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THEORY OF THE HEAVY ION FUSION CROSS SECTION

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ABSTRACT

A general reaction theory of the heavy-ion fusion cross section that exhibits clearly the possible effects of directly coupled channels is developed. Our expression for the fusion cross section shows how the presence of coupled channels enhances its value in general. The result may be used to standardize the discussion of, e.g., sub-barrier heavy-ion fusion in general.

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Heavy-ion fusion reactions have been customarily treated within simple one-degree-of-freedom dynamical models. In recent years several experiments and their subsequent analysis have clearly shown that in many instances these simple models do not suffice for a satisfactory description. In particular, systems such as $^{16}O + {}^{A}Sm$, $^{A_1}Ni + {}^{A_2}Ni$, exhibit sub-barrier fusion cross sections several orders of magnitude larger than predicted by simple one-dimensional barrier penetration models[1-3]. These experimental findings clearly called for a more appropriate description involving explicit reference to several nuclear structure aspects of nuclei such as deformation, single-particle motion, etc..

A natural framework for incorporating these structure features is the coupled-channels theory as was done recently in Refs. 4-6 . The general conclusion reached by these authors, is that the heavy-ion fusion cross section considered as an inclusive quantity, is given by

$$\sigma_F = \sum_\lambda P_{o\lambda}\, \sigma_F(\lambda) \quad , \tag{1}$$

with $\sum_\lambda P_{o\lambda} = 1$

and $\sigma_F(\lambda)$ refers to the fusion cross sections attached to the appropriate eigenchannels labeled by λ , which are defined through the diagonalization of the channel-coupling matrix performed with a matrix U , with $P_{o\lambda} = |U_{o\lambda}|^2$.

It would be of value, though, to exhibit the effects of these coupled channels on the fusion cross section explicitly, as exemplifying a genuine multistep process. This can be accomplished formally within Feshbach's reaction theory.

The purpose of the present Letter is to develop a general theory of the fusion cross section that shows clearly the effect arising from directly coupled channels. This is done by first calculating the total reaction cross section, σ_R, and then perform its natural decomposition into the different contributing physical processes. Within a model involving several coupled inelastic channels, the fusion cross section, σ_F, (or total compound nucleus cross section) is defined to be $\sigma_F \equiv \sigma_R - \sigma_D$, where σ_D refers to the total inelastic cross section (a sum over all inelastic channels).

It is worth mentioning that such a decomposition of σ_R has been used rather extensively in other branches of nuclear physics such as pion-nucleus reactions, where the analog to σ_F is referred to as the absorption cross section[7].

Following Feshbach[8], we introduce two projection operators, P and Q, defined to project out of the total wave function of the nuclear system, Ψ, the direct component, $P\Psi$ and the compound nucleus component (fusion component) $Q\Psi$. We allow several orthogonal channels to be present in P, and insist that $P+Q=1$.

The effective equation of $P\Psi$ is given, as usual, by

$$\left(E - PH_0P - P\mathcal{V}QG(E)Q\mathcal{V}P\right)P\Psi = 0 \tag{2}$$

with the conventional notation of H_0 denoting the diagonal piece of the total Hamiltonian ($PH_0Q = 0$), and $\mathcal{V}$ the part of H that couples P to Q. It is to be understood at this point that Eq. (2) describes the optical model piece of $P\Psi$ in the sense of Ref. 9). The compound nucleus propagator QGQ

is assumed to represent the energy-averaged propagation of the fused system. We ignore altogether the fluctuating piece of the $P\Psi$-wave function.

Since our starting point for a one-channel fusion (no coupled channels effects) is one-dimensional barrier penetration (namely we consider the effect arising from the coupling between a one-channel $P\Psi$ and $Q\Psi$, through to be representable by total absorption for the contributing partial waves), we suppress any explicit reference ot the Q-space and write $P(H_0 + QGQ)P = P(H' + V')P$. The effect of V', assumed to represent direct couplings among channels contained in P, is mainly centered in the surface region, and therefore we take it to be real. Of course H' is not Hermitian due to the implicit Q-space (fusion) coupling.

