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DYNAMICS OF THE NUCLEAR ONE-BODY DENSITY: SMALL AMPLITUDE REGIME

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ABSTRACT

We present a microscopic treatment for the small amplitude limit of the equations of motion for the nuclear one-body density. These were derived previously by means of projection techniques, and allow for the explicit separation of mean-field and collision effects which result from the dynamics of many-body correlations. The form of the nuclear response in the presence of collision effects is derived. An illustrative application to a soluble model is discussed.

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1. INTRODUCTION

In a previous work[1] (henceforward called I) we studied the dynamics of the one-body density matrix of a many-fermion (nuclear) system, including the effects of many-body correlations. We showed there that correlations can be handled in terms of memory integrals which on the one hand, modify the Hartree-Fock mean field to generate the unitary time-displacement of the one-body density; and which, on the other hand, give rise to a contribution causing the occupation probabilities of the natural orbitals (eigenvectors of the one-body density) to change in time. The latter effect constitutes, therefore, a non-unitary aspect of the effective time evolution. The corresponding contribution has been shown in I to reduce to a collision term of the Uehling-Uhlembeck type[2] in a weak coupling, markovian regime.

A way of handling the non-unitary (collisional) corrections to an effective Hartree-Fock mean field description, in the particular case of small amplitude collective motion, has been described in ref. (3) (see also ref. (4)). It led to modified RPA equations whose solutions involve, in general, complex frequencies. Correlation corrections to the mean field evolution have not been treated explicitely in (3), however, on the grounds that they were already included in the effective Hartree-Fock mean field. While this can in fact be maintained to some extent[5], it is clear that rapid energy dependence which occurs in correlation corrections to the mean field, in particular, is being left out of the picture. This energy dependence will be related below to that of the modified mass operators considered in refs. (6-8), where its importance as a source of the spreading of nuclear giant resonances has been demonstrated. The approach described in I will enable us to derive an expression for the one body linear response function containing both unitary and collisional correlation corrections explicitely. In order to keep the discussion as transparent as possible, we will assume throughout a two-body Hamiltonian with a potential soft enough to guarantee the relevance of a one-body, Hartree-Fock type mean field. The scheme to be given can also be adapted for use in connection with phenomenological or semi-phenomenological effective forces.

The rest of the paper is organized as follows. In section 2 we review the main results of I in the context of a one-body linear response problem. Section 3 deals with the specialization to small amplitude motion of the responding system and with the presentation of working approximations to the various formal objects. An illustrative application of the equations to the soluble model of Lipkin, Meshkov and Glick[9] is given in section 4. The last section 5 is devoted to a final discussion and conclusions.

2. ONE-BODY LINEAR RESPONSE

The equation governing the time evolution of the one-body density operator $\hat{\rho}(t)$ in a weak one-body external field is written as (see Appendix A)

$$i\dot{\hat{\rho}} = \hat{\hat{\ell}}(t)\hat{\rho}(t) + \hat{\hat{\ell}}_{ext}(t)\,\hat{\rho}(t) + \hat{r}(t) + \hat{r}_{ext}(t) \qquad (2.1)$$

where the objects acting on $\hat{\rho}(t)$ on the r.h.s. are Liouville operators (sometimes called superoperators), and $\hat{r}$ and

$\hat{r}_{ext}$ are operators associated with initial correlations, as described below. The external field liouvillian $\hat{\hat{\ell}}_{ext}$ is assumed to be associated with a one-body field $\hat{H}_{ext}(t)$, so that

$$\hat{\hat{\ell}}_{ext}(t)\,\hat{\rho}(t) = \left[\hat{H}_{ext}(t),\hat{\rho}(t)\right].$$

The internal, effective one-body liouvillian can be split as

$$\hat{\hat{\ell}}(t)\hat{\rho}(t) = \left[\hat{\hat{\ell}}_0(t) + \hat{\hat{\ell}}_1(t)\right]\hat{\rho}(t)$$

where

$$\left[\hat{\hat{\ell}}_0(t)\hat{\rho}(t)\right]_{\lambda\mu} = \mathrm{Tr}\left(c^+_\mu c_\lambda\left[H, F_0(t)\right]\right) \qquad (2.2a)$$

and

$$\left[\hat{\hat{\ell}}_1(t)\hat{\rho}(t)\right]_{\lambda\mu} = -i\,\mathrm{Tr}\left(c^+_\mu c_\lambda\int_0^t dt'\left[H, G(t,t')Q(t')\left[H,F_0(t')\right]\right]\right) \qquad (2.2b)$$

The uncorrelated Fock space density $F_0(t)$ is best written in terms of fermion operators c_λ, c^+_λ associated with the time dependent natural orbitals $|\lambda(t)>$, which make $\hat{\rho}(t)$ diagonal:

$$\hat{\rho}(t) = \sum_\lambda |\lambda(t)\rangle\, p_\lambda(t)\,\langle\lambda(t)|$$

Then[10]

$$F_0(t) = \prod_\lambda\left[(1-p_\lambda)c_\lambda c^+_\lambda + p_\lambda c^+_\lambda c_\lambda\right] \qquad (2.3)$$

and the diagonal ($\lambda=\mu$) matrix elements (2.2a) vanish. The contribution (2.2b), on the other hand, still exists both for $\lambda\neq\mu$, when it gives correlation corrections to the Hartree-Fock-like mean field of eq. (2.2a), and for $\lambda=\mu$, giving the collision corrections. This shows that these two types of correlation corrections, although appearing in a completely symmetric way in eq. (2.1), are readily disentangled when one adopts the natural orbitals representation.

