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THE TWO-PION EXCHANGE THREE-NUCLEON POTENTIAL AND D-WAVES IN TRINUCLEON SYSTEMS

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TWO-PION EXCHANGE THREE-NUCLEON POTENTIAL AND D-WAVES IN TRINUCLEON SYSTEMS*

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ABSTRACT

Equipotential plots are employed in the qualitative study of the coupling between S and D waves in trinucleon systems. The contributions of a redefined version of the two-pion exchange three-nucleon potential derived by means of chiral symmetry are shown to be comparable to those of two-nucleon interactions. Therefore the usual treatment of the former by means of perturbation theory is not justified.

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I. INTRODUCTION

One of the interesting features of trinucleon systems is that their static properties cannot be entirely attributed to the nucleon-nucleon interaction. This conclusion arises from various studies that, although based on different calculation techniques and realistic two-body potentials, have produced results that disagree consistently with experiment. For instance, the ^3H and ^3He nuclei are typically underbound by 1 MeV(1-3). The most concentrated effort to explain this and other discrepancies has been centred on the idea of three-nucleon potentials, with especial emphasis on that due to the exchange of two pions(4,5). Indeed, recent calculations have shown that the net effects of this force are attractive and that its magnitude may be compatible with the missing 1 MeV in the binding energy(2,3,6). These results give us confidence that the right direction is being pursued and pave the way for other refinements such as the inclusion of shorter range forces(7,8) and relativistic corrections.

The ground state wave-functions of ^3H and ^3He contain, in principle, eleven components, namely three S, four P and four D states(9,10). More than 90% of the normalization is due to the so called principal S state, which is totally symmetric under the exchange of the spatial coordinates of the nucleons. The large probability associated with this principal S state allows us to generate all the other states from it through the action of two and three-body potentials. The tensor parts of these potentials deserve particular attention, because they are responsible for the presence of D waves, which account for about 7% of the normalization and contribute significantly to the binding energy[2,3,6].

The two-pion exchange three-nucleon potential (ππE-3NP) contains tensor operators[4,5] and hence both two and three-body interactions may be effective in generating D waves. Nevertheless, many recent calculations consider only the contributions of the former when constructing the wave-functions[6,11,12]. The usual motivation for approaching the problem in this way is twofold: first, the inclusion of three-body interactions is associated with considerable technical difficulties in most formalisms; second, there is the suspicion that the effects of three-body forces are small, and hence the calculation of energy shifts by means of perturbation theory would seem to be justified.

The purpose of the present work is to investigate to what extent this last statement is correct, by studying the action of two and three-body potentials on the principal S state. This study is performed in a qualitative way, by means of equipotential plots. We restrict ourselves to S and D waves, which are the most important ones. The role of P waves will be analysed elsewhere. In section II we construct the explicit forms of the wave-functions while the potentials are discussed in section III. The action of these potentials on the principal S state is given in section IV. In section V we display the equipotential plots and present our conclusions.

II. WAVE FUNCTIONS

In this section we display the construction of the wave-function in order to set the notation. We follow here the works of Cohen and Willis[10] and of Simonov[13].

The ground states of the ^3H and ^3He nuclei are characterized by the total angular momentum $J=\frac{1}{2}$ and form a doublet in isospin space when electromagnetic interactions are not considered. Another important property of these states is that they must be totally antisymmetric under permutations of pairs of particles. This requirement suggests that we should use the properties of the group of permutations of three objects in the construction of trinucleon wave-functions.