Denoting the entrance channel projection operator by p_1 and $\mathbb{P}_1 = P - p_1$, we have (with $\mathbb{P}_1 p_1 = p_1 \mathbb{P}_1 = 0$),

$$(E - p_1 H' p_1)\, p_1\Psi = p_1 V' \mathbb{P}_1 \Psi \tag{3}$$

$$(E - \mathbb{P}_1 H' \mathbb{P}_1)\, \mathbb{P}_1 \Psi = \mathbb{P}_1 V' p_1 \Psi \; . \tag{4}$$

With boundary conditions of only outgoing waves in $\mathbb{P}_1\Psi$, the above two coupled equations can be reduced to an effective equations for $p_1\Psi$

$$\left(E - p_1 H' p_1 - p_1 V' \mathbb{P}_1 \mathcal{G}^{(+)}_{\mathbb{P}_1}(E)\, \mathbb{P}_1 V' p_1\right) p_1\Psi = 0 \tag{5}$$

where $\mathcal{G}^{(+)}_{\mathbb{P}_1}(E) = \left(E^{(+)} - \mathbb{P}_1 H' \mathbb{P}_1\right)^{-1}$ denotes the matrix propagator of the $\mathbb{P}_1$ channels.

The total reaction cross section extracted from $p_1\Psi$ is given by

$$\sigma_R = -\frac{2}{v}\langle p_1\Psi^{(+)}|\,\mathrm{Im}\big(p_1 H' p_1 + p_1 V' \mathbb{P}_1 \mathcal{G}^{(+)}_{\mathbb{P}_1}(E)\,\mathbb{P}_1 V' p_1\big)\,|p_1\Psi^{(+)}\rangle , \quad (6)$$

$v = \sqrt{\frac{2E}{\mu}}$, μ being the reduced mass of the system.

In Eq. (6), $p_1\Psi^{(+)}$ is the exact solution of Eq. (5) with the appropriate outgoing wave boundary condition. With our assumption that V' is Hermitian, the imaginary part of the second part inside the round brackets in Eq. (6) becomes proportional to $\mathrm{Im}\,\mathcal{G}^{(+)}_{\mathbb{P}_1}(E)$, which is given by

$$\mathrm{Im}\,\mathcal{G}^{(+)}_{\mathbb{P}_1}(E) = \Omega^{(-)}_{\mathbb{P}_1}\,\mathrm{Im}\,G^{(+)}_{\mathbb{P}_1}(E)\,\Omega^{(-)\dagger}_{\mathbb{P}_1} + \mathcal{G}^{(+)\dagger}_{\mathbb{P}_1}(E)\,\mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1}\,\mathcal{G}^{(+)}_{\mathbb{P}_1}(E) \quad (7)$$

where $\Omega^{(-)}_{\mathbb{P}_1}$ is the Möller operator defined in the $\mathbb{P}_1$ sub-space ($\Omega^{(-)}_{\mathbb{P}_1} = \mathbb{P}_1 + \mathcal{G}^{(-)}_{\mathbb{P}_1}\mathbb{P}_1\mathcal{U}^{\dagger}_{\mathbb{P}_1}$, $\Omega^{(+)}_{\mathbb{P}_1} = \mathbb{P}_1 + \mathcal{G}^{(+)}_{\mathbb{P}_1}\mathbb{P}_1\mathcal{U}_{\mathbb{P}_1}$), $G^{(+)}_{\mathbb{P}_1}$ is the "free" $\mathbb{P}_1$-propagator (free except for long-range Coulomb repulsion effects, $G^{(+)}_{\mathbb{P}_1}(E) = (E^{(+)} - (\mathbb{P}_1 H' \mathbb{P}_1 - \mathcal{U}_{\mathbb{P}_1}))^{-1}$, and $\mathcal{U}_{\mathbb{P}_1}$ is the optical potential contained in $\mathbb{P}_1 H' \mathbb{P}_1$. Eq. (7) is the usual unitarity identity, generalized to non-Hermitian interactions[7].

