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by

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Abstract

The total reaction cross section for intermediate energy nucleon-nucleus scattering systems is calculated within the Dirac-eikonal formalism. Comparison with data indicates that the recently proposed impulse-approximation Dirac optical potential for nucleon-nucleus scattering, is not absorptive enough.

I - Introduction

In the last few years the Dirac equation used with a mixture of phenomenological scalar and vector interaction, has been shown to provide a greatly improved starting point for understanding intermediate energy proton-nucleus scattering[1]. Calculation based on an impulse-approximation optical potential gave excellent agreement with data on elastic scattering differential cross-section, spin polarization and spin rotation for systems such as 500 Mev $p_{pol} + {}^{16}O$ and ${}^{40}Ca$.

Parallel to the above developments several attempts to derive the relativistic nucleon-nucleus optical potential have been made. These range from extending the familiar impulse approximation "$t\rho$" - type derivation to include explicitly scalar and vector components[3], to more ambitious plans starting from a relativistic many-body field theory of interacting nucleons and mesons. One such theory, which is extensively cited, is that of Walecka[4]. Though originally constructed to describe nuclear matter as a system of interacting nucleons and isoscalar scalar and vector mesons, it is also adequate for the description of spin saturated, isoscalar (closed-shell) nuclei such as ${}^{16}O$ and ${}^{40}Ca$. The inclusion of the isovector π and ρ mesons in the theory was subsequently performed by Serot[5]. In most of the applications of the theory, special emphasis was placed on deriving the real part of the nucleon-nucleus optical potential (the single particle potential).

In a recent work Horowitz[6], calculated the relativistic imaginary potential to lowest order in nuclear matter for the exchange of σ, ω and π-mesons. Of course the relativistic "$t\rho$" potential refered to earlier does supply a well-defined imaginary potential, which is directly related to the scalar and the time component of the vector nuclear density. It would be important to check these potentials in a direct way.

An important observable quantity that is directly related to the imaginary part of the optical potential is the total reaction cross section σ_R. Though obtainable from an optical model analysis of the elastic scattering data, it is, nevertheless, of value to calculate σ_R directly. Such a calculation would supply a further test of the adequacy of the theoretical imaginary potential and help analyzing its reactive content.

The propose of the present paper is to develop a theory of σ_R within a Dirac description of the elastic scattering of nucleons of nuclei. We use the eikonal approximation in our discussion of the nucleon-nucleus elastic scattering amplitude. Such a Dirac-eikonal approximation has recently been put forward by Amado et.al[7] and Friar and Wallace[8].

The paper is organized as follows. In Section II we present a denivation of σ_R from the Dirac equation that describes the scattering of nucleons from nuclei affected by complex scalar and vector interactions. We then use the eikonal model in Section II to express σ_R in terms of an impact parameter integral involving relativistic nuclear transmission coefficients. In Section IV we present the results of our calculation of σ_R for $p + {}^{40}Ca$ and $p + {}^{208}Pb$ in the energy range $100 < E_p < 1000$ Mev, and make a comparison with the data, as well as with the non-relativistic calculation of Digiacomo, De Vries and Peng[8]. Finally, in Section V, we present several concluding remarks.

II - The Total Reaction Cross Section Obtained From The Dirac Equation

The Dirac equation that describe the elastic scattering of a nucleon, treated as a Dirac particle, from a spin-saturated nucleus, is usually written in the form, using a time-independent description,

$$\left[\vec{\alpha}\cdot\vec{p} + \beta(m+V_s) + V_o \right] \psi = E\,\psi \qquad (1)$$

where it is assumed that the average, complex, nucleon-nucleus potential is a sum of a scalar component, V_S , and the fourth (time) component of a vector potential, V_0 . The matrices $\vec{\alpha}$ and β are Dirac's, and ψ is the four-component vector wave-function.

Let us write V_S and V_0 as

$$V_S = U_S - i\,W_S$$

$$V_o = U_o - i\,W_o \qquad (2)$$

Equation (1) can be rewritten as

$$\left[\gamma_4(E - V_o) - i\,\vec{\gamma}\cdot\vec{p} - (m + V_s)\right] \psi = 0 \qquad (3)$$

obtained from the usual relations, $i\vec{\gamma} \equiv \gamma_4\vec{\alpha}$, $\gamma_4 \equiv \beta$

We now perform the usual manipulations of multiplying Eq.(3) from the left by $\bar{\psi} = \psi^{\dagger}\gamma_4$ and constructing its conjugate with the subsequent multiplication from the left by ψ, to obtain finally

$$\bar{\psi}\left[(\gamma_4(E - V_o) - i\,\vec{\gamma}\cdot\vec{p} - (m + V_s) \right] \psi = 0 \qquad (4)$$

$$\bar{\psi}\left[(\gamma_4(E - V_o^{+}) - i\,\vec{\gamma}\cdot\vec{p} - (m + V_s^{+}) \right] \psi = 0 \qquad (5)$$

the usual Wronskian argument now supplies us with the continuity equation

$$-\vec{\nabla}\cdot\vec{j} = \frac{2}{\hbar}\left(\psi^{\dagger} W_o \psi + \psi^{\dagger}\gamma_4 W_s \psi \right) \qquad (6)$$

with

$$\vec{j} = i\,\bar{\psi}\,\vec{\gamma}\,\psi \qquad (7)$$

the hadronic current.

