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DERIVATION OF A PION-RHO EXCHANGE THREE-BODY FORCE AND APPLICATION TO THE TRINUCLEON SYSTEM

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ABSTRACT

The pion-rho exchange three-body force is derived by means of Lagrangians which are approximately invariant under chiral and gauge transformations. The leading contribution to the potential arises from a seagull diagram, which corresponds to forces that are dominantly repulsive and comparable to those due to the exchange of two pions. The qualitative features of our results are analysed by means of plots of the energy of the trinucleon system.

I. INTRODUCTION

The problem of three-body forces (3BF) has a long history. Neverthless, only recently one has achieved a good understanding of the dynamical origins of the force due to the exchange of two pions ($\pi\pi$E-3BF), which is the most important class of forces of this type[1-4]. This success is a consequence of the application of Chiral symmetry to the problem.

The $\pi\pi$E-3BF is based on an intermediate πN scattering amplitude for off-shell pions that cannot, of course, be directly measured. It becomes necessary the use of a theoretical amplitude which should reproduce on-shell πN data and also be suitable for off-shell extrapolation. Chiral symmetry allows us to construct such an amplitude. This symmetry describes the interactions of low-energy pions with other hadrons by assuming that they are approximately invariant under transformations of the group $SU(2) \times SU(2)$. The symmetry becomes exact in the unphysical limit in which the four-momenta of the pions vanish.

A further advantage associated with the employment of Chiral symmetry is that we are entitled to use all the nice features of a covariant field theory when we choose to implement it by means of effective Lagrangians. In this case a clear dynamical meaning is ascribed to the terms of the amplitude describing a particular process.

In this work we study the 3BF due to the exchange of a pion and a rho-meson ($\pi\rho$E-3BF), which involves an intermediate off-shell amplitude for the process $\pi N \to \rho N$. The interactions of rho-mesons with other hadrons are approximately gauge invariant. Hence the intermediate amplitude is calculated by means of an effective Lagrangian which is approximately invariant under both chiral and gauge transformations. This choice strongly influences our final results.

The theoretical amplitude for the process $\pi N \to \rho N$ can be tested in several intermediate energy reactions such as $\pi N \to \pi\pi N$, $NN \to \pi NN$, $\pi d \to NN$ and others. The assumption that the isovector part of the hadronic electromagnetic current is dominated by neutral rho mesons allows us to relate this theoretical amplitude to electromagnetic form factors and magnetic moments, as well as to photo and electro-production of pions from nucleons. These last two reactions are somewhat simpler than the others and allow a more direct test of the amplitude $\pi N \to \rho N$. In this work we rely on phenomenological parameters extracted from these two processes[5-7].

The theoretical amplitude for the process $\pi N \to \rho N$ is derived in the next section. This amplitude is employed in section III in order to construct the 3BF. In section IV we study the features of this force by means of diagrams and in section V we summarize our conclusions.

II. THE $\pi N \to \rho N$ AMPLITUDE

The amplitude for the process $\pi^a(k)N(P) \to \rho^b_\mu(q)N(P')$ involving virtual bosons is denoted by T^{ab}_μ. The dynamical content of this amplitude, as tested in the reactions $\gamma N \to \pi N$ and $eN \to e\pi N$, is depicted in Fig. 1. The nucleon-pole, seagull and pion-exchange diagrams are not gauge invariant when considered in isolation. When they are taken together, however cancellations of large terms occur and the symmetry is achieved. In the

calculation of 3BF, on the other hand, a cancellation of this type does not occur, since here we need only the part of the first diagram describing backward propagation in time. This contribution is negligible as we show below. The pion-exchange diagram has already been included in the $\pi\pi$E-3BF and it should not be considered in the $\pi\rho$E-3BF in order to avoid double counting. So, out of the first three diagrams, only the seagull contributes significantly to the $\pi\rho$E-3BF. In fact, this contribution is so large that to some extent the short range of the force is offset. Finally, the last diagram corresponds to the excitation of the Δ-resonance. Recently it has been claimed[8] that this diagram would be responsible for the leading contribution to the $\pi\rho$E-3BF. Our study of the trinucleon system, however, shows that the excitation of the Δ amounts to about 10% of the total force.

In the remainder of this section we evaluate the diagrams of Fig. 1. For the sake of completeness we consider even the pion-exchange diagram, which does not contribute to the $\pi\rho$E-3BF. The vertices in these diagrams are extracted from the following terms of a Lagrangian which is approximately invariant under gauge and chiral transformations[9].

$$\mathcal{L}_{\pi NN} = \frac{g}{2m}\,\bar{N}\,\vec{\tau}\cdot\left[\gamma^\mu\gamma_5\partial_\mu\vec{\phi}\right]N \tag{1}$$

$$\mathcal{L}_{\rho NN} = \frac{\gamma_0}{2}\,\bar{N}\,\vec{\tau}\cdot\left[\gamma^\mu\vec{\rho}_\mu + \frac{\mu_p-\mu_n}{2m}\,i\,\sigma^{\mu\nu}(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\right]N \tag{2}$$

$$\mathcal{L}_{\pi\rho NN} = \frac{g}{2m}\,\gamma_0\,\bar{N}\,\vec{\tau}\cdot\left[\gamma^\mu\gamma_5\vec{\rho}_\mu\times\vec{\phi}\right]N \tag{3}$$

$$\mathcal{L}_{\pi\pi\rho} = \gamma_0\,\vec{\rho}_\mu\cdot(\vec{\phi}\times\partial^\mu\vec{\phi}\,) \tag{4}$$

$$L_{\pi N\Delta} = g_\Delta \bar{\Delta}_\mu \vec{M} \cdot \left[(g^{\mu\nu} - \frac{\eta}{4}\gamma^\mu\gamma^\nu)\partial_\nu\vec{\phi}\right]N + h.c. \tag{5}$$

$$L_{\rho N\Delta} = i\,\gamma_\Delta \bar{\Delta}_\mu \vec{M} \cdot \left[(g^{\mu\nu} - \frac{\xi}{4}\gamma^\mu\gamma^\nu)\gamma^\lambda\gamma_5(\partial_\lambda\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\lambda)\right]N \quad . \tag{6}$$

The symbols $\vec{\phi}$, $\vec{\rho}$, N and Δ_μ correspond to the fields of pion, rho, nucleon and delta, whereas $\vec{\tau}$ and $\vec{M}$ are matrices that produce isovectors when sandwiched between two nucleons or a nucleon and a delta. The parameters η and ξ represent the possibility of spin 1/2 components in the off-pole delta wave-function. The universal rho coupling constant is denoted by γ_o whereas μ_p and μ_n are the proton and neutron anomalous magnetic moments.