Inserting Eq. (7) into (6), and using $\mathrm{Im}\,p_1 H' p_1 = \mathrm{Im}\,\mathcal{U}_{p_1}$, we find

$$\sigma_R = \Big[-\frac{2}{v}\langle p_1\Psi^{(+)}|\,p_1 V' \mathbb{P}_1\,\Omega^{(-)}_{\mathbb{P}_1}(E)\,\mathrm{Im}\,G^{(+)}_{\mathbb{P}_1}(E)\,\Omega^{(-)\dagger}_{\mathbb{P}_1}(E)\,\mathbb{P}_1 V' p_1\,|p_1\Psi^{(+)}\rangle\Big]$$

$$+\Big[-\frac{2}{v}\langle p_1\Psi^{(+)}|\,\mathrm{Im}\,\mathcal{U}_{p_1} + p_1 V'\mathbb{P}_1\,\mathcal{G}^{(+)\dagger}_{\mathbb{P}_1}(E)\,\mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1}\,\mathcal{G}^{(+)}_{\mathbb{P}_1}(E)\,\mathbb{P}_1 V' p_1\,|p_1\Psi^{(+)}\rangle\Big] \quad (8)$$

We call the first term in Eq. (8), the total summed inelastic cross section, σ_D, as that what it precisely is, since $\Omega^{(-)}_{\mathbb{P}_1}\,\mathrm{Im}\,G^{(+)}_{\mathbb{P}_1}\,\Omega^{(-)\dagger}_{\mathbb{P}_1} = -\pi \sum\!\!\!\!\!\!\int_j |\Phi^{(-)}_j(E_q)\rangle\langle\Phi^{(-)}_j(E_q)|\,\delta(E-E_q-Q_j)\frac{d^3q}{(2\pi)^3}$ which when used, gives for $\sigma_D = \frac{2\pi}{v}\sum\!\!\!\!\!\!\int_j |\langle\Phi^{(-)}_j|\mathbb{P}_1 V' p_1|p_1\Psi^{(+)}\rangle|^2\,\delta(E-E_q-Q_j)\frac{d^3q}{(2\pi)^3}$. Therefore we identify the total fusion cross section to be $\sigma_F = \sigma_R - \sigma_D$,

$$\sigma_F = -\frac{2}{v}\langle p_1\Psi^{(+)}|\,\mathrm{Im}\,\mathcal{U}_{p_1} + p_1 V'\mathbb{P}_1\,\mathcal{G}^{(+)\dagger}_{\mathbb{P}_1}\,\mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1}\,\mathcal{G}^{(+)}_{\mathbb{P}_1}\,\mathbb{P}_1 V' p_1\,|p_1\Psi^{(+)}\rangle . \quad (9)$$

Equation (9) above clearly exhibits the multi-step nature of σ_F when strongly coupled direct channels are involved, as exemplified by the second term. This can be made more clear by first writing for $|p_1\Psi^{(+)}\rangle$, which of course, by definition, contains the effect of channel-coupling to all orders,

$$|p_1\Psi^{(+)}\rangle = \Omega'^{(+)}_{p_1}\,|p_1\mathring{\Psi}^{(+)}\rangle \quad (10)$$

where $\Omega'^{(+)}_{p_1} = p_1 + \mathcal{G}^{(+)}_{p_1} p_1 V' \mathbb{P}_1\,\mathcal{G}^{(+)}_{\mathbb{P}_1}\,\mathbb{P}_1 V' p_1 + \cdots$, $|p_1\mathring{\Psi}^{(+)}\rangle$ is the entrance channel wavefunction in the limit $\mathbb{P}_1 V' p_1 = 0$, and using

$$\mathcal{G}^{(+)}_{\mathbb{P}_1}\,\mathbb{P}_1 V' p_1\,\Omega'^{(+)}_{p_1} = \mathscr{G}^{(+)}_{\mathbb{P}_1}\,\mathbb{P}_1 V' p_1 \quad (11)$$

In Eq. (11) $\mathscr{G}^{(+)}_{\mathbb{P}_1}(E)$ is the full (exact) $\mathbb{P}_1$-matrix propagator which contains the effect of the coupling $\mathbb{P}_1 V' p_1$, to all orders.