Interating Eq.(6) over a large volume and using Gauss's theorem, gives us

$$-\int_S \vec{j}\cdot d\vec{A} = \frac{2}{\hbar}\langle \psi^{(+)} | (W_o + \gamma_4 W_S) | \psi^{(+)} \rangle \qquad (8)$$

where the integral is over a surface surrounding the potential, in a region where the potential has completely vanished, and describes the net inward flux due to absorption $(W_o \neq 0 , W_S \neq 0)$.

Dividing this flux by the incident current $\dfrac{v}{(1-(v/c)^2)^{1/2}} \equiv v\gamma$ (assuming that $\psi^{(+)}$ in normalized to unity), gives the total reaction cross section

$$\sigma_R \equiv \frac{-\int_{S\to\infty} \vec{j}\cdot d\vec{A}}{v\gamma} = \frac{2}{\hbar v\gamma}\langle \psi^{(+)} | W_o + \gamma_4 W_S | \psi^{(+)} \rangle \qquad (9)$$

We remind the reader again that $\psi^{(+)}$ is a scattering vector wave function.

Equation (9) can be further reduced to a form more convenient for numerical evaluation. We do this by explicitly writing $\psi^{(+)}$ in terms of its upper (large) and lower (small) components,

$$\psi^{(+)} = \left(\frac{E+m}{2m}\right)^{1/2} \begin{pmatrix} 1 \\ \frac{1}{A}\vec{\sigma}\cdot\vec{p} \end{pmatrix} u_S \qquad (10)$$

where

$A = E + m + V_S - V_0$, and u_s satisfies the reduced Dirac equation

$$\left(\vec{\sigma}\cdot\vec{p}\ \frac{1}{A}\ \vec{\sigma}\cdot\vec{p}\ - E - m - V_S - V_0\right) u_s = 0 \tag{11}$$

with Eq.(10), σ_R , Eq.(9), becomes

$$\sigma_R = \frac{E+m}{\hbar v \gamma m}\left[\int d^3r \left(W_0 + W_s\right) u_s^\dagger u_s - \left(W_s - W_0\right)\left(\frac{1}{A}\vec{\sigma}\cdot\vec{p}\, u_s\right)^\dagger \left(\frac{1}{A}\vec{\sigma}\cdot\vec{p}\, u_s\right)\right] \tag{12}$$

Using the fact that $(W_s - W_0)/|A|^2 = \frac{1}{2i}\left(A^{-1} - A^{\dagger -1}\right)$, we can, after performing one integration by parts ad using Gauss' theorem on the second term on the RHS of Eq.(12), write for σ_R the following surface integral

$$\sigma_R = -\frac{1+\gamma}{2v\gamma}\int_S dA \left[u_s^\dagger \left(\frac{\vec{\sigma}\cdot\hat{n}}{A}\ \vec{\sigma}\,\vec{p}\, u_s\right) + \left(\frac{\vec{\sigma}\cdot\vec{n}}{A}\ \vec{\sigma}\cdot\vec{p}\, u_s\right)^\dagger u_s\right] \tag{13}$$

which reduces, in the appropriate $S \to \infty$ limit, where $A \to E + m = (1+\gamma)m$, to the final expression

$$\sigma_R = -\frac{1}{m v \gamma}\int_{S\to\infty} dA\ \mathrm{Re}\left(u_s^\dagger\, \vec{\sigma}\cdot\hat{n}\ \vec{\sigma}\cdot\vec{p}\ u_s\right) \tag{14}$$

Eq.(14) could have been obtained directly from the first part of Eq.(9), namely from the identification $\sigma_R = \frac{-1}{\gamma v}\int_{S\to\infty} d\vec{A}\cdot\vec{J}$. Our derivation above serves as a check of the correctness of Eq.(9). In the next section, we evaluate Eq.(14) in the eikonal (small-angle) limit.

III – The Total Reaction Cross Section In The Dirac Eikonal Approximation

The eikonal approximation to $\psi^{(+)}$ or u_s of Equation (1) Or (11), has been recently discussed by Amado et.al . In this section we derive an eikonal form for σ_R, starting with Eq.(14). We follow the notation of Ref.(7).

Within the eikonal approximation to u_s, we have, as $r \to \infty$,

$$\vec{p}\, u_s \xrightarrow[r\to\infty]{} m v \gamma\ \hat{z}\ u_s \tag{15}$$

Using Eq.(15) in Eq.(14), we obtain

$$\sigma_R = -\int_{S\to\infty} dA\ u_s^\dagger\ \hat{n}\cdot\hat{z}\ u_s \tag{16}$$

Since S is any large surface surrounding the interaction potential, we may take for it two planes perpendicular to the z-axis at $z = \pm\infty$. We thus have

$$\sigma_R = \int d^2b \left[|u_s|^2(\vec{b}, z\to-\infty) - |u_s|^2(\vec{b}, z\to+\infty)\right] \tag{17}$$

Equation (17) exhibits very nicely the physical meaning of in terms of the probability densities $|u_s|^2(\vec{b}, z\to-\infty)$ and $|u_s|^2(\vec{b}, z\to\infty)$.

