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PHASE TRANSITION IN AN ABELIAN GAUGE THEORY AT FINITE TEMPERATURE

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PHASE TRANSITION IN AN ABELIAN GAUGE THEORY AT FINITE TEMPERATURE

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ABSTRACT

Within the context of an Abelian Gauge Theory, we discuss phase transition driven by the spontaneous generation of domain walls. We calculate, semiclassically, the critical temperature. The results are very close to those obtained via the effective potential approach.

RESUMO

No contexto de uma Teoria de Gauge Abeliana, discutimos transição de fase induzida pela geração espontânea de paredes de Bloch. Calculamos, semiclassicamente, a temperatura crítica. Os resultados são bastante próximos daqueles obtidos pela técnica do potencial efetivo.

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I. INTRODUCTION

Field theories whose symmetry is broken spontaneously at zero temperature are expected to exhibit symmetry restoration at high temperatures. One expects then that the system will exhibit two phases: the symmetric and the broken symmetry one (the symmetry is restored for temperatures higher than a critical T_c)[1]. Each phase of the system will be characterized, as usual, by an order parameter $\langle\Phi\rangle$ such that for the unbroken (Φ_s) and broken symmetry (Φ_b) phases one has

$$\langle\Phi_s\rangle = 0 \qquad (T > T_c)$$

$$\langle\Phi_b\rangle = a(T) \neq 0 \qquad (T < T_c)$$

The order parameter for theories whose breakdown of symmetry proceeds via the Higgs mechanism is the vacuum expectation value of a scalar field.

One approach to the study of the phase transition in Field Theory is based on the behavior of the effective potential at finite temperature. By looking at the temperature dependence of the effective potential[2,3] one can convince oneself that a model whose symmetry is spontaneously broken indeed exhibits two phases. At high temperatures the symmetry is manifest, whereas for T close to zero the symmetry is spontaneously broken. From the effective potential one can also compute, for instance, the critical temperature[2,3].

An alternative to the study of symmetry restoration and phase transition was proposed by Ventura[4]. Within this scheme the symmetry restoration takes place as the result of the condensation of topological defects[5]. This picture of the phase transition is similar to the one proposed by Kosterlitz and Thouless[6]. Actually this is just an explicit realization of such a phenomenon in Field Theory.

This paper is devoted to a discussion of the basic ingredients of such an alternative to symmetry restoration within the context of an Abelian Gauge Theory. We present the model and the functional integration formalism of such a Field theoretical model at finite temperature in section II. Section III is devoted to semiclassical approximations to the functional integral and to the computation of the free energy in the vacuum sector. In section IV we compute the free energy in the kink sector. The picture of the phase transition and the computation of the critical temperature in the high temperature limit, are presented in section V. We end the paper with some discussions in section VI.

II. ABELIAN GAUGE MODEL AT FINITE TEMPERATURE

We will study the phase transition occurring in the Abelian gauge model defined by the Lagrangian density

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - U[\Phi^*\Phi] \tag{2.1}$$

where $D_\mu = \partial_\mu - ieA_\mu$, A_μ is a gauge field, Φ a complex scalar field and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.2}$$

$$U[\Phi^*\Phi] = \frac{m^4}{4\lambda} [1 - \frac{2\lambda}{m^2} (\Phi^*\Phi)]^2 \quad . \tag{2.3}$$

By introducing the field variables ϕ and ξ as

$$\Phi \to e^{i(\xi/a)} \frac{\phi + a}{\sqrt{2}}$$

$$\Phi^* \to e^{-i(\xi/a)} \frac{\phi + a}{\sqrt{2}} \tag{2.4}$$

where $a = m/\sqrt{\lambda}$ and defining

$$B_\mu = A_\mu - \frac{1}{ea} \partial_\mu \xi \tag{2.5}$$

one gets the following expression for the Lagrangian density (2.1)

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 a^2 B_\mu B^\mu + \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - m^2\phi^2 -$$

$$- \lambda a \phi^3 - \frac{\lambda}{4} \phi^4 + \frac{1}{2} e^2 \phi(\phi + 2a) B_\mu B^\mu \quad . \tag{2.6}$$