The propagator $G(t,t')$ is formally written as

$$G(t,t') = T\exp\left[-i\int_{t'}^{t} d\tau\, Q(\tau)L\right] \tag{2.4}$$

where L is the liouvillian generator associated with the hamiltonian H, and the superoperator $Q(t)$ essentially eliminates uncorrelated parts (in the sense of $F_0(t)$) of the objects upon which it acts. It is discussed in detail in I, where, in particular it is given explicitely in terms of the ingredients appearing in $F_0(t)$, eq. (2.3). It appears also in the initial correlation terms which are given by

$$\hat{r}_{\lambda\mu}(t) = Tr\left(c^+_\mu c_\lambda \left[H, G(t,0)F'_I\right]\right) \tag{2.5}$$

and

$$\hat{r}_{ext\,\lambda\mu}(t) = -i\,Tr\left(c^+_\mu c_\lambda \int_0^t dt' \left[H, G(t,t')Q(t')\left[H_{ext}(t'), F'_I\right]\right]\right) \tag{2.6}$$

where F'_I is the correlation part of the initial full density matrix $F(t)$, i.e.,

$$F'_I = F(0) - F_o(0)$$

and it has been assumed that, in the absence of $H_{ext}(t)$, $F(t)$ is stationary.

The initial correlation terms contain in general both non-diagonal and diagonal contributions, analogously to eq. (2.2b), which thus represent unitary and non-unitary contributions to $i\hat{\rho}$, respectively. In particular, the time dependence of eq. (2.5) cancels that of eq. (2.2b) for a stationary state in the absence of the external force. This gives, together with eq. (2.2a), $i\dot{\hat{\rho}}=0$ in this case, and illustrates the fact that, when including explicitely correlation effects on the dynamical law for $\hat{\rho}(t)$, one must always include, also explicitely, the appropriate initial correlations, if the generation of unwanted transients is to be avoided. Needless to say that approximations should also be consistently made on the internal and on the initial correlation contributions. A further discussion of this point is given in section 3 below. It is also illustrated by the example presented in section 4.

3. SMALL AMPLITUDE REGIME: THE RPA RESPONSE AND BEYOND

The assumption of a stationary initial state, stable against the external perturbation represented by $H_{ext}(t)$, allows us to linearize the equations describing the motion of the system in the following way. We first adopt the initial natural orbitals $|\lambda(0)\rangle$ as the basic representation and write current natural orbitals at time t as

$$|\lambda(t)\rangle = \left[1 - i\hat{\mathcal{F}}(t)\right]|\lambda(0)\rangle + \mathcal{O}(\hat{\mathcal{F}}^2) \tag{3.1}$$

where $\hat{\mathfrak{f}}(t)$ is the infinitesimal hermitean one-body generator of the changes of the natural orbitals. If we associate the states $|\lambda(0)\rangle$ to fermion operators a_λ^+, a_λ, then

$$c_\lambda^+(t) = a_\lambda^+ - i[\hat{\mathfrak{f}}, a_\lambda^+] + O(\hat{\mathfrak{f}}^2)$$

This same type of relation will also apply to other objects such as uncorrelated Fock space density, eq. (2.3). Second, we write current occupation probabilities $p_\lambda(t)$ as

$$p_\lambda(t) = p_\lambda(0) + p'_\lambda(t) \tag{3.2}$$

where $p'(t)$ is to be considered a small quantity of the order $\hat{\mathfrak{f}}$. It is clear that $\dot{p}_\lambda(t) = \dot{p}'_\lambda(t)$, and that the sum of all the occupation fluctuations $p'_\lambda(t)$ vanishes at all times. Under these circumstances the contribution eq. (2.2a) to $i\dot{\hat{\rho}}_{\lambda\mu}$ can be written up to and including first order quantities as

$$[\hat{\hat{\ell}}_0(t)\hat{\rho}(t)]_{\lambda\mu} \cong \left[1 + \sum_\alpha p'_\alpha(t)\frac{\partial}{\partial p_\alpha(0)}\right] \mathrm{Tr}\{a_\mu^+ a_\lambda [H, F_0(0)]\}$$

$$- i\,\mathrm{Tr}\{[a_\mu^+ a_\lambda, F_0(0)][\hat{\mathfrak{f}}(t), H]\}\,. \tag{3.3}$$

We can also express the time derivative of the one-body density in terms of $\hat{\mathfrak{f}}(t)$ and of the $p'_\lambda(t)$:

$$i\,\dot{\hat{\rho}}_{\lambda\mu} = i\,\dot{p}'_\lambda(t) + [\hat{\mathfrak{f}}(t), \hat{\rho}(0)]_{\lambda\mu}\,. \tag{3.4}$$

Equations (3.3) and (3.4) can be combined to give an approximation to the small amplitude time evolution of the

one body density in which all correlation effects are ignored. This amounts to just the familiar Random Phase approximation to the linear response problem. Since neither eqs. (3.3) nor the external driving term contain contributions to the diagonal part of $\dot{\hat{\rho}}$, it follows that occupation probabilities are in this case constant. This implies the vanishing of the first term on the r.h.s. of eq. (3.3) (the vanishing of the zeroth order part expresses just the stationarity of F_0 in this approximation), and

$$\dot{\hat{\mathfrak{f}}}_{\lambda\mu}(t)(p_\mu - p_\lambda) + i\sum_{\sigma\tau} \Gamma_{\lambda\mu;\sigma\tau}\, \hat{\mathfrak{f}}_{\sigma\tau}(t) = H^{ext}_{\lambda\mu}(t)(p_\mu - p_\lambda) \tag{3.5}$$

where

$$\Gamma_{\lambda\mu;\sigma\tau} = \mathrm{Tr}\{[a_\mu^+ a_\lambda, F_0(0)][a_\sigma^+ a_\tau, H]\} \tag{3.6}$$

is the dynamical matrix which characterizes the RPA approximation.

