\nu_o - \nu - 2\pi^2 i\ \nu^{1/2}|g(\nu^{1/2})|^2} \tag{13}$$

If $D_o(\nu) = (\nu-\nu_o)\ D_o'(\nu_o)$ is replaced by a more realistic expression such as the one given by equation (13) the procedure described in section II has to be completely redone. A complicated expression results which does not present a pole in $W = 4\nu_o$ due to the extra factor $2\pi^2 i\ \nu^{1/2}|g(\nu^{1/2})|^2$ in equation (13) above. We thus conclude in agreement with Simonov and Badalyan(3) and Simonov(4) that a resonance in a two-particle subsystem does not produce a resonance in the three-particle system at this energy.

V. A NEW INTEGRAL EQUATION

In this section we show how to obtain from the representation (9) a new integral equation.

We first note that if t is replaced by $\phi_-(x)$ we have a new integral equation for $\tau^\pm(\phi_-(x),x';W)$.

If this equation is iterated an infinite number of times then it is easy to see that the resulting series can be resummed to give

$$\tau^\pm(t,x';W) = Z^\pm(t,x';W) + \int_{\Gamma_1(W)} dz\ \beta_\pm(t,z) \frac{(2gg^*)}{2(W-\frac{3}{4}z^2)^{1/2}} \frac{D_\pm(\phi_-(z),x',W)}{D_o(W-\frac{3}{4}\phi_-^2(z))} +$$

$$+(1+\theta(E,\nu_o)) \frac{(2gg^*)(2\pi i)\ \tau^\pm(E^{1/2},x';W)\ E_\pm(t,\phi_+(E^{1/2});W)}{2\left(W-\frac{3}{4}\phi_+^2(E^{1/2})\right)^{1/2} E^{1/2}\ D_o'(\nu_o)}$$

where E and D obey the following new integral equations

$$D_{\pm}(\phi_-(z),x';W) = Z^{\pm}(\phi_-(z),x';W) + $$

$$+ \int_{\Gamma_1(W)} dz' \frac{\beta_{\pm}(\phi_-(z),z')(2gg^*)\, D_{\pm}(\phi_-(z'),x';W)}{(W-\frac{3}{4}z'^2)^{1/2}\, D_o(W-\frac{3}{4}\phi_-^2(z'))} \tag{14}$$

and

$$E_{\pm}(z,\phi_+(E^{1/2});W) = \beta_{\pm}(z,\phi_+(E^{1/2}) + $$

$$+ \int_{\Gamma_1(W)} dz' \frac{\beta_{\pm}(t,z')}{(W-\frac{3}{4}z'^2)^{1/2}} \frac{(2gg^*)}{D_o(W-\frac{3}{4}\phi_-^2(z'))} E^{\pm}(\phi_-(z'),\phi_+(E^{1/2});W) \tag{15}$$

and

$$\beta_{\pm}(x,y) = \frac{1}{x-y} \pm \frac{1}{x+y} \tag{16}$$

These equations should be numerically easier to solve than the original equation (3) as can be seen from inspection of the kernel. We have shown here that the Brayshaw[9] method results in appreciable simplification of the solution of an integral equation. We hope that the detailed presentation made in this paper will stimulate an interest in this method so that it will receive the proper attention we feel it deserves.

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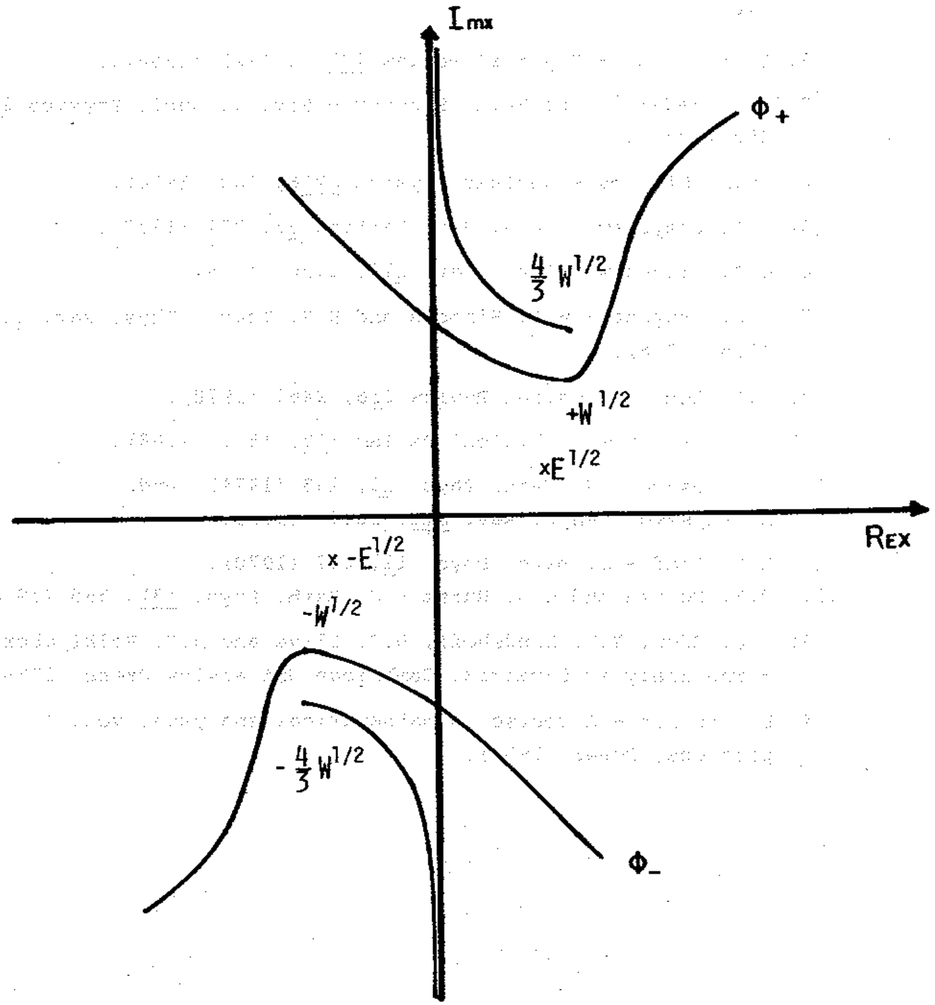

FIGURE 1