The $\pi N \to \rho N$ amplitude can be decomposed as

$$T^{ab}_\mu = \delta_{ab}\, T^+_\mu + i\,\varepsilon_{abc}\,\tau_c\, T^-_\mu \quad . \tag{7}$$

The amplitudes $T^\pm_\mu$ receive contributions from the diagrams of Fig. 1 and hence they are written as

$$T^\pm_\mu = T^\pm_\mu)_N + T^\pm_\mu)_S + T^\pm_\mu)_\pi + T^\pm_\mu)_\Delta \tag{8}$$

where the subscripts N , S , π and Δ stand for nucleon-pole, seagull, pion-exchange and delta-pole. The evaluation of these amplitudes produces the following results:

$$i\,T^+_\mu)_N = -\frac{\gamma_o g}{4m}\,\bar{u}\gamma_5\left\{\left[m(1+\mu_p-\mu_n)\left(\frac{1}{s-m^2}+\frac{1}{u-m^2}\right)+\frac{\mu_p-\mu_n}{2m}\right](\gamma_\mu\not{q}-\not{q}\gamma_\mu) + \left[\frac{(P+k)_\mu}{(s-m^2)}+\frac{(P+k)_\mu}{(u-m^2)}\right]\left[2m+(\mu_p-\mu_m)\not{q}\right]\right\}u \tag{9}$$

$$iT^-_\mu)_N = \frac{\gamma_o g}{4m}\,\bar{u}\,\gamma_5\left\{m(1+\mu_p-\mu_m)\left(\frac{1}{s-m^2}-\frac{1}{u-m^2}\right)(\gamma_\mu\not{q}-\not{q}\gamma_\mu) + (\mu_p-\mu_n)\left[-2\gamma_\mu+\left(\frac{(P+k)_\mu}{s-m^2}-\frac{(P-k)_\mu}{u-m^2}\right)\not{q}\right]-2\gamma_\mu+2m\left[\frac{(P+k)_\mu}{s-m^2}-\frac{(P+k)_\mu}{u-m^2}\right]\right\}u \tag{10}$$

$$iT^+_\mu)_S = 0 \tag{11}$$

$$iT^-_\mu)_S = \frac{\gamma_o g}{2m}\,\bar{u}\,\gamma_5\,\gamma_\mu\,u \tag{12}$$

$$iT^+_\mu)_\pi = 0 \tag{13}$$

$$iT^-_\mu)_\pi = \frac{\gamma_o g}{2m}\,\bar{u}\,\gamma_5\left[\frac{2m(2k_\mu-q_\mu)}{t-\mu^2}\right]u \tag{14}$$

$$\begin{aligned} iT^+_\mu)_\Delta = \frac{\gamma_\Delta g_\Delta}{9M^2_\Delta}\,\bar{u}\gamma_5 \Bigg\{\Bigg[\Bigg[&\left(\frac{1}{s-M^2_\Delta}+\frac{1}{u-M^2_\Delta}\right)\left(m\alpha+(q^2-2m^2)\beta+3M^2_\Delta(k^2-q.k-2m^2-2mM_\Delta)\right)+ \\ &+\left(\frac{1}{s-M^2_\Delta}-\frac{1}{u-M^2_\Delta}\right)\not{q}\alpha + 2m(m+M_\Delta) + 2q.k\left(1-\frac{\eta}{2}\right)(1-\xi) + 2k^2(1-\xi) + 2q^2\left(1-\frac{\eta}{2}\right) - \\ &- 4m^2\left(1-\frac{\eta\xi}{2}\right) + 4m\,M_\Delta(1-\eta)(1-\xi)\Bigg]\frac{\gamma^\lambda q^\nu}{2} + \beta(m\gamma^\lambda - q^\lambda)\left[\frac{(P+k)^\nu}{s-M^2_\Delta}+\frac{(P+k)^\nu}{u-M^2_\Delta}\right] + \\ &+ 6M^2_\Delta\left[\frac{1}{s-M^2_\Delta}+\frac{1}{u-M^2_\Delta}\right]k^\lambda P^\nu + 3M^2_\Delta(m+M_\Delta)\gamma^\lambda\left[\frac{(P-k)^\nu}{s-M^2_\Delta}+\frac{(P+k)^\nu}{u-M^2_\Delta}\right] + \\ &+ 2\left(1-\frac{\eta}{2}\right)P^\lambda q^\nu + 2\left[m\left(1-\frac{\eta\xi}{2}\right) - M_\Delta(1-\eta)(1-\xi)\right]\gamma^\lambda P^\nu\Bigg\}(g_{\mu\lambda}q_\nu - g_{\mu\nu}q_\lambda)u \end{aligned} \tag{15}$$

$$iT^{-}_{\mu})_{\Delta} = \frac{\gamma_\Delta g_\Delta}{18M_\Delta^2}\,\bar{u}\,\gamma_5 \left\{ \left[\left(\frac{1}{s-M_\Delta^2} - \frac{1}{u-M_\Delta^2} \right) \left(m\alpha + (q^2-2m^2) - 3M_\Delta^2 (k^2-2q.k-2m^2-2m\,M_\Delta) \right) + \right.\right.$$

$$+ \left(\frac{1}{s-M_\Delta^2} + \frac{1}{u-M_\Delta^2} \right) \not{q}\alpha + 2q.P \left(1 - \frac{\eta}{2}\right)(1-\xi) \Bigg] \frac{\gamma^\lambda q^\lambda}{2} +$$

$$+ \beta(m\gamma^\lambda - q^\lambda) \left[\frac{(P+k)^\nu}{s-M_\Delta^2} - \frac{(P-k)^\nu}{u-M_\Delta^2} \right] + 6M_\Delta^2 \left(\frac{1}{s-M_\Delta^2} - \frac{1}{u-M_\Delta^2} \right) k^\lambda P^\nu +$$

$$+ 3M_\Delta^2 (m+M_\Delta)\gamma^\lambda \left[\frac{(P-k)^\nu}{s-M_\Delta^2} - \frac{(P+k)^\nu}{u-M_\Delta^2} \right] + 2\left(1 - \frac{\eta}{2}\right) k^\lambda q^\nu +$$

$$\left. + 2(m+M_\Delta)\ \gamma^\lambda q^\nu + 2\left[m\left(1 - \frac{\eta\xi}{2}\right) - M_\Delta (1-\eta)(1-\xi) \right] \gamma^\lambda k^\nu \right\} (g_{\mu\lambda} q_\nu - g_{\mu\nu} q_\lambda) u \qquad (16)$$

In the above expressions we have used the following variables:

$$s = (p+k)^2 = (p'+q)^2 \qquad (17)$$

$$u = (p-k)^2 = (p'-k)^2 \qquad (18)$$

$$t = (k-q)^2 \qquad (19)$$

$$P = (p+p') \qquad (20)$$

$$\alpha = (m+M_\Delta)(M_\Delta^2 - m^2) + k^2(m+2M_\Delta) \qquad (21)$$

$$\beta = (2M_\Delta^2 + mM_\Delta - m^2 + k^2) \qquad (22)$$

It is worth noting that the gauge invariance of

the interactions requires the amplitudes to be conserved, that is, to satisfy the condition

$$q^\mu T^{\pm}_{\mu} = 0 \quad . \qquad (23)$$

The amplitudes $T^{+}_{\mu})_N$ and $T^{\pm}_{\mu})_\Delta$ are individually conserved. The gauge condition for $T^{-}_{\mu})_N$, $T^{-}_{\mu})_S$ and $T^{-}_{\mu})_\pi$, on the other hand, holds only when these amplitudes are taken together, since the seagull and pion-exchange contributions cancell the last two terms in eq. (10).