In order to go beyond this context in the treatment of the time evolution of $\hat{\rho}$ we must handle the correlation contributions to eq. (2.1), which contain the complicated many-body propagator $G(t,t')$. We presently do this by treating this object in the simplest possible approximation, which can be motivated as follows. In eqs. (2.2b), (2.5) and (2.6) $G(t,t')$ acts on correlated states creating new correlations or absorbing them. As shown in eq. (2.4), furthermore, this propagator contains the filtering operator $Q(\tau)$ which prevents the system from reverting to an uncorrelated state at intermediate times. Eventually the correlations must evolve to forms which are simple enough to give nonvanishing contributions to the overall trace. The approximation we use consists in neglecting intermediate creation and absorption of new correlations within $G(t,t')$, which is assumed to merely propagate the starting correlations from time t' to time t by means of the mean field dynamics, i.e., following through the time evolution of the natural orbitals. Formally, this is a weak coupling approximation which we implement replacing $G(t,t')$ by $g(t,t')$, the unitary time-displacement operator associated with the correlation free mean-field propagation prescribed by the ℓ.h.s. of eq. (3.5).

We now deal first with the internal correlation corrections, eq. (2.2b). Using the approximation above for $G(t,t')$ we obtain in a straightforward way (see also refs. (1) and (3))

$$\left[\hat{\ell}_1(t)\hat{\rho}(t)\right]_{\lambda\mu} \simeq \frac{i}{2}\sum_{\beta\gamma\delta}\int_0^t dt' \Big[\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_t \langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_{t'} \left(p_\lambda p_\beta q_\gamma q_\delta - p\leftrightarrow q\right)_{t'}$$

$$+\langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_t \langle\gamma\delta|\tilde{v}|\mu\beta\rangle_{t'} \left(p_\mu p_\beta q_\gamma q_\delta - p\leftrightarrow q\right)_{t'}\Big]. \tag{3.7}$$

In this equation $q = 1-p$, the $\tilde{v}$ matrix elements are the antisymmetrized two-body potential matrix elements of H evaluated with natural orbitals at times indicated by the subscript. As shown, occupation probabilities correspond to time t'. In obtaining eq. (3.7) we also neglected one-particle transitions caused by the two-body force, as these are essentially canceled by the effect of the filtering operator $Q(t')$ (see I for details on this point).

The structure of eq. (3.7) is conveniently illustrated by means of the Feynman-Goldstone like diagrams of fig. (3.1), drawn under the assumption that the natural orbital

occupation probabilities are either zero or one at t'. The lines represent time-dependent[11] natural orbitals. The contributions represented by these diagrams constitute, for $\lambda\neq\mu$, time dependent corrections to the Hartree-Fock mean field. For $\lambda\neq\mu$, on the other hand, they relate to changes of the "vacuum" occupation probabilities.

Since the time-dependence of the natural orbitals appearing in eq. (3.7) is chosen as that which results from eqs. (3.5) and (3.6), the time propagation represented by the particle and hole lives in fig. (3.1) will contain all the complexity of the RPA response (fonons) at intermediate times. This could in fact be made explicit by rewriting eq. (3.7) in terms of some fixed single-particle base (e.g. the static Hartree-Fock base). The multiple rescattering of particle-hole states of the static base is an important effect to be taken into account in the "quasidegenerated limit" in which different particle-hole energies differ little in comparison with typical two-body matrix elements. In the opposite limit of widely spaced particle-hole energies one can, as an approximation, leave out the two-body effects in eq. (3.6) for the purpose of evaluating eq. (3.7). For notational simplicity we use the latter option in the following development. Conversion of the results to the form appropriate to the quasidegenerate limit is discussed in Appendix C.

We are now left the task of writing down the linearized form of eq. (3.7). This is again straightforward use of eqs. (3.1) and (3.2). One obtains a zeroth order term identical to eq. (3.7) itself but involving stationary orbitals and occupation probabilities and collects furthermore other terms which are linear either in the infinitesimal unitary generator $\hat{\mathfrak{F}}(t)$ or in the occupation fluctuations $p'(t)$. The complete expressions are too lengthy to be profitably quoted at this point. We give them in Appendix B and limit the discussion here to a description of the structure and content of the resulting dynamical equations.

Since in the presence of correlation effects the occupation probabilities are no longer constant, we now get contributions from the first term on the r.h.s. of eqs. (3.3) and (3.4). As a result of this we get additional contributions involving $p'_\lambda(t)$ in (3.5) and another set of equations involving the time-derivatives of these quantities. Eq. (3.5) then becomes replaced by

$$\dot{\hat{\mathfrak{F}}}_{\lambda\mu}(t)(\rho_\mu-\rho_\lambda) + i\sum_{\sigma\tau}\Gamma_{\lambda\mu;\sigma\tau}\left(\hat{\mathfrak{F}}_{\sigma\tau}(t)+i\delta_{\sigma\tau}p'_\sigma(t)\right) + \sum_{\sigma\tau}\int_0^t dt'\,\Delta_{\lambda\mu;\sigma\tau}(t,t')\,\hat{\mathfrak{F}}_{\sigma\tau}(t') = H^{ext}_{\lambda\mu}(t)(\rho_\mu-\rho_\lambda) \tag{3.8a}$$

$$i\dot{p}'_\lambda(t) = \sum_{\sigma\tau}\int_0^t dt'\,\Delta_{\lambda\lambda;\sigma\tau}(t,t')\,\hat{\mathfrak{F}}_{\sigma\tau}(t') + \sum_{\sigma}\int_0^t dt'\,\Lambda_{\lambda\sigma}(t-t')\,p'_\sigma(t') \tag{3.8b}$$