This Lagrangian density exhibits the well known phenomenon of spontaneous breakdown of the Abelian gauge invariance via the Higgs mechanism. The Euler-Lagrange equations which follow from (2.6) are

$$\Box \phi + 2m^2\phi + 3\lambda a \phi^2 + \lambda\phi^3 - e^2(\phi + a) B_\mu B^\mu = 0 \tag{2.7}$$

$$\Box B_\mu + e^2(\phi + a)^2 B_\mu = 0 \quad . \tag{2.8}$$

In order to investigate the effects of finite temperature we couple the aforementioned systems weakly to a heat bath at temperature T. Under these conditions the thermodynamical properties of such a system should be inferred from the partition function which, in the functional integration formalism, can be written as

$$Z = \int DB_\mu \int D\phi \; e^{-\int_0^\beta d\tau \int d^3x \, L_E[B_\mu,\phi]} \qquad (2.9)$$

where $\beta = 1/T$, L_E is the Euclidean version of (2.6) and the sums in (2.9) are over field theoretical configurations satisfying periodic boundary conditions

$$\begin{aligned} B_\mu(0,\vec{x}) &= B_\mu(\beta,\vec{x}) \\ \phi(0,\vec{x}) &= \phi(\beta,\vec{x}) \quad . \end{aligned} \qquad (2.10)$$

III. SEMICLASSICAL APPROXIMATIONS - THE VACUUM SECTOR

Here we shall develop the basic scheme needed for the computation of the partition function within a semi-classical approximation. In order to proceed in this way we denote by ϕ_c and B_μ^c a classical solution of the Euclidean version of equation (2.7) and (2.8). The interesting classical solutions for our purposes are those which do not depend on time.

As usual we consider as the classical solution of (2.8) the trivial one

$$B_\mu^c = 0 \quad . \qquad (3.1)$$

In this case equation (2.7) reduces to

$$-\nabla^2\phi_c + 2m^2\phi_c + 3\lambda a\phi_c^2 + \lambda\phi_c^3 = 0 \quad . \qquad (3.2)$$

Let us consider fluctuations around a solution of (3.2) and (3.1), that is, take the field configuration

$$\phi = \phi_c + \eta = \phi_o - a + \eta \qquad (3.3)$$

$$B_\mu = B_\mu^c + b_\mu = b_\mu \qquad (3.4)$$

where we write $\phi_c = \phi_o - a$ for simplicity. After substituting into (2.9) and keeping up to quadratic terms in the fluctuations one can write an approximate expression for the partition function.

$$Z = e^{-\beta S_E^c[\phi_o]} \int_{\eta(0,\vec{x})=\eta(\beta,\vec{x})} D\eta \; e^{-S_E[\eta]} \int_{b_\mu(0,\vec{x})=b_\mu(\beta,\vec{x})} Db_\mu \; e^{-S_E[b_\mu]} \qquad (3.5)$$

where $S_E^c[\phi_o]$, $S_E[\eta]$ and $S_E[b_\mu]$ are the Euclidean version of

$$S^c[\phi_o] = \int d^4x \left\{\frac{1}{2}(\partial_\mu\phi_o)(\partial^\mu\phi_o) - U[\phi_o]\right\} \qquad (3.6)$$

$$S[\eta] = \int d^4x \left\{\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \frac{1}{2}U''[\phi_o]\eta^2\right\} \qquad (3.7)$$

$$S[b_\mu] = \int d^4x \left\{-\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}e^2\phi_o^2 b_\mu b^\mu\right\} \qquad (3.8)$$

and

$$U[\phi_o] = \frac{m^4}{4\lambda}\left[1 - \frac{\lambda}{m^2}\phi_o^2\right]^2 \qquad (3.9)$$

The prime denotes differentiation with respect to the argument and we write $f_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$.

For the free energy $(F = - T \ln Z)$ one gets formally

$$F = S_E^C[\phi_o] + \frac{1}{2} T \operatorname{tr} \ln \hat{\Omega}_H + \frac{3}{2} T \operatorname{tr} \ln \hat{\Omega}_V \qquad (3.10)$$

where the factors 1/2 and 3/2 are associated to the number of degrees of freedom of the scalar field (1) and the vector meson field (3). The operators $\hat{\Omega}_H$ and $\hat{\Omega}_V$ are given respectively by

$$\hat{\Omega}_H = - \sum_{i=1}^{4} \partial_i^2 + U''[\phi_o] \qquad (3.11)$$

$$\hat{\Omega}_V = - \sum_{i=1}^{4} \partial_i^2 + e^2\phi_o^2 \qquad (3.12)$$

These are formal expressions because we have to take into account forthe surface contributions (remember that we are working at finite temperature). We also remark here that the functional integration for the Higgs field is a trivial one, whereas for the vector meson field we have to take some care in order to account for the physical degrees of freedom only. In the appendix the calculations are shown explicitly.

