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IFUSP/P-335

THE NESTED-DOORWAY MODEL OF MULTISTEP COMPOUND PROCESSES

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(To be presented at the Third Varenna Conference on Nuclear Reaction Mechanisms - Varenna, Italy - June, 1982)

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ABSTRACT

The multistep compound contribution to preequi - librium reactions are discussed within the nested-doorway model. Emphasis is placed on the generalized cross-section auto-correlation function. Several of the more widely used concepts in the conventional, one-class, statistical analysis are discussed and generalized to the multiclass case. A summary of the formal results of the nested-doorway model, obtained within Feshbach's projection operator theory is given.

MAY/82

* Supported in part by the CNPq

I. Introduction

In the past few years several attempts have been made to develop a workable theory of multistep compound processes. These processes constitute an important, and eventually the dominant contribution, to the preequilibrium emission cross section at lower energies. The major aim of all these approaches is to derive an expression for the average fluctuation cross section. With this expression one may then calculate both the continuum as well as the discrete portions of the spectrum of the emitted particle. At low incident energies the discrete spectrum is usually dominated by compound as well as multistep compound processes.

Recently[1] it was recognized that in analysing the discrete transitions dominated by compound processes, the cross-section auto-correlation function $C(\varepsilon)$ is of a more fundamental importance than the fluctuation cross-section, σ^{fl}, in so far as its being related directly to the average life times that characterize the compound system. In the nested doorway model proposed in[2] these life times are related to the average correlation widths of the different classes of overlapping doorway resonances populated in the system. It is therefore of importance to actually calculate both σ^{fl} and $C(\varepsilon)$.

In the model of Ref. (2) these are simply related by

$$\sigma^{fl} = \sum_n \sigma_n^{fl} \quad , \quad C(\varepsilon) = \left| \sum_n \frac{\sigma_n^{fl}}{1 + i\,\varepsilon/\tilde{\Gamma}_n} \right|^2 \quad , \text{ where}$$

n refers to a given class of doorways with a corresponding correlation width $\tilde{\Gamma}_n$. Given the $\tilde{\Gamma}_n$, s one would therefore require a knowledge of the σ_n^{fl}, s so that both σ^{fl} and $C(\varepsilon)$ are calculated. In Ref. (3) a derivation of σ_n^{fl} was given using Feshbach's projection operator theory.

The aim of the present paper is to present a more

thorough discussion of the results of Ref. (3) in so far as the application to the analysis of experimental data is concerned. In section II the S-matrix auto-correlation function $C^S(\varepsilon)$, is calculated within the nested doorway model. The condition of the validity of the sum-over poles approximation to $C^S(\varepsilon)$ is then discussed. Several of the methods and concepts used in the analysis of the conventional one-class Ericson fluctuations, are discussed in connection with the nested doorway model. In section III a summary of the formal results of Ref. (3) is presented, and finally, in section IV, several comments are made with regards to the comparison between the ND model and other models of multistep compound processes.

II. Statistical Analysis of Multistep Compound Processes

II.1. Transitions to discrete states

For completeness as well as for the discussion to follow, we provide in this section the full details of Ref.(3). We calculate below the S-matrix auto-correlation function, $C^S_{cc'}(\varepsilon)$, for a given partial wave. The fluctuation cross section is then easily obtained as $\sigma^{fl}_{cc'} = C^S_{cc'}(0)$. The function $C^S_{cc'}(\varepsilon)$ is defined as

$$C^S_{cc'}(\varepsilon) \equiv \left\langle S^{fl}_{cc'}(E)\ S^{fl\,*}_{cc'}(E+\varepsilon) \right\rangle_I \qquad (2.1)$$

In Eq. (2.1), $S^{fl}_{cc'}(E)$ is the fluctuating component of the full S-matrix and I denotes the energy interval inside which the average is taken.

The fundamental assumption of the nested-doorway model is that the intermediate $(A_1 + A_2)$ system through which the reaction proceeds exhibits overlapping doorway levels, and that these levels clearly separate themselves into distinct "classes", each identified by the correlation width $\tilde{\Gamma}_n$ of the levels of class n with the widths arranged in the "well-nested" sequence

$$\tilde{\Gamma}_1 \gg \tilde{\Gamma}_2 \gg \cdots \gg \tilde{\Gamma}_N \qquad (2.2)$$

where $\tilde{\Gamma}_N$ is the correlation width of the fine-structure levels of the system. The states of class n contribute to the reaction that portion of the pre-equilibrium flux which appears with an average time-delay of $\sim \hbar/\tilde{\Gamma}_n$. It should be emphasized that by cor-

relation widths we mean those widths extracted from a fit of the cross-section auto-correlation function to the experimental data. The correlation widths, $\tilde{\Gamma}_n$, are generally not equal to the "true" total widths of the overlapping resonances of class n. They do, however, become equal when the number of open channel is large compared to (Γ_n/D_n), where D_n is the average level spacing of class n, (see below).

Because of the condition of Eq. (2.2), we can define a nested sequence of energy-averaging intervals I_n by interpolating the I_n,s between the $\tilde{\Gamma}_n$,s

$$\tilde{\Gamma}_{n-1} \gg I_n \gg \tilde{\Gamma}_n \tag{2.3}$$

This permits us to define N different optical S-matrices, $\bar{S}_n \equiv \langle S \rangle_{I_n}$, by averaging the total S-matrix successively over these N intervals; $\bar{S}_n(E)$ will contain the poles describing the doorways of class (n-1), $\bar{S}_{n-1}(E)$, those of class n-2, and so on.

In all the discussion that follows, the assumption of overlapping resonances in each class is made in the sense

$$\frac{\overline{\Gamma_n}}{D_n} \gg 1 \tag{2.4}$$

where $\bar{\Gamma}_n$ is the average total width of the doorways in class n. The above condition is necessary for the statistical treatment to be applicable.

It has been demonstrated in Ref. (3), a summary of whose results is given in the next section of the present paper, that the contribution of the resonances of class n to the fluctuating part of S may be written in the sum-over-poles form

$$S^{fl}_{n,cc'} = -i \sum_i \frac{g_{ni,c}\, g_{ni,c'}}{E - \varepsilon_{ni}} \tag{2.5}$$

In obtaining Eq. (2.5) the Kawai, Kerman and McVoy (KKM) prescription[4] is employed in a successive way. Namely averaging first over the smallest energy interval I_N defines an "optical" S matrix which we call $\bar{S}_N \equiv \langle S \rangle_{I_N}$,

$$\underset{\sim}{S} = \underset{\sim}{\bar{S}}_N + \underset{\sim}{S}^{fl}_N \tag{2.6}$$

with S^{fl}_N given by an expression analoguous to (2.5), and, via the KKM manipulation

$$\langle \underset{\sim}{S}^{fl}_N \rangle_{I_N} = 0 \tag{2.7}$$

Since there are other doorway classes present in the system with widths larger than I_N (see Eq. (2.5)). The I_N-optical S-matrix, $\bar{S}_N$, must exhibit intermediate structure. Thus averaging over and interval I_{N-1} defines a second "optical" S-matrix, $\bar{S}_{N-1}$, so that

$$\underset{\sim}{\bar{S}}_N = \underset{\sim}{\bar{S}}_{N-1} + \underset{\sim}{S}^{fl}_{N-1} \tag{2.8}$$

Continuing in this way, namely extracting "absorption" due to the coupling to the compound system, in a nested fashion, we can decompose S into contributions from all distint classes of overlapping doorways in addition to the really slowly

varying optical $\bar{S}_1$,

$$\underset{\sim}{S} = \overline{\underset{\sim}{S}}_1 + \sum_{n=1}^{N} \underset{\sim}{S}_n^{fl} \qquad (2.9)$$

with S_n^{fl} given by Eq. (2.5) and satisfying, conditions similar to Eq. (2.7). This last feature of S_n^{fl} permits a decompositon of the S-matrix auto-correlation function, as well as the energy averaged fluctuation cross-section into an incoherent seem of contributions from the different classes of doorways.

Generalizing the KKM manipulations slightly we obtain

$$\begin{aligned} C^{S}_{cc'}(\varepsilon) &= \left\langle S^{fl}_{cc'}(E)\, S^{fl}_{cc'}(E+\varepsilon)\right\rangle_{I_1} \\ &\simeq \sum_{n=1}^{N} \left\langle S^{fl}_{n,cc'}(E)\, S^{fl\,*}_{n,cc'}(E+\varepsilon)\right\rangle_{I_1} \\ &\simeq \sum_{n=1}^{N} \left\langle \sum_{q} \frac{|g_{nq,c}|^2 |g_{nq,c'}|^2}{\left(E-E_{nq}+i\frac{\Gamma_{nq}}{2}\right)\left(E-E_{nq}+\varepsilon-i\frac{\Gamma_{nq}}{2}\right)} \right\rangle_{I_1} \\ &\simeq \sum_{n=1}^{N} \frac{2\pi}{\mathcal{D}_n} \left\langle \frac{|g_{nq,c}|^2 |g_{nq,c'}|^2}{\Gamma_{nq}+i\varepsilon} \right\rangle_{I_1} \\ &\simeq \sum_{n=1}^{N} \frac{2\pi}{\mathcal{D}_n} \left\langle \frac{1}{\Gamma_{nq}} \right\rangle_q \left\langle |g_{nq,c}|^2 |g_{nq,c'}|^2 \right\rangle \\ &\quad \times \left\langle \frac{1}{\Gamma_{nq}+i\varepsilon} \right\rangle_q \left(\left\langle \frac{1}{\Gamma_{nq}} \right\rangle_q \right)^{-1} \end{aligned} \qquad (2.10)$$

or

$$C^{S}_{cc'}(\varepsilon) = \sum_{n=1}^{N} \sigma^{fl}_{n,cc'} \left\langle \frac{1}{\Gamma_{nq}+i\varepsilon} \right\rangle_q \left(\left\langle \frac{1}{\Gamma_{nq}} \right\rangle_q \right)^{-1} \qquad (2.11)$$

where $\sigma^{fl}_{n,cc'} = \langle X_{n,cc} X_{n,c'c'} + X_{n,cc'} X_{n,c'c} \rangle_I$ (2.12)

In writing the final form of Eq. (2.11), we have used the defining equation for the X_n - matrix, (Eq. (3.26)), and the statistical properties of the form factors $g_{nq,c}$.