III. THE PION-RHO EXCHANGE THREE BODY POTENTIAL

The three-body potential is extracted from the scattering amplitude for three unbounded nucleons, which corresponds to permutations of the diagrams of Fig. 2. In this figure v^a and v^b_μ represent the πN and ρN vertices and $\hat{T}^{ab}_{\mu}$ denotes the amplitude for the reaction $\pi N \to \rho N$ in which we have suppressed the diagrams corresponding to pion-exchange and nucleons propagating forward in time. This suppression is required since the former process has already been included in the $\pi\pi$E-3BF and the latter corresponds to the iteration of a two-body force.

The amplitude for the process depicted in Fig.2 is denoted by $T^{\pi\rho}_{3N}$ and is given by

$$iT^{\pi\rho}_{3N} = \left[u(\vec{p}_2{}') \not{k}\, \gamma_5\, u(\vec{p}_2) \right] \frac{g}{2m} \frac{1}{k^2-\mu^2} \left[\vec{\tau}^{(2)} . \vec{\tau}^{(3)} \hat{T}^{+}_{\mu} + i\, \vec{\tau}^{(1)} . \vec{\tau}^{(2)} \times \vec{\tau}^{(3)}\, \hat{T}^{-}_{\mu} \right]$$

$$\frac{1}{q^2-m_\rho^2} \frac{\gamma_o}{2} \left[\bar{u}(\vec{p}_3{}') \left(\gamma^\mu + \frac{(\mu_p - \mu_n)}{2m}\, i\, \sigma^{\mu\lambda} q_\lambda \right) u(\vec{p}_3) \right] . \qquad (24)$$

The momenta of the nucleons bound in nuclei are assumed to be of order of the pion mass. Thus we use the non-relativistic limit of eq. (24), that is given by

$$t_{2N}^{\pi\rho} = i \frac{g\gamma_o}{2} \frac{1}{\vec{k}^2+\mu^2} \frac{1}{\vec{q}^2+m_\rho^2} \vec{\sigma}^{(2)}.\vec{k} \left[\vec{\tau}^{(2)}.\vec{\tau}^{(3)} t_\mu^+ j^\mu + i\, \vec{\tau}^{(1)}.\vec{\tau}^{(2)} \times \vec{\tau}^{(3)} t_\mu^- j^\mu\right] \tag{25}$$

where $t_\mu^\pm$ and j_μ denote, respectively, the non-relativistic limits of the amplitudes $\hat{T}_\mu^\pm$ and of the isovector nucleon current. This last four vector is

$$j_\mu = \left[2m;\ -i\,(1+\mu_p-\mu_n)\ \vec{q} \times \vec{\sigma}^{(3)} + \vec{P}_3\right] . \tag{26}$$

The amplitudes $t_\mu^\pm$ receive contributions from nucleons propagating backward in time, seagull term and delta pole. The first of these contribution is obtained by means of the following decomposition of the numerator of the nucleon propagator:

$$(\not{p}+m) = \frac{1}{2E}\left[(E+p_o)\bar{u}(\vec{p})\,u(\vec{p}) - (E-p_o)\bar{v}(-\vec{p})\,v(-\vec{p})\right] \tag{27}$$

where p_o is the energy of the propagating nucleon and $E = \sqrt{m^2+\vec{p}^2}$.

A backward propagating nucleon is thus described by the nucleon pole amplitude in which the nucleon propagator is replaced by

$$\frac{(\not{p}+m)}{p_o^2-E^2} \rightarrow \frac{\gamma_o}{2E} - \frac{(\not{p}+m)}{2E\,(p_o+E)} \tag{28}$$

Using this new propagator we obtain the contribution of nucleons propagating backward in time, which is denoted by $\bar{t}_N^\pm$ and given by

$$j.\bar{t}_N^+ = -i \frac{\gamma_o g}{2m} \frac{1}{4m^2} \Big\{ j_o(1+\mu_p-\mu_n) \left[\vec{q}^2\vec{\sigma}^{(1)}.\vec{k} + (\vec{k}^2-\vec{q}.\vec{k})\vec{\sigma}^{(1)}.\vec{q} + i\vec{k}\times\vec{q}.\vec{P}_1\right] +$$

$$+ 2m\left[\vec{j}.\vec{P}_1\vec{\sigma}^{(1)}.\vec{k} + 2mk_o\vec{\sigma}.^{(1)}\vec{j} - \vec{j}.\vec{k}\vec{\sigma}^{(1)}.\vec{P}_1 - i\,\vec{k}\times\vec{q}.\vec{j}\right]\Big\} \tag{29}$$

$$j.\bar{t}_N^- = -i \frac{\gamma_o g}{2m} \frac{1}{4m^2} \Big\{ j_o(1+\mu_p-\mu_n) \left[\vec{q}.\vec{P}_1\vec{\sigma}^{(1)}.\vec{k} + 2mk_o\vec{\sigma}^{(1)}.\vec{q} + \vec{q}.\vec{k}\vec{\sigma}^{(1)}.\vec{P}_1\right] +$$

$$+ 2m\left[\vec{q}.\vec{j}\vec{\sigma}^{(1)}.\vec{k} + \vec{k}^2\vec{\sigma}^{(1)}.\vec{j} - \vec{k}.\vec{j}\vec{\sigma}^{(1)}.\vec{q} + i\,\vec{k}\times\vec{j}.\vec{P}_1\right]\Big\} \tag{30}$$

The seagull contribution is obtained from eqs. (11,12)

$$j.t_s^- = -i \frac{\gamma_o g}{2m}\left[-j_o\vec{\sigma}^{(1)}.\vec{P}_1 + 2m\,\vec{\sigma}^{(1)}.\vec{j}\right] . \tag{31}$$

Finally, the delta-pole corresponds to the following non-relativistic amplitudes:

$$j.t_\Delta^+ = i\frac{2\gamma_\Delta g_\Delta}{9M_\Delta^2}\frac{1}{M_\Delta^2-m^2}\Big\{ j_o 2m\left[(m^2+mM_\Delta-(M_\Delta^2-m^2)(1-\eta/2)(1-\xi))\vec{q}.\vec{k} +\right.$$

$$\left. + (3M_\Delta^2+2mM_\Delta+m^2-(M_\Delta^2-m^2)(1-\xi))\vec{k}^2\right]\vec{\sigma}^{(1)}.\vec{q} + j_o 2m\left[-6M_\Delta^2\,\vec{q}.\vec{k} +\right.$$

$$\left. + (2M_\Delta^2+mM_\Delta-m^2-(M_\Delta^2-m^2)(1-\eta/2))\vec{q}^2\right]\vec{\sigma}^{(1)}.\vec{k} + \left[3M_\Delta^3+5mM_\Delta^2+m^2M_\Delta-m^3 -\right.$$

$$\left. - (M_\Delta^2-m^2)\left(m(1-\eta\xi/2) - M_\Delta(1-\eta)(1-\xi)\right)\right]\left[j_o\vec{q}.\vec{P}_1\vec{\sigma}^{(1)}.\vec{P} + 2m\,i\,\vec{k}\times\vec{q}.\vec{j}\right]\Big\} \tag{32}$$