In order to introduce some important expressions for section V and as a simple application of the results just obtained, let us compute the free energy up to quadratic fluctuations around the trivial solution

$$\phi_o = a \qquad (\phi_c = \phi_o - a = 0)$$
$$B_\mu^C = 0 \quad . \qquad (3.13)$$

In this case we are referred to the vacuum sector.

For this vacuum configuration one can write for the operators $\hat{\Omega}_H$ and $\hat{\Omega}_V$

$$\hat{\Omega}_H = - \sum_{i=1}^{4} \partial_i^2 + 2m^2 \qquad (3.14)$$

$$\hat{\Omega}_V = - \sum_{i=1}^{4} \partial_i^2 + e^2a^2 \quad . \qquad (3.15)$$

Due to the periodic boundary conditions satisfied by the fluctuations and from the previous expressions one gets the following set of eigenvalues for the operators $\hat{\Omega}_H$ and $\hat{\Omega}_V$

$$\Omega_H(n,\vec{k}) = \left(\frac{2\pi n}{\beta}\right)^2 + \vec{k} + 2m^2 \qquad (3.16)$$

$$\Omega_V(n,\vec{k}) = \left(\frac{2\pi n}{\beta}\right)^2 + \vec{k}^2 + e^2a^2 \quad . \qquad (3.17)$$

The free energy obtained in (3.8) is then written as (we have assumed that the cosmological constant is zero: $V(a) = 0$)

$$F_{Vac}(T) = \frac{T}{2} \sum_{\vec{k}} \sum_n \ln \left[\left(\frac{2\pi n}{\beta}\right)^2 + \vec{k}^2 + 2m^2\right] + \frac{3T}{2} \sum_{\vec{k}} \sum_n \ln \left[\left(\frac{2\pi n}{\beta}\right)^2 + \vec{k}^2 + e^2a^2\right] \quad . \qquad (3.18)$$

By using the identity

$$\sum_n \ln \left[\left(\frac{2\pi n}{\beta}\right)^2 + b^2\right] = \beta b + 2 \ln \left[1 - e^{-\beta b}\right] \qquad (3.19)$$

one gets for the free energy in the vacuum sector

$$F_{Vac}(T) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\varepsilon^H(k) + \frac{3}{2}\int \frac{d^3k}{(2\pi)^3}\,\varepsilon^V(k) +$$

$$+ TV \int \frac{d^3k}{(2\pi)^3}\,\ell n\left[1 - e^{-\beta\varepsilon^H(k)}\right] + 3TV\int \frac{d^3k}{(2\pi)^3}\,\ell n\left[1 - e^{-\beta\varepsilon^V(k)}\right] \tag{3.20}$$

where $V = L^3$ is the volume and

$$\varepsilon^H(k) = \sqrt{k^2+2m^2}$$

$$\varepsilon^V(k) = \sqrt{k^2+e^2a^2} \quad . \tag{3.21}$$

The two first terms in the above expression are divergent and come from the zero point energy contributions.

IV. THE KINK SECTOR

In this section we will compute the partition function associated to a non-trivial field theoretical configuration referred to as the kink sector. Postponing its physical implication until the next section we just recall that the kink is a classical static solution of the form

$$\phi_o(x) = a\tanh(mx/\sqrt{2}) \quad . \tag{4.1}$$

With this solution (remember that $B^c_\mu = 0$) one gets for the operators $\hat{\Omega}_H$ and $\hat{\Omega}_V$

$$\hat{\Omega}_H = -\sum_{i=1}^{4}\partial_i^2 + 2m^2 - 3m^2\,\mathrm{sech}^2(mx/\sqrt{2}) \tag{4.2}$$

$$\hat{\Omega}_V = -\sum_{i=1}^{4}\partial_i^2 + e^2a^2 - e^2a^2\,\mathrm{sech}^2(mx/\sqrt{2}) \tag{4.3}$$

For simplicity we will decompose the free energy in the kink sector as the sum over the Higgs field contribution (F^H_{kink}) plus the vector meson contribution (F^V_{kink})

$$F_{kink} = F^H_{kink} + F^V_{kink} \quad . \tag{4.4}$$

The Higgs field contribution is a straighforward calculation and the result is just that quoted in the work of Ventura[4]. Since the one dimensional potential given in (4.2) admits just two bound states[8] (one of which being the zero mode associated to the translacional invariance of the theory) one gets

$$\Omega_H = \left(\frac{2\pi n}{\beta}\right)^2 + \omega^2_{\vec{k}} + \omega^2_{\left[\begin{smallmatrix}\ell\\ q\end{smallmatrix}\right]} \tag{4.5}$$

where

$$\omega^2_{\vec{k}} = k_y^2 + k_z^2$$

$$\omega^2_\ell = \begin{cases} 0 & \text{for } \ell=0 \\ \frac{3}{2}\,m & \text{for } \ell=1 \end{cases} \tag{4.6}$$

$$\omega^2_q = \left(\frac{1}{2}\,q^2 + 2\right)m^2 \quad \text{in the continuum.}$$