II.1. The Permutation Group:

The group of permutations of three objects contains three odd elements, P_{12}, P_{23} and P_{31}, and has three irreducible representations, denoted by s, m and a. The representations s and a are one-dimensional and, respectively, symmetric and antisymmetric under the permutation of any pair of particles. These properties correspond to

$$P_{12}s = P_{23}s = P_{31}s = s \quad , \tag{1}$$

$$P_{12}a = P_{23}a = P_{31}a = -a \quad . \tag{2}$$

The representation m, on the other hand, has a mixed symmetry and is bidimensional. Its elements, m^+ and m^-, have the following properties[12]

$$P_{12} \begin{bmatrix} m^+ \\ m^- \end{bmatrix} = \begin{bmatrix} m^+ \\ -m^- \end{bmatrix}$$

$$P_{23} \begin{bmatrix} m^+ \\ m^- \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} m^+ \\ m^- \end{bmatrix} \tag{3}$$

$$P_{31} \begin{bmatrix} m^+ \\ m^- \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} m^+ \\ m^- \end{bmatrix}$$

The wave-functions are constructed by means of products of representations that can be reduced to forms behaving as s, m and a. The reduction of products involving the one-dimensional representations is trivial and is not shown explicitly. In the case of two mixed-symmetry representations m_1 and m_2 we have[10]

$$m_1 \otimes m_2 = s \oplus m \oplus a \tag{4}$$

where

$$\begin{aligned} s &= \sqrt{\tfrac{1}{2}}\,(m_1^- m_2^- + m_1^+ m_2^+) \\ m^+ &= \sqrt{\tfrac{1}{2}}\,(m_1^- m_2^- - m_1^+ m_2^+) \\ m^- &= \sqrt{\tfrac{1}{2}}\,(m_1^- m_2^+ + m_1^+ m_2^-) \\ a &= \sqrt{\tfrac{1}{2}}\,(m_1^- m_2^+ - m_1^+ m_2^-) \quad . \end{aligned} \tag{5}$$

The trinucleon wave-functions contain elements in configuration, spin and isospin spaces. The behaviour of each of these elements under permutations can be classified as s, m or a and must be combined in such a way as to produce a totally antisymmetric final result.

II.2. Spin States:

We consider first the spin wave-functions. The total spin of a trinucleon system may be either 1/2 or 3/2. The former value corresponds to states with permutation symmetry of type m. They are represented by $|m\mu\rangle_S$, where the subscript S indicates a spin state-vector and μ is the magnetic quantum number. Their explicit form is

$$\begin{aligned} |m^+ \tfrac{1}{2}\rangle_S &= \sqrt{\tfrac{1}{6}}\,(2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) \\ |m^- \tfrac{1}{2}\rangle_S &= \sqrt{\tfrac{1}{2}}\,(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) \quad . \end{aligned} \tag{6}$$

The corresponding values for $\mu = -\frac{1}{2}$ are obtained by exchanging $\uparrow$ and $\downarrow$ everywhere and multiplying the result by (-1) in order to follow the Condon-Shortley convention[14].

The states with total spin 3/2, denoted by $|s\mu\rangle_S$, are totally symmetric under nucleon permutations

$$\begin{aligned} |s \tfrac{3}{2}\rangle_S &= |\uparrow\uparrow\uparrow\rangle \\ |s \tfrac{1}{2}\rangle_S &= \sqrt{\tfrac{1}{3}}\,(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle) \quad . \end{aligned} \tag{7}$$

The state vectors for negative values of μ are obtained by exchaning $\uparrow$ and $\downarrow$ everywhere.

II.3. Isospin States:

The isospin of the trinucleon system is 1/2 and hence we have only mixed-symmetry states. They are written as $|mi\rangle_I$, where the subscript I indicates the isospin space and i may be $\pm 1/2$, corresponding to either ^3He or ^3H. The explicit form of the isospin state is totally analogous to eqs. (6).

II.4. Spin-Isospin States:

The expressions displayed above allow us to construct the $\Gamma^{\sigma\mu}_{\frac{1}{2}i}(\lambda)$, the spin-isospin state with total spin σ and permutation symmetry of the type λ. Using eqs. (6) and (7) we obtain

$$\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(s) = \sqrt{\frac{1}{2}}\,(|m^-\mu\rangle_S\,|m^-i\rangle_I + |m^+\mu\rangle_S\,|m^+i\rangle_I) \tag{8}$$

$$\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(m^+) = \sqrt{\frac{1}{2}}\,(|m^-\mu\rangle_S\,|m^-i\rangle_I - |m^+\mu\rangle_S\,|m^+i\rangle_I)$$