$$j.t_\Delta^- = -i\,\frac{\gamma_\Delta g_\Delta}{9M_\Delta^2}\,\frac{1}{M_\Delta^2-m^2}\Big\{j_o 2m(2mq_o-\vec{q}.P_1)\left[m^2+m\,M_\Delta-(M_\Delta^2-m^2)(1-\eta/2)(1-\xi)\right]\vec{\sigma}^{(1)}.\vec{q}\,+$$

$$+\left[3M_\Delta^3+mM_\Delta^2-m^2M_\Delta+m^3+(M_\Delta^2-m^2)(m(1-\eta\xi/2)-M_\Delta(1-\eta)(1-\xi))\right]$$

$$\left[-j_o 2mk_o\vec{\sigma}^{(1)}.\vec{q}+j_o\vec{q}.\vec{k}\vec{\sigma}^{(1)}.\vec{P}_1-2m\vec{q}.\vec{k}\vec{\sigma}^{(1)}.\vec{j}+2m\,\vec{k}.\vec{j}\vec{\sigma}^{(1)}.\vec{q}\right]\Big\}\,. \qquad (33)$$

Using the form of j_μ given by eq. (26) we see that the nucleon contribution is of order μ^2/m^2 smaller than the seagull one, for one has

$$j.\bar{t}_N^+ \sim j\,.\,\bar{t}_N \sim \gamma_o . g\mu\,\frac{\mu^2}{m^2}$$

$$j.t_S^- \sim \gamma_o\, g\mu \quad .$$

These results allow us to neglect the contribution of backward propagating nucleons.

The three-body potential in momentum space is defined as

$$<\vec{p}_1'\vec{p}_2'\vec{p}_3'|w_{123}^{\pi\rho}|\vec{p}_1\vec{p}_2\vec{p}_3> = -\left(\frac{2\pi}{2m}\right)^3 \delta^3\,(\vec{p}_f-\vec{p}_i)\; t_{3N}^{\pi\rho} \quad . \qquad (34)$$

In this work we consider only the local part of the potential and therefore the velocity dependent terms in eqs. (31-33) are neglected. Moreover, in order to apply this potential to the trinucleon problem we evaluate its expectation value between totally antisymmetric spin and isospin states[10]. Denoting this expectation value by (...), we have

$$(<\vec{p}_1'\vec{p}_2'\vec{p}_3'|w_{123}^{\pi\rho}|\vec{p}_1\vec{p}_2\vec{p}_3>) = -\left(\frac{2\pi}{2m}\right)^3 \delta^3\,(\vec{p}_f-\vec{p}_i)\,(t_{3N}^{\pi\rho})$$

$$= (2\pi)^3\,\delta^3\,(\vec{p}_f-\vec{p}_i)\left(\frac{g}{2m}\right)^2 \frac{1}{\vec{k}^2+\mu^2}\;\frac{1}{\vec{q}^2+m_\rho^2}\;2\gamma_o^2\,\frac{(1+\mu_p-\mu_n)}{2m}\;\Big[\vec{q}.\vec{k}\;+$$

$$+\,(\delta_1+\delta_2)(\vec{q}.\vec{k})^2+\delta_3\,\vec{k}^2\vec{q}^2+\delta_4\,\vec{k}^2\vec{q}.\vec{k}\Big] \qquad (35)$$

where

$$\delta_1 = -\frac{2}{3}\,\frac{\gamma_\Delta g_\Delta}{\gamma_o g}\,\frac{m}{M_\Delta-m} \qquad (36)$$

$$\delta_2 = -\frac{2}{9}\,\frac{\gamma_\Delta g_\Delta}{\gamma_o g(1+\mu_p-\mu_n)}\,\frac{m^2\left[m-(M_\Delta-m)(1-\xi)\right]}{M_\Delta^2(M_\Delta-m)} \qquad (37)$$

$$\delta_3 = \frac{2}{9}\,\frac{\gamma_\Delta g_\Delta}{\gamma_o g}\,\frac{m\{2mM_\Delta-m^2-(M_\Delta-m)\left[m(1-\eta\xi/2)-M_\Delta(1-\eta)(1-\xi)\right]\}}{M_\Delta^2(M_\Delta-m)} \qquad (38)$$

$$\delta_4 = \frac{2}{9}\,\frac{\gamma_\Delta g_\Delta}{\gamma_o g(1+\mu_p-\mu_n)}\,\frac{m^2\left[3M_\Delta+m+(M_\Delta+m)(1-\xi)\right]}{M_\Delta^2(M_\Delta+m)} \qquad (39)$$

These expressions allow us to assess the relative importance of the seagull and delta contributions to the potential. In order to do so we need the numerical values of the parameters entering the equations above. We adopt the following values for the masses: $\mu = 139.57$ MeV , $m_\rho = 776.0$ MeV, $m = 938.28$ MeV , $M_\Delta = 1220.0$ MeV[7]. The coupling constants are $g = 13.39$, $g_\Delta = 1.84\ \mu^{-1}$[7], $\gamma_o = 6.00$, $\gamma_\Delta = 2.00\ \mu^{-1}$. The value of γ_o has been extracted from the relation $\gamma_o = m\rho/\sqrt{2}\,f\pi$[11] , where $f\pi$ is the pion decay constant whose value is taken to be $f\pi$ = 91 MeV. The $\rho N\Delta$ coupling constant is related to the $\gamma N\Delta$ form factor C by $\gamma_\Delta = C\gamma_o$. The value of the parameter C can be extracted from electroproduction amplitudes and in this work we adopt the value $C = 0.34\ \mu^{-1}$ (7). The study of electroproduction processes also yield the values $\eta=2$ and $\xi=3$ for the off shell delta coupling constants.

These experimental parameters produce the following values for the strength of the delta relative to the seagull: $\delta_1 = -0.10\ \mu^{-2}$, $\delta_2 = -0.01\ \mu^{-2}$, $\delta_3 = -0.06\ \mu^{-2}$, $\delta_4 < 0.01\ \mu^{-2}$. These figures show that the delta contribution is rather smaller than that of the seagull. Hence in the remiander of the work we consider only the latter contribution to the 3BF. Using the above mentioned relation between γ_o and $f\pi$ we obtain our final form for the potential in momentum space.

$$(\langle \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}'|W^{\pi\rho}_{123}|\vec{p}_1\vec{p}_2\vec{p}_3\rangle) =$$

$$= (2\pi)^3\delta^2(\vec{p}_f-\vec{p}_i)\left(\frac{g\mu}{2m}\right)^2 \frac{1}{\vec{k}^2+\mu^2}\,\frac{1}{\vec{q}^2+m\rho^2}\,\frac{m_\rho^2}{\mu^2}\,\frac{1}{f\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m}\,\vec{q}\cdot\vec{k} \qquad (40)$$