Using the usual substitution for the upper component

$$u^{(+)}_{\vec{k},s} = e^{i\vec{k}\cdot\vec{r}}\ e^{iS(\vec{r})}\ \chi_s \tag{18}$$

Where χ_s are Dirac spinors and $S(\vec{r})$ satisfies the differential equation

$$\vec{k}\cdot\vec{\nabla}\ S(\vec{r}) = -m\left\{V_c(r) + V_{so}(r)\left[\vec{\sigma}\cdot\vec{r}\times\vec{k} - i\,\vec{r}\cdot\vec{k}\right]\right\} \tag{19}$$

In eq.(19) the central, $V_c(r)$ and spin-orbit, V_{so} interaction are given by (see Eq. (11))

$$V_c(r) = V_S(r) + \frac{E}{m} V_0(r) + \frac{V_S^2(r) - V_0^2(r)}{2m} \tag{20}$$

$$V_{SO}(r) = \frac{1}{2mA} \frac{1}{r} \frac{d}{dr}\left(V_0(r) - V_S(r)\right) \tag{21}$$

and $\vec{k} = \frac{1}{2}(\vec{k} + \vec{k}')$, the average of the initial and final momenta.

Defining the z-axis to be along the direction of $\vec{k}$, the eikonal phase $S(\vec{r})$, can be written as

$$i S(\vec{r}) = -\frac{m}{k}\int_{-\infty}^{z} dz' \left\{ V_c(\vec{b}, z') + V_{so}(\vec{b}, z')\left[\vec{\sigma}\cdot\vec{b}\times\vec{k} - i k z'\right]\right\} \tag{22}$$

Using Eq.(22) in Eq.(18), we can write down immediately for $|u_s|^2(\vec{b}, z)$ the following

$$|u_s|^2(\vec{b}, z) = \chi_s^{\dagger} \exp\left[-2\, \mathrm{Im}\, S(\vec{b}, z)\right] \chi_s \tag{23}$$

We remind the reader that S is an operator in spin space.

Let us introduce the quantities

$$F = E - V_0(\vec{b}, z) \tag{24}$$

$$N = m - V_S(\vec{b}, z) \tag{25}$$

Then Eqs.(17), (20) and (21) give us for

$$\sigma_R = \chi_s^{\dagger}\left[\int d^2b \left(1 - e^{\phi(\vec{b})}\right)\right]\chi_s \tag{26}$$

where

$$\phi(\vec{b}) \equiv \phi_c(\vec{b}) - \phi_{so}(\vec{b})\, \vec{\sigma}\cdot(\hat{b}\times\hat{k}) \tag{27}$$

and

$$\phi_c(\vec{b}) = \frac{1}{(\hbar c)^2 k}\int_{-\infty}^{+\infty} dz\, \mathrm{Im}\left(N^2 - F^2\right) \tag{28}$$

$$\phi_{so}(\vec{b}) = b \int_{-\infty}^{+\infty} dz\, \frac{1}{r}\, \mathrm{Im}\left[\frac{1}{F+N}\frac{\partial}{\partial r}(F+N)\right] \tag{29}$$

At this point it is worth mentioning that the quantities $\phi_c(\vec{b})$ and $\phi_{so}(\vec{b})$, are related to the thickness functions, $t_c(\vec{b})$ and $t_{so}(\vec{b})$ of Amado et. al[7], defined by

$$t_c(b) = \frac{-i}{2(\hbar c)^2 k}\int_{-\infty}^{+\infty} dz' \left(N^2 - F^2 + E^2 - m^2\right) \tag{30}$$

$$t_{so}(b) = \frac{-ib}{2}\int_{-\infty}^{+\infty} dz' \frac{1}{F+N}\frac{1}{r}\frac{\partial}{\partial r}(F+N) \tag{31}$$

Thus

$$\phi_c(\vec{b}) = 2\, \mathrm{Re}\, t_c(b) \tag{32}$$

$$\phi_{so}(\vec{b}) = 2\, \mathrm{Re}\, t_{so}(b) \tag{33}$$

Going back to Eq.(26), we note first that we can write it as

$$\sigma_R = \chi_s^{\dagger}\left[\int d^2b \left(1 - e^{\phi_c(b)} e^{-\phi_{so}(b)\, \vec{\sigma}\cdot\hat{b}\times\hat{k}}\right)\right]\chi_s$$

$$= \chi_s^{+}\left[\int d^2b\left(1-e^{\phi_c(b)}\cosh\phi_{so}(b)+\vec{\sigma}\cdot(\hat{b}\times\hat{k})e^{\phi_c(b)}\cdot\sinh\phi_{so}(b)\right)\right]\chi_s$$

$$= 2\pi\,\chi_s^{+}\left[\int b\,db\left(1-e^{\phi_c(b)}\cosh\phi_{so}(b)\right)\right]\chi_s$$

$$= 2\pi\int b\,db\left(1-e^{\phi_c(b)}\cosh\phi_{so}(b)\right) \tag{34}$$

The term involving $\vec{\sigma}\cdot\hat{b}\times\hat{k}$ does not contribute to the b-integral due to symmetry about the z-axis. Eq.(34) can also be written as (Eqs.(32) and (33))

$$\sigma_R = 2\pi\int b\,db\left(1-e^{2\mathrm{Re}\,t_c(b)}\cosh 2\mathrm{Re}\,t_{so}(b)\right) \tag{35}$$