We remark first that zeroth order contributions have been left out of these equations. This is in keeping with the linear response framework in which an initially stationary state is assumed. Actually the zeroth order contributions resulting from the linearization of eq. (3.7) must in this case cancel against corresponding zeroth-order contributions from the initial correlation term $\hat{r}(t)$, eq. (2.5). First order contributions from eq. (3.7) are collected in the memory kernels $\Delta(t,t')$ and $\Lambda(t-t')$ appearing in eqs. (3.8). As displayed

explicitely in Appendix B, $\Delta(t,t')$ contains both a delta function part involving the time integral from 0 to t of a zeroth order expression and a memory part depending on $t-t'$ only (to be called the frequency-local part because of the convolution theorem for Laplace transforms). The delta function part comes from taking into account first order disturbance of the natural orbitals, involved in the two-body matrix element, at time t (see eq. (3.7)). This matrix element "closes" a correlation process started at t'. The time dependence associated with the upper limit of the integration interval must actually cancel a corresponding first order contribution present in $\hat{r}(t)$, eq. (2.5), in which a preexisting correlation in the (stationary) initial state is similarly closed at time t. These linear terms with time-dependent coefficients arising from the delta-function parts should therefore be left out when dealing with the linear response of a stationary state. However, together with the zeroth order terms, they generate the appropriate transients which follow from a small initial disturbance of a stationary state in an initial condition problem. These features are illustrated in the example discussed in section 4. A final remark concerns the source term $\hat{r}_{ext}(t)$, eq. (2.6), which contains interference of the external field $H_{ext}(t)$ with initial correlations. The evaluation of this term involves the problem of actually specifying the initial correlations $F'(0)$. Our weak coupling approximation involving the many-body propagator $G(t,t')$, however, already considerably restricts that part of $F'(0)$ which leads to nonvanishing contributions to $\hat{r}_{ext}(t)$. A working approximation for this consists then in assuming the minimal $F'(0)$ which leads to the cancellation of the delta-function part of $\hat{\hat{\Delta}}(t,t')$ by the corresponding initial correlation contribution from $\hat{F}(t)$, eq. (2.5). This allows us to deal with the response to a specified one-body external field $\hat{H}_{ext}(t)$, subject to the validity of the weak coupling approximation, in a consistent, if not complete way.

Before concluding this section we mention briefly a further possible approximation which simplifies somewhat the coupled equations (3.8). It has also been used in ref. (3) and consists in dropping the last term in eq. (3.8b) on grounds of the weak coupling approximation, noting that for $p'_\sigma(0) \equiv 0$ its simplest contribution involves already the product of four two-body matrix elements[3]. With this approximation we may integrate eq. (3.8b) and obtain, from eq. (3.8a), a closed equation for the unitary generator $\hat{\mathfrak{F}}(t)$. Since the memory kernels of this equation are all frequency-local, it can be usefully rewritten in a frequency representation by means of a Laplace transform. Defining the constant liouvillian matrix $\hat{\hat{G}}$ as

$$\hat{\hat{G}}_{\lambda\mu;\sigma\tau} = (\rho_\mu - \rho_\lambda)\,\delta_{\lambda\sigma}\,\delta_{\mu\tau}$$

we get in matrix notation

$$\tilde{\mathfrak{F}}(s) = \left[s\hat{\hat{G}} + i\,\hat{\hat{\Gamma}}\left(1+\frac{1}{s}\hat{\hat{\Delta}}_c(s)\right) + \hat{\hat{\Delta}}_u(s)\right]^{-1}\hat{\hat{G}}\,\tilde{H}^{ext}(s) \qquad (3.9)$$

where s is the Laplace frequency variable, $\hat{\hat{\Delta}}_c$ and $\hat{\hat{\Delta}}_u$ being respectively the $\lambda=\mu$ and the $\lambda\neq\mu$ parts of $\hat{\hat{\Delta}}_{\lambda\mu,\sigma\tau'}$, see eqs. (3.8a) and (3.8b). Equation (3.9) identifies the linear response operator and its standard RPA limit, which is obtained

just by letting $\hat{\hat{\Delta}} \to 0$. In general, eq. (3.9) contains what amounts to a frequency dependent mass operator. The frequency dependence comes both from unitary correlation contributions (through $\hat{\hat{\Delta}}_u(s)$) and from collisional effects (through $\hat{\hat{\Delta}}_c(s)$).

4. SIMPLE ILLUSTRATIVE EXAMPLE

In order to make the content of at least some of the equations given in the preceding section more concrete we describe here briefly their use in the context of the soluble model of Lipkin, Meshkov and Glick[9]. We write the hamiltonian of the model in the form

$$H = \frac{\epsilon}{2}\sum_{m\sigma}\sigma c^{+}_{m\sigma}c_{m\sigma} + \frac{V}{2}\sum_{mm'\sigma} c^{+}_{m\sigma}c^{+}_{m'\sigma}c_{m'-\sigma}c_{m-\sigma} \qquad (4.1)$$

where $\sigma = +1, -1$ (up, down) and $m = 1,\ldots,N$. We will consider only states within a definite multiplet $J = N/2$ of the quasi-spin group associated with the model. In this case all sublevels m have always identical occupations for each σ. Since moreover $p_\sigma + p_{-\sigma} = 1$ we have

$$F_c(0) = \prod_{m\sigma}\left[p_\sigma c^{+}_{m\sigma}c_{m\sigma} + (1-p_\sigma)c_{m\sigma}c^{+}_{m\sigma}\right]. \qquad (4.2)$$