In this work we are mostly concerned with the qualitative features of the $\pi\rho$E-3BF. Hence, before going to coordinate space, it is worthwhile to compare this potential with the $\pi\pi$E-3BF, that is given by (3,4)

$$(\langle \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}'|W^{\pi\pi}_{123}|\vec{p}_1\vec{p}_2\vec{p}_3\rangle) =$$

$$= (2\pi)^3\delta^3(\vec{p}_f-\vec{p}_i)\left(\frac{g\mu}{2m}\right)^2 \frac{1}{\vec{k}^2+\mu^2}\,\frac{1}{\vec{k}'^2+\mu^2}\,\frac{1}{\mu^2}\left\{\alpha_\sigma\,\vec{k}\cdot\vec{k}' + \right.$$

$$+ \left[\frac{1}{f\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{4g_\Delta^2}{9(M_\Delta-m)} - \beta_\sigma\right]\left[(\vec{k}\cdot\vec{k}')^2 - \frac{\vec{k}^2\vec{k}'^2}{3}\right] +$$

$$\left. + \left[\frac{2}{3f\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{\beta_\sigma}{3}\right]\vec{k}^2\vec{k}'^2\right\} \qquad (41)$$

where $\vec{k}$ and $\vec{k}'$ denote the pion momenta. The parameters α_σ and β_σ describe the πN σ-term and their role in the $\pi\pi$E-3BF is discussed in detail in refs. (3,4).

Comparing both potentials one notes that the $\pi\rho$E-3BF has the same structure as the term proportional to α_σ in eq. (41), which corresponds to S-waves in the intermediate πN system and is mainly repulsive, as we will see below. Their relative magnitude is displayed by the ratio

$$\frac{\dfrac{m_\rho{}^2}{\vec{q}^2+m_\rho{}^2}\;\dfrac{(1+\mu_p-\mu_n)}{f\pi^2\;2m}}{\dfrac{\mu^2}{\vec{k}'^2+\mu^2}\;\dfrac{\alpha_\sigma}{\mu^2}} = \frac{1+\vec{k}'^2/\mu^2}{1+\vec{q}^2/m_\rho{}^2}\times\frac{0.82}{1.05} \qquad (42)$$

where we have used the numerical values of refs. (3,4). This ratio shows that for low momenta both potentials are comparable, since the short range nature of the $\pi\rho$E-3BF is offset by its magnitude.

The potential in coordinate space is given by

$$(\langle \vec{r}_1{}'\vec{r}_2{}'\vec{r}_3{}'|W^{\pi\rho}_{123}|\vec{r}_1\vec{r}_2\vec{r}_3\rangle) =$$

$$= \int \frac{d\vec{p}_1{}'}{(2\pi)^3}\cdots\frac{d\vec{p}_3}{(2\pi)^3}\,\delta^3(\vec{p}_1{}'+\ldots-\vec{p}_3)\,e^{-i\vec{p}_1{}'\cdot\vec{r}_1{}'}\cdots e^{i\vec{p}_3\cdot\vec{r}_3}\left[-\frac{(2\pi)^3}{(2m)^3}\,(t^{\pi\rho}_{3N})\right] =$$

$$= \delta^3(\vec{r}_1-\vec{r}_1{}')\,\delta^3(\vec{r}_2-\vec{r}_2{}')\,\delta^3(\vec{r}_3-\vec{r}_3{}')\int\frac{d\vec{k}}{(2\pi)^2}\,\frac{d\vec{q}}{(2\pi)^2}\,e^{-i\vec{k}\cdot\vec{r}_{12}}\,e^{-i\vec{q}\cdot\vec{r}_{31}}\left[-\frac{(t^{\pi\rho}_{3N})}{(2m)^3}\right] \equiv$$

$$\equiv \delta^3(\vec{r}_1-\vec{r}_1{}')\,\delta^3(\vec{r}_2-\vec{r}_2{}')\,\delta^3(\vec{r}_3-\vec{r}_3{}')\,W^{\pi\rho}(1,23) \qquad (43)$$

where we have used the notation

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j \quad . \tag{44}$$

Using the explicit form of $(t_{3N}^{\pi\rho})$ and replacing momentum variables in the numerator by derivatives we obtain

$$w^{\pi\rho}(1,23) = -\left(\frac{\mu}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \frac{m_\rho^2}{\mu^2} \frac{1}{f\pi^2} \frac{(1+\mu_p-\mu_n)}{2m} \left[\vec{\nabla}_{12}\cdot\vec{\nabla}_{31}\, U^\pi(r_{12})U^\rho(r_{31})\right] \tag{45}$$

where

$$U^\pi(r) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\vec{k}.\vec{r}}}{\vec{k}^2 + \mu^2} = \frac{e^{-\mu r}}{\mu r} \tag{46}$$

$$U^\rho(r) = \frac{4\pi}{\mu} \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}.\vec{r}}}{\vec{q}^2 + \mu^2} = \frac{e^{-m_\rho^2}}{\mu r} \quad . \tag{47}$$

The potential in which the intermediate $\pi N \to \rho N$ reaction involves nucleon 1 is given by

$$w^{\pi\rho}(1) = w^{\pi\rho}(1,23) + w^{\pi\rho}(1,32) \quad . \tag{48}$$

Evaluating the action of the derivatives on the functions U we obtain the following expression for $w^{\pi\rho}(1)$

$$w^{\pi\rho}(1) = -\left(\frac{\mu}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \frac{m_\rho^2}{\mu^2} \frac{1}{f\pi^2} \frac{(1+\mu_p-\mu_n)}{2m} \cos\theta_1\ \mu\, m\rho$$

$$\left[U_1^\pi(r_{12})U_1^\rho(r_{31}) + U_1^\rho(r_{12})U^\pi(r_{31})\right] \tag{49}$$

where

$$\cos\theta_1 = \frac{\vec{r}_{12}.\vec{r}_{31}}{r_{12}\ r_{31}} \tag{50}$$

$$U_1^\pi(r) = -\left(1 + \frac{1}{\mu r}\right) \frac{e^{-\mu r}}{\mu r} \tag{51}$$

$$U_1^\rho(r) = -\left(1 + \frac{1}{m_\rho r}\right) \frac{e^{-m_\rho r}}{m\ r} \tag{52}$$

The final form of the $\pi\rho$E-3BF is given by

$$w^{\pi\rho} = C_1^{\pi\rho}\{\cos\theta_1 \left[U_1^\pi(r_{12})U_1^\rho(r_{31}) + U_1^\rho(r_{12})U_1^\pi(r_{31})\right] +$$

$$+ \text{cyclic permutations}\} \quad . \tag{53}$$

The coefficient $C_1^{\pi\rho}$ is

$$C_1^{\pi\rho} = -\left(\frac{g\mu}{2m}\right)^2 \left(\frac{1}{4\pi}\right)^2 \frac{m_\rho^2}{\mu^2} \frac{1}{f\pi^2} \frac{(1+\mu_p-\mu_n)}{2m}\ \mu\, m_\rho \tag{54}$$

and its numerical value is

$$C_1^{\pi\rho} = -\ 118.7\ \text{MeV} \quad . \tag{55}$$

This is a huge number compared to the corresponding coefficient of the $\pi\pi$E-3BF which is[3,4] $C_1^{\pi\pi} = -0.92$ MeV. Both potentials are comparable even for internucleon distances of order of μ^{-1}, whereas for smaller distances the $\pi\rho$E-3BF dominates. These remarks bring the problem of assessing how realistic are the results of the present calculation.