$$\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(m^-) = \sqrt{\frac{1}{2}}\,(|m^-\mu\rangle_S\,|m^+i\rangle_I + |m^+\mu\rangle_S\,|m^-i\rangle_I) \tag{9}$$

$$\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) = \sqrt{\frac{1}{2}}\,(|m^-\mu\rangle_S\,|m^+i\rangle_I - |m^+\mu\rangle_S\,|m^-i\rangle_I) \tag{10}$$

$$\Gamma^{\frac{3}{2}\mu}_{\frac{1}{2}i}(m^+) = |s\mu\rangle_S\,|m^+i\rangle_I$$

$$\Gamma^{\frac{3}{2}\mu}_{\frac{1}{2}i}(m^-) = |s\mu\rangle_S\,|m^-i\rangle_I \quad . \tag{11}$$

The total angular momentum of the trinucleon is 1/2 and hence the $\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(\lambda)$ are used in the construction of S waves, whereas the $\Gamma^{\frac{3}{2}\mu}_{\frac{1}{2}i}(\lambda)$ contribute to D waves.

II.5. Configuration States:

The spatial coordinates of the three nucleons are $\vec{R}_1$, $\vec{R}_2$ and $\vec{R}_3$. The relative position of particles i and j is described by the vector $\vec{r}_{ij}$, defined as

$$\vec{r}_{ij} = \vec{R}_j - \vec{R}_i \quad . \tag{12}$$

The various $\vec{r}_{ij}$ obey the relation

$$\vec{r}_{12} + \vec{r}_{23} + \vec{r}_{31} = 0 \quad . \tag{13}$$

Thus the spatial wave-functions of the trinucleon depend on two vectors, which are usually taken to be the Jacobi variables, given by

$$\vec{x} \equiv \vec{r}_{12}$$

$$\vec{y} \equiv \sqrt{\frac{1}{3}}\,(\vec{r}_{23} - \vec{r}_{31}) \quad . \tag{14}$$

These two vectors are the elements of a mixed-symmetry representation of the permutation group, $\vec{x}$ and $\vec{y}$ transforming as m^- and m^+ respectively.

In order to construct the spatial states we consider first the scalar functions of $\vec{x}$ and $\vec{y}$, which are tensors of rank zero. Using eqs. (5) we obtain the following totally symmetric combination

$$\rho^2 \equiv \vec{x}^2 + \vec{y}^2$$

$$= \frac{2}{3} (\vec{r}_{12}^2 + \vec{r}_{23}^2 + \vec{r}_{31}^2) = -\frac{4}{3} (\vec{r}_{31} . \vec{r}_{12} + \vec{r}_{12} . \vec{r}_{23} + \vec{r}_{23} . \vec{r}_{31}) \quad . \qquad (15)$$

The variable ρ is the hyper-radius in the hyperspherical formalism[15,5]. We can also define two dimensionless second order polynomials transforming as m^+ and m^-

$$\Pi_2(m^+) \equiv \frac{\vec{x}^2 - \vec{y}^2}{\rho^2}$$

$$= \frac{2}{3\rho^2} (2\vec{r}_{12}^2 - \vec{r}_{23}^2 - \vec{r}_{31}^2) = -\frac{2}{3\rho^2} (\vec{r}_{31} . \vec{r}_{12} + \vec{r}_{12} . \vec{r}_{23} - 2\vec{r}_{23} . \vec{r}_{31}) \qquad (16)$$

$$\Pi_2(m^-) \equiv \frac{2\vec{x}.\vec{y}}{\rho^2}$$

$$= -\frac{2}{\sqrt{3}\,\rho^2} (\vec{r}_{23}^2 - \vec{r}_{31}^2) = -\frac{2}{\sqrt{3}\,\rho^2} (\vec{r}_{31} . \vec{r}_{12} - \vec{r}_{12} . \vec{r}_{23}) \quad . \qquad (17)$$

These functions and eqs. (5) allow us to construct all the polynomials $\Pi_n^\alpha(\lambda)$ of order n on $\vec{x}$ and $\vec{y}$, where λ indicates the type of symmetry under the permutation group and α is an index that accounts for possible degeneracies[13].