The hermitean one-body generator $\hat{\mathfrak{F}}(t)$ can be generally written as

$$\hat{\mathfrak{F}}(t) = \sum_{\sigma\sigma'}\mathfrak{F}_{\sigma\sigma'}(t)\sum_m c^{+}_{m\sigma}c_{m\sigma'} \qquad (4.3)$$

and with these elements we may evaluate directly the left-hand side of eq. (3.5). The RPA modes and frequencies result immediately from the homogeneous version of this equation which in this particular case reads (assuming $p_+ = 0$)

$$\dot{\mathcal{F}}_{+-}(t) = -i\epsilon\,\mathcal{F}_{+-}(t) + i(N-1)V\,\mathcal{F}_{-+}(t) \tag{4.4}$$

and its complex conjugate. The RPA frequencies are given by

$$\omega^2 = \epsilon^2 - (N-1)^2 V^2 .$$

They vanish at the onset of instability for the underlying Hartree-Fock stationary state.

Looking now into the correlation corrections for the model, eq. (3.7), we find first that frequency-local terms are zero in the present approximation when $p_t(0) = 0$. Besides this, the zeroth order non-diagonal delta function contribution (i.e., with $\lambda \neq \mu$) and the first order in $\hat{\mathcal{F}}$ diagonal contribution also vanish identically. Collecting therefore the zeroth order diagonal contribution and the first order in p' contribution we are led to the initial value problem for the occupations (cf. eq. (3.8b))

$$\dot{p}' = 2(N-1)V^2 \int_0^t \cos 2\epsilon(t-t')\left[1-4p'(t')\right] dt' \tag{4.5}$$

which can be solved by means of a Laplace transform. We shall however retain just effects which are not more than quadratic in V, in which case the approximate solution of eq. (4.5) is, for the initial condition $p'(0) = 0$,

$$p'(t) \simeq \frac{(N-1)V^2}{\epsilon^2} \sin^2 \epsilon t . \tag{4.6}$$

This displays, in particular, transient oscillations of the occupation amplitudes resulting from the nonstationarity of the assumed initial state at t=0. The oscillations can in fact be eliminated by introducing appropriate initial correlations into the problem. These are, in this case, perturbative two-particle, two-hole amplitudes added to the noninteracting ground state of the model, which have no other effect up to order V^2. The stationary value of the occupations, namely

$$\overline{p}' \simeq \frac{(N-1)V^2}{2\epsilon^2}$$

agrees to this order with the occupation probabilities of the exact stationary state. The non-diagonal contributions from eq. (3.7) on the other hand, give (cf. eq. (3.8a))

$$\dot{\mathcal{F}}_{+-}(t) = -i\left[\epsilon + \frac{(N-1)V^2}{\epsilon}\right]\mathcal{F}_{+-} + i(N-1)V\mathcal{F}_{-+} + i\frac{(N-1)V^2}{\epsilon} e^{-2i\epsilon t}\mathcal{F}_{+-} \tag{4.7}$$

and its complex conjugate. The time-dependent coefficient in the last term is again a result of the particular initial state that has been assumed, and can be absorbed by introducing initial correlations in the form of perturbative two-particle, two-hole amplitudes. This will also change other contributions only at order V^3 or higher. Dropping therefore this term one gets the corrected frequencies

$$\omega^2 \simeq \epsilon^2 - (N-1)(N-3)V^2 + \mathcal{O}(V^4) \tag{4.8}$$

which reproduce the exact value to this order in V.

5. DISCUSSION AND CONCLUDING REMARKS

In the preceding sections we presented and illustrated the use of a set of equations governing the time-evolution of the one-body density of the nuclear many-fermion system, including dynamical effects due to many-body correlations. The time evolution is in general non unitary, with the non-unitarity expressing itself in a simple and complete way in terms of the time variation of the occupation probabilities (eigenvalues of the one-body density) of the one-body natural states (eigenvalues of the one-body density). This neat separation of the nonunitary (collisional) effects also provides for the simultaneous identification of an extended mean field, which also includes correlation effects. This extended mean field is also characterized in a simple and complete way as generating the time evolution of the natural states.

Our main concern in this paper has been the specialization of the general equations to the context of small amplitude nuclear collective motion. We did this by a straight-forward linear expansion of the general equations about a stable stationary state. The resulting equations (3.8a) and (3.8b) are appropriate to a linear-response situation in which effects beyond those involved in the usual RPA approach are included. They can however be used also as the equations for an initial condition problem, as shown explicitely in the illustration given in section 4.

Treatments of small amplitude motion including dynamical effects beyond those in the usual RPA have been given before. In a first general line of approach, particle-phonon coupling effects have been added as complicators of the microscope structure of the vibrations[7,8,12]. This can be

both motivated and controlled in terms of the inclusion of bubbles containing phononlike self-energy and "induced interaction"[12] insertions (see fig. 3, Appendix C) in the irreducible mass operator appearing in the integral equation for the polarization propagator[13]. This leads eventually to a response operator which has basically the form of eq. (3.9) <u>without</u> $\frac{1}{s}$ term related to the fluctuations of the occupation probabilities. This indicates that the frequency-dependent mass operator which causes in this case the spreading of collective RPA modes does not contain linear response effects of the contribution to the effective mean field represented by fig. 5.1, whose importance has been pointed out in other contexts[5]. In eq. (3.9) these effects come from the fluctuating part of the occupation probabilities and are therefore of an essentially collisional character.

