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by

F.A.B. Coutinho

Instituto de Física, Universidade de São Paulo
São Paulo, Brasil

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ABSTRACT

It is shown that when a system of two identical bosons moving in one dimension have a bound state of energy ν_0, then the N body system will also have a bound state at a specific energy given by equation (3).

In a recent paper[1] the connection between bound-states of a three-particle system and bound-states of a two-particle subsystem was investigated for three identical bosons moving in one dimension. It was found that in general if the two particle subsystems has a bound-state of energy ν_0 then the three-particle system will have a bound-state at the energy $4\nu_0$. This was done by using a completely off-shell method introduced by Brayshaw[2].

In this paper we show that this result follows from on-shell considerations[3] and generalize it to show that a system of N bosons has a bound-state at a given energy if the subsystem of two bosons has a bound-state.

We consider the Feynman diagram show in Fig. 1 for N = 2. The Landau[4], Cutkosky[5] rules gives that the amplitude it represents has a pole if the intermediate particle is on shell, that is, if

$$E \left[\frac{4}{3} (1 - \cos\theta) + \frac{1}{3}\right] + \nu_0 = 0 \qquad (1)$$

were E is the total kinetic energy in the center of mass system and ν_0 is the bound-state energy of the two-particle subsystem.

In one dimension however, θ is only 0 (forward scattering) and π (backward scattering). So we get two singularities at $E = -3\nu_0$ (for forward scattering) and $E = -\frac{1}{3}\nu_0$ (for backward scattering).

However we know from scattering theory in one dimension[6,7], that poles on the physical sheet of the transmission coefficient, that is, in the forward amplitude, correspond to bound-states of the system. Furthermore from the work of Coleman and Norton[8] it is easy to show that the pole $E = -3\nu_0$ is on the physical sheet. This pole occurs therefore for the total energy $W(3) = -4\nu_0$.

In the general case of N particles we consider again the diagram of Fig. 1. The singularity in the forward amplitude occurs at

$$E = \frac{N+1}{1-N}\left[|W(N)| - |W(N-1)|\right] \qquad (2)$$

where W(N) means the total binding energy of the bound-state for the N-particle system. So we have

$$W(N+1) = \frac{2N}{1-N}\,|W(N)| - \frac{N+1}{1-N}\,|W(N-1)| \qquad (3)$$

Equation (3) gives the energy of the bound-state in the system with (N+1) particles associated with the energy of the bound-state with (N-1) and N particles. Equation (3) reduces to the previous result[1] if one remember that $W(1) = 0$.

The bound-states energies calculated with formula(3) agrees with the bound-states energies calculated by MacGuire[9] who used a separable delta function potential between the particles. Our claim is that any potential that binds the two body system will bind the N body system. This claim is verified in the case of N = 3 in two[10,11] other calculations using different potentials.

It is not known if the many body system can have other bound-states than the ones predicted by the above mechanism. An investigation of this point, using the method described in reference 1 is in progress.

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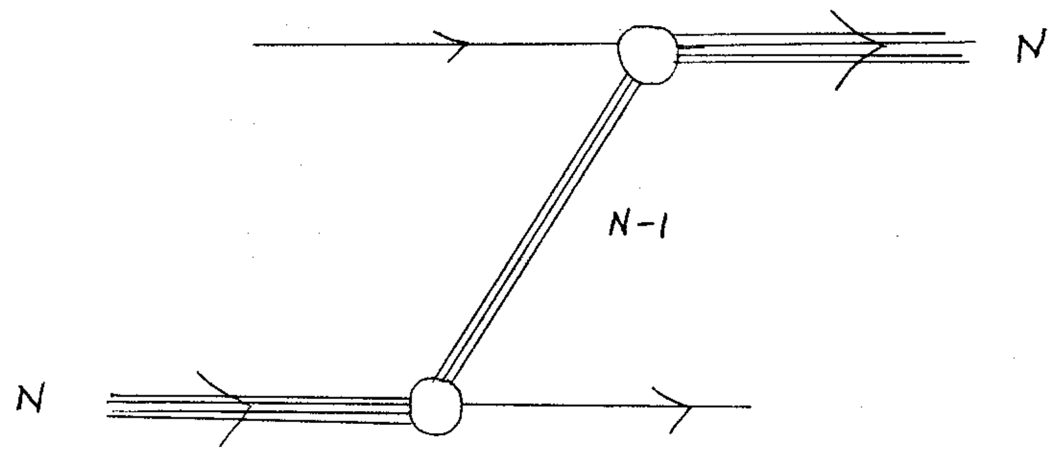

FIG. 1