The energy-averaged fluctuation cross-section for the transition c -> c' is easily obtained from Eq. (2.12) by setting $\varepsilon = 0$, vis

$$\sigma^{fl}_{cc'} \equiv C^{S}_{cc'}(0) = \sum_{n=1}^{N} \sigma^{fl}_{n,cc'} \qquad (2.13)$$

Explicit expressions for the individual $\sigma^{fl}_{n,cc'}$, in terms of generalized transmission matrices P_n, is accomplished by further reducing the average $\langle X_{n,cc} X_{n,c'c'} + X_{n'cc'} X_{n,c'c} \rangle_I$ into products of averages $\langle X_n \rangle_I$, and using the following relation involving P_n and $\langle X_n \rangle_I$ obtained in Ref. (2)

$$\underset{\sim}{P}_n - \overline{\underset{\sim}{P}}_{n+1} = \langle \underset{\sim}{X}_n \rangle \, \mathrm{Tr} \langle \underset{\sim}{X}_n \rangle + \langle \underset{\sim}{X}_n \rangle^2 \qquad (2.14)$$

In the limit of large number of open channels, the quadratic term in Eq. (2.15) may be dropped. This approximation permits finding the explicit form of $X_n(P_n)$, which, when inserted into (2.14) results in the multi-class Hauser-Feshbach expression for $\sigma^{fl}_{cc'}$ [2,3]

$$\sigma_{cc'}^{fl} = \sum_{n=1}^{N} \frac{(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})_{cc}(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})_{c'c'} + (\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})_{cc'}(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})_{c'c}}{Tr(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})} \qquad (2.15)$$

with $P_{N+1} = 0$ (2.16)

It is clear that in order to write Eq. (2.12) in the sum-over-poles form and thus identify the correlation widths of the different classes we have to specify the distribution of widths of the different classes of overlapping resonances. In the original Ericson paper[5] where the discussion of the one-class case is given, the assumption of narrow width distribution was invoked. This, if used in connection with Eq. (2.12), would result in the following simple generalization of Ericson's formula

$$C_{cc'}^{S}(\varepsilon) \cong \sum_{n=1}^{N} \sigma_{n,cc'}^{fl} \frac{\bar{\Gamma}_n}{i\varepsilon + \bar{\Gamma}_n} \qquad (2.17)$$

where $\bar{\Gamma}_n$ is the average width of the n^{th} class of overlapping resonances.

It is, however, quite well-known that the assumption of narrow width distribution for overlapping resonances is not quite valid[6]. In fact, one finds from Moldauer's numerical study[6], quite a wide distribution. This would certainly make the validity of Eq. (2.13) doubtful.

On the other hand, Agassi, Weidenmüller and Mantzouranis[7] have shown, at least in the one-class case, that the one pole form of $C_{cc'}^{S}(\varepsilon)$ is quite adequate in the limit of large number of channels, as long as the average width,

$\bar{\Gamma}_n$, that appears in Eq. (2.13), is substituted by $\frac{D}{2\pi}$ TrP, where P is the optical transmission matrix and D is the average level distance. No explicit reference to the resonance width distribution was made in their treatment. One would therefore expect the general validity of Eq. (2.13) with $\bar{\Gamma}_n$ replaced by an appropriately defined correlation width $\tilde{\Gamma}_n$

$$C_{cc'}^{S}(\varepsilon) = \sum_{n=1}^{N} \sigma_{n,cc'}^{fl} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\varepsilon} \qquad (2.18)$$

Recently, McVoy, Mello and Tang[8] have taken a fresh look at the derivation of $C_{cc'}^{S}(\varepsilon)$ in the one-class case. We expect this discussion to be easily generalized to the multiclass case. The principal result of McVoy et al. may be summarized as follows.

Within the spirit behind the derivation of $C_{cc'}^{S}(\varepsilon)$, one should really consider Eq. (2.12), in the one-class case, as given in terms of the Stieltjes transform of the width distribution $P(\Gamma)$,

$$C_{cc'}^{S}(\varepsilon) = \frac{\sigma_{cc'}^{fl}}{\langle \Gamma_q^{-1} \rangle_q} \int_0^{\infty} \frac{P(\Gamma)}{\Gamma + i\varepsilon} d\Gamma \qquad (2.19)$$

With the use of a reasonable form of $P(\Gamma)$ obtained by imposing several constraints that guarantee a) the finiteness of all inverse moments of Γ, b) the validity of the Moldauer/Simonious theorem[9], $\bar{\Gamma} = -\frac{D}{\pi} \ln|\det \bar{S}|$, and c) the relation $\tilde{\Gamma} = \frac{\langle \Gamma_q^{-1} \rangle_q}{\langle \Gamma_q^{-2} \rangle_q}$, they evaluated the cross section auto-correlation

function $C_{cc'}(\varepsilon) \equiv |C^S_{cc'}(\varepsilon)|^2$ (ignoring direct reactions contribution). The resulting $C_{cc'}(\varepsilon)$ was found to be very close to the more approximate one-pole form

$$C_{cc'}(\varepsilon) = \frac{\sigma^{fl}_{cc'}}{1 + \varepsilon^2/(\tilde{\Gamma})^2} \tag{2.20}$$

with $\tilde{\Gamma}$ given as above i.e.

$$\tilde{\Gamma} = \langle \Gamma_q^{-1} \rangle_q \Big/ \langle \Gamma_q^{-2} \rangle_q \tag{2.21}$$

We mention in passing that Eq. (2.21) is the KKM equivalent of the more widely known, $\tilde{\Gamma} = \frac{D_n}{2\pi} \mathrm{Tr}\underline{\underline{P}}$.

To make the above discussion more transparent, we use a simple form for $P(\Gamma)$ which approximate reasonably well the numerically generated $P(\Gamma)$ worked out by Moldauer[6] and discussed by McVoy et al.[8].

$$P(\Gamma) = \frac{27}{2} \frac{1}{\bar{\Gamma}} \left(\frac{\Gamma}{\bar{\Gamma}}\right)^2 \exp\left(-3\frac{\Gamma}{\bar{\Gamma}}\right) \tag{2.22}$$

With (2.22), the correlation width, given by Eq. (2.21), comes out to be

$$\tilde{\Gamma} = \frac{\bar{\Gamma}}{3} \tag{2.23}$$

and the S-matrix auto-correlation function obtained from Eq. (2.12) becomes[10]

$$C^S_{cc'}(\varepsilon) = \sigma^{fl}_{cc'} \left[1 - \frac{i3\varepsilon}{\bar{\Gamma}} - \left(\frac{3\varepsilon}{\bar{\Gamma}}\right)^2 \exp\left(\frac{i3\varepsilon}{\bar{\Gamma}}\right) E_1\left(\frac{i3\varepsilon}{\bar{\Gamma}}\right)\right] \tag{2.24}$$

where $E_1(X)$ is the exponential integral[11].

In figure (1) we show the cross-section auto-correlation function normalized to unit at $\varepsilon = 1$, $|C^S_{cc'}(\varepsilon)/\sigma^{fl}_{cc'}|^2$, plotted vs. the quantity $\frac{3\varepsilon}{\bar{\Gamma}}$ (Eq. (2.24)).

The extracted value of the correlation width is about $\frac{5}{4}\frac{\bar{\Gamma}}{3}$, slightly larger than that given in Eq. (2.23). This is reasonable in view of the fact that, in contrast to McVoy et al.[8], the condition $\tilde{\Gamma} = \frac{\langle \Gamma_q^{-1} \rangle_q}{\langle \Gamma_q^{-2} \rangle_q}$ was not imposed as a constraint in the construction of $P(\Gamma)$, Eq. (2.22). For comparison, we also show the results obtained with the one-pole approximation to $|C^S_{cc'}(\varepsilon)/\sigma^{fl}_{cc'}|^2$, i.e. $\frac{1}{1+\varepsilon^2/(\tilde{\Gamma})^2}$ with $\tilde{\Gamma}$ obtained from the exact result above, Eq. (2.24). It is clear that the one pole expression approximates very well the exact one in the small ε region. Of course this is the region accessible to unambiguous experimental studies. The fact that the $\tilde{\Gamma}$ above comes out to be very close to $\tilde{\Gamma}$ of Eq. (2.23) indicates that the one-pole approximation applied to the one-class auto-correlation function should be quite reasonable in general.

Certainly all of the above discussion may be easily extended to the multi-class case discussed here within the nested-doorway model. We therefore consider Eq. (2.18) to supply a reasonable approximation to $C^S_{cc'}(\varepsilon)$. Notice that, generally, $\tilde{\Gamma}_n \leq \bar{\Gamma}_n$.

The more general form of $C^S_{cc'}(\varepsilon)$, valid in the presence of direct reaction, may be easily worked out[3]. The result for the cross-section auto-correlation function is

$$C^{S}_{cc'}(\varepsilon) = \left\langle \sigma_{cc'}(\varepsilon)\,\sigma_{cc'}(\varepsilon+\varepsilon) - \left(\left\langle \sigma_{cc'}(\varepsilon)\right\rangle_{I}\right)^{2} \right\rangle_{I}$$

$$= \left| C^{S}_{cc'}(\varepsilon)\right|^{2} + \left[2\,\mathrm{Re}\; C^{S}_{cc'}(\varepsilon)\right]\sigma^{DIR}_{cc'} \qquad (2.25)$$

where $\sigma^{DIR}_{cc'}$ is the direct reaction partial cross-section (we remind the reader that in all our discussion we refer to a given partial wave contribution).

Equations (2.14), (2.16), (2.18) and (2.25) constitute the principal results of the nested-doorway model of the multistep compound components of the pre-equilibrium emission cross section. Explicit expressions for $\sigma^{fl}_{n,cc'}$ in terms of generalized and model transmission matrices have been derived and discussed in Refs. (2,3). A summary of this discussion is given in the next section.