Equation (35) is the principal result of this section. It expresses σ_R in the usual form of an impact parameter integral involving "relativistic" transmission coefficients given by

$$T(b) = 1 - e^{2\mathrm{Re}\,t_c(b)}\cosh 2\,\mathrm{Re}\,t_{so}(b) \tag{36}$$

It is clear that the exact form and details of $T(b)$ would be irrelevant if the nucleon-nucleus scattering is dominated by a black disk-type absorption. In such a case $T(b)$ would be representable as

$$T(b) = \Theta\,(b - R_c) \tag{37}$$

where Θ is the step function, and R_c is a characteristic absorption radius. If Eq.(37) is used, σ_R becomes the simple geometrical limit,

$$\sigma_R = \pi R_c^2 \tag{38}$$

If the above were true, not too much physics would be extracted from σ_R. Luckily total reaction cross section data of proton-nucleus systems at intermediate energies exhibit major deviations from the black disk result Eq.(38). Nuclei become quite transparent to nucleons at intermediate energies [9], and the quantity that measures this nuclear transparency in details is given by $T(b)$ of Eq.(36). Therefore detailed evaluation and discussion of $T(b)$ and the resulting σ_R is clearly called for. This has been done using the conventional non-relativistic theory by Digiacomo, De Vries and Peng. In the next section we present our result for and discussion of σ_R within the Dirac-eikonal treatment presented in this section.

Before ending this section, we warn the reader that $t_c(b)$ is ill-defined for proton scattering because of the presence of the long range Coulomb potential which is present in $V_0(r)$. This difficulty can be dealt with easily by some appropriate modification of the integral involved. In Appendix I we present the details. Here we only cite the final Coulomb-midified, but finite, σ_R

$$\sigma_R = 2\pi\int db\,b\left(1-e^{2\mathrm{Re}\,\tilde{t}_c(b)}\cosh 2\,\mathrm{Re}\,t_{so}(b)\right) \tag{39}$$

$$\tilde{t}_c(b) = \frac{-i}{2(\hbar c)^2 k}\int_{-\infty}^{+\infty} dz\left(N^2 - F^2 + E^2 - m^2 - \frac{2EZ_1Z_2e^2}{r(b,z)}\right) \tag{40}$$

Finally, a word about the optical theorem and its generalized version for charged particle scattering. For neutral particles the usual form of the optical theorem

$$\sigma_R = \frac{4\pi}{k}\,\mathrm{Im}\,F(\vec{k},\vec{k};E) - \int |F(\vec{k},\vec{k}';E)|^2\,d\Omega_{\hat{k}'} \tag{41}$$

should yield the correct expression for σ_R . In fact, with the elastic scattering amplitude $F(\vec{k}',\vec{k}';E)$ derived by Amado et. al[7].

$$F(\vec{k},\vec{k}';E) = F_1 + \vec{\sigma}\cdot\hat{n}\,F_2 \tag{42}$$

$$F_1 = -i\,k\int_0^\infty db\; b\; J_0(qb)\left(e^{t_c(b)}\cosh t_{so}(b) - 1\right) \tag{43}$$

$$F_2 = -\,k\int_0^\infty db\; b\; J_1(qb)\, e^{t_c(b)}\sinh t_{so}(b) \tag{44}$$

$\vec{q}\equiv|\vec{k}-\vec{k}'|$, and J_0 and J_1, are ordinary Bessel functions, Eq.(41) results in exactly the expression for σ_R given in Eq. (35).

For charged particle scattering, Eq. (41) yields infinite values for both terms on the RHS. However a generalized optical theorem can be derived for the purpose and it does supply a means of calculating σ_R ,

$$\sigma_R = \frac{4\pi}{k}\,\mathrm{Im}\left[F(\vec{k},\vec{k};E) - F_c(\vec{k},\vec{k};E)\right] \tag{45}$$

$$-\int\left[|F_c(\vec{k}\,\vec{k}';E)|^2 - |F(\vec{k},\vec{k}';E)|^2\right]d\Omega$$

where F_c is the point Coulomb scattering amplitude. In a way, the procedure we employ in the Appendix amounts to basically calculating the difference $F-F_c$ in the form of an impact-parameter integral, which yields completely convergent results.