Therefore, the Higgs field contribution to the free energy is

$$F^H_{kink} = AM + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\varepsilon^H(k) + \frac{1}{2}\sum_{\ell=0}^{1}\int \frac{d^2k}{(2\pi)^2}\,\varepsilon^H_\ell(k) +$$

$$+ TA\sum_{\ell=0}^{1}\int \frac{d^3k}{(2\pi)^2}\,\ell n\left[1-e^{-\beta\varepsilon^H_\ell(k)}\right] + TA\int \frac{d^2k}{(2\pi)^2}\,\frac{d\ell}{dq}\,dq\,\ell n\left[1-e^{-\beta\varepsilon^H_q(k)}\right] \tag{4.7}$$

where $A=L^2$ is the area and M is the classical mass of the one-dimensional soliton

$$M = 2\sqrt{2}\,m^3/3\lambda \tag{4.8}$$

and

$$\varepsilon^H(k) = \sqrt{k^2+2m^2}$$

$$\varepsilon^H_\ell(k) = \sqrt{\omega_k^2+\omega_\ell^2} \tag{4.9}$$

$$\varepsilon^H_q(k) = \sqrt{\omega_k^2 + (\tfrac{1}{2}q^2+2)m^2}\quad .$$

The divergent terms are the contribution of the zero point energy.

For the contribution in the continuum $d\ell/dq$ should be obtained from the phase-shifts of the one-dimensional potential[4]. One gets

$$\frac{d\ell}{dq} = \frac{1}{2\pi}\left[\frac{mL}{\sqrt{2}} - 6\,\frac{q^2+2}{(q^2+1)(q^2+4)}\right] \tag{4.10}$$

After substituting (4.10) into (4.7) and introducing the variable

$$p^2 = \frac{1}{2}m^2q^2 \tag{4.11}$$

one finally gets

$$F^H_{kink} = AM + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,\varepsilon^H(k) + \frac{1}{2}\sum_{\ell=0}^{1}\int \frac{d^2k}{(2\pi)^2}\,\varepsilon^H_\ell(k) +$$

$$+ TA\sum_{\ell=0}^{1}\int \frac{d^2k}{(2\pi)^2}\,\ell n\left[1-e^{-\beta\varepsilon^H_\ell}\right] + TV\int \frac{d^3k}{(2\pi)^3}\,\ell n\left[1-e^{-\beta\varepsilon^H}\right] -$$

$$- 2\sqrt{2}\,m\,TA\int \frac{d^2k}{(2\pi)^2}\,\frac{dp}{(2\pi)}\left\{\frac{1}{p^2+2m^2} + \frac{1}{2p^2+m^2}\right\}\ell n\left[1-e^{-\beta\varepsilon^H_p}\right] \tag{4.12}$$

where

$$\varepsilon^H_p = \sqrt{k^2+p^2+2m^2}\quad . \tag{4.13}$$

We will compute now the vector meson field contribution to the free energy. The eigenvalues of the operator $\hat{\Omega}_V$ defined in (4.3) can be obtained analytically[8]. The resulting expressions are simple only in the case in which the coupling constants satisfy the relationship

$$\frac{e^2}{\lambda} = \frac{1}{2}S(S+1) \qquad S = 1,2,3,\ldots \tag{4.14}$$

and we will give the explicit expressions for this case only. The parameter S introduced above has a simple interpretation: it gives the number of bound states exhibited by the one-dimensional potential given in (4.3). Under these circumstances one can write

$$F^V_{kink} = \frac{3}{2}\int \frac{d^3k}{(2\pi)^3}\, \varepsilon^V(k) + \frac{3}{2}\sum_{\ell=0}^{s-1}\int \frac{d^2k}{(2\pi)^2}\, \varepsilon^V_\ell(k) +$$

$$+ 3TA \sum_{\ell=0}^{s-1}\int \frac{d^2k}{(2\pi)^2}\, \ell n\left[1 - e^{-\beta\varepsilon^V_\ell(k)}\right] + 3TV\int \frac{d^3k}{(2\pi)^3}\, \ell n\left[1 - e^{-\beta\varepsilon^V(k)}\right] -$$