The most obvious non-realistic aspects of the above results concerns the behaviour of the functions U_1^π and U_1^ρ at short distances, since their diverge as $1/r^2$. This shortcomming could be avoided by recalling that there is a short distance repulsion between nucleons which could be simulated by a hard core of radius r_o. This would amount to setting $U^\pi(r) = U^\rho(r) = 0$ for $r<r_o$, making the potential discontinuous at this point.

A more popular choice for the solution of this short distance problem is the introduction of form factors in the boson-nucleon vertices. In the case of the $\pi\pi$E-3BF, for example, the inclusion of form factors modify the function $U^\pi(r)$ as follows[3,4]

$$U^\pi(r) = \frac{4\pi}{\mu} \int \frac{d\vec{K}}{(2\pi)^3} \frac{e^{-i\vec{k}.\vec{r}}}{\vec{k}^2+\mu^2} \left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{k}^2}\right)^2 \qquad (56)$$

where Λ is a phenomenological parameter. This integral is well behaved at the origin and apparently we are left only with the task of finding suitable values for Λ. This use of form factors is not, however, unproblematic from a theoretical view point, since realistic values of Λ are of order of the nucleon mass. Hence form factors introduce corrections which are of the same order of magnitude as those due to relativity, which have been neglected throughout the calculation.

The real problem associated with the functions $U^\pi(r)$ and $U^\rho(r)$ is that the non-relativistic nature of bound nucleons has not been implemented mathematically in eqs. (46,47). This fact makes it disputable, for instance, the result of the action of the momentum operator on $U^\rho(r)$, for we have

$$(\vec{k})_{operator}\, U^\rho(r) = -i\vec{\nabla}\, U^\rho(r) = i\, m_\rho\, U_1^\rho \frac{\vec{r}}{r} \qquad (57)$$

This operation shows that derivatives correspond to momenta of order of m_ρ, which are not compatible with the non-relativistic assumption that pervades this work. This example suggests that, if this assumption were properly implemented in eq. (47), the last factor in eq. (54) would be replaced by another of order μ, producing the value $C_1^{\pi\rho} = -21.3$ MeV. The problem of the mathematical implementation of the non-relativistic assumption into eqs. (46,47) will be discussed elsewhere. In this work we limit ourselves to the use of a hard core, since here we are mostly interested in the qualitative aspects of the $\pi\rho$E-3BF, which are discussed in the next section.

IV. GRAPHICAL STUDY

In this section we study the qualitative features of the $\pi\rho$E-3BF by means of energy diagrams, following the work of Brandenburg and Glöckle[12]. The trinucleon energy due to two-body interactions is given by

$$(\langle \vec{r}_1'\vec{r}_2'\vec{r}_3' | \sum_{pairs} w_{ij}^{2B} | \vec{r}_1\vec{r}_2\vec{r}_3\rangle) =$$

$$= \delta^3(\vec{r}_1-\vec{r}_1')\,\delta^3(\vec{r}_2-\vec{r}_2')\,\delta^3(\vec{r}_3-\vec{r}_3')\; w^{2B} \quad . \qquad (58)$$

The two-body energy w_{ij}^{2B} is assumed to be well described by the Reid soft-core potential. Using Reid's notation, we have

$$w^{2B} = \sum_{pairs} \frac{1}{2}\left[V(^1S) + V(^3S_1)\right] \qquad (59)$$

where $V(^3S_1)$ receives contribution from the central potential only.

The energy diagrams are constructed by fixing the positions of two of the nucleons and using the third as a probe. The coordinate system used to describe the positions of the nucleons is represented in Fig. 3, where x is the distance between the fixed nucleons. In Fig. 4, we show the equipotentials derived from eq. (59) for the value $x = 0.88$ fm, corresponding to the minimum of W_{ij}^{2B}.

The $\pi\pi$E-3BF is usually divided into two pieces, associated to the s and p waves of the intermediate πN amplitude. These partial contributions are denoted by $W_s^{\pi\pi}$ and $W_p^{\pi\pi}$ and their energy diagrams are given in Figs. 5 and 6. The figure describing $W_s^{\pi\pi}$ is rather different from the corresponding one in the work of Brandenburg and Glöckle. Indeed, the s-wave potential displayed in that work contains an attractive region which is comparable to that due to p-waves, whereas in our calculation we have obtained a $W_s^{\pi\pi}$ that is mostly repulsive and at least one order of magnitude smaller than the p-wave contribution. The origin of this discrepancy can be traced back to the way the contribution of the πN σ-term has been treated in both works.

In the paper of Coelho, Das and Robilotta, which serves as basis for the present study, the σ-term contribution has been parametrized in such a way that its contribution to the 3BF is proportional to

$$W_s^{\pi\pi} \alpha - \frac{g}{\vec{k}^2+\mu^2} \frac{g}{\vec{k}'^2+\mu^2} [\alpha_\sigma] \quad . \tag{60}$$

In the work of Brandenburg and Glöckle, on the other hand, the

results of ref. (1) are used, and the term corresponding to eq. (60) is given by

$$W_s^{\pi\pi})_{BG} \; \alpha \left\{\frac{g}{\vec{k}^2+\mu^2} \frac{g}{\vec{k}'^2+\mu^2} [a + c(\vec{k}^2+\vec{k}'^2)]\right\} \tag{61}$$

This term can be rewritten as

$$W_s^{\pi\pi}.)_{BG} \; \alpha \left\{\frac{g}{\vec{k}^2+\mu^2} \frac{g}{\vec{k}'^2+\mu^2} [a-2\mu^2 c] + c\left[g \frac{g}{\vec{k}'^2+\mu^2} + \frac{g}{\vec{k}^2+\mu^2} g\right]\right\} . \tag{62}$$

The first bracket this equation is equivalent to eq. (60) since

$$- \alpha_\sigma = a - 2\mu^2 c = - \frac{\sigma}{f\pi^2} \tag{63}$$

where σ is the πN σ-term. The second bracket, however, describes a 3BF in which the propagation of one of the pions is replaced by a contact interaction that corresponds to a δ-function in configuration space. This situation does not change much when we allow g to become the πN form-factor, for this term would represent "contact" interactions between nucleons that are not point like. These "contact" interactions are not realistic because very small internucleon distances are presented by the repulsive core of the two-nucleon interaction. Contact terms should, therefore, be neglected before one goes to configuration space.

The combined contribution of two-body and $\pi\pi$E forces represents the background for the $\pi\rho$E-3BF and it is displayed in Fig. 7. Inspecting this contour plot we note that the introduction of the $\pi\pi$E-3BF favours the triangular configuration.