These polynomials can be used in the expansion of spatial wave-functions having definite symmetry properties.

II.6. S-Waves:

The wave-function for the principal S state is given by

$$|S\rangle = S(\vec{x},\vec{y})\, \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) \qquad (18)$$

where $\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a)$ is the totally antisymmetric spin-isospin state-vector given by eq. (10) and $S(\vec{x},\vec{y})$ is a fully symmetric function of the nucleon coordinates. Its general form is

$$S(\vec{x},\vec{y}) = \sum_{\alpha,n} S_{n\alpha}\, \Pi_n^\alpha(s) \qquad (19)$$

where the $S_{n\alpha}$ are coefficients determined by the Schrödinger equation.

The state S' has mixed symmetry and is given by

$$|S'\rangle = S'_-(\vec{x},\vec{y})\, \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(m^+) - S'_+(\vec{x},\vec{y})\, \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(m^-) \quad . \qquad (20)$$

The functions $S'_+(\vec{x},\vec{y})$ and $S'_-(\vec{x},\vec{y})$ transform as m^+ and m^- and have the following general form

$$S'_\pm(\vec{x},\vec{y}) = \sum_{\alpha,n} S'_{n\alpha}\, \Pi_n^\alpha(m^\pm) \qquad (21)$$

where $S'_{n\alpha}$ are dynamical coefficients and $\Pi_n^\alpha(m^\pm)$ are the polynomials introduced above. They are constructed using the combination rule given by eqs. (5) and $\Pi_2(m^{\pm})$, that vanish in the triangular configuration. Thus the same happens for $S'_{\pm}(\vec{x},\vec{y})$.

The third possible S state is totally antisymmetric in coordinate space and will be ignored here, since its probability is very small.

II.7. D-Waves:

The spatial parts of the D waves are tensors of rank two. Using the vectors $\vec{x}$ and $\vec{y}$ it is possible to construct three such tensors, namely

$$T^M_{xx} \equiv \sqrt{\frac{1}{4\pi}}\; Y^M_2(\hat{x}) \tag{22}$$

$$T^M_{yy} \equiv \sqrt{\frac{1}{4\pi}}\; Y^M_2(\hat{y}) \tag{23}$$

$$T^M_{xy} \equiv \sqrt{\frac{5}{6}} \sum_{m,m'} \left[\langle 11mm' | 2M \rangle\, Y^m_1(\hat{x})\; Y^{m'}_1(\hat{y}) \right] \tag{24}$$

where the functions Y are the usual spherical harmonics and the notation $\hat{x}$ and $\hat{y}$ means $\hat{x} \equiv (\theta_x, \phi_x)$, $\hat{y} \equiv (\theta_y, \phi_y)$.

The wave-functions with total angular momentum 1/2 are the result of the coupling of these tensors with spin 3/2 states, given by

$$\Omega^{\mu}_{xx} \equiv 4\pi \sum_{mn} \left[\langle 2\, \tfrac{3}{2}\, Mm | \tfrac{1}{2}\, \mu \rangle\; T^M_{xx}\; |sm\rangle_S \right] . \tag{25}$$

The definitions of Ω^{μ}_{yy} and Ω^{μ}_{xy} are totally analogous.

Multiplying the functions Ω by x and y we obtain expressions that can be combined as in eqs. (5) in order to form representations of the permutation group. We get

$$\Omega^{\mu}(s) \equiv \frac{1}{\rho^2}\,(x^2\Omega^{\mu}_{xx} + y^2\Omega^{\mu}_{yy}) \tag{26}$$

$$\Omega^{\mu}(m^+) \equiv \frac{1}{\rho^2}\,(x^2\Omega^{\mu}_{xx} - y^2\Omega^{\mu}_{yy}) \tag{27}$$

$$\Omega^{\mu}(m^-) \equiv \frac{2xy}{\rho^2}\,\Omega^{\mu}_{xy} \tag{28}$$

The permutation symmetry of the various objects becomes more explicit when we introduce two new sets of functions Ω, associated with the coordinates of the various pairs of particles. They are defined as