A different approach to this problem can be found in ref. (4), which consists in adding in a phenomenological way collision effects to time-dependent Hartree-Fock mean field equations written semiclassically in a phase-space representation. The treatment given there of a collision term of the Uehling-Uhlembeck type produces, with a sensible two-body effective force, spreading widths for the isoscalar quadrupole giant resonances which are of the correct order of magnitude. This is also the case for the treatments reviewed before, especially when continuum effects, which turn out to be nonaditive, are also included. Given this situation, it is our view that a correct assessment of the relative importance of the two types of contribution is still lacking. The numerical implementation of the results of this work for realistic cases, now in progress, should contribute to clarify this issue.

APPENDIX A - LINEAR RESPONSE OF THE ONE-BODY DENSITY

The linear response to the external one-body field $H^{ext}(t)$ of a stationary state of H associated with a density operator $F_I = |\psi_I\rangle\langle\psi_I|$ is obtained from the equation

$$i\dot{F}(t) = \left[H, F(t)\right] + \left[H^{ext}(t), F_I\right] \tag{A.1}$$

with the initial condition $F(0) = F_I$, since $F(t)$ differs from F_I by terms which are at least of the first order in H^{ext}. Applying to eq. (A.1) the projection techniques presented in I we are led in a straightforward way to

$$F(t) = F_0(t) + F'(t) \tag{A.2}$$

where $F_0(t)$ has the form (2.3) and $F'(t)$ is given by

$$F'(t) = G(t,0)F'_I + \int_0^t dt'\, G(t,t')Q(t')\left[H, F_0(t')\right] + \\
+ \int_0^t dt'\, G(t,t')Q(t')\left[H^{ext}(t'), F_I\right] . \tag{A.3}$$

The last term in eq. (A.3) contains $F_I = F_0(0) + F'_I$. Since $F_0(0)$ differs from $F_0(t')$ by terms which are again at least of the first order in H^{ext}, and since the external field is assumed to be a one-body field, the contribution due to $F_0(0)$ vanishes to first order. Using this expression, together with (A.2) and

$$i\dot{\rho}_{\lambda\mu}(t) = \mathrm{Tr}\left(c^+_\mu c_\lambda \left[H, F(t)\right]\right)$$

one obtains immediately eq. (2.1).

APPENDIX B - EXPLICIT EXPRESSIONS FOR THE COEFFICIENTS OF THE LINEARIZED EQUATIONS

The time-dependent coefficients Δ and Λ in eqs. (3.8a) and (3.8b) are obtained from a straightforward linearization of eq. (3.7) in $\hat{\mathfrak{F}}$ and p' (see eqs. (3.1) and (3.2)). We get

$$\Delta_{\lambda\mu;\sigma\tau}(t,t') = \delta(t-t')\,\Delta^{t.\ell.}_{\lambda\mu;\sigma\tau}(t) + \Delta^{f.\ell.}_{\lambda\mu;\sigma\tau}(t-t')$$

with

$$\Delta^{t.\ell.}_{\lambda\mu;\sigma\tau}(t) = -\sum_{\alpha\beta}{}' \int_0^t dt_1\, e^{i(\epsilon_\beta+\epsilon_\sigma-\epsilon_\mu-\epsilon_\alpha)(t-t_1)} \langle\tau\beta|\tilde{v}|\lambda\alpha\rangle\langle\mu\alpha|\tilde{v}|\sigma\beta\rangle \times \\
\times (p_\lambda p_\alpha q_\beta q_\sigma - p \leftrightarrow q) + \\
+ \frac{1}{2}\sum_{\alpha\beta}{}' \int_0^t dt_1\, e^{i(\epsilon_\alpha+\epsilon_\beta-\epsilon_\mu-\epsilon_\tau)(t-t_1)} \langle\alpha\beta|\tilde{v}|\lambda\sigma\rangle\langle\mu\tau|\tilde{v}|\alpha\beta\rangle(p_\lambda p_\tau q_\alpha q_\beta - p\leftrightarrow q) + \\
+ \frac{1}{2}\sum_{\alpha\beta\gamma}{}' \delta_{\tau\lambda} \int_0^t dt_1\, e^{i(\epsilon_\alpha+\epsilon_\beta-\epsilon_\mu-\epsilon_\gamma)(t-t_1)} \langle\alpha\beta|\tilde{v}|\sigma\gamma\rangle\langle\mu\gamma|\tilde{v}|\alpha\beta\rangle(p_\lambda p_\gamma q_\alpha q_\beta - p\leftrightarrow q)$$

$$+ \left(\lambda \leftrightarrow \mu, \text{ complex conjugate}\right). \tag{B.1}$$

and

$$
\begin{aligned}
\Delta^{f.\ell.}_{\lambda\mu;\sigma\tau}(t-t') = \\
= \sum_{\alpha\beta}{}' \, e^{i(\epsilon_\beta+\epsilon_\tau-\epsilon_\alpha-\epsilon_\mu)(t-t')} & \langle\tau\beta|\tilde{v}|\lambda\alpha\rangle\langle\mu\alpha|\tilde{v}|\sigma\beta\rangle(p_\lambda p_\alpha q_\beta q_\tau - p\leftrightarrow q) - \\
- \frac{1}{2}\sum_{\alpha\beta}{}' \, e^{i(\epsilon_\alpha+\epsilon_\beta-\epsilon_\sigma-\epsilon_\mu)(t-t')} & \langle\alpha\beta|\tilde{v}|\lambda\sigma\rangle\langle\mu\tau|\tilde{v}|\alpha\beta\rangle(p_\lambda p_\sigma q_\alpha q_\beta - p\leftrightarrow q) - \\
- \frac{1}{2}\sum_{\alpha\beta\gamma}{}' \, \delta_{\sigma\mu} e^{i(\epsilon_\alpha+\epsilon_\beta-\epsilon_\gamma-\epsilon_\tau)(t-t')} & \langle\alpha\beta|\tilde{v}|\lambda\gamma\rangle\langle\tau\gamma|\tilde{v}|\alpha\beta\rangle(p_\alpha p_\beta q_\lambda q_\gamma - p\leftrightarrow q) + \\
+ (\lambda\leftrightarrow\mu, \text{ complex conjugate}). &
\end{aligned}
\tag{B.2}
$$