Thus

$$\sigma_F = -\frac{2}{v}\langle \mathcal{P}_1\Psi^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathcal{P}_1} | \mathcal{P}_1\Psi^{(+)}\rangle$$
$$-\frac{2}{v}\langle \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)} | \mathcal{P}_1 V' \mathbb{P}_1 \mathcal{G}^{(+)\dagger}_{\mathbb{P}_1}(E)\,\mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1}\mathcal{G}^{(+)}_{\mathbb{P}_1}(E)\mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle \quad (12)$$

Let us introduce now the fusion cross section in the absence of channel coupling, $\overset{\circ}{\sigma}_F = -\frac{2}{v}\langle \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathcal{P}_1} | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle$. Then the first term in Eq. (12), would in general be smaller than σ_F, due to the fact that $\left|\mathcal{P}_1\Psi^{(+)}\right\rangle| < \left|\mathcal{P}_1\overset{\circ}{\Psi}_1\right\rangle|$, owing to loss of flux form $\mathcal{P}_1$ to $\mathbb{P}_1$ due to channel coupling in $|\mathcal{P}_1\Psi^{(+)}\rangle$. However, if there are several strongly coupled channels present in $\mathbb{P}_1$, the second term in Eq. (12) could more than compensate for this reduction in the value of the first term, giving an over-all enhancement in σ_F.

To show this explicitly, we assume that $\mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1}$ is diagonal, $\langle \Psi_j^{(+)}(\vec{q}) | \mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1} | \Psi_{j'}^{(+)}(\vec{q}')\rangle = \delta_{jj'}\delta(\vec{q}-\vec{q}')\langle \Psi_j^{(+)}(\vec{q}) | \mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1} | \Psi_j^{(+)}(\vec{q})\rangle$ and consider only the on-energy-shell parts of $\mathcal{G}^{(+)}_{\mathbb{P}_1}(E)$ and $\mathcal{G}^{(+)\dagger}_{\mathbb{P}_1}(E)$. We obtain

$$\sigma_F = -\frac{2}{v}\langle \mathcal{P}_1\Psi^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathcal{P}_1} | \mathcal{P}_1\Psi^{(+)}\rangle$$
$$-\frac{2\pi^2}{v}\sum_j\int\frac{d^3q}{(2\pi)^3}\langle \mathbb{P}_1\Psi_j^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1} | \mathbb{P}_1\Psi_j^{(+)}\rangle\,\delta(E-E_j-Q_j)\,\left|\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}} | \mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle\right|^2 \quad (13)$$

where we have implicitly diagonalized $\mathcal{G}^{(+)}_{\mathbb{P}_1}(E)$. Calling $\sigma_F(1) = -\langle \mathcal{P}_1\Psi^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathcal{P}_1} | \mathcal{P}_1\Psi^{(+)}\rangle\frac{2}{v}$ and $\sigma_F(j) = -\langle \mathbb{P}_1\Psi_j^{(+)} | \mathrm{Im}\,\mathcal{U}_{\mathbb{P}_1} | \mathbb{P}_1\Psi_j^{(+)}\rangle\frac{2}{v_q}$ we may recast Eq. (13) into the following more attractive form

$$\sigma_F = \sigma_F(1) + \sum_j \mathcal{P}_{1j}\,\sigma_F(j) \quad (14)$$

where

$$\mathcal{P}_{1j}\,\sigma_F(j) \equiv \frac{\pi^2}{k}\int q\frac{d^3q}{(2\pi)^3}\delta(E-E_j-Q_j)\left|\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}}(q) | \mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle\right|^2\sigma_F(j;q)$$
$$k = \frac{\mu v}{\hbar}.$$

The factor q appearing in the integrand comes about as a result of our definition of σ_R and σ_F, Eqs. (6) and (9). The $\mathcal{P}_{1j}$ are not the usual inelastic probabilities since they are constructed from the matrix element $\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}} | \mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle$ and not $\langle \mathbb{P}_1\Psi_j^{(-)} | \mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}\rangle$. The difference between the two matrix elements resides in the fact that $|\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}} | \mathbb{P}_1\Psi_j^{(-)}\rangle|$ is different from unity because of absorption in the j,th eigenchannel. Actually $\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}} | \mathbb{P}_1\Psi_j^{(-)}\rangle$ is just the inverse of the elastic S-matrix element of the j,th eigenchannel, $(S^{-1})_{jj}$, and thus it could be quite larger than 1 under conditions of strong absorption[10].