IV - <u>Numerical Results</u>

In this section we present the results of our calculation of σ_R, Eq. (39), for $p+{}^{40}Ca$ and $p+{}^{208}Pb$, in the proton energy range

$10 < E_p < 1000$ MeV . We take for the proton-nucleus optical potential, the impulse-approximation Dirac optical interaction for spin-saturated nuclei, has the general form[3]

$$\langle\vec{k}'|U_{oo}|\vec{k}\rangle = -\frac{4\pi i k}{m}\left[F_S(q)\rho_S(q) + \gamma_4^0 F_V(q)\rho_V(q)\right] \tag{46}$$

$$\equiv V_S(q) + V_0(q)$$

In Eq. (46), F_S and F_V are the scalar and vector pieces of the Lorentz-invariant $N-N$ amplitude, respectively, and ρ_S and ρ_V are the scalar and vector form factors of the target nucleus, given by

$$\rho_S(q) = \langle 0|\sum_i e^{i\vec{q}\cdot\vec{r}_i}|0\rangle \tag{47}$$

$$\rho_V(q) = \langle 0|\sum_i \gamma_4^i\, e^{i\vec{q}\cdot\vec{r}_i}|0\rangle \tag{48}$$

The above densities can be better visualized when written in configuration space,

$$\rho_S(r) = \langle 0|\sum_i \gamma_4^i\,\delta(\vec{r}-\vec{r}_i)|0\rangle \equiv \sum_\alpha^{occ}\bar{\psi}_\alpha(r)\,\psi_\alpha(r) \tag{49}$$

$$\rho_V(r) = \langle 0|\sum_i \delta(\vec{r}-\vec{r}_i)|0\rangle \equiv \sum_\alpha^{occ}\psi_\alpha^\dagger(r)\,\psi_\alpha(r) \tag{50}$$

where the find α-sums are over occupied single particle states. Writing in terms of its upper and lower components,

$$\psi_\alpha = \begin{pmatrix}\psi_\alpha^U\\ \psi_\alpha^L\end{pmatrix}\quad,\quad \bar{\psi} = \left(\psi_\alpha^{U*},\; -\psi_\alpha^{L*}\right) \tag{51}$$

we can express $\rho_S(r)$ and $\rho_V(r)$ as

$$\rho_S(r) = \sum_{\alpha}^{occ} |\psi_\alpha^U(r)|^2 - \sum_{\alpha}^{occ} |\psi_\alpha^U(r)|^2 \equiv \rho_U(r) - \rho_L(r) \tag{52}$$

$$\rho_V(r) = \sum_{\alpha}^{occ} |\psi_\alpha^U(r)|^2 + \sum_{\alpha}^{occ} |\psi_\alpha^L(r)|^2 \equiv \rho_U(r) + \rho_L(r) \tag{53}$$

Therefore, the difference $\rho_V(r) - \rho_S(r)$, measures the strength of the lower component density $2\rho_L(r)$, and accordingly, the degree to which the optical potential is relativistic.

The potential calculated bv McNeil et. al., is obtained by setting $F_S(q) \simeq F_S(0)$ and $F_V(q) \simeq F_V(0)$ in Eq. (46). In this limit, which is quite reasonable in the energy range considered, the Fourier transform of Eq. (46) yields a local potential in configuration space, with its r-dependence completely specified by $\rho_S(r)$ and $\rho_V(r)$. We therefore write

$$V_S(r) = V_S^0(E)\,\hat{\rho}_S(r) = \left(U_S^0(E) - i W_S^0(E)\right)\hat{\rho}_S(r) \tag{54}$$

$$V_0(r) = V_0^0(E)\,\hat{\rho}_V(r) = \left(U_0^0(E) - i W_0^0(E)\right)\hat{\rho}_V(r) \tag{55}$$

where ρ_S and ρ_V represent the shape of the densities and they are both normalized to unity in the central region. McNeil et. al.[3] presented their results for $U_S^0(E)$, $W_S^0(E)$, $U_0^0(E)$ and $W_0^0(E)$ at a radius where ρ_S and ρ_V are both $0.16\,fm^{-3}$. These values of the densities, correspond to a Fermi momentum, $k_F = 1.37\,fm^{-1}$. It is found that W_S^0 is negative, implying, using our convention, Eq. (2), that the scalar interaction is regenerative whereas the vector one is absorptive. Their values come out comparable, with

$W_0^0(E)$ a bit larger than W_S^0. All of these results are is accord with phenomenological findings. The above results were also confirmed by Horowitz[6], in his nuclear matter calculation of W_S and W_0.

Armed with the above facts, we evaluated σ_R, Eq. (39), using the results of McNeil et. al. , as presented in their figure 1 . For the density shape of ^{208}Pb we have used Saxon-Wood forms with parameters fixed in accordance with results obtained from electron scattering, which basically supplies ρ_V for protons. We have, however set $\hat{\rho}_S(r) = \hat{\rho}_V(r)$ for all r. The radius, R, and diffuseness, a, parameters for ^{208}Pb, are[10]

$$R = 6.624\,fm, \qquad a = 0.549\,fm.$$

The density shape of ^{40}Ca is usually parametrized as

$$\rho_{^{40}Ca}(r) = \left(1 + \omega r^2/R^2\right) / \left[1 + \exp\,(r - R)/a\right] \tag{56}$$

with $\omega = -0.1017$, $R = 3.669\,fm$, $a = 0.584\,fm$

The results are presented in Figures 1 and 2. It is clear from the figures that the comparison of our σ_R in the energy range 100 < E < 1000 MeV, where one expects our theory based on the impulse and eikonal approximation to be quite adequate, with the data shows only qualitative agreement. There is a clear indication that the impulse-approximation Dirac optical potential is not absorptive enough. The quality of our calculation coincides with those of Di Giacomo et al.[9].