$$\Omega^{\mu}_{ij} \equiv 4\pi \sum_{m,n} \left[\langle 2\, \tfrac{3}{2}\, Mm | \tfrac{1}{2}\, \mu \rangle\; T^M_{ij}\; |sm\rangle_S \right] \tag{29}$$

$$\Omega^{\mu}_{k} \equiv 4\pi \sum_{m,n} \left[\langle 2\, \tfrac{3}{2}\, Mm | \tfrac{1}{2}\, \mu \rangle\; T^M_{k}\; |sm\rangle_S \right] \tag{30}$$

where

$$T^M_{ij} \equiv \sqrt{\frac{1}{4\pi}}\; Y^M_2(\hat{r}_{ij}) \tag{31}$$

and

$$T^M_{k} \equiv \sqrt{\frac{5}{6}} \sum_{m,n} \left[\langle 11mm' | 2M \rangle\, Y^m_1(\hat{r}_{jk})\; Y^{m'}_1(\hat{r}_{ki}) \right] . \tag{32}$$

The action of the odd permutation operators on these new functions is given by

$$P_{ij}\,\Omega^{\mu}_{ij} = \Omega^{\mu}_{ij}$$
$$P_{jk}\,\Omega^{\mu}_{ij} = \Omega^{\mu}_{ik} = \Omega^{\mu}_{ki} \qquad (33)$$

$$P_{ij}\,\Omega^{\mu}_{k} = \Omega^{\mu}_{k}$$
$$P_{jk}\,\Omega^{\mu}_{k} = \Omega^{\mu}_{j} \quad . \qquad (34)$$

With future purposes in mind it is worth pointing out that eqs. (29) and (31) correspond to the following form for the bilinear products

$$(\Omega^{\mu}_{ij})^{\dagger}\,\Omega^{\mu'}_{k\ell} = \delta_{\mu\mu'}\,P_2\left(\frac{\vec{r}_{ij}\cdot\vec{r}_{k\ell}}{r_{ij}\ \ r_{k\ell}}\right) + \text{imaginary term} \qquad (35)$$

where P_2 is the usual Legendre polynomial and the imaginary term is proportional to the z component of $(\vec{r}_{ij}\times\vec{r}_{k\ell})$. This term vanishes when the orientations of the system are integrated[10] and hence is not written explicitly below.

The relationship between the variables $\vec{x}$, $\vec{y}$ and $\vec{r}_{ij}$, given by eqs. (14), allows us to express eqs. (26-28) in terms of the Ω^{μ}_{ij} and Ω^{μ}_{k}. We have

$$\Omega^{\mu}(s) = \frac{2}{3\rho^2}\,(r^2_{12}\,\Omega^{\mu}_{12} + r^2_{23}\,\Omega^{\mu}_{23} + r^2_{31}\,\Omega^{\mu}_{31})$$
$$= -\frac{4}{3\rho^2}\,(r_{31}\,r_{12}\,\Omega^{\mu}_{1} + r_{12}\,r_{23}\,\Omega^{\mu}_{2} + r_{23}\,r_{31}\,\Omega^{\mu}_{3}) \quad . \qquad (36)$$

$$\Omega^{\mu}(m^{+}) = \frac{2}{3\rho^2}\,(2r^2_{12}\,\Omega^{\mu}_{12} - r^2_{23}\,\Omega^{\mu}_{23} - r^2_{31}\,\Omega^{\mu}_{31})$$
$$= -\frac{2}{3\rho^2}\,(r_{31}\,r_{12}\,\Omega^{\mu}_{1} + r_{12}\,r_{23}\,\Omega^{\mu}_{2} - 2r_{23}\,r_{31}\,\Omega^{\mu}_{3}) \qquad (37)$$

$$\Omega^{\mu}(m^{-}) = -\frac{2}{\sqrt{3}\,\rho^2}\,(r^2_{23}\,\Omega^{\mu}_{23} - r^2_{31}\,\Omega^{\mu}_{31})$$
$$= -\frac{2}{\sqrt{3}\,\rho^2}\,(r_{31}\,r_{12}\,\Omega^{\mu}_{1} - r_{12}\,r_{23}\,\Omega^{\mu}_{2}) \quad . \qquad (38)$$