Though obtained using the on-energy-shell approximation for $\mathbb{P}_1$-channels Green function, Eq. (14) could very well be more general. In fact numerical studies done by the U. of Texas group on related approximations in incomplete heavy ion fusion reactions[11], have shown that the off-shell part of the channel Green function, in cases where Coulomb repulsion is strong, is practically identical to the on-shell part. Thus, to take into account the off-shell propagation effects, it suffices to multiply the second term of Eq. (13) by a factor of 4. Therefore, with P_{1j} given approximately by

$$\mathcal{P}_{1j} = \frac{4\pi^2}{k}\left|\langle \widetilde{\mathbb{P}_1\Psi_j^{(+)}}(q) | \mathbb{P}_1 V' \mathcal{P}_1 | \mathcal{P}_1\overset{\circ}{\Psi}{}^{(+)}(k)\rangle\right|^2 q \quad (15)$$

Eq. (14) supplies a reasonable expression for the heavy-ion fusion cross section which contains explicitly the effects of coupled channels. The enhancement in σ_F over the one-channel barrier penetration value, σ_F, is quite apparent in Eq. (14).

For all practical purposes, Eq. (14) is identical to Eq. (1), discussed in Ref. 5). We consider our theory, however, more advantageous, as it supplies a rather well-defined and simple expression for the P_{1j}, which can be calculated from, e.g. a DWBA code, after paying due attention to the differences in the matrix elements by multiplying the final result by S_{jj}^{-1}.

We emphasize that features such as the presence of different "eigen-barriers" discussed in Refs. (5) and (6), are certainly present in our $\sigma_F(j)$. This is the case, since by definition, these fusion cross sections are calculated from the <u>exact</u> eigenchannel wave-function $|\mathbb{P}_1\Psi_j^{(+)}\rangle$ which contains the effect of the direct channel coupling to all orders and thus must be dominated by the physics of the penetration of an effective <u>eigen-barrier</u>.

It is worth mentioning at this point that our theory is not restricted to sub-barrier fusion. We endevour to suggest that the directly coupled channels are always present in σ_F. They seem to have noticeable effect at sub-barrier energies, where small changes in the eigen-barriers seem to lead to large effects (enhancement) in σ_F, and at higher energies, where many inelastic channels (and particle transfer) start competing with fusion. This is the region in energy, usually referred to as region II when one finds $\frac{\sigma_F}{\sigma_R}$ becoming smaller than unity as the energy is increased. At these relatively higher energies quantum barrier penetration effects become unimportant and purely geometrical features dominate σ_F.

Using such a geometrical picture of $\sigma_F(j)$, in the sense

$$\sigma_F(j) = \pi R_j^2 \left[\frac{E - Q_j - V_j}{E - Q_j}\right] \tag{16}$$

where R_j, Q_j and V_j are the radius, Q-value and effective barrier of the jth. eigenchannel, we envisage a simple expression for σ_F, from our Eq. (14), after treating the P_{1j} statistically, in the sense of, e.g., Ref. 13),

$$\sigma_F \cong \pi \overline{R^2} \left[1 - \frac{\overline{V}}{E - \overline{\Delta E}}\right] \tag{17}$$

where, the average quantities $\overline{R^2}$, $\overline{V}$ and $\overline{\Delta E}$ are in general functions of the center of mass energy, E.

It is our opinion that a consistent parametrization of σ_F must involve a minimum of three parameters; an average radius parameter, $(\overline{R^2})^{1/2}$, an average fusion barrier, $\overline{V}$ and an average energy loss, $\overline{\Delta E}$.

In conclusion, we have formally analysed the reactive content of the heavy-ion fusion cross section and derived a simple expression that shows clearly the effect of directly coupled channels. Our unified reaction theory of heavy-ion fusion, may help devise approximation procedures to be used in the analysis of e.g. sub-barrier data that exhibit marked deviation from the prediction of one-dimensional barrier penetration model calculation. It is hoped that the discussion presented in this Letter would help unify the different theoretical approaches to the above, as well as to other, fusion problems.

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