$$- 6TA \int \frac{d^2k}{(2\pi)^2}\frac{dq}{2\pi}\left\{\frac{D\frac{dN}{dq} - N\frac{dD}{dq}}{N^2 + D^2}\right\} \ell n\left[1 - e^{-\beta\varepsilon^V_q(k)}\right] \quad . \qquad (4.15)$$

where

$$\varepsilon^V(k) = \sqrt{k^2+e^2a^2} = \sqrt{k^2 + \frac{1}{2}\, s(s+1)m^2}$$

$$\varepsilon^V_\ell(k) = \sqrt{k^2 + \frac{1}{2}\, (s+2s\ell-\ell^2)m^2} \qquad (4.16)$$

$$\varepsilon^V_q(k) = \sqrt{k^2 + \frac{1}{2}\, [q^2+s(s+1)]m^2}$$

and

$$N = \sum_{n=0}^{\frac{s-b}{2}} (-1)^n\, P_s(s-1-2n)\, q^{2n+1}$$

$$(4.17)$$

$$D = \sum_{n=0}^{\frac{s-a}{2}} (-1)^n\, P_s(s-2n)\, q^{2n}$$

with

$a = b = 1$ for odd s

(4.18)

$a = 0$; $b = 2$ for even s

and $P_s(\ell)$ is such that

$$P_s(0) = 0! = 1$$

$$P_s(1) = 1 + 2 + 3 + \ldots + s \qquad (4.19)$$

$$P_s(2) = 1.2 + 1.3 + \ldots + 1.s + 2.3 + \ldots + 2.s + \ldots + (s-1)s$$

$$\vdots$$

$$P_s(s) = 1.2.3.\ldots.\; s = s!$$

The divergent terms are the zero point energy contribution to the free energy in the kink sector.

V. PHASE TRANSITION - THE CRITICAL TEMPERATURE

One notes first of all that the kink solution represents a Block wall which separates the space into two regions. In each region of the space there leaves one of the two degenerate vacua ($\pm a$). In the case of a condensate of such walls being formed, one can predict that

$$\langle\Phi\rangle \equiv \frac{1}{V}\int dx\, \langle\Phi(x)\rangle = 0$$

and consequently the symmetry is (globally) restored.

At first sight one might think that there is no chance for a Block wall of the form (4.1) to appear spontaneously in the system since it has infinite energy. However, as emphasized by Kosterlitz and Thouless, as the temperature increases the entropy might take over the energy and topologically non-trivial field theoretical configurations having infinite energy might sprout in the system .

In order to see that explicitly in our case let us analyze the difference between the free energy of the kink sector and the vacuum sector per unit of area. We will compute in this way

$$f_{wall}(T) = \frac{F_{kink} - F_{vac}}{A} \tag{5.1}$$

which can be interpreted as the free energy associated to a Bloch wall.

From the thermodinamical view point, the condition for a Bloch wall to sprout spontaneously in the system is

$$f_{wall}(T) < 0 \tag{5.2}$$

for, under this condition, the situation in which space is divided by a Bloch wall is favoured.

It is not difficult to check that for low temperatures $f_{wall} > 0$ and there is no chance for a wall to appear in the system. Thus, one can introduce a critical temperature T_c defined by

$$f_{wall}(T_c) = 0 \tag{5.3}$$

that is, just the point where above it f_{wall} changes sign and the walls can sprout spontaneously in the system.

For our model, the full expression for f_{wall} can be obtained from (3.20), (4.4), (4.12) and (4.15). It is important to remark that in the expression for f_{wall} some of

the zero point energy contributions will remain. These contributions are divergent and come from the well known ultraviolet divergences of the Quantum Field Theory. The way of getting rid of them is the usual one: we just add appropriate countertems to the Lagrangean in order to get a renormalized theory. One of the effects of the renormalization scheme is just to introduce a quantum correction to the energy per unit of area of the wall[4]. In the semiclassical limit these correction are small and do not affect appreciably the result of the critical temperature. In this way we will neglect all these correction in the following.