The symmetry properties of these objects do not change when we make the replacements: $(r^2_{ij}/\rho^2)\Omega^{\mu}_{ij} \rightarrow \Omega^{\mu}_{ij}$ and $(r_{jk}r_{ki}/\rho^2)\Omega^{\mu}_{k} \rightarrow \Omega^{\mu}_{k}$. Thus, using the combination rule given by eqs. (5), we can construct the following wave-functions with orbital angular momentum $\ell = 2$

$$|D\rangle = D(\vec{x},\vec{y})\left[(2\Omega^{\mu}_{12} - \Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{-}i\rangle_I + \sqrt{3}\,(\Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{+}i\rangle_I\right] \qquad (39)$$

$$|D'\rangle = (\Omega^{\mu}_{12} + \Omega^{\mu}_{23} + \Omega^{\mu}_{31})\left[D'_{+}(\vec{x},\vec{y})\,|m^{-}i\rangle_I - D'_{-}(\vec{x},\vec{y})\,|m^{+}i\rangle_I\right] \qquad (40)$$

$$|D''\rangle = D''_{+}(\vec{x},\vec{y})\left[(2\Omega^{\mu}_{12} - \Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{-}i\rangle_I - \sqrt{3}\,(\Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{+}i\rangle_I\right]$$
$$- D''_{-}(\vec{x},\vec{y})\left[-\sqrt{3}\,(\Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{-}i\rangle_I - (2\Omega^{\mu}_{12} - \Omega^{\mu}_{23} - \Omega^{\mu}_{31})\,|m^{+}i\rangle_I\right]$$
$$(41)$$

where the spatial wave-functions may be written as

$$D(\vec{x},\vec{y}) = \sum_{\alpha,n} d_{n\alpha}\ \Pi^{\alpha}_{n}(s) \qquad (42)$$

$$D'_{\pm}(\vec{x},\vec{y}) = \sum_{\alpha,n} d'_{n\alpha} \; \Pi^{\alpha}_{n}(m^{+}) \tag{43}$$

$$D''_{\pm}(\vec{x},\vec{y}) = \sum_{\alpha,n} d''_{n\alpha} \; \Pi^{\alpha}_{n}(m^{\pm}) \quad . \tag{44}$$

The coefficients $d_{n\alpha}$, $d'_{n\alpha}$ and $d''_{n\alpha}$ can be obtained from the dynamical equation describing the trinucleon system. The dependence of $|D'\rangle$ and $|D''\rangle$ on the polynomials $\Pi^{\alpha}_{n}(m^{\pm})$ means that they vanish in the triangular configuration, analogously to the case of $|S'\rangle$. It is also worth pointing out that in the colinear configuration all Ω^{μ}_{ij} become identical, causing both $|D\rangle$ and $|D''\rangle$ to vanish.

In this work we do not consider the D wave whose spatial part is totally antisymmetric.

III. POTENTIALS

III.1. The Nucleon-Nucleon Potential:

The general spin-isospin structure of $V(r_{ij})$, the potential for nucleons i and j, is

$$V(\vec{r}_{ij}) = V_{11}(r_{ij})P_1I_1 + V_{13}(r_{ij})P_1I_3 + V_{31}(r_{ij})P_3I_1 + V_{33}(r_{ij})P_3I_3 \tag{45}$$

where P and I are spin and isospin projection operators and the subscripts 1 and 3 denote singlet and triplet states. Each of the components $V_{\alpha\beta}(r_{ij})$ is assumed to contain only central

and tensor terms

$$V_{\alpha\beta}(r_{ij}) = V^{C}_{\alpha\beta}(r_{ij}) + V^{T}_{\alpha\beta}(r_{ij})\, S_{ij}\,(\vec{r}_{ij},\vec{r}_{ij}) \quad . \tag{46}$$

The tensor S_{ij} for two general vectors $\vec{u}$ and $\vec{v}$ is defined as

$$S_{ij}(\vec{u},\vec{v}) \equiv 3(\vec{\sigma}^{(i)}\cdot\hat{u})(\vec{\sigma}^{(j)}\cdot\hat{v}) - \hat{u}\cdot\hat{v}\;\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)} \tag{47}$$

where $\vec{\sigma}^{(i)}$ is the Pauli operator acting on nucleon i. We adopt the alternate soft core values given by Reid[16] for the various components of $V(r_{ij})$.