Of course, at lower energies, nuclear medium effects (Pauli blocking, multiple scattering contributions, etc.), which were completely left out here, are certainly important. Thus it is pointless to dwell into a detailed discussion of our result in the energy range 10 < E < 100 MeV.

V - Concluding Remarks

In this paper we have developed a theory for the total reaction cross section of nucleon-nucleus scattering systems at intermediate energies. Our theory is based on the Dirac-equation description of nucleon-nucleus scattering. Our evaluation of the resulting expression for σ_R, was performed using the eikonal approximation to the Dirac scattering amplitude. The comparison of our results, obtained with an impulse-approximation Dirac optical potential, with experimentally deduced σ_R for $p+^{40}Ca$ and $p+^{208}Pb$ convinced us that the impulse-approximation Dirac optical potential (IDOP) is not absorptive enough. The missing absorptive component should be such, as not to affect other observables which are extremely well described by the IDOP, and spin polarization and rotation.

In a future publication we shall extend our calculations to lower energies and to other nuclear systems. Further, the connection between our relativistic transmission coefficient and its non-relativistic counterpart (also obtained from a "tρ"-type interaction), was not studied here and will be dealt with aposteriori. It is hoped that our study will encourage experimentalists to pay more attention to σ_R than it had so far received.

Appendix I

In this appendix, we discuss the modifications on σ_R as well as the elastic scattering amplitude $F(\vec{k},\vec{k}';E)$ needed to render them calculable in the presence of the Coulomb interaction.

The idea, is to write the amplitude, (Eq. (42) as

$$F(\vec{k},\vec{k}';E) = F_C(\vec{k},\vec{k}';E) + \left[F(\vec{k},\vec{k}';E) - F_C(\vec{k};\vec{k}';E)\right] \qquad \text{(AI.1)}$$

Let us call the second term on the RHS of Eq. I.1 $F_N(\vec{k},\vec{k}';E)$. Loosely

speaking F_N represents the nuclear part of F. We now write the closed form expression for the first term in AI.1 and represent F_N as an impact-parameter integral .

Using the eikonal form of F given by Amado et. al.' and expressing F_C in a similar manner we can write

$$F_N = ik \int \frac{d^2b}{2\pi}\, e^{i\vec{q}\cdot\vec{b}} \left(e^{\chi_C(b)} - e^{\chi(b)} \right) \qquad \text{(AI.2)}$$

$$\chi(b) = t_c(b) + t_{SO}(b)\, \vec{\sigma}\cdot(\hat{b}\times\hat{k}) \qquad \text{(AI.3)}$$

and

$$\chi_C = -i\frac{Z_1Z_2e^2E}{(\hbar c)^2 k}\int_{-\infty}^{\infty}\frac{dz}{r(z,b)} = -i\frac{Z_1Z_2e^2E}{(\hbar c)^2 k}\int_{-\infty}^{\infty}\frac{dz}{\sqrt{z^2+b^2}} \qquad \text{(AI.4)}$$

Clearly, the integral in I.4 is infinite. To get a finite result we introduce a screening radius a such that

$$\chi_C(b) = -i\frac{Z_1Z_2e^2E}{(\hbar c)^2 K}\int_{-a}^{a}\frac{dz}{\sqrt{z^2+b^2}}$$

$$\approx -\frac{2iZ_1Z_2e^2E}{(\hbar c)^2 k}\ln\frac{2a}{b} \qquad \text{(AI.5)}$$

$$= -\frac{2iZ_1Z_2e^2E}{(\hbar c)^2 K}\ln\frac{2a}{b_0} + \frac{2iZ_1Z_2e^2E}{(\hbar c)^2 K}\ln\frac{b}{b_0}$$

where b_0 is taken to be 1fm, and is introduced only to make the argument of the ln's dimensionless. Recognizing that the factor $\frac{Z_1Z_2e^2E}{(\hbar c)^2K}$ is just the Sommerfeld parameter, η , we have finally

$$\chi_C(b) = -2i\eta\ln\frac{2a}{b_0} + 2i\eta\ln\frac{b}{b_0} \qquad \text{(AI.6)}$$