In order to be more explicit let us write the explicit expressions for f_{wall} in the $s=1$ and $s=2$ case. For $s=1$ one gets

$$\begin{aligned} f^{(1)}_{wall} = M &+ T\int \frac{d^2k}{(2\pi)^2}\left\{\ell n\left[1 - e^{-\beta\varepsilon^H_o}\right] - 2\sqrt{2}\, m \int \frac{dp}{2\pi}\, \frac{1}{2p^2+m^2}\, .\, \ell n\left[1 - e^{-\beta\varepsilon^H}\right]\right\} + \\ &+ T\int \frac{d^2k}{(2\pi)^2}\left\{\ell n\left[1 - e^{-\beta\varepsilon^H_1}\right] - 2\sqrt{2}\, m \int \frac{dp}{2\pi}\, \frac{1}{p^2+2m^2}\, \ell n\left[1 - e^{-\beta\varepsilon^H}\right]\right\} + \\ &+ 3T\int \frac{d^2k}{(2\pi)^2}\left\{\ell n\left[1 - e^{-\beta\varepsilon^V_o}\right] - 2\sqrt{2}\, m \int \frac{dp}{2\pi}\, \frac{1}{2p^2+m^2}\, \ell n\left[1 - e^{-\beta\varepsilon^V}\right]\right\}. \end{aligned} \tag{5.4}$$

where

$$\begin{aligned} &\varepsilon^H_o = \sqrt{k^2} \;;\quad \varepsilon^H_1 = \sqrt{k^2 + \tfrac{3}{2} m} \;;\quad \varepsilon^V_o = \sqrt{k^2 + \tfrac{1}{2} m^2} \\ &\varepsilon^H = \sqrt{k^2+p^2+2m^2} \;;\quad \varepsilon^V = \sqrt{k^2+p^2+m^2} \end{aligned} \tag{5.5}$$

This expression can be further simplified if we rescale the integration variables: $s = k^2$, $t = k^2 + \frac{3}{2} m^2$, $u = k^2 + \frac{1}{2} m^2$, $r = k^2 + p^2 + 2m^2$ and $v = k^2 + p^2 + m^2$. After the integration in p one gets

$$f^{(1)}_{wall} = M + g^H_o(T) + g^H_1(T) + 3g^V_{01} \qquad (5.6)$$

where

$$g^H_o(T) = \frac{T}{4\pi} \int_o^\infty ds\ \ell n\left[1 - e^{-\beta\sqrt{s}}\right] - \frac{T}{4\pi} \int_{2m^2}^\infty dr\ \frac{2}{\pi} \text{ arc tan}\left[2\left(\frac{r}{2m^2} - 1\right)^{\frac{1}{2}}\right] \times$$

$$\times\ \ell n \left[1 - e^{-\beta\sqrt{r}}\right] \qquad (5.7)$$

$$g^H_1(T) = \frac{T}{4\pi} \int_{\frac{3}{2} m^2}^\infty dt\ \ell n\left[1 - e^{-\beta\sqrt{t}}\right] - \frac{T}{4\pi} \int_{2m^2}^\infty dr\ \frac{2}{\pi} \text{ arc tan}\left[\left(\frac{r}{2m^2} - 1\right)^{\frac{1}{2}}\right] \ell n\left[1 - e^{-\beta\sqrt{r}}\right] \qquad (5.8)$$

$$g^V_{01}(T) = \frac{T}{4\pi} \int_{\frac{1}{2} m^2}^\infty du\ \ell n \left[1 - e^{-\beta\sqrt{u}}\right] - \frac{T}{4\pi} \int_{m^2}^\infty dv\ \frac{2}{\pi} \text{ arc tan}\left[2\left(\frac{v}{2m^2} - 1\right)^{\frac{1}{2}}\right] \times$$

$$\times\ \ell n \left[1 - e^{-\beta\sqrt{v}}\right] \qquad (5.9)$$

Following an analogous procedure we obtain for the case $s = 2$.

$$f^{(2)}_{wall} = M + g^H_o(T) + g^H_1(T) + g^V_{02}(T) + g^V_{12}(T)$$

where $g^H_o(T)$ and $g^H_1(T)$ are given by (5.7) and (5.8), respectively,

and

$$g^V_{02}(T) = \frac{T}{4\pi} \int_{m^2}^\infty du\ \ell n\left[1 - e^{-\beta\sqrt{u}}\right] - \frac{T}{4\pi} \int_{3m^2}^\infty dv\ \frac{2}{\pi} \text{ arc tan}\left[6\left(\frac{v}{3m^2} - 1\right)\right]^{\frac{1}{2}} \times$$

$$\times\ \ell n\left[1 - e^{-\beta\sqrt{v}}\right] \qquad (5.10)$$

$$g^V_{12}(T) = \frac{T}{4\pi} \int_{\frac{5}{2} m^2}^\infty d\ell\ \ell n\left[1 - e^{-\beta\sqrt{\ell}}\right] - \frac{T}{4\pi} \int_{3m^2}^\infty dv\ \frac{2}{\pi} \text{ arc tan}\left[\frac{3}{2}\left(\frac{v}{3m^2} - 1\right)\right]^{\frac{1}{2}} \times$$