It should be realized that in the actual analysis of the data at lower energie, where well-defined transitions to specific final states are dominated by compound nuclear processes, one should use for the fluctuation cross-section and the cross-section auto correlation function, the full partial wave summed expression evaluated at a given angle. However the simple form of $C_{cc'}(\varepsilon)$

$$C_{cc'}(\varepsilon) = \left| \sum_{n=1}^{N} \sigma_{n,cc'} \frac{\tilde{\Gamma}_n}{\tilde{\Gamma}_n + i\varepsilon} \right|^{2},$$

Eq. (2.18), is usually found to be adequate.

In Figs (2) and (3) we show sample fits using Eqs. (2.25) and (2.18) of several transitions induced by light and heavy ions respectively. More than one correlation width

was extracted indicating clearly the multistep nature of these transitions. Further, the correlation widths extracted from the $^{27}Al(^{3}He,p)^{29}Si$ reaction, were compared with the ones calculated within the exciton model and the agreement was found to be quite reasonable.

In view of the upsurge of interest in the multistep compound processes, we discuss below briefly the applicability of some of more widely used concepts in the conventional statistical analysis[14].

a) Counting of maxima method[15]:

This method could be nicely incorporated into the nested-doorway model. In its original form[15], the method predicts for the average number of maxima per unit energy the following

$$K = \frac{\beta}{2\pi\alpha} \qquad (2.26)$$

where α and β are determined from knowledge of the variance, $C^{(S)}(0)$ as well as its higher derivatives. For the one-class case worked out by Brink and Stephen[15], the ratio β/α comes out to be $\sim \pi/\tilde{\Gamma}$,thus resulting in a K value of $0.5/\tilde{\Gamma}$

Allowing for the presence of several correlation widths would them result in a more complicated expression for K[16]. This renders the method less useful if used for the extraction of the $\tilde{\Gamma}$,s. However it does supply an important independent check on the values of these $\tilde{\Gamma}$,s extracted form auto-correlation studies, as was demonstrated recently in Ref.16 for the reaction $^{25}Mg(^{3}He,p)$ investigated by Bonetti et.al.[18].

b) The variance of the cross-section:

It has been recently pointed out by Bonetti and Mello Mello[17] that within the nested-doorway model described above,

the approximate Ericson's formula for the variance of cross section fluctuations, originally derived for the one-class resonance model, remains valid independentally of the number of doorway classes present in the system.

This formula, in its original form[4] is given by

$$C(0) = 2\left[(2i+1)(2I+1)(2i'+1)(2I'+1)\right]^{-1} \qquad (2.27)$$

where i(i') and I(I') are the intrinsic spins of projectile and target respectively in the incident (final) channels. The condition of validity of Eq. (2.27) is that the spins are not all equal to zero. Further it is assumed the channel spins[5] (e.g. i + I) should be small compound to the orbital angular momenta and that $L << \ell_i, \ell_f$ where $\vec{L} = \vec{\ell}_i + \vec{\ell}_f$.

Since in deriving Eq. (2.27) above no reference is made with regard to the structure of $\sigma^{fl}_{cc'}(J)$, where $\vec{J} = \vec{\ell} + \vec{s}$, it is therefore expected that Ericson's results above are also applicable in the multiclass case. This conclusion is quite useful as Eq. (2.27) may be used to estimate the contribution of direct reaction (e.g. pick-up) to the particular transition studied, vis,

$$C^{(0)}_{exp} = C^{(0)}_{th}\left[1 - y_D^2\right] \qquad (2.28)$$

where y_D^2 represents the relation contribution of direct reactions.

Bonetti and Mello have applied the above results to the reactions $^{25}Mg(^3He,p)$ and $^{25}Mg(^3He,\alpha)$ studied recently by Bonetti et al.[18]. They were able to reproduce reasonably well the experimental values of C(0). The descrepancies found in several cases were attributed to direct reaction contribution (Eq. (2.28)).

c) Sum Rules

It is clear from the preceeding discussion that MSCP may be characterized by several distinct coherence widths $\tilde{\Gamma}_n$. One might then wonder whether there in any natural constraining relation involving all these correlation widths.

Using as a guide the one-class unitarity sum rule

$$Tr\,\underset{\sim}{P} = 2\pi\frac{\tilde{\Gamma}}{D} \qquad (2.29)$$

where $\underset{\sim}{P}$ is the optical transmission matrix, it was suggested in Ref. (20) that the generalization to the multiclass case is simply

$$Tr\left[\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1}\right] = 2\pi\frac{\tilde{\Gamma}_n}{D_n} \qquad (2.30)$$

$$Tr\,\underset{\sim}{P}_N = 2\pi\frac{\tilde{\Gamma}_N}{D_N} \qquad (2.31)$$

where the transmission matrices $\underset{\sim}{P}_n$ are defined in Eq. (2.14).

The N sum rules Eq. (2.30) were recently justified in Ref. (21), using the theory of Agassi et al., (Ref. (7)).

A more useful sum rule involving the trace of the optical transmission matrix only $Tr\underset{\sim}{P}_1$, can be easily obtained by summing Eq. (2.30) over all n,

$$Tr\,\underset{\sim}{P}_1 = 2\pi\sum_{n=1}^{N}\frac{\tilde{\Gamma}_n}{D_n} \qquad (2.32)$$

It is quite possible that in actual application to data analysis, Eq. (2.32) will supply an upper bound for the ex-

perimental $\sum_n 2\pi \frac{\tilde{\Gamma}_n}{D_n}$, since not all the $\tilde{\Gamma}_n$, s will likely be extracted.

Another sum rule involving the Γ_n^{corr}, may be easily obtained for the Heidelberg formulation of MSCP. This second sum rule, which was also derived in Ref. (20), has the form

$$\sum_{h=1}^{N} \left(\Gamma_h^{(o)\uparrow} + \Gamma_h^{(o)\downarrow} \right) = \sum_{n=1}^{N} \tilde{\Gamma}_n \quad , \qquad (2.33)$$

where $\Gamma_n^{(0)\uparrow}$ and $\Gamma_n^{(0)\downarrow}$ are, respectively, the bare escape and damping widths of class n.
Although the RHS of Eq.(2.33) is not directly related to any optical quantity, as is the case in Eq.(2.32),it does however, present a possibility of relating $\sum_n \tilde{\Gamma}_n$ to simple global properties of the nucleus.

d) The angular cross-correlation function

In the presence of several distinct classes of doorway resonances, the angular cross correlation function, defined by[21],

$$C_{cc'}(\theta,\theta') = \frac{\left\langle \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta) \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta') \right\rangle_I - \left\langle \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta) \right\rangle_I \left\langle \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta') \right\rangle_I}{\left\langle \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta) \right\rangle_I \left\langle \frac{d\sigma_{cc'}}{d\Omega_{c'}}(\theta') \right\rangle_I} , \qquad (2.34)$$

is expected to exhibit several coherence angles. This can be easily seen by restricting the calculation to pure compound processes and ignoring spin effects, in which case one obtains[22]

$$C_{cc'}(\theta-\theta') \cong \frac{\left| \sum_n \left(\sum_J (2J+1)\, \sigma^{fl}_{n,cc'}(J) \cos\left(J_n(\theta-\theta')\right) \right) \right|^2}{\left(\sum_J (2J+1) \sum_n \sigma^{fl}_{n,cc'}(J) \right)^2} \qquad (2.35)$$

$$\simeq \left| \sum_{n=1}^{N} \cos\left(J_n(\theta-\theta')\right) F_{cc'}\left(\Delta_n(\theta-\theta')\right) \right|^2 \qquad (2.36)$$

where J_n and Δ_n represent the position of center of gravity and the width of the partial fluctuation cross section σ_n^{fl} of class n respectively. The function $F_{cc'}(\Delta_n(\theta-\theta'))$ attains a unit value at $\theta=\theta'$ and drops gradually to zero at large values of the difference $\theta-\theta'$. In the particular case of channels c,c' couple strongly to a given class of doorways, n, then only one term in the expression for $C(\theta-\theta')$ (Eq. (2.36)) would contribute. In this case the coherence angles may be determined from

$$\left(F\left(\Delta_n(\theta-\theta')_{coh.} \right) \right)^2 = \frac{1}{2} \qquad (2.37)$$

leading to

$$(\theta-\theta')_{coh.,n} = \frac{A}{\Delta_n} \qquad (2.38)$$

with A being a constant determined by the details of the form of $\sigma_n^{fl}(J)$.

In the more general case of several classes that couple equally strongly to the channels, than the coherence angle, which is defined as the angle at which the envelope of $C(\theta-\theta')$ becomes 0.5, is determined by J_n and Δ_n of all the classes contributing. This would result in a more difficult interpretation of the physical significance of the coherence angle.

II.2. Analysis of the continuum region of the spectrum

So far we have discussed the application of the nested-doorway model to the analysis of the discrete portion of the spectrum of emitted particles. At low incident energies this portion is dominated by compound processes (for a given transition the cross section is symmetric about $\theta=90^o$). As the incident energy is increased the transitions to well-separated final states become predominantly direct in nature. The compound and multistep compound processes will contribute mostly to the continuum region of the spectrum. Under such circumstances one must deal with partially inclusive cross sections. This entails the use of an appropriately averaged cross section multiplied by the density of states of the residual nucleus.

The construction of this inclusive cross section has been made by Feshbach, Kerman and Koonin[23]. However, their expression for $\sigma_{n,cc'}^{fl}$ is different form ours in that they impose several restrictions related primarily to the nature of the coupling among the doorway classes (their "chaining hypothesis"), on the one hand, and between the classes and the open-channels subspace on the other. Detailed discussion concerning the differences between the FKK and the nested-doorway models may be found in Refs. (2) and (3). For the calculation of the inclusive cross-section in the nested doorway model one may use the $\sigma_{n,cc'}$ that appears in Eq. (2.40) to construct

$\frac{d\sigma_\gamma^{fl}}{dU}$ and $\frac{d^2\sigma_\gamma^{fl}}{d\epsilon\, d\Omega}$ given in Eqs. (3.60) and (5.1) of FKK. An important modification should be kept in mind: since the nes nested-doorway model does not make the chaining assumption, the restriction of the ν-sum in FKK to three terms should be relaxed. The number of terms to be included depends on the detailed nature of the final channels to which the compound system decays.