Using Eq. (AI.6), Glauber[12] obtained the following closed expression for F_C

$$F_C(\vec{k},\vec{k}';E) = \frac{2\eta k}{q^2}\, e^{-2i\eta \ln \frac{2a}{b_0}} \exp\left[-2i\eta \ln \frac{q b_0}{2} + 2i\sigma_0\right] \tag{AI.7}$$

where

$$\sigma_0 = \arg \Gamma(1+i\eta) \tag{AI.8}$$

Writing for $\chi(b)$, Eq, (AI.3),

$$\chi(b) = \chi(b) - \chi_C(b) + \chi_C(b) \tag{AI.9}$$

$$\equiv \chi_N(b) + \chi_C(b) \tag{AI.10}$$

we express F_N as

$$F_N = i\kappa \int \frac{d^2 b}{2\pi}\, e^{i\vec{q}\cdot\vec{b}}\, e^{\chi_C(b)} \left(1 - e^{\chi_N(b)}\right) \tag{AI.11}$$

$$= i\kappa\, e^{-2i\eta \ln \frac{2a}{b_0}} \int \frac{d^2 b}{2\pi}\, e^{i\vec{q}\cdot\vec{b}}\, e^{2i\eta \ln \frac{b}{b_0}} \left(1 - e^{\chi_N(b)}\right) \tag{AI.12}$$

We see clearly that the constant phase $\exp\left[-2i\eta \ln \frac{2a}{b_0}\right]$ completely factors and both in F_N and F_C and therefore can be dropped altogether. This amounts to taking $a \rightarrow \infty$

The final expression for F and σ, can now be written down straightforwardly

$$F(\vec{k},\vec{k}';E) = \left(\tilde{F}_1 + \vec{\sigma}\cdot\hat{n}\, \tilde{F}_2\right) \tag{AI.13}$$

$$\tilde{F}_1(q) = F_C(q) + i\kappa \int_0^\infty db\, b\, J_0(qb)\, e^{2i\eta \ln \frac{b}{b_0}} \left[1 - e^{\tilde{t}_c(b)} \cosh t_{so}(b)\right] \tag{AI.14}$$

$$\tilde{F}_2(q) = -\kappa \int_0^\infty b\, db\, J_1(qb)\, e^{2i\eta \ln \frac{b}{b_0}}\, e^{\tilde{t}_c(b)} \sinh t_s(b) \tag{AI.15}$$

$$\sigma_R = 2\pi \int_0^\infty b\, db \left(1 - e^{2\,\mathrm{Re}\, \tilde{t}_c(b)} \cosh 2\,\mathrm{Re}\, t_s(b)\right), \tag{AI.16}$$

where $\tilde{t}_c(b)$ is given by

$$\tilde{t}_c(b) \equiv t_c(b) - \chi_C(b) \tag{AI.17}$$

$$= \frac{-i}{2(\hbar c)^2 \kappa} \int_{-\infty}^{\infty} dz \left(N^2 - F^2 + E^2 - m^2 - \frac{2 E Z_1 Z_2 e^2}{r}\right) \tag{AI.18}$$

All integrals appearing in the above formulae are finite.

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Figure Caption

Figure 1. The total reaction cross section for p + ^{40}Ca. The data points were collected from Ref. 11. (See Text for details)