We show, in Fig. 8, the equipotentials for the $\pi\rho$E-3BF derived in this work using the value of $C_1^{\pi\rho}$ given by eq. (55). This value corresponds, as we pointed out above, to a 3BF that could be unrealistically large because the non-relativistic condition for the nucleons has not been properly implemented. This fact does not prevent, however, the possibility of a qualitative study. The main characteristic of the $\pi\rho$E-3BF is that it is mostly repulsive, acting in the opposite direction of $W_p^{\pi\pi}$. In order to have a feeling of the joint effect of all the forces considered in this work we display in Figs. 9 and 10 the total potential for $C_1^{\pi\rho} = -118.7$ MeV and $C_1^{\pi\rho} = -21.3$ MeV. The latter value is motivated by the hope that the non-relativistic consistency of the calculations would be roughly simulated when we replace μ for the last m_ρ in eq. (54).

V. SUMMARY

In this work we have derived a pion-rho exchange three-body potential using effective Lagrangians that are approximately chiral and gauge symmetric. This approach allows a clear identification of the dynamical origin of the various terms of the three-body force. The leading contribution to the potential comes from the seagull term, which is a typical product of a gauge Lagrangian. This term is one order of magnitude larger than those arising from the excitation of the Δ-resonance.

The three-body force derived here is comparable to that due to the exchange of two pions, having the same structure as its component related to s-waves in the intermediate πN amplitude. The pion-rho force is mainly repulsive and its contribution tends to cancell that of the two-pion force. The extent of this cancellation is difficult to assess since it is related to the use of hard-cores, form factors and the delicate problem of how to implement mathematically the non-relativistic condition for bound nucleons.

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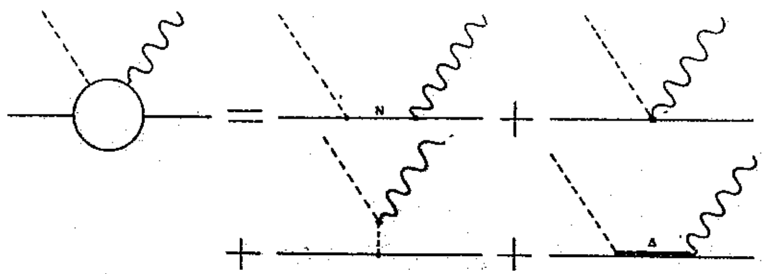

FIG. 1 - Diagrams that contribute to the process πN→ρN .

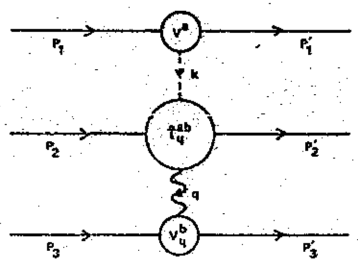

FIG. 2 - Diagram corresponding to the πρE-3BF

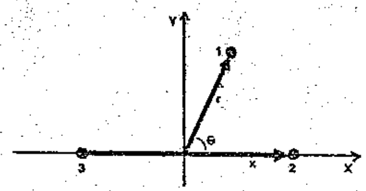

FIG. 3 - The coordinates of the trinucleon system.

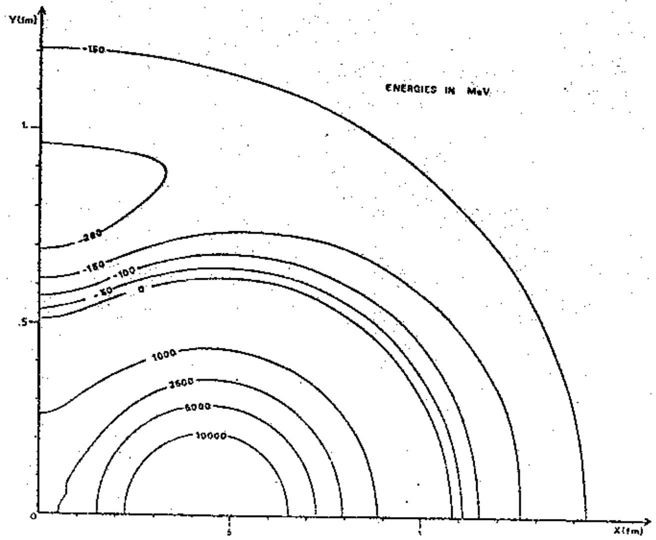

FIG. 4 - The two body potential ; x = 0.88 fm.

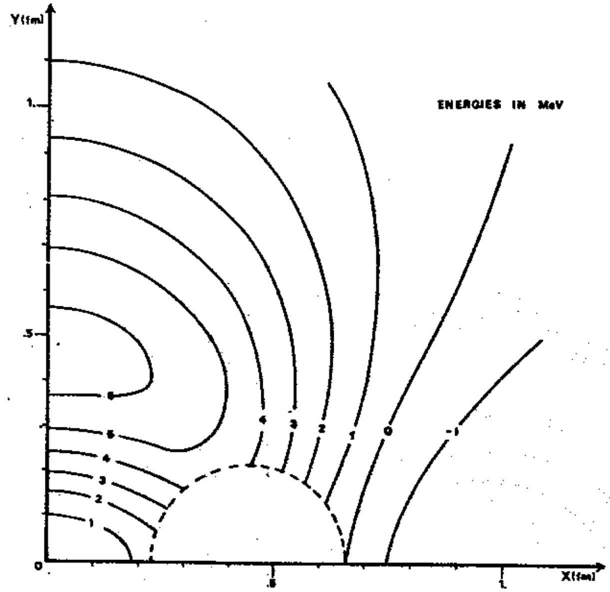

FIG. 5 - The s-wave two-pion exchange potential ; x = 0.88 fm.

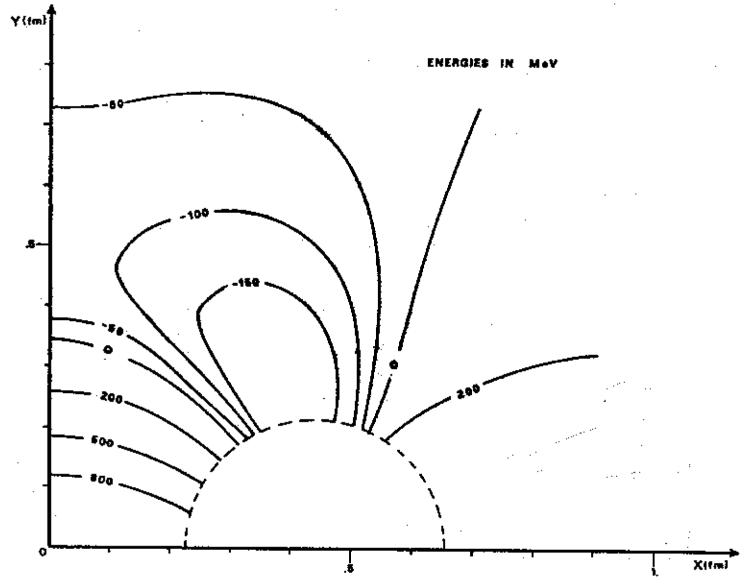

FIG. 6 - The p-wave two-pion exchange potential ; x = 0.88 fm.