III. A Summary of the Nested-Doorway Model

The crucial input into the nested-doorway model and the rest of the discussion in the previous section is the sum-over-pole expression for $\underline{S}_n^{f\ell}$, Eq. (2.5). A complete derivation of Eq. (2.5) was given in Ref. (3). In the present paper we shall outline the major steps needed to be taken in order to arrive at the sum-over-poles form for $\underline{S}_n^{f\ell}$, with the constraint $\langle \underline{S}_n^{f\ell} \rangle_{I_n} = 0$ built in clearly through KKM.

III.1 KKM-ing the Compound Nucleus

The derivation of Eq. (2.4) can be most expediently accomplished with the help of Fig. 4. As a result of the nested-doorway assumption, Eq. (2.2), the decomposition of the Hilbert space, as shown in Fig. 1, into a doorway subspace, represented by the projection operator, d_n, an "open channel" subspace, $P_n \equiv p + d_1 + \ldots + d_{n-1}$, and a fine structure subspace, $Q_{n+1} \equiv d_{n+1} + \ldots, d_N$, is unique. Within the averaging interval, I_{n+1} with $\Gamma_n >> I_{n+1} >> \Gamma_{n+1}$, we may treat the subspace spanned by Q_{n+1} on the average as a "sink" for the flux that reaches it. This implies the introduction of an effective complex Hamiltonian

$$\mathcal{H}(Q_{n+1}) = H + H Q_{n+1} \left\langle \frac{1}{E - Q_{n+1} H Q_{n+1}} \right\rangle_{I_{n+1}} Q_{n+1} H \qquad (3.1)$$

that operates in the remaining subspace $d_n + P_{n-1}$.

In order to obtain the fluctuation component of the wave function, due to the doorway class, d_n, we have to solve for $P_{n-1} \langle P_n \psi \rangle_{I_{n+1}}$ and $d_n \langle P_n \psi \rangle_{I_{n+1}}$, where ψ in the

total ware function of the system. This amounts to solving the following set of coupled equations

$$\left[E - \mathcal{P}_{n-1} \mathcal{H}(Q_{n+1}) \mathcal{P}_{n-1}\right] \mathcal{P}_{n-1} \Phi_{\mathcal{P}_n} = \mathcal{P}_{n-1} \mathcal{H}(Q_{n+1}) d_n \Phi_{\mathcal{P}_n} \qquad (3.2)$$

$$\left[E - d_n \mathcal{H}(Q_{n+1}) d_n\right] d_n \Phi_{\mathcal{P}_n} = d_n \mathcal{H}(Q_{n+1}) \mathcal{P}_{n-1} \Phi_{\mathcal{P}_n} \qquad (3.3)$$

where we have introduced the definition

$$\Phi_{\mathcal{P}_n} \equiv \langle \mathcal{P}_n \Psi \rangle_{I_n} \qquad (3.4)$$

The set of equations (3.2) and (3.3) is similar to the set usually considered in the simpler (P,Q) theories. The only difference is the oppearance in Eqs. (3.2) and (3.3) of the non-Hermitian Hamiltonian operator $\mathcal{H}(Q_{n+1})$. This however does not alter the KKM procedure, which furnishes the fluctuation S-matrix $S_n^{f\ell}$.

$$S^{f\ell}_{n,cc'} = -2\pi i \left\langle \Phi^{(-)c'}_{\mathcal{P}_{n-1}} \middle| V_{\mathcal{P}_{n-1} d_n} \left[E - d_n \mathcal{H}(\mathcal{P}_{n-1}) d_n\right]^{-1} \times V_{d_n \mathcal{P}_{n-1}} \middle| \Phi^{(+)c}_{\mathcal{P}_{n-1}} \right\rangle \qquad (3.5)$$

where

$$\mathcal{H}(\mathcal{P}_{n-1}) = \mathcal{H}(Q_{n+1}) + V_{d_n \mathcal{P}_{n-1}} \mathcal{G}^{(+)}_{\mathcal{P}_{n-1}} V_{\mathcal{P}_{n-1} d_n} \qquad (3.6)$$

$$\mathcal{L}^{(+)}_{P_{n-1}} = \left[E^{(+)} - P_{n-1}\, \mathcal{H}(Q_n) P_{n-1} \right]^{-1} \tag{3.7}$$

$$V_{P_{n-1} d_n} = P_{n-1}\, \mathcal{H}(Q_{n+1})\, d_n \left[\frac{i\, I_n/2}{E - d_n\, \mathcal{H}(Q_{n+1})\, d_n + i\, I_n/2} \right]^{1/2} \tag{3.8}$$

$$V_{d_n P_{n-1}} = \left[\frac{i I_n/2}{E - d_n\, \mathcal{H}(Q_{n+1})\, d_n + i I_n/2} \right]^{1/2} d_n\, \mathcal{H}(Q_{n+1})\, P_{n-1} \tag{3.9}$$

and $\varphi^{(\pm)c}_{P_{n-1}}$ is a solution of

$$\left[E - P_{n-1}\, \mathcal{H}(Q_n)\, P_{n-1} \right] \varphi^{(\pm)c}_{P_{n-1}} = 0 \tag{3.10}$$

Having derived a formal expression for $\underset{\sim}{S}_n^{fl}$ in Eq. (3.5) we now cast it in the sum-over-poles form by expanding the d_n-propagator, $(E - d_n \mathcal{H}(P_{n-1}) d_n)^{-1}$, in terms of biorthogonal basis states $|\psi_{ni}\rangle$ and $|\tilde{\psi}_{ni}\rangle$ which satisty the following eigenvalue equations

$$\left(\mathcal{E}_{ni} - d_n\, \mathcal{H}(P_{n-1})\, d_n \right) |\psi_{ni}\rangle = 0 \tag{3.11}$$

$$\left(\mathcal{E}^{*}_{ni} - d_n\, \mathcal{H}(P_{n-1})^{\dagger}\, d_n \right) |\tilde{\psi}_{ni}\rangle = 0 \tag{3.12}$$

Using the fact

$$\theta\, V_{P_{n-1} d_n}\, \theta^{-1} = \left(V_{d_n P_{n-1}} \right)^{\dagger} \tag{3.13}$$

where θ is the time-reveral operator, we can then prove

$$g_{ni} \equiv \sqrt{2\pi}\, \langle \varphi^{(-)c}_{P_{n-1}} | V_{P_{n-1} d_n} | \psi_{ni} \rangle$$

$$= \sqrt{2\pi}\, \langle \tilde{\psi}_{ni} | V_{d_n P_{n-1}} | \varphi^{(+)c}_{P_{n-1}} \rangle \tag{3.14}$$

where we have used

$$\theta\, |\varphi^{(-)c}_{P_{n-1}}\rangle = |\varphi^{(+)c}_{P_{n-1}}\rangle \tag{3.15}$$

Using Eq. (3.14), we finally obtain the desired expression for S_n^{fl}

$$\underset{\sim}{S}^{fl}_{n,cc'} = -i \sum_{i \in \{d_n\}} \frac{g_{ni,c}\, g_{ni,c'}}{E - \mathcal{E}_{ni}} \tag{3.16}$$

with the KKM condition $\langle S_n^{fl} \rangle_{I_n} = 0$, satisfied by construction. We should stress that the residue factors $g_{ni,c}$ still contain modulations which would be "seen" if the averaging interval is increased. This is clear from its definition, Eq. (3.14), with the "optical" wave function containing all doorway classes with correlation widths larger than $\tilde{\Gamma}_n$.

III.2 The Average Doorway Width

Of interest is the average total width of doorways of class n. One may obtain an idea concerning this by analysing the Bell-Steinberger relation, which in our case is given by

$$\langle \psi_{ni} | \psi_{ni} \rangle = \frac{2i \langle \psi_{ni} | \mathrm{Im}\, d_n \mathcal{H}(P_{n-1})\, d_n | \psi_{ni} \rangle}{2i\, \mathrm{Im} \langle \tilde{\psi}_{ni} | d_n \mathcal{H}(P_{n-1}) d_n | \psi_{ni} \rangle}$$

$$\equiv \frac{-2 \langle \psi_{ni} | \mathrm{Im}\, d_n \mathcal{H}(P_{n-1}) d_n | \psi_{ni} \rangle}{\Gamma_{ni}} \tag{3.17}$$

Due to the appearance of $\tilde{\psi}_{ni}$ in the definition of the total width above, it becomes rather difficult to find a simple expression for it. However, we may use the equation above to find the over all structure of Γ_{ni}. This may be accomplished by the calculation of the numerator in Eq. (3.17). To this end we use the defining equation for the operator $d_n \mathcal{H} (P_{n-1}) d_n$, Eq. (3.6), and the following identity for the elements of the matrix propagator $\mathcal{G}_{P_n}$

$$p \mathcal{G}^{(+)}_{P_{n-1}} p = \mathcal{G}^{(+)}_{opt} + \mathcal{G}^{(+)}_{opt} V_{p D_{n-1}} \mathcal{G}^{(+)}_{D_{n-1}} V_{D_{n-1} p} \mathcal{G}^{(+)}_{opt} \tag{3.18}$$

$$p \mathcal{G}^{(+)}_{P_{n-1}} D_{n-1} = \mathcal{G}^{(+)}_{opt} V_{p D_{n-1}} \mathcal{G}^{(+)}_{D_{n-1}} \tag{3.19}$$