III.2. The Three-Nucleon Potential:

The final form of the two-pion exchange three-nucleon potential derived by means of chiral symmetry has recently become the subject of some controversy[2,17,18], which has motivated a better understanding of the problem. One of the important implications of the critical assessment of the ππE-3NP is that the expressions derived by the Tucson-Melbourne group[4,19] and by ourselves[5] have to be modified in order to be used in realistic calculations. We discuss here very briefly the most important points that have led to the form of the potential to be adopted in this work.

The ππE-3NP is denoted by W and has the following generic form

$$W = W_s + W_p + W'_p \quad , \tag{48}$$

the subscripts s and p indicating partial waves in the intermediate pion-nucleon amplitude. The description of this amplitude by means of chiral symmetry allows the potential to be written as[4,19;5,18]

$$W = \left(\frac{C_s}{\mu^2}\right)(\vec{\tau}^{(1)}.\vec{\tau}^{(2)})(\vec{\sigma}^{(1)}.\vec{\nabla}_{31})(\vec{\sigma}^{(2)}.\vec{\nabla}_{23}) \times$$

$$\times \left\{1 + \frac{1}{\mu^2}(\vec{\nabla}^2_{31} - \mu^2) + \frac{1}{\mu^2}(\vec{\nabla}^2_{23} - \mu^2)\right\} U(r_{31})\, U(r_{23})$$

$$+ \left(\frac{C_p}{\mu^2}\right)(\vec{\tau}^{(1)}.\vec{\tau}^{(2)})(\vec{\sigma}^{(1)}.\vec{\nabla}_{31})(\vec{\sigma}^{(2)}.\vec{\nabla}_{23})(\vec{\nabla}_{31}.\vec{\nabla}_{23})\, U(r_{31})\, U(r_{23})$$

$$- \left(\frac{C'_p}{\mu^2}\right)(\vec{\tau}^{(1)} \times \vec{\tau}^{(2)}.\vec{\tau}^{(3)})(\vec{\sigma}^{(1)}.\vec{\nabla}_{31})(\vec{\sigma}^{(2)}.\vec{\nabla}_{23})(\vec{\sigma}^{(3)}.\vec{\nabla}_{31} \times \vec{\nabla}_{23})\, U(r_{31})\, U(r_{23})$$

$$+ \text{c.p.} \tag{49}$$

where c.p. indicates "cyclic permutations of the indices 1 , 2 , 3" and U(r) is a Yukawa function modified by the πNN form factor

$$U(r) = \frac{4\pi}{\mu}\int \frac{d\vec{k}}{(2\pi)^3}\,\frac{e^{-i\vec{k}.\vec{r}}}{\vec{k}^2 + \mu^2}\left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 + \vec{k}^2}\right)^2$$

$$= \frac{e^{-\mu r}}{\mu r} - \frac{\Lambda}{\mu}\frac{e^{-\Lambda r}}{\Lambda r} - \frac{1}{2}\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2} - 1\right) e^{-\Lambda r} \quad . \tag{50}$$

The coefficients C_s , C_p and C'_p were determined by means of chiral dynamics and have the following numerical values[5]: $C_s = 0.92$ MeV , $C_p = -1.99$ MeV , $C'_p = -0.67$ MeV .

The terms proportional to C_p and C'_p in eq. (49) are the same as those of eq. (61) of our derivation of the ππE-3NP[5]. The term with coefficient C_s, on the other hand, has been modified by the inclusion of the factors $(\vec{\nabla}^2 - \mu^2)$ within the curly brackets. This modification is needed, as pointed out by McKellar and Glöckle[2], in order to make the σ-contribution to the intermediate πN scattering amplitude compatible with the Adler consistency condition[20]. A more detailed discussion of the problem can be found in ref. (18).