Primed sums are over sets of different one-particle states. The symbols $p \leftrightarrow q$ and $\lambda \leftrightarrow \mu$ indicate interchange of the corresponding labels in the preceding expressions and matrix elements are now evaluated between static natural orbitals. The various terms of (B.1) and (B.2) can perhaps be best visualised in terms of the Feynman-Goldstone-like diagrams of fig. B.1. There the dots stand for antisymmetrized two-body matrix elements, and the lines correspond to mean-field time evolution. Upward (downward) arrows indicate the occupation factors $q(p)$ in a typical term. When $\lambda \neq \mu$ the frequency-local terms are seen to be associated with the so called induced interaction terms and with self-energy corrections which have been considered e.g. in connection with the damping of nuclear giant resonances[12]. The corresponding terms with $\lambda = \mu$, on the other hand, when associated with the $p'_\sigma(t)$ terms in eq. (3.8a), give rise to more exotic contributions to an effective two-body interaction

akin to those obtained by functional differentiation of corrections involving the change of ground state occupation probabilities in density dependent mean field theories[15]. (See also fig. 5.1).

The coefficients in the last term of eq. (3.8b) are given by[3]

$$
\begin{aligned}
\Lambda_{\lambda\sigma}(t-t') = \\
= 2i\sum_{\alpha\beta}{}' |\langle\sigma\alpha|\tilde{v}|\lambda\beta\rangle|^2 \cos\left[(\epsilon_\sigma+\epsilon_\alpha-\epsilon_\lambda-\epsilon_\beta)(t-t')\right](p_\sigma p_\lambda q_\beta + p\leftrightarrow q) - \\
- i\sum_{\alpha\beta}{}' |\langle\alpha\beta|\tilde{v}|\lambda\sigma\rangle|^2 \cos\left[(\epsilon_\alpha+\epsilon_\beta-\epsilon_\lambda-\epsilon_\sigma)(t-t')\right](p_\alpha p_\beta q_\lambda + p\leftrightarrow q) - \\
- i\sum_{\alpha\beta\gamma}{}' \delta_{\lambda\sigma} |\langle\alpha\beta|\tilde{v}|\lambda\gamma\rangle|^2 \cos\left[(\epsilon_\alpha+\epsilon_\beta-\epsilon_\lambda-\epsilon_\gamma)(t-t')\right](p_\alpha p_\beta q_\gamma + p\leftrightarrow q).
\end{aligned}
\tag{B.3}
$$

APPENDIX C - THE QUASIDEGENERATE CASE

When particle-hole excitations are quasidegenerate it is not allowed to ignore the effects of the residual interaction in eq. (3.6) when evaluating eq. (3.7). The density fluctuations resulting from distortions of the stationary state will in this case occur with characteristic frequencies which may be very different from the particle-hole excitation energies. This means that one should substitute phonons for the uncorrelated particle-hole pairs in the intermediate propagation between t' and t.

In order to show in a simple way how this substitution can be done in connection with the linearized equations (3.8a) and (3.8b) we consider as an example the first frequency-local term on the r.h.s. of eq. (B.2). Introducing the notation $||\alpha\beta\rangle$ for the operator $|\alpha\rangle\langle\beta|$, to be considered as a vector in Liouville space, with the norm defined through the internal product $\langle\gamma\delta||\alpha\beta\rangle = \mathrm{Tr}[(|\gamma\rangle\langle\delta|)^{+}|\alpha\rangle\langle\beta|]$, we can cast the matrix elements appearing there in the form

$$\langle\tau\beta|\tilde{v}|\lambda\alpha\rangle\langle\mu\alpha|\tilde{v}|\sigma\beta\rangle = \langle\mathrm{Tr}(\tilde{v}|\tau\rangle\langle\lambda|)||\alpha\beta\rangle\langle\alpha\beta||\mathrm{Tr}(\tilde{v}|\sigma\rangle\langle\mu|)\rangle$$

Since, moreover

$$||\alpha\beta\rangle\, e^{i(\epsilon_\beta-\epsilon_\alpha)(t-t')}\,\langle\alpha\beta|| = e^{i\hat{\hat{\Gamma}}_{sp}(t-t')}\,||\alpha\beta\rangle\langle\alpha\beta||,$$

where $\hat{\hat{\Gamma}}_{sp}$ stands for the mean field Liouvillian of eq. (3.6) with the particle-hole two-body part ommited, we can rewrite the entire term as

$$\sum_{\alpha\beta}{}' e^{i(\epsilon_\tau-\epsilon_\mu)(t-t')}\langle\mathrm{Tr}(\tilde{v}|\tau\rangle\langle\lambda|)||\, e^{i\hat{\hat{\Gamma}}_{sp}(t-t')}\,||\alpha\beta\rangle\langle\alpha\beta||\mathrm{Tr}(\tilde{v}|\sigma\rangle\langle\mu|)\rangle \times (p_\lambda p_\alpha q_\beta q_\tau - p\leftrightarrow q)\,. \tag{C.1}$$

The substitution of the phonons for the uncorrelated particle-hole pairs can now be carried out in a straightforward way replacing $\hat{\hat{\Gamma}}_{sp}$ by tge full RPA operator $\hat{\hat{\Gamma}}$. When a Laplace transform is used to convert the time variable to a frequency variable, the Liouville matrix element appearing in (C.1) will be given in terms of the familiar RPA response function.