With Eqs. (3.18) and (3.19) we find for $\langle \psi_{ni} | \psi_{ni} \rangle$ the following

$$\langle \psi_{ni} | \psi_{ni} \rangle = \frac{\Gamma_{ni}^{\uparrow} + \Gamma_{ni}^{\downarrow} - 2 \langle \psi_{ni} | Im\, v_{d_n D_{n-1}} \mathcal{G}^{(+)}_{D_{n-1}} v_{D_{n-1} d_n} | \psi_{ni} \rangle}{\Gamma_{ni}} \tag{3.20}$$

where $D_{n-1} = P_{n-1} - p$, and v is a coupling potential given by

$$v = V + V \mathcal{G}^{(+)}_{opt} V \tag{3.21}$$

The propagators $\mathcal{G}^{(+)}_{D_{n-1}}$ and $\mathcal{G}^{(+)}_{opt}$ are the D_{n-1} subspace matrix propagator and the optical propagator, respectively. The widths $\Gamma_{ni}^{\uparrow}$ and $\Gamma_{ni}^{\downarrow}$ are defined by

$$\Gamma_{ni}^{\uparrow} \equiv -2 \langle \psi_{ni} | Im\, V_{d_n p} \mathcal{G}^{(+)}_{opt} V_{p d_n} | \psi_{ni} \rangle \tag{3.22}$$

$$\Gamma_{ni}^{\downarrow} \equiv -2 \langle \psi_{ni} | Im\, d_n \mathcal{H} (Q_{n+1}) d_n | \psi_{ni} \rangle \tag{3.23}$$

The last term in the numerator of Eq. (3.20) is a genuine multi-class modification of the usual expression for the average width of a doorway. Due to the presence of doorway classes with correlation widths larger than that of class n, the average width of doorway class n contains a piece that resembles the escape width, however it involves the mixing with the wider-width doorway classes. To emphasize this point we call this piece $\Gamma_{ni}^{\uparrow}(D_{n-1})$. Note that the mixing of n with D_{n-1} is determined by the potential v which contains a direct mixing (internal mixing) term, V, and an indirect one (external mixing) given by $V \mathcal{G}^{(+)}_{opt} V$.

Owing to the fact that $\langle \overline{\psi_{ni} | \psi_{ni}} \rangle \geq 1$, Eq. (3.20) supplies us with an upper value for the average width $\langle \Gamma_{ni} \rangle_i$, of class-n doorways,

$$\Gamma_n \equiv \langle \Gamma_{ni} \rangle_i \leq \Gamma_n^{\uparrow} + \Gamma_n^{\downarrow} + \Gamma_n^{\uparrow}(D_{n-1}) \tag{3.24}$$

The equality sign would hold in the weak coupling (absorption) limit.

III.3 The X_n and τ_n matrices

Having obtained the sum-over-poles form for $S_n^{f\ell}$, Eq. (3.16), with the KKM condition $\langle S_n^{f\ell} \rangle_{I_n}$, satisfied by construction for each class of doorways, the calculation of $\sigma_{cc'}^{f\ell}$ becomes straightforward. The result is

$$\sigma_{cc'}^{f\ell} = \sum_n \left[X_{n,cc} X_{n,c'c'} + X_{n,cc'} X_{n,c'c} \right] \tag{3.25}$$

with

$$X_{n,cc'} \equiv \sqrt{\frac{2\pi}{\tilde{\Gamma}_n' D_n}} \left\langle g_{ni,c} \, g_{ni,c'}^{*} \right\rangle_{i \in I_n} \tag{3.26}$$

and

$$\tilde{\Gamma}_n' \equiv \left(\left\langle \Gamma_{ni}^{-1} \right\rangle_i \right)^{-1} \tag{3.27}$$

Using unitarity, Eqs. (2.17) and (2.21) are then easily obtained.

In this sub-section we discuss the formal structure of the $\underset{\sim}{X}_n$ - matrix defined above is Eq. (3.26). This is necessary in order to make contacts with the results obtained within other models of MSCP.

In order to calculate the $\underset{\sim}{X}_n$ matrix, we need to develop a theory for the form factors $g_{ni,c}$. This is easily accomplished by using the KKM reduction procedure on the "continuum" wave function $\varphi_{\mathcal{P}_{n-1}}$ that appears in the definition

of $g_{ni,c}$, Eq. (3.14). The result is

$$g_{ni,c} = g_{ni,c}^{opt} + \sum_{m=1}^{n-1} g_{ni,c}^{f\ell}(d_m) \tag{3.28}$$

where $g_{ni,c}^{opt}$ the optical form factor given by

$$g_{ni,c}^{opt} = \sqrt{2\pi} \left\langle \tilde{\Psi}_{ni} \middle| V_{d_n p} \middle| \varphi_{opt}^{(+)c} \right\rangle \tag{3.29}$$

and $g_{ni,c}^{f\ell}(d_m)$ are the fluctuating parts of $g_{ni,c}$, that arise from the modulations due to the doorway classes above n

$$g_{ni,c}^{f\ell}(d_n) = \sqrt{2\pi} \left\langle \tilde{\Psi}_{ni} \middle| V_{d_n d_m} \mathcal{G}_{d_m} V_{d_m \mathcal{P}_{n-1}} \middle| \Psi_{ni} \right\rangle \tag{3.30}$$

with V_{dndm} given by the following expression

$$V = v + v \mathcal{G}_{\mathcal{D}_{m-1}} v \tag{3.31}$$

and v defined in Eq. (3.21).

With the help of Eqs. (3.28) and (3.30), a recursion equation for $\langle X_n \rangle_{I_1}$ may be obtained[3],

$$\left\langle X_{n,cc'} \right\rangle_{I_1} = X_{n,cc'}^{opt} + \sum_{m=1}^{n-1} X_{n,cc'}^{f\ell}(m) \tag{3.32}$$

with

$$X_{n,cc'}^{f\ell}(m) = \sqrt{\frac{D_n / \tilde{\Gamma}_n'}{D_m / \tilde{\Gamma}_m'}} \left\langle X_{m,cc'} \right\rangle_{I_1} \eta'_{mn} \tag{3.33}$$

and

$$\eta'_{mn} = \frac{2\pi}{\tilde{\Gamma}'_m D_n} \left\langle \left| \langle \tilde{\Psi}_{mj} | V_{d_m d_n} | \Psi_{ni} \rangle \right|^2 \right\rangle_{I_1} \qquad (3.34)$$

The downward branching ratio η'_{mn} dictates how class n is coupled to class m above it. This coupling is both direct (internal mixing) and indirect (external mixing) as clear from the definition of V_{dmdn}, Eq. (3.31). The optical X_n-matrix, $\underset{\sim}{X}_n^{opt}$ refers to the direct coupling of class n to the open channels subspace p.

The X_n-matrices are of course not directly observable. In order to derive an equation similar to (3.32) for observable quantities, we have to calculate the differences between the generalized satchler transmission matrices, P_n, Eq. (2.21) One step before actually doing this is to introduce two more sets of transmission matrices, T'_n and τ'_n defined through

$$T'_{n,cc} = \sqrt{\frac{2\pi \tilde{\Gamma}'_n}{D_n}} \left\langle X_{n,cc} \right\rangle_{I_1} \qquad (3.35)$$

$$\tau'_{n,cc} = \sqrt{\frac{2\pi \tilde{\Gamma}'_n}{D_n}} \left\langle X^{opt}_{n,cc} \right\rangle_{I_1} \qquad (3.36)$$

with Eqs. (3.35) and (3.36), we can now recast Eq. (3.32) into the more appealing form

$$T'_{n,cc} = \tau'_{n,cc} + \sum_{m=1}^{n-1} T'_{m,cc}\, \eta'_{mn} \qquad (3.37)$$

the transmission coefficient $T_{n,cc}$ specifies the way class n is populated from channel c both directly and through the doorways above n. The only reference in Eq. (3.37) to the presence of doorway classes with widths smaller than that of class n is through the complex potentials that should enter into the calculation of τ' and η'_{mn} (see previous discussion). A more explicit reference to the doorway classes "below" n should be found in the differences $\langle P_n - P_{n+1} \rangle_I$ of Eq. (2.21). In Ref. (3) a relation between $\langle P_n - P_{n+1} \rangle_I$ and T'_n was obtained after performing a subtle renormalization of $T'_n \to T_m$, $\tau'_n \to \tau_n$ and $\eta'_{mn} \to \eta_{mn}$. This renormalization is necessary in view the fact that in our discussion the inequality, Eq. (3.24), does not permit a simple identification of $\tilde{\Gamma}'_n$ with $\mathrm{Tr}\, \underset{\sim}{X}_n$ (see Ref. (3) for fuller discussion).

Using the renormalized transmission matrices (T, τ) and downward branching ratios (η_{mn}), one could show[3]

$$\left\langle P_{n,cc'} - P_{n+1,cc'} \right\rangle_{I_1} = T_{n,cc'} \left(1 - \sum_{m>n} \eta_{mn} \right) \qquad (3.38)$$

The interpretation of Eq. (3.38) is quite simple. The first factor, $T_{n,cc}$ represents the "probability" of reaching class n, from channel c (see discussion following Eq. (3.37). A part of this flux will continue "downward" to other doorway classes (m > n). This is exemplified by the second term on the RHS of Eq. (3.38), $T_{n,cc} \sum_{m>n} \eta_{mn}$. The difference, $T_{n,cc} - T_{n,cc} \sum_{m>n} \eta_{mn}$, therefore represents the net flux that has reached class n and remained there.

With the help of Eqs. (2.22), (3.35) and (3.38), we may write several forms for the fluctuation cross section $\sigma^{fl}_{cc'}$ (in the absence of direct reactions)

$$\sigma_{cc'}^{fl} = \sum_{n=1}^{N} \sigma_{n,cc'}^{fl}$$

$$= \sum_{n=1}^{N} \frac{\mathcal{D}_n}{2\pi \tilde{\Gamma}_n'} T_{n,cc}' T_{n,c'c'}' \qquad (3.39)$$

$$= \sum_{n=1}^{N} \frac{T_{n,cc}\, T_{n,c'c'}}{Tr\, T_{mn}} \left(1 - \sum_{m>n} \eta_{mn}\right) \qquad (3.40)$$

The expression for $\sigma_{cc'}^{fl}$, closest in structure to the original one-class Hauser-Feshbach expression is given in Eq. (2.16). However for actual model calculation, Eqs. (3.39) and (3.37) should be more convenient. For fuller details we refer the reader to Ref. (3).

