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AN ALTERNATIVE VIEW OF COSMOLOGICAL PHASE TRANSITIONS

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ABSTRACT

We present a description of cosmological phase transitions, which differs from the one based on the effective potential approach. In our scheme, the phase transition is driven by the spontaneous generation of domain walls (solitons), and below the critical point, statistical mechanics prohibits the existence of these walls.

There is today a widespread belief that the History of the Universe can be told, by extending the unified (spontaneously broken) gauge theories of electro-weak and strong interactions to high temperatures, and to the genesis of the Universe[1-6]. It is also conjectured that field theory phase transitions play a crucial role in delimitating the various evolutionary stages, which the Universe went across[1-6].

Field theory phase transitions exist, because any symmetry that, at absolute zero is spontaneously broken, comes to be recovered at high temperatures. The critical point is just the onset of the symmetric phase[1-3].

The standard way of studying these phenomena, is analysing the effective potential dependence on temperature[1-3].

This paper proposes another description of the phase transition mechanism, which differs from the effective potential approach in many respects, and leads us to reinterpret the standard picture. As we will show, this new scheme has far reaching consequences on the study of cosmological evolution, and is free from some apparent puzzles and contradictions inherent in the orthodox interpretation.

In order to compare the two pictures we shall analyze the model describing a scalar boson self-interaction, whose potential is

$$- \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \qquad (1)$$

It is argued[1-3,6] that the character of phase transitions is insensitive to the details of the theory, and that all relevant physics is already present in the simple model above. Let us adopt this philosophy.

The model defined by expression (1) has a critical temperature T_c, such that, $\langle\phi\rangle \neq 0$ below T_c, and $\langle\phi\rangle = 0$ above T_c

In the one loop approximation, effective potential calculations give[1,2]

$$\tilde{T}_c = 2\frac{m}{\sqrt{\lambda}} \tag{2}$$

This one loop result is reliable only if $\lambda \ll 1$, i.e., the semi-classical regime.

As noticed by Ventura[7], in a semiclassical analysis of this ϕ^4 theory, above a certain temperature $T_c(\lambda)$, macroscopic solitons (sheets of infinite area and infinite energy) can be generated spontaneously, thanks to the fluctuations' entropy.

Below $T_c(\lambda)$, when there are no solitons, one has $|\langle\phi\rangle| = \frac{m}{\sqrt{\lambda}}$. On the other hand, above $T_c(\lambda)$, one must have $\langle\phi\rangle = 0$, because the system is then a soup of solitons, full of $+\frac{m}{\sqrt{\lambda}}$ and $-\frac{m}{\sqrt{\lambda}}$ domains randomly distributed in space. The soliton is a Bloch wall, dividing the system into locally ordered domains[7].

We interpret the calculations of Ref. (7) as an alternative (and more realistic) approximation to computing $T_c(\lambda)$, that is, we view the phase transition of Ref. (7), as being the same transition discovered by means of the effective potential method.

In fact, when $\lambda \ll 1$, our soliton scheme leads to[8]:

$$T_c(\lambda) = \sqrt{\frac{8}{3}}\,\frac{m}{\sqrt{\lambda}} = 1.63\,\frac{m}{\sqrt{\lambda}} \tag{3}$$

This number results very close to the one shown in Eq. (2), and gives support to the interpretation proposed above.

It has been observed[9] and stressed[10] that topologically non-trivial field configurations, having divergent action (or energy) can play an essential role in driving a phase transition. Our approach to the cosmological phase transition is along this line of thought.

According to the modern description of phase transitions[11], the critical point is the lowest temperature which can afford an infinite bubble. An infinite bubble in turn requires an infinite Bloch wall, and therefore our proposal (that $T_c(\lambda)$ should be the lowest temperature which allows the spontaneous formation of solitons, or Bloch walls) is, in a sense, nothing more than a rephrasing of that renormalization group statement.

Above T_c (or $\tilde{T}_c$) one has a condensate of solitons, so that, introducing fluctuations around this sophisticated background field is a much more appropriate procedure, to study the excitation spectrum, than fluctuating around $\phi = 0$, as recommended by the effective potential method.

Taking the results of ref. (7) for large values of $\frac{1}{\lambda}$ and $\frac{T}{m}$, one gets the following expression for the free energy a soliton carries per unit of area:

$$f_{sol}(T) = \frac{(\sqrt{2}\,m)^3}{3\,\lambda} - \frac{m\,T^2}{2\,\sqrt{2}}\ , \tag{4}$$

showing explicitly that the critical temperature $T_c(\lambda)$ is such that $f_{sol}(T) < 0$ if $T > T_c(\lambda)$, the situation where the fluctuations' entropy overcomes the soliton internal energy, and solitons sprout in the system.

To get an insight about what a multisoliton configuration looks like, we will now consider parallel solitons configuration. We take our system to be a cubic box of volume $V = L^3$, and define: (1) The Nth configuration has 3N solitons; (2) there exist N solitons parallel to each face of the cube; (3) the average distance between any soliton and its nearest parallel neighbor is $\Delta = \frac{L}{N}$; and (4) Δ is much larger than the soliton width, that is $\Delta \gg \frac{1}{2m}$, so that we are thinking of a dilute condensate of solitons.

We think about parallel solitons configurations with the purpose of simplifying the subsequent discussion. If we wished, we could take into account non parallel solitons, as well. But that would only make our argumentation duller, without improving either, its accuracy or rigor.