REFERENCES

(1) M.C. Nemes and A.F.R. de Toledo Piza, Phys. Rev. C27, 862 (1983) (refered to as I in the text).

(2) See e.g. L.D. Kadanoff and G. Baym, Quantum Statistical Mechanics, Benjamin, N.Y. 1962

(3) M.C. Nemes and A.F.R. de Toledo Piza, Zeit. Phys. A310, 339 (1983).

(4) P. Schuck and J. Winter, in Time Dependent Hartree-Fock and Beyond (Lecture Notes in Physics vol. 171), K. Goeke and P.-G. Reinhard, eds., Springer-Verlag 1982, p. 190.

(5) J.W. Negele, Proc. of the Tucson Int. Topical Conference on Nuclear Physics (Lecture Notes in Physics vol. 40), B.R. Barret, ed., Springer-Verlag 1975, p. 270; Revs. Mod. Phys. 54, 913 (1982).

(6) V. Bernard and N.V. Giai, Nucl. Phys. A348, 75 (1980).

(7) J. Wambach, V.K. Mishra and Li Chu-Hsia, Nucl. Phys. A380, 285 (1982).

(8) R. de Haro Jr., J. Wambach, S. Krewald and J. Speth, Jülich preprint, 1983; R. de Haro Jr., Doctoral Thesis, Jülich Report ISSN 0366-0885, July 1982 (unpublished).

(9) A. Glick, H.J. Lipkin and N. Meshkov, Nucl. Phys. 62, 118 (1965); 62, 199 (1965); 62, 211 (1965).

(10) J. Des Cloiseaux, in Many-Body Physics (Les Houches 1967), C. de Witt and R. Balian, eds., Gordon and Breach, 1968.

(11) M. Baranger and I. Zahed, Phys. Rev. C29, 1005 (1984).

(12) G.F. Bertsch, P.F. Bortignon and R.A. Broglia, Revs. Mod. Phys. 55, 287 (1983).

(13) A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill, 1971.

FIGURE CAPTIONS

Fig. 3.1 - Feynman-Goldstone like picture of eq. (3.7). The lines stand for natural orbitals propagating with the one-body mean field.

Fig. 5.1 - Contribution to the effective one-body mean field involving corrections to ground-state occupation probabilities.

Fig. B.1 - Feynman-Goldstone like picture of contributions to eqs. (B.1) (top row) and (B.2) (bottom row).

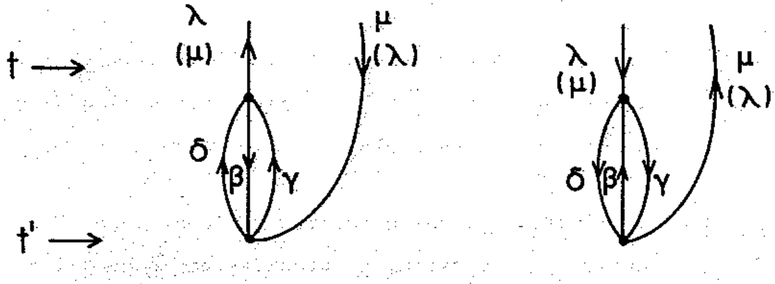

fig 3.1

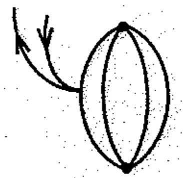

fig 5.1

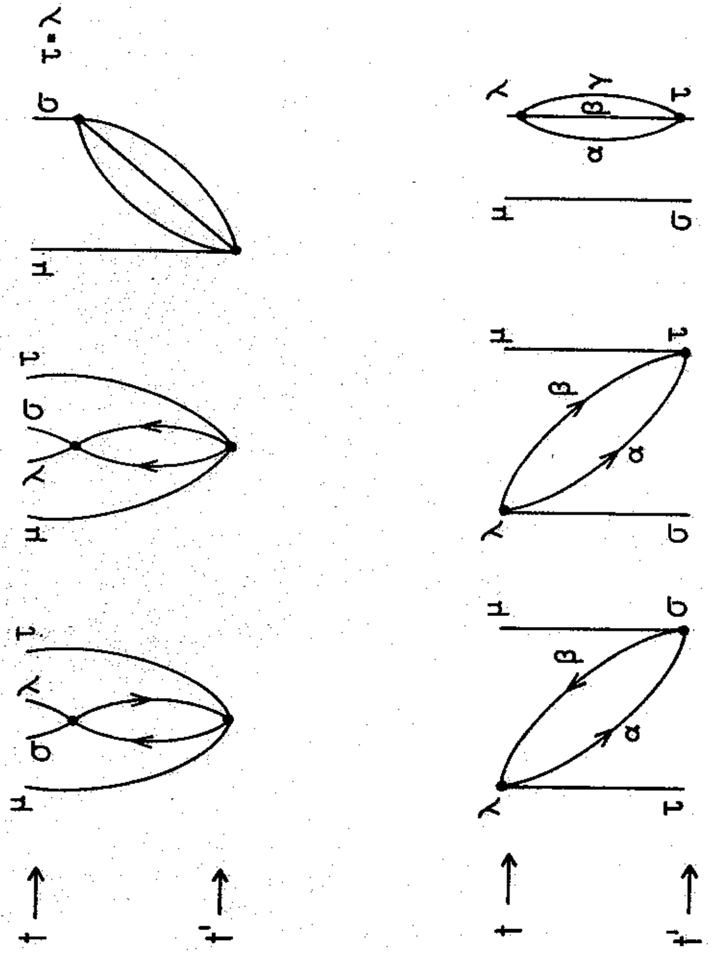

fig B.1