IV. Discussion

The results of the previous sections exhibited clearly the general structure of the fluctuation cross section and the S-matrix autocorrelation function for the multistep compound process. Although these results were obtained within the nested doorway model, we believe that structurally they should be quite general. To clarify this point we discuss below another, seemingly different, approach to the MSCP, namely the one developed by Weidenmüller and collaborators[7),12)].

Summarizing the ND model results; we have obtained the following expressions for $\sigma_{cc'}^{fl}$ and $c_{cc'}^{s}(\varepsilon)$ (we shall exclude direct reactions from our consideration)

$$\sigma_{cc'}^{fl} = \sum_{n} X_{n,cc}\, X_{n,c'c'} \qquad (4.1)$$

$$C_{cc'}^{S}(\varepsilon) = \sum_{n} \frac{X_{n,cc}\, X_{n,c'c'}}{1 + i\,\varepsilon/\tilde{\Gamma}_n} \qquad (4.2)$$

with $X_{n,cc}$ given by a linear conbination of the optical $X_{n,cc}^{opt}$ (or equivalently the T_n being a linear combination of the τ_n's)

In the Heidelberg approach, the fluctuation cross section and the S-matrix correlation function are expressed as

$$\sigma_{cc'}^{fl} = \langle \hat{\tau}_c \,|\, M^{-1} \,|\, \hat{\tau}_{c'} \rangle \qquad (4.3)$$

$$C_{cc'}^{S}(\varepsilon) = \langle \hat{\tau}_c \,|\, (M + 2\pi i \varepsilon \mathbf{1})^{-1} \,|\, \hat{\tau}_{c'} \rangle \qquad (4.4)$$

where $\langle\hat{\tau}| = (\hat{\tau}_1, \hat{\tau}_2, \ldots)$ is a row vector and $\hat{\tau}_i = \sqrt{D_i}\ \tau_i$ with τ_i the optical penetration factor and D_i the average spacing for level class i. The matrix M is given by the following expression (ignorning external mixing for simplicity)

$$M_{mn} = \delta_{mn}\,\Gamma_n^{\downarrow} - \sqrt{\Gamma_n^{\downarrow}\,\Gamma_m^{\downarrow}} \tag{4.5}$$

The widths $\Gamma_n^{\downarrow}$ and $\Gamma_n^{\prime\downarrow}$ are model widths and should not be confused with the average widths appearing in Eq. (3.24).

Clearly in order to set Eqs. (4.3) and (4.4) into the more transparent form of our Eqs. (4.1) and (4.2), we have to diagonalize both the matrix M^{-1} and the matrix $(M + 2\pi i\varepsilon I)^{-1}$. This is accomplished through an orthogonal transformation, O with $OO^T = 1$. Denoting the eigenvalue, of the matrix M by λ_n, we immediately obtain[1)]

$$\sigma_{cc'}^{fl} = \sum_n X_{n,cc}^W\, X_{n,c'c'}^W \tag{4.6}$$

$$C_{cc'}^S(\varepsilon) = \sum_n \frac{X_{n,cc}^W\, X_{n,c'c'}^W}{1 + i\varepsilon/\lambda_n} \tag{4.7}$$

where the Weidenmüller X_n - matrix, X_n^W is simply

$$\underset{\sim}{X}_n^W = \frac{1}{\sqrt{2\pi\lambda_n}} \sum_k O_{nk}\,\hat{\tau}_k$$

$$= \frac{1}{\sqrt{2\pi\lambda_n}} \sum_k O_{nk}\sqrt{D_k}\ \tau_k \tag{4.8}$$

a linear combination of the τ_k, s.

Of course we do not expect the above equation for X_n^W (in terms of Weidenmüller's τ_k,s) to coincide with our equation for X_n (in terms of our optical τ_k,s) since the basic assumptions entering in the two models are quite different. In the nested doorway model, the statistical assumptions are invoked on the S-matrix ($\langle S_n^{fl}\rangle_{I_n} = 0$), which refers to the physical states, whereas in Weidenmüller's theory these assumptions are invoked on the residual interaction that connects model states. It is of interest to find appropriate limits in which Weidenmüller's results (rewritten as Eqs. (4.6), (4.7) and (4.8))coincide with ours. To simplify the algebra we work out below the two-class case discussed in Ref. (12) in connection with isospin mixing.

The matrix M is then given by

$$M = \begin{pmatrix} \Gamma_1^{\downarrow} & -\sqrt{\Gamma_1^{\prime\downarrow}\,\Gamma_2^{\prime\downarrow}} \\ -\sqrt{\Gamma_1^{\prime\downarrow}\,\Gamma_2^{\prime\downarrow}} & \Gamma_2^{\downarrow} \end{pmatrix} \tag{4.9}$$

The eigenvalues λ_n, s are easily obtained

$$\lambda_1 = \tfrac{1}{2}(\Gamma_1^{\downarrow} + \Gamma_2^{\downarrow}) + \tfrac{1}{2} R$$

$$\lambda_2 = \tfrac{1}{2}(\Gamma_1^{\downarrow} + \Gamma_2^{\downarrow}) - \tfrac{1}{2} R$$

$$R = \tfrac{1}{2}\left[(\Gamma_1^{\downarrow} - \Gamma_2^{\downarrow})^2 + 4\,\Gamma_1^{\prime\downarrow}\,\Gamma_2^{\prime\downarrow}\right]^{1/2} \tag{4.10}$$

the elements of the orthogonal matrix, O, can also be worked out straightforwardly

$$O_{11} = \sqrt{2}\ \frac{\sqrt{\Gamma_1'^{\downarrow}\,\Gamma_2'^{\downarrow}}}{R^{1/2}\left[R-(\Gamma_1'-\Gamma_2')\right]^{1/2}} \tag{4.11}$$

$$O_{12} = -\frac{1}{\sqrt{2}}\ \frac{\left[-(\Gamma_1'-\Gamma_2')+R\right]^{1/2}}{R^{1/2}} \tag{4.12}$$

$$O_{21} = \sqrt{2}\ \frac{\sqrt{\Gamma_1'^{\downarrow}\,\Gamma_2'^{\downarrow}}}{R^{1/2}\left[R+(\Gamma_1'-\Gamma_2')\right]^{1/2}} \tag{4.13}$$

$$O_{22} = \frac{1}{\sqrt{2}}\ \frac{\left[(\Gamma_1'-\Gamma_2')+R\right]^{1/2}}{R^{1/2}} \tag{4.14}$$

With the above expressions for O, one can then write down immediately the explicit form of X_1^W and X_2^W of Eq. (4.8).

To discuss the appropriate limits mentioned above, we introduce the following quantities

$$\mu \equiv \Gamma_1'^{\downarrow}/\Gamma_1'$$

$$s \equiv \Gamma_2'/\Gamma_1'$$

$$r \equiv \left(D_2/D_1\right)^{1/2} \tag{4.15}$$

If we now set $s \ll 1$ and $r \ll 1$ (but with both $\frac{\Gamma_1'}{D_1}$ and $\frac{\Gamma_2'}{D_2}$ larger than unity) we immediately obtain to zeroth order in r and s, the following

$$X^W_{1,cc} \cong \frac{1}{\sqrt{2\pi}}\,\frac{\sqrt{2}}{\sqrt{\frac{\Gamma_2'}{D_2}}}\,\frac{1}{\left[1+2\frac{r^2}{s}\mu^2\right]^{1/2}}\left[\tau_{1,c}-\frac{1}{2}(s+2r^2\mu^2)\tau_{2,c}\right]$$

$$\cong \frac{1}{\sqrt{2\pi}}\,\frac{\sqrt{2}}{\sqrt{\frac{\Gamma_2'}{D_2}}}\,\frac{1}{\left[1+2\frac{r^2}{s}\mu^2\right]^{1/2}}\,\tau_{1,c} \tag{4.16}$$

and

$$X^W_{2,cc} \cong \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sqrt{\frac{\Gamma_2'}{D_2}}}\,\frac{1}{\left(1-\frac{r^2}{s}\mu^2\right)^{1/2}}\left[\mu\tau_{1,c}+\tau_{2,c}\right] \tag{4.17}$$

Which, with proper renormalizations, has exactly the same form as our $X_{1,c}$ and $X_{2,c}$, Eq. (3.32).

Similar considerations as above can be made with regard to the MIT approach[6] to MSCP. In this section, we have concentrated our discussion on the Heidelberg model since the MIT model seems to be close in spirit to the nested doorway model. For further discussion of this point we refer the reader to Ref. (3).

ACKNOWLEDGEMENT

I would like to thank Brett Carlson, Pier Mello and Alex Szanto de Toledo for very useful discussion, comments and suggestions.

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FIGURE CAPTIONS

Figure 1. The cross-section auto-correlation function, Eq. (2.24), plotted vs. $3\varepsilon/\bar{\Gamma}$ (solid curve). The dashed curved represents the one-pole approximation to $C(\varepsilon)$.

Figure 2. Correlation function for the reaction $^{27}Al(^{3}He,p)^{29}Si$, leading to the (a) second and (b) fifth excited state of ^{29}Si. The continuous curves are Ericson's formula $C(\varepsilon) = \frac{1}{\varepsilon^2 + \Gamma^2}$, with a correlation width of 55 keV and 230 keV, respectively (from Ref. (12)).

Figure 3. Autocorrelation functions for four of the excitation functions for $^{15}N(^{12}C,\alpha)^{23}Na$ by J. Gomez del Campo et al., Nucl.Phys. A297 (1978) 125. The fits were obtained with Eq. (2.18). a) Ground state transition. Full line was obtained with (3). Dashed and dashed-dotted lines were obtained with Eq. (2) (one class). b) Summed transitions to states in ^{23}Na with excitation energies in the range 7.180 - 7.272 MeV. c) Same as b) for the range 7.386 - 7.446 MeV. d) Same as b) for the summed transitions to the 8.555 MeV - 8.602 MeV excited states (form Ref. (13)).