The energy per unit length, stored in the intersection of two perpendicular solitons, is $\alpha\, m^2/\lambda$ ($\alpha \simeq 2.68$)[12], where α is a positive dimensionless number.

Hence, the solitons contribution to the Nth configuration free energy will be[7]

$$VF_N = 3V\left[f_{sol}(T)\,\frac{1}{\Delta} + \frac{\alpha\, m^2}{\lambda\Delta^2}\right] \qquad (5)$$

Here, we are discarding contributions given by excitations of the intersection lines.

Eq. (5) also does not include the <u>unbound</u> elementary particles contribution. In the high temperature region, elementary particles gives a well known T^3 term to the specific heat (the T^3 law).

Among all those parallel configurations, statistical mechanics chooses the one having the lowest free energy.

When $T < T_c$, the minimum of F_N is attained at $\Delta = \infty$ (which means zero solitons) because $f_{sol}(T)$ is <u>positive</u>.

Above T_c, $f_{sol}(T)$ is <u>negative</u>, and the average distance between next neighbor walls Δ_0, which minimizes the free energy, is given by

$$\frac{1}{\Delta_0(T)} = \frac{\lambda}{4\sqrt{2}\,\alpha}\left[\frac{T^2}{m} - \frac{8m}{3\lambda}\right] = \frac{1}{d_0}\left[\left(\frac{T}{T_c}\right)^2 - 1\right], \qquad (6)$$

where T_c is shown in Eq. (3), and d_0 is a characteristic length

of the model,

$$d_0 = \frac{3\alpha}{\sqrt{2}\, m} \simeq \frac{5.7}{m} \qquad (7)$$

The solitons' contribution to the free energy density is, therefore,

$$F_{sol}(T) = \begin{cases} 0 & ,\ \text{if } T < T_c \\ -\dfrac{3\lambda}{32\alpha}\,(T^2 - T_c^2)^2 & ,\ \text{if } T > T_c \end{cases} \qquad (8)$$

This part of the free energy leads to a jump in the specific heat at $T = T_c$, and the jump discontinuity is

$$\frac{1}{\sqrt{6}}\left(\frac{2m}{\sqrt{\lambda}}\right)^3 .$$

Of course, if the distance between next neighbor walls is similar to the soliton width (i.e., if $\Delta_0 \simeq \frac{1}{2m}$), one can no longer use the soliton approximation to describe our system. According to Eq. (6), that happens at higher temperatures,

$$T^2 \geq T_c^2 + 8\sqrt{2}\,\frac{\alpha\, m^2}{\lambda} = (1 + 3\sqrt{2}\,\alpha)T_c^2 \simeq (3.5\,T_c)^2 \qquad (9)$$

and in this domain, fluctuations should move around much more complex background fields, that would reflect the extreme chaos of high entropy states.

One important quantity in the study of cosmological phase transitions is[1,2] the density of domains at a given temperature, $D(T)$. In the present approach, one gets:

$$D(T) = \frac{1}{\Delta_0^3} = \frac{1}{d_0^3}\left[\left(\frac{T}{T_c}\right)^2 - 1\right]^3 \quad , \quad \text{for} \quad T > T_c \qquad (9a)$$

and

$$D(T) = 0 \quad , \quad \text{for} \quad T < T_c \qquad (9b)$$

Result (9b) is a consequence of our discarding of anharmonic fluctuations, which are corrections to the semiclassical methods, and would produce finite bubbles in the nonsymmetric phase. In other words, shape preserving finite bubbles are not classical solutions of theory (1), so that finite bubble effects (which could modify Eq. (9b)) can only be generated by anharmonic corrections (third and fourth powers of fluctuations), that are beyond the scope of a semiclassical treatment. The only kind of Bloch wall a semiclassical scheme allows, is the infinite planar soliton considered here.

In this regard, we also point out that the neglecting of anharmonic corrections made our phase transition to be of first order. We believe, however, that a more complete treatment (envolving higher powers of fluctuations) might turn it to be of second order.

We now list the main features of our approach, comparing them with the effective potential method predictions:-

(1) Thermodynamics does not allow the existence of macroscopic solitons below T_c. They do not have enough statistical weight to survive in the nonsymmetrical phase. Therefore, the Universe today is not expected to have macroscopic Bloch walls, contrarily to the usual interpretation[1-3,5] based on the effective potential method.

(2) False vacuum is not an issue in our scheme, because elementary excitations (particles) move through a realistic condensate of solitons, when $T > T_c$. In the symmetric phase, instead of formulating

perturbation theory around $\phi = 0$, we understand that the basic configuration must be a more sophisticated (though harder to treat) background field.

(3) Above T_c , there is an ammount of energy that cannot be reduced or associated to elementary particles. It is the solitons' energy:

$$V \frac{m^2}{2\alpha} (T^2 - T_c^2)$$

This energy of macroscopic nature can be viewed as a Linde type cosmological constant[1,2] , at $T > T_c$. But no such a thing exists below T_c .

Our calculation thus indicate that the primordial Universe had a finite cosmological constant, that disappeared after the phase transition. Therefore, the rate of expansion of the early Universe should be slower than the standard picture prediction.

Although it was our interest in cosmology that motivated this investigation, we think our results migh also be useful in other areas, where dynamical systems phase transitions play a role.

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