$$\times\ \ell n \left[1 - e^{-\beta\sqrt{v}}\right] \quad . \qquad (5.11)$$

In the high temperature limit $(T >> m)$ the integrations are simple and the explicit expressions for $f_{wall}(T)$ in the case $s = 1$ $(e^2 = \lambda)$ and $s = 2$ $(e^2 = 3\lambda)$ are given by

$$f^{(1)}_{wall}(T) = M - \frac{\sqrt{2}}{2} m T^2 \qquad (5.12)$$

$$f^{(2)}_{wall}(T) = M - \frac{5}{6} \sqrt{2} m T^2 \qquad (5.13)$$

By using the definition (5.3), the critical temperature are given respectively by

$$T^{(1)}_c = \frac{2}{\sqrt{3}} \frac{m}{\sqrt{\lambda}} = 1.15 a \qquad (5.14)$$

$$T^{(2)}_c = \frac{2}{\sqrt{5}} \frac{m}{\sqrt{\lambda}} = 0.89 a \quad . \qquad (5.15)$$

The above results can be compared with those based on the behavior of the effective potential at finite temperature[2,3]. By adapting those results to our language one gets

$$T_c^{(1)} = \sqrt{\frac{12}{7}}\ \frac{m}{\sqrt{\lambda}} = 1.31\ a \qquad (5.16)$$

$$T_c^{(2)} = \sqrt{\frac{12}{13}}\ \frac{m}{\sqrt{\lambda}} = 0.96\ a \qquad (5.17)$$

VI. DISCUSSIONS

We have extended to an Abelian Gauge Theory the picture of symmetry restoration proposed by Ventura. Within this scheme the symmetry restoration follows as a result of the condensation of topologically non-trivial excitations. The mathematical tool we have worked with is the functional integration formalism of Field Theory at finite temperature.

The temperature above which Bloch walls will arise spontaneously in the system (the critical temperature) is explicitly computed for some ratios of the coupling constants. The results we get for the critical temperature are very close to those obtained within the effective potential approach and give support to the present method.

We just want to stress that due to the rich structure of the vacuum of the theory at high temperature, there are far reaching consequences of this mechanism of phase transition. Some of them have been reported in reference 9. The extension to realistic models will follow the lines indicated in this paper and will be our next step.

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APPENDIX

This appendix is devoted to the computation of the determinant involving the vector meson field, in order to justify the factor 3 in expressions (3.10) and (4.15).

We start with the computation of the determinant in the vacuum sector. Up to quadratic terms in the fluctuations b_μ one can write

$$\mathcal{L} = -\frac{1}{4}\, f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}\, e^2a^2b_\mu b^\mu \quad . \qquad (A.1)$$

The equation of motion is

$$\Box b_\mu - \partial_\mu(\partial^\nu b_\nu) + e^2a^2b_\mu = 0 \qquad (A.2)$$

and, if one takes the divergence of this expression, one gets the supplementary condition

$$\partial_\mu b^\mu = 0 \qquad (A.3)$$

which ensures us that the massive vector field describes spin 1 particles.

In order to compute the partition function taking into account condition (A.3) one can use the Faddeev-Popov method. Since this method is easy to implement in a theory in which the Lagrangian exhibit local gauge invariance, one first use the Stueckelberg trick in order to gain gauge invariance[10]. The Lagrangian (A.1) is equivalent to the Lagrangian

$$L = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{1}{2} e^2 a^2 a_\mu a^\mu - e\, a\, a_\mu (\partial^\mu \rho) + \frac{1}{2} (\partial_\mu \rho)(\partial^\nu \rho) \qquad (A.4)$$

if one defines

$$b_\mu = a_\mu - \frac{1}{ea} \partial_\mu \rho \qquad (A.5)$$

and imposes the supplementary condition

$$\partial_\mu a^\mu + e\, a\, \rho = 0 \quad . \qquad (A.6)$$

One notes now that (A.4) is invariant under the gauge transformations

$$a^\omega_\mu = a_\mu + \partial_\mu \omega$$
$$\rho^\omega = \rho + e\, a\, \omega \qquad (A.7)$$

and one can use the Faddeev-Popov method.

The partition function will be written as

$$Z = \int Da_\mu \int D\rho \exp\left[i\, S_{eff}\right] \det\left(\frac{\delta F}{\delta \omega}\right) \qquad (A.8)$$

where $F = \partial_\mu a^\mu + e\, a\, \rho - f$ and the effective action is given by

$$S_{eff} = \int d^4x \left\{ -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} (\partial_\mu a^\mu)^2 + \frac{1}{2} e^2 a^2 a_\mu a^\mu + \frac{1}{2} (\partial_\mu \rho)(\partial^\nu \rho) - \frac{1}{2} e^2 a^2 \rho^2 \right\} \quad . \qquad (A.9)$$

Remember that we have to take periodic boundary conditions in the imaginary time direction $x_o \to -i\tau$.