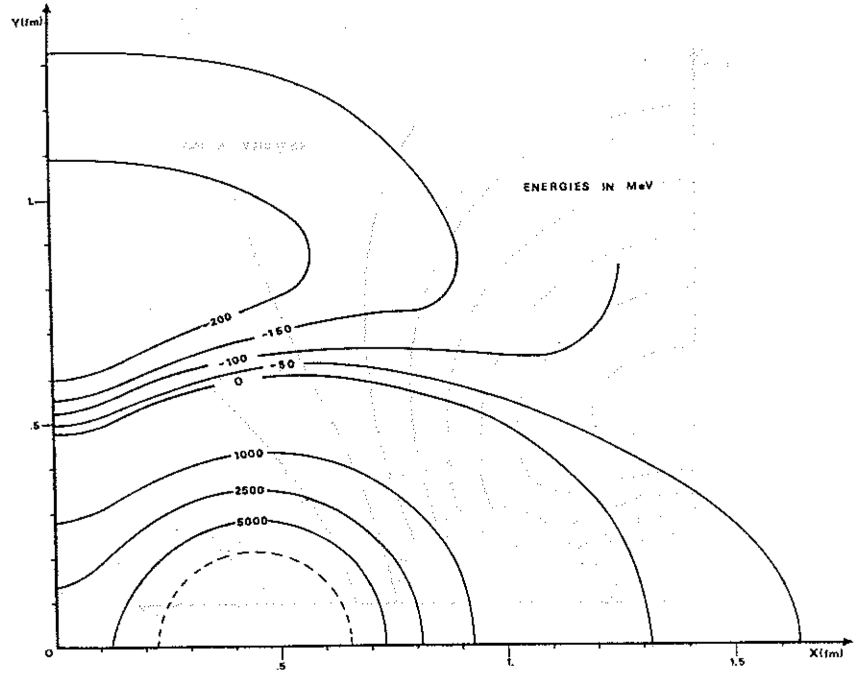

FIG. 7 - Combined two-body and two-pion exchange potentials; x = 0.88 fm.

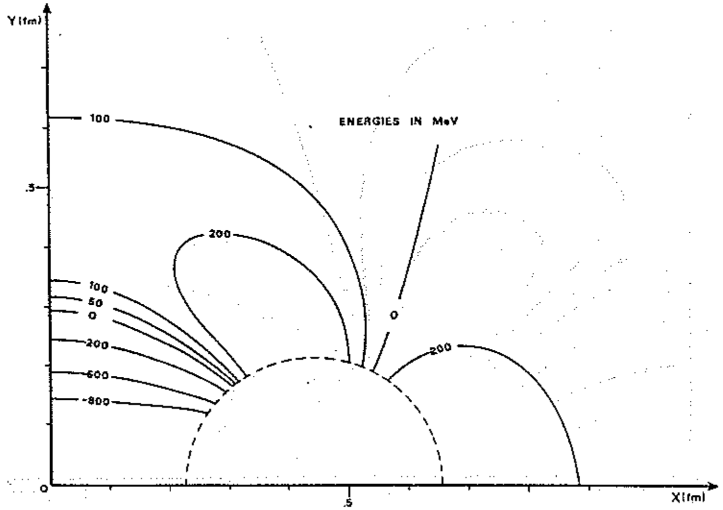

FIG. 8 - The pion-rho exchange potential ; x = 0.88 fm. $C_1^{\pi\rho}$ = - 118.7 MeV .

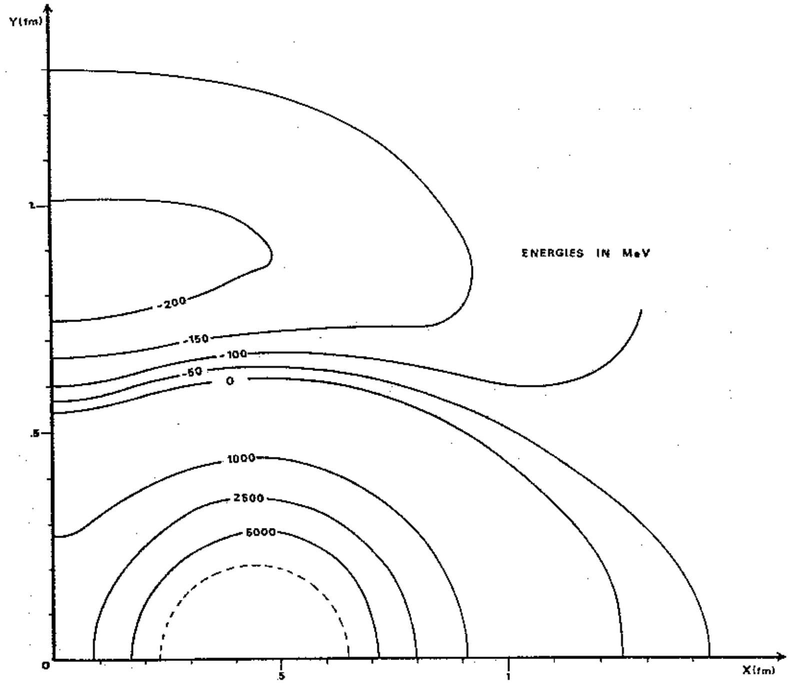

FIG. 9 - The total potential ; x = 0.88 F ; $C_1^{\pi\rho}$ = - 118.7 fm.

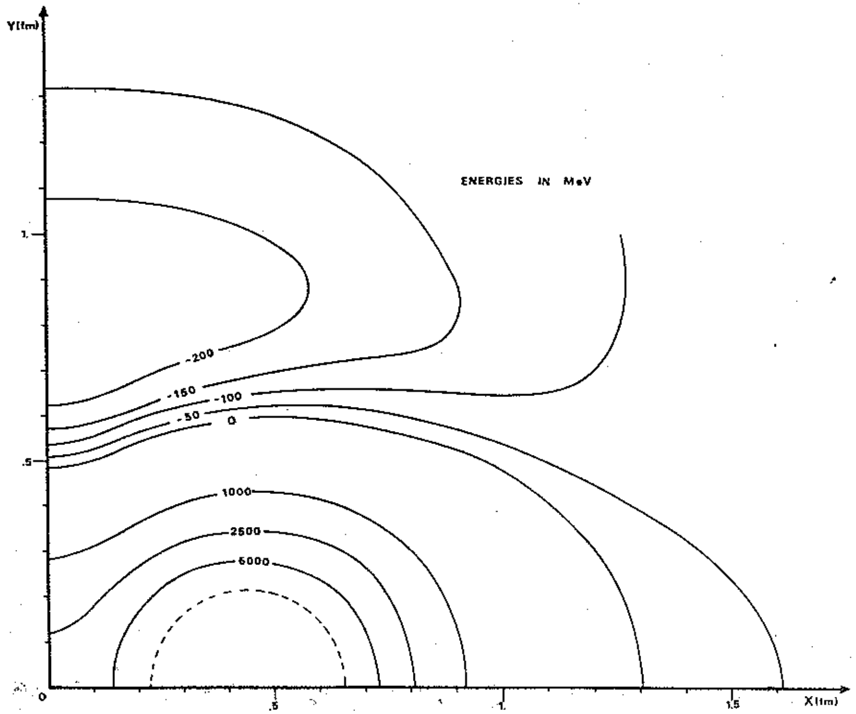

FIG. 10 - The total potential ; x = 0.88 F ; $C_1^{\pi\rho}$ = - 21.3 fm.