Figure 4. Partition of Hilbert space. Left: usual Hauser-Feshbach partition. Center: Nested-doorway partition. Right: Generalized Hauser-Feshbach used in the evaluation of $\sigma_{cc'}^{fl}$.

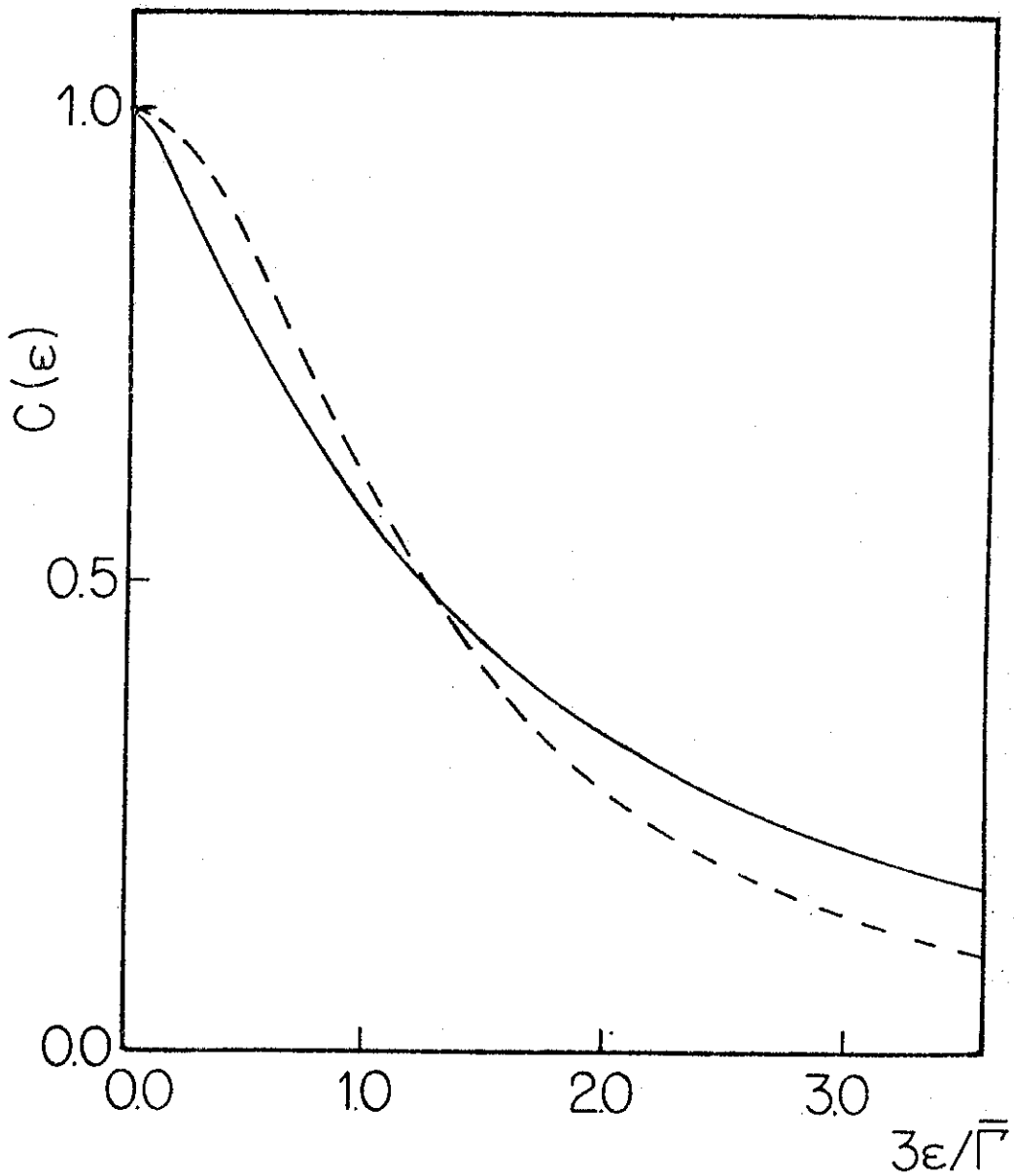
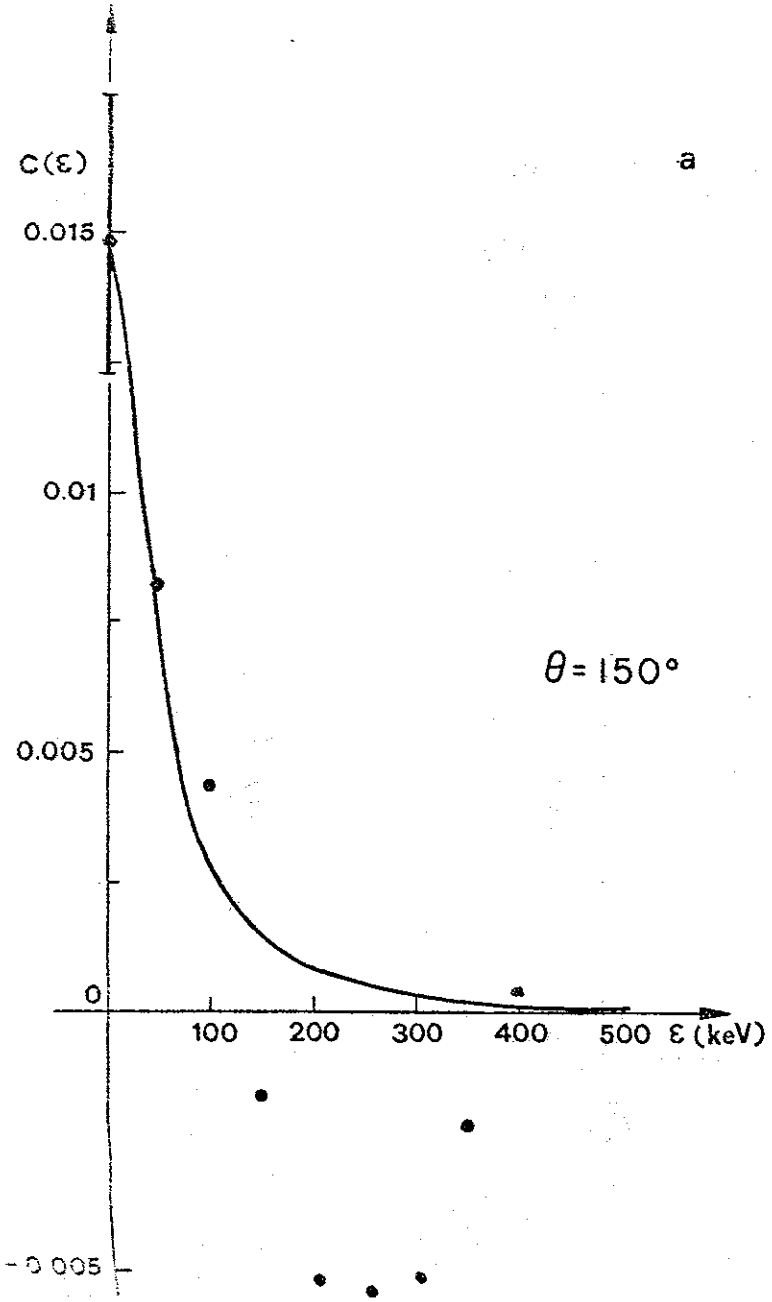


Fig. 1


C(ε)
a
0.015
0.01
0.005
0
-0.005
θ = 150°
100
200
300
400
500 ε (keV)

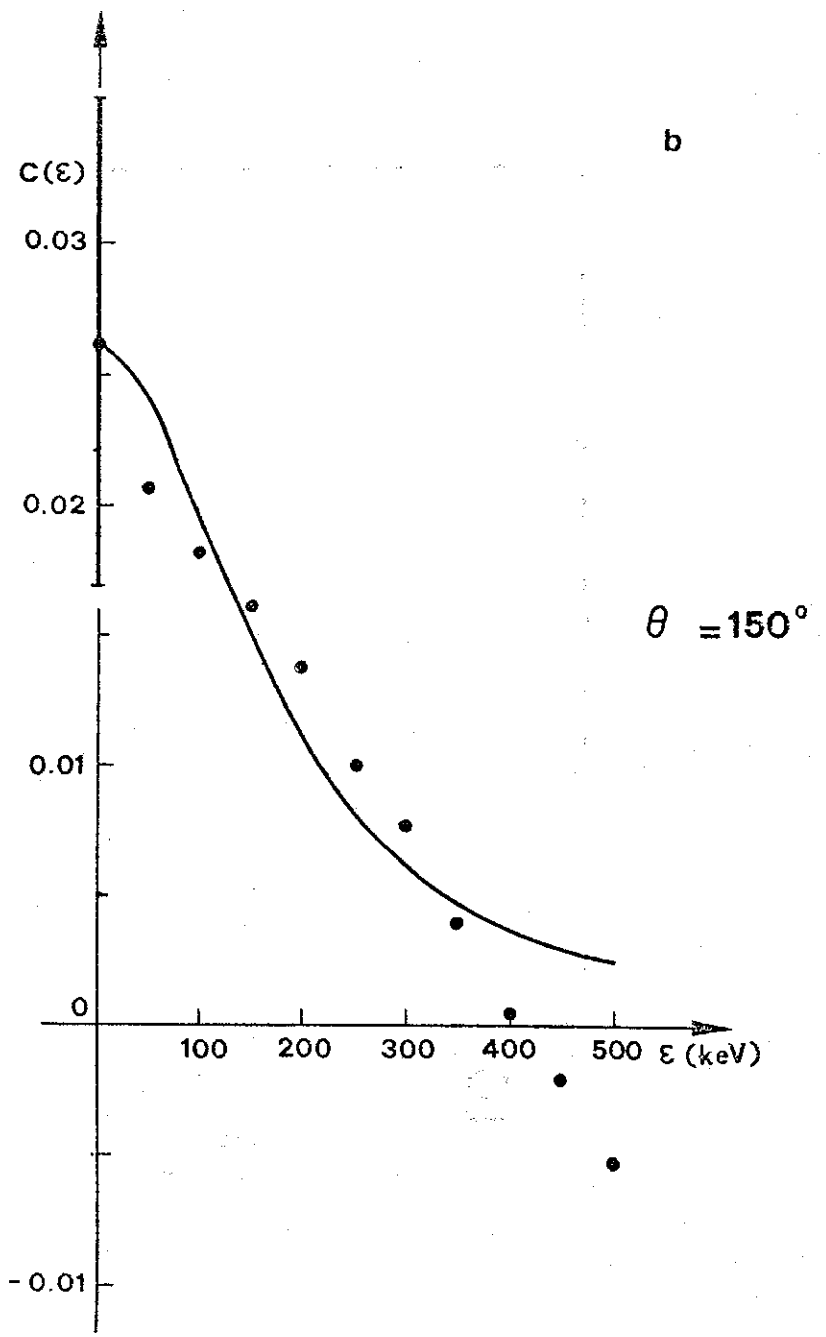

C(ε)
b
0.03
0.02
0.01
0
-0.01
θ = 150°
100
200
300
400
500 ε (keV)