Figure 2. Same as above for p + ^{208}Pb

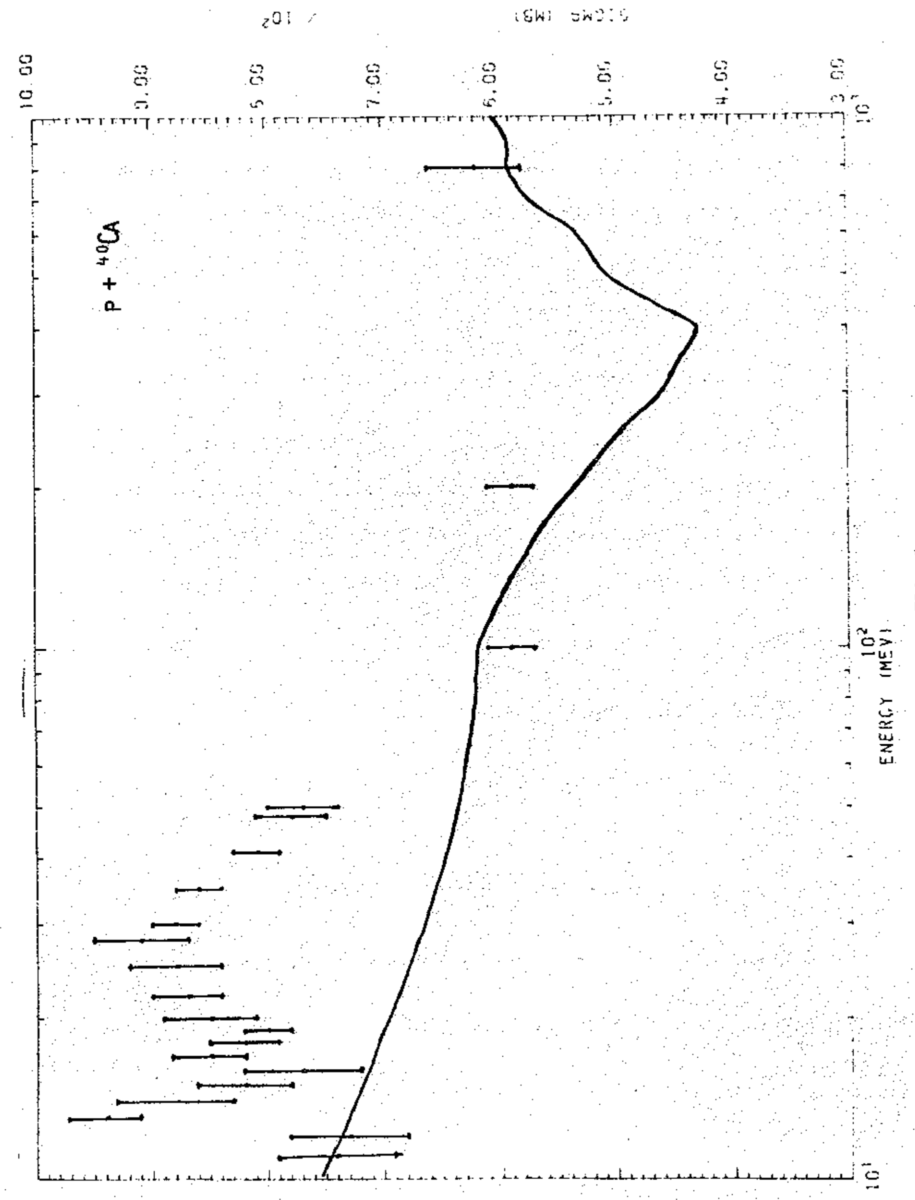

FIG. 1

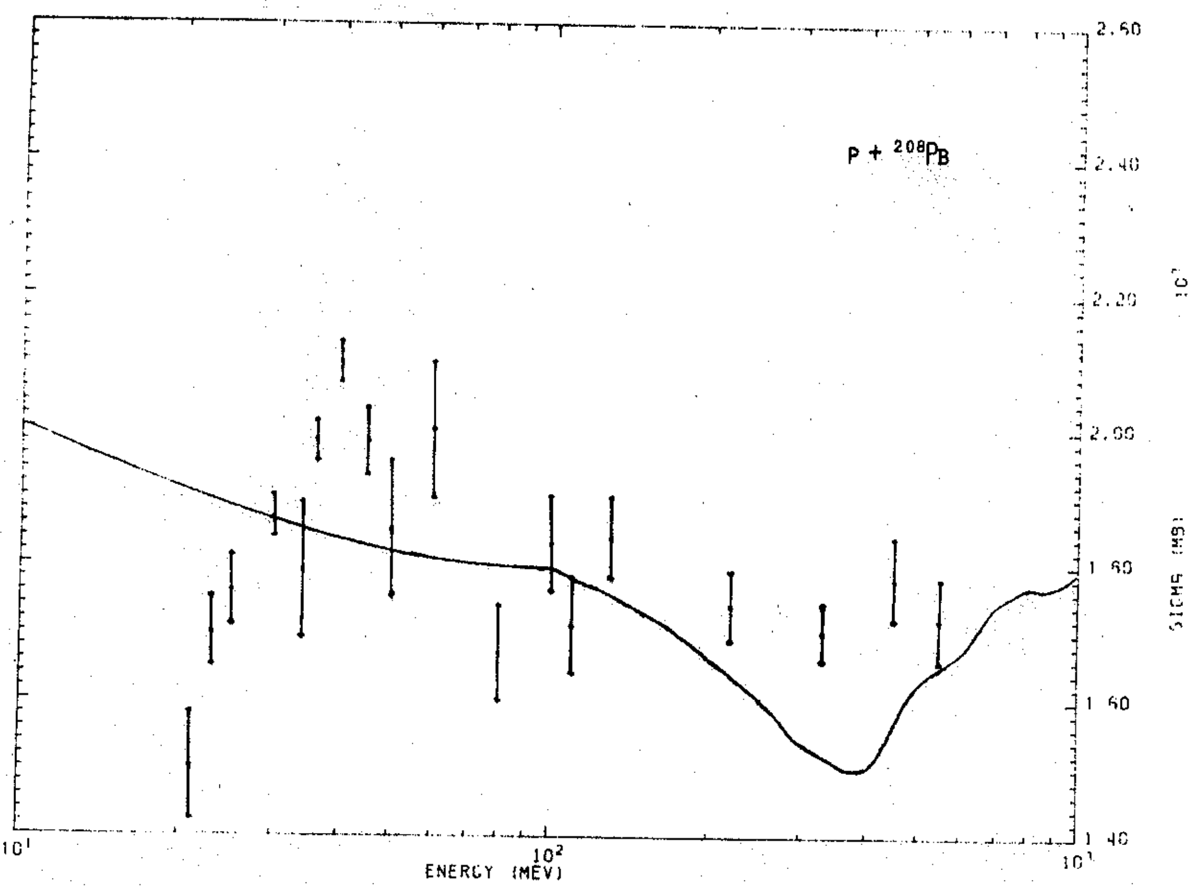
P + 208PB
2.60
2.40
2.20
2.00
1.80
1.60
1.40
SIGMA (MB)
10
10¹
10²
10³
ENERGY (MEV)

FIG. 2