One can write further

$$Z = \int Da_\mu \exp\left[i\, S_{eff}(a)\right] \int D\rho \exp\left[i\, S_{eff}(\rho)\right] \det\left(\frac{\delta F}{\delta \omega}\right) \qquad (A.10)$$

where

$$S_{eff}(a) = -\frac{1}{2} \int d^4x\, a^\nu \left[\partial_\mu \partial^\mu + e^2 a^2\right] a^\nu$$

$$S_{eff}(\rho) = -\frac{1}{2} \int d^4x\, \rho \left[\partial_\mu \partial^\mu + e^2 a^2\right] \rho \qquad (A.11)$$

$$\frac{\delta F}{\delta \omega} = \partial_\mu \partial^\mu + e^2 a^2 \quad .$$

In calculating $S_{eff}(a)$ we make the substitution $a_o \to i\, a_o$, which change only the normalization constant[7].

Therefore, the partition function is

$$Z = \left[\det \hat{\Omega}_V\right] \left[\det^{-\frac{1}{2}} \hat{\Omega}_V\right] \left[\det^{-\frac{1}{2}} \hat{\Omega}_V\right]^4 \qquad (A.12)$$

and consequently

$$Z = \exp\left[3\left(-\frac{1}{2} \operatorname{Tr} \ln \hat{\Omega}_V\right)\right] \qquad (A.13)$$

where $\hat{\Omega}_V = - \sum_{i=1}^{4} \partial_i + e^2 a^2$. This is the desired result, as given in (3.10).

In the soliton sector it is easier to get the factor 3 in the gauge $b_3=0$. In this case one gets, up to quadratic fluctuations

$$L = - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{1}{2} e^2 \phi_o^2(z) \, b_\mu b^\mu \quad . \tag{A.14}$$

The equation of motion is

$$\Box b_\mu - \partial_\mu (\partial^\nu b_\nu) + e^2 \phi_o^2(z) \, b_\mu = 0 \tag{A.15}$$

and, by taking the divergence of this expression, one gets the supplementary condition

$$\partial_\mu \left[\phi_o^2(z) \, b^\mu \right] = 0 \quad . \tag{A.16}$$

One notes first that in the gauge $b_3=0$ everything is analogous to the Lorentz gauge discussed previously. In this gauge one can write

$$A = \int Db_\mu \exp\left[i \, S_{eff} \right] \tag{A.17}$$

where

$$S_{eff} = - \frac{1}{2} \int d^4x \, b^\nu \left[\partial_\mu \partial^\mu + e^2 \phi_o^2(z) \right] b^\nu \quad . \tag{A.18}$$

Remember that the desired expression is the Euclidean version of the above one.

By recalling that $b_3=0$ one finally get

$$Z = \left[\det^{-\frac{1}{2}} \hat{\Omega}_V \right]^3 = \exp\left[3 \left(- \frac{1}{2} \operatorname{tr} \ell n \, \hat{\Omega}_V \right) \right] \tag{A.19}$$

where $\hat{\Omega}_V = - \sum_{i=1}^{4} \partial_i^2 + e^2 \phi_o^2(z)$.

REFERENCES

1. D.A. Kirznits and A.D. Linde, Phys. Lett. 42B, 471 (1972).
2. L. Dolan and R. Jackiw, Phys. Rev. D9, 3320 (1974).
3. A.D. Linde, Rep. Prog. Phys. 42, 389 (1979).
4. I. Ventura, Phys. Rev. B24, 2812 (1981).
5. J. Fröhlich, G. Morchio and F. Strocchi, Nucl. Phys. B190, 553 (1981).
6. J.M. Kosterlitz and D.J. Thouless, J. Phys. C6, 1181 (1973).
7. C.W. Bernard, Phys. Rev. D9, 3312 (1974).
8. P.M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill, N.Y., 1953) p.1650; S. Flügge, Practical Quantum Mechanics (Springer-Verlag, N.Y., 1974) p.94.
9. G.C. Marques and I. Ventura, "An Alternative View of Cosmological Phase Transition", Preprint IFUSP/P-342 (1982).
10. See, for instance, K. Nishijima, Fields and Particles (Benjamin, N.Y., 1969) p.78.