Fig. 2

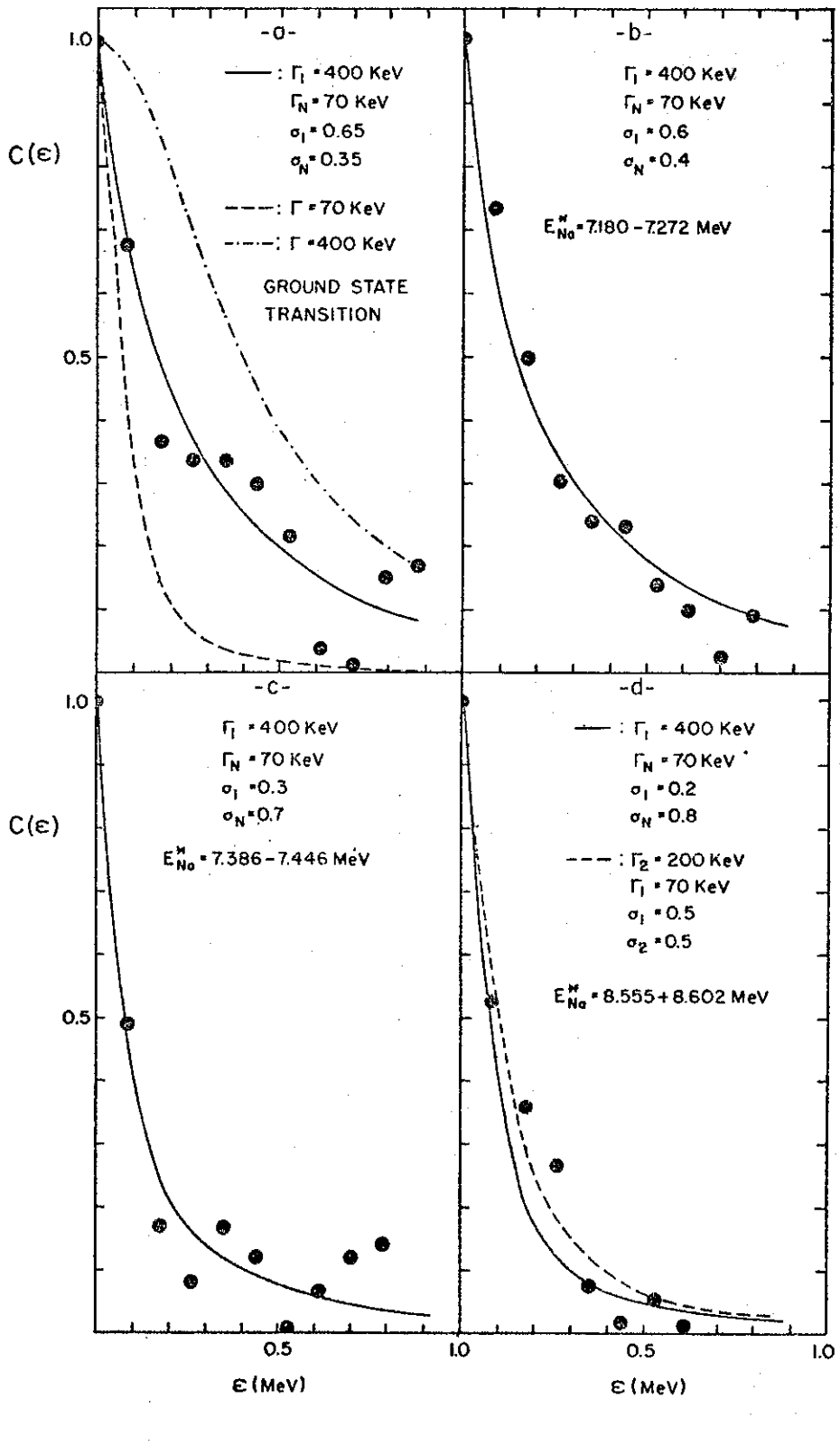

-a-
——: Γ1 = 400 KeV
ΓN = 70 KeV
σ1 = 0.65
σN = 0.35
– – –: Γ = 70 KeV
-·-·-: Γ = 400 KeV
GROUND STATE
TRANSITION
-b-
Γ1 = 400 KeV
ΓN = 70 KeV
σ1 = 0.6
σN = 0.4
E*Na = 7.180 – 7.272 MeV
-c-
Γ1 = 400 KeV
ΓN = 70 KeV
σ1 = 0.3
σN = 0.7
E*Na = 7.386 – 7.446 MeV
-d-
——: Γ1 = 400 KeV
ΓN = 70 KeV
σ1 = 0.2
σN = 0.8
– – –: Γ2 = 200 KeV
Γ1 = 70 KeV
σ1 = 0.5
σ2 = 0.5
E*Na = 8.555 + 8.602 MeV
C(ε)
C(ε)
1.0
0.5
0.5
1.0
ε (MeV)
ε (MeV)

Fig. 3

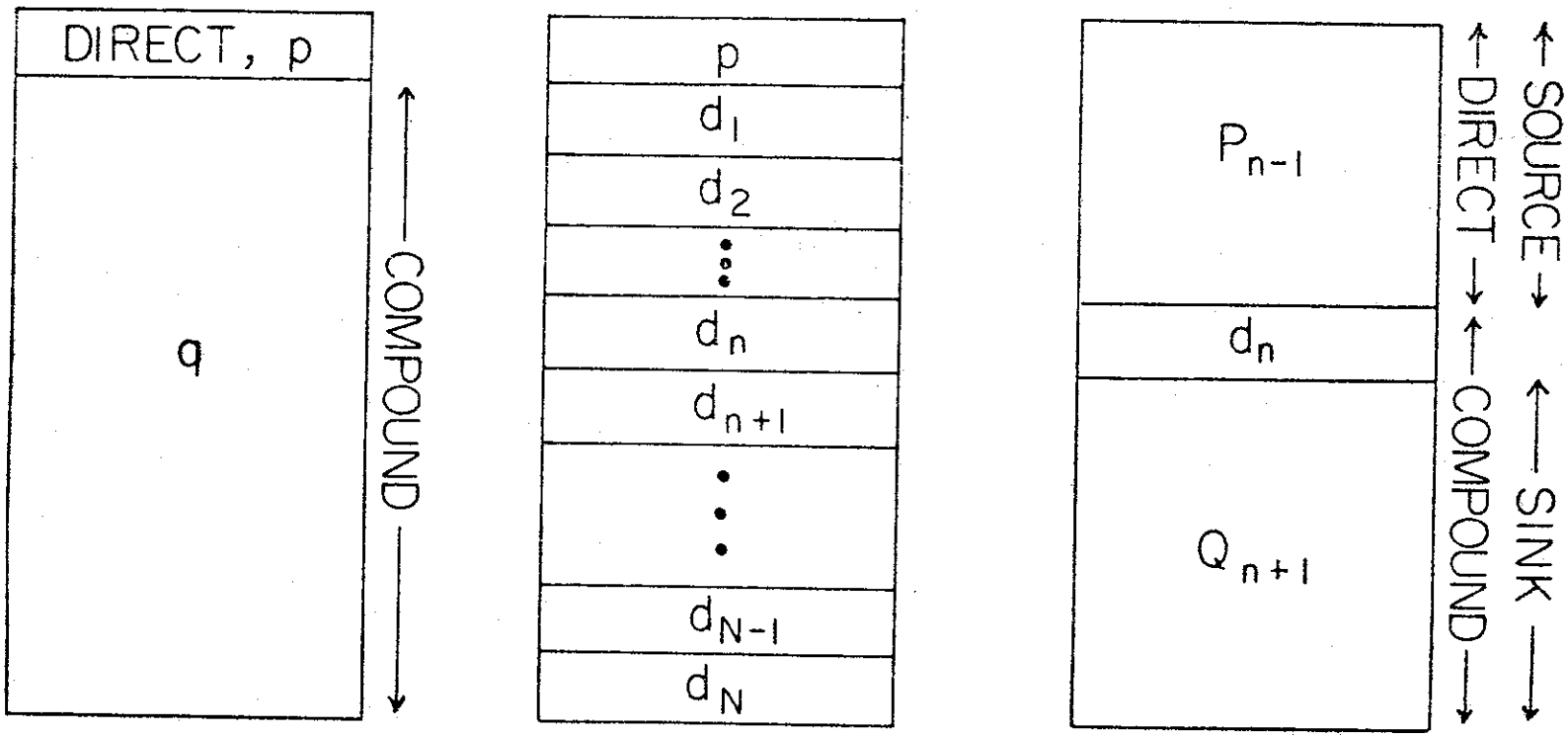

DIRECT, p
q
COMPOUND
p
d_1
d_2
d_n
d_{n+1}
d_{N-1}
d_N
P_{n-1}
d_n
Q_{n+1}
SOURCE
DIRECT
COMPOUND
SINK

Fig. 4