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S U M M A R Y

Assuming that quarks obey general statistics, we propose a non-relativistic approach that can describe several properties of hadrons : quark confinement, baryonic number conservation and 3-quark saturation in baryons. In our formalism, which is different from parastatistics, the assumption of three triplets of quarks is not necessary.

About four decades ago, Gentile [1 - 3] deduced within a thermodynamical context, a general quantum statistical distribution function for a system of N identical particles. He assumed that the quantum states of an individual particle can be occupied by a finite arbitrary number, $\underline{d}$, of particles. The Fermi and Bose statistics would correspond to $\underline{d} = 1$ and $\underline{d} = \infty$, respectively.

We have shown [4,5], using the irreducible representations of the symmetric group S_N in Hilbert space that, besides the usual one-dimensional boson (Y_S) and fermion (Y_A) states, also general intermediate states (Y), corresponding to subspaces with dimensions going from 2^2 up to $(N - 1)^2$, are compatible with the postulates of quantum mechanics. There was established a one-to-one correspondence between the Young shapes and the wavefunctions with well defined symmetries in Hilbert space. The first to note the need of extending the definition of the one-dimensional wavefunctions was Okayama [6]. However, he has obtained multi-dimensional wavefunctions where the over-all implied symmetries are not clearly displayed. Improving their results, we have written, in a τ -dimensional subspace, the state vector Y, as an orthonormalized τ -vector. Thus, there is a quantization of the system for each shape. For these subspaces there is a Geometric Superselection Rule (G S R) [7] : " transitions between different irreducible subspaces are forbidden ". Then, by adopting a somewhat new second quantization procedure, it was also established that :

1) Boson and fermion creation and annihilation operators obey the

usual bilinear commutation relations.

2) For the general states, the commutation relations have a multilinear matricial form governed by matrices depending on the structure of the irreducible manifolds. These relations indicate that N particles described by Y states are strongly correlated, forming a single cluster.

3) The state vector Y does not have a pure fermionic or bosonic behaviour, but it is a fermion-boson hybrid. The occupation number $\underline{d}$ for Y states runs from 2 up to N - 1.

The quantum mechanical and field theoretical approaches are known to be equivalent in the case of Bose and Fermi fields [7] . We are not aware, at the moment, of a method for extending our general states approach to the study of relativistic phenomena. Anyway, we must expect that, in a non-relativistic limit, the quantum field theoretical results should be reproduced in terms of our wavefunctions and commutation relations. Nevertheless, if we try to translate the usual parastatistics field theoretical results [8] into our quantum mechanical language, we verify that, from a symmetric group point of view, it would be hard to accept [5] the paraboson and parafermion concepts in quantum mechanics.

At this point a natural question arises : the remaining shapes associated to the hybrid states Y correspond to what kind of particles (by particles we mean a particle or a quasi-particle) ?

In this note, particles represented by Y states, will be named gentileons. Although there is a wide collection of possible intermediate states, many internal quantum numbers such as spin, isospin and others arising from internal symmetries or dynamical arguments can be used to drastically reduce the available number of states (9). These selection rules would depend on the specific gentileons constituting the system. If we have only $N = 3$ gentileons, there is only one intermediate four-dimensional state, which was carefully analysed in our previous work (4,5). For $N = 2$, there is no intermediate state and the system is represented by Y_A or Y_S.

Let us consider now the collision problem of two systems with gentilionic internal structures. System 1 is composed by N_1 gentileons with internal symmetries defined by the Young shape $S_p (N_1)$ whereas the system 2, composed by N_2 gentileons, is characterized by the Young shape $S_q (N_2)$. The gentileons are assumed to be identical and their total number $N = N_1 + N_2$ is conserved during the collision. By taking into account the Geometric Superselection Rule (G S R), we verify that the symmetries of the internal states are conserved :

$$S_p (N_1) + S_q (N_2) \rightleftarrows S_p (N_1) + S_q (N_2)$$

The ensuing consequences follow from this symmetry conservation law: two systems cannot coalesce and a free gentileon cannot be absorbed or emitted by a system. This suggests that, at least in a non-relativistic approach, gentileons cannot escape from a system. They could be, for instance, dynamical entities as quantum collective states or particles so strongly correlated that they would be unable to appear freely. Anyway, they could be understood as " confined entities " and it is with this spirit that we pursue this note. This would explain why only bosons and fermions have been observed in laboratories and why gentileons have never been detected as free elementary particles in the physical world. It is implied that, if we have a set of identical systems, each one consisting of N gentileons, and if we identify the evolution space with the group itself with respect to which the systems are elementary (10), only two descriptions are possible : bosonic or fermionic.

As an application of the geometric reasoning developed above, let us consider now the standard SU(3) model of strongly interacting particles in a non-relativistic approximation for the internal dynamics (11). If we assume that the fundamental triplet (n p λ) associated with a baryon is constituted by spin-half gentileons described by a four-dimensional hybrid Y defined on SU(3) space, several interesting possibilities are suggested. Naturally, since Y is not necessarily symmetric or antisymmetric under permutations, no specific symmetrisation is required for its radial part. Also, by adopting a Y state for the description of (n p λ) in SU(3) space, it is easy to see (4, 5) that we get the possibility of accomodating two identical particles in the same quantum state, without assuming parastatistics (12) or the existence of three triplets of quarks (13).

It is worthwhile to note that according to GSR, this choice for Y could automatically account for :

(a) baryonic number conservation

(b) quark confinement and

(c) 3-quark saturation in baryons.

Summarizing, we see that in a non-relativistic approach, several fundamental properties of baryons would thus be ascribed to the impossibility of transitions between equivalence classes defined by the action of the symmetric group on SU(3) components.

Next, we want to specialize the preceding discussion to mesons. To this effect, we point out that the set of accessible states of a system composed of 3 gentileons is completely inequivalent to the set which corresponds to a system composed by 2 gentileons. This extremely strong condition is the basis of the entire discussion on meson states. Structural differences between baryon and meson quark contents are expected to occur. The mesons could not be constructed with two flavours coming from the baryonic set (n p λ). Thus we would be compelled to construct a meson by introducing a new set of states. This new set is naturally generated by the $\bar{3}$ representation of SU(3). It is worthy to observe that quark confinement in mesons should also be a consequence of GSR.

As a final remark, it must be emphasized that our general results are not modified when the symmetry is extended to SU(6).

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