After having introduced the factors $(\vec{\nabla}^2 - \mu^2)$ into the potential W, we argue that it has to be redefined before being used in realistic calculations. This second modification is required to prevent huge distortions in W when form-factors are introduced, as discussed extensively in ref. (18) for the case of W_s. One interesting feature of this redefinition of W is that it prescribes the exclusion of factors $(\vec{\nabla}^2 - \mu^2)$. So, the factors introduced for one reason into the potential will have to be dropped for another reason, as we discuss below.

The action of the various derivatives on the functions U(r) can be evaluated with the help of the following results

$$\frac{\partial U(r)}{\partial x_i} = \frac{x_i}{r}\,\frac{\partial U(r)}{\partial r}$$

$$\equiv \frac{x_i}{r}\,\mu\, U_1(r) \tag{51}$$

$$\frac{\partial^2 U(r)}{\partial x_i \partial x_j} = \delta_{ij} \frac{1}{r} \frac{\partial U(r)}{\partial r} + \frac{x_i x_j}{r^2} \left[\frac{\partial^2 U(r)}{\partial r^2} - \frac{1}{r} \frac{\partial U(r)}{\partial r} \right]$$

$$\equiv \delta_{ij} \frac{\mu}{r} U_1(r) + \frac{x_i x_j}{r^2} \mu^2 U_2(r) \qquad (52)$$

where the x_i are the Cartesian components of $\vec{r}$ and

$$U_1(r) = -\left(1 + \frac{1}{\mu r}\right) \frac{e^{-\mu r}}{\mu r} + \frac{\Lambda^2}{\mu^2} \left(1 + \frac{1}{\Lambda r}\right) \frac{e^{-\Lambda r}}{\Lambda r} + \frac{1}{2} \left(\frac{\Lambda^2}{\mu^2} - 1\right) e^{-\Lambda r} \qquad (53)$$

$$U_2(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} - \frac{\Lambda^2}{\mu^3} \left(1 + \frac{3}{\Lambda r} + \frac{3}{\Lambda^2 r^2}\right) \frac{e^{-\Lambda r}}{\Lambda r}$$

$$- \frac{1}{2} \frac{\Lambda}{\mu} \left(\frac{\Lambda^2}{\mu^2} - 1\right) \left(1 + \frac{1}{\Lambda r}\right) e^{-\Lambda r} \quad . \qquad (54)$$

The action of the Laplacian can be obtained from eq. (52)

$$\vec{\nabla}^2 U(r) = 3 \frac{\mu}{r} U_1(r) + \mu^2 U_2(r) \quad . \qquad (55)$$

An alternative expression can be obtained directly from eq. (50)

$$\vec{\nabla}^2 U(r) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{(-\vec{k}^2 - \mu^2 + \mu^2)}{(\vec{k}^2 + \mu^2)} e^{-i\vec{k}.\vec{r}} \left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 + \vec{k}^2}\right)^2$$

$$= \mu^2 (U(r) - G(r)) \qquad (56)$$

where $G(r)$ is a function proportional to the Fourier transform of the form factor

$$G(r) = \frac{4\pi}{\mu^3} \int \frac{d\vec{k}}{(2\pi)^3} e^{-i\vec{k}.\vec{r}} \left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 + \vec{k}^2}\right)^2$$

$$= \frac{1}{2} \frac{\mu}{\Lambda} \left(\frac{\Lambda^2}{\mu^2} - 1\right)^2 e^{-\Lambda r} \quad . \qquad (57)$$

In ref. (5) we have denoted the right hand side of eq. (56) by a function $U_0(r)$, defined as

$$U_0(r) \equiv U(r) - G(r) \quad . \qquad (58)$$

Comparing eqs. (55) and (56) we obtain

$$\frac{3}{\mu r} U_1(r) = U(r) - G(r) - U_2(r) \quad . \qquad (59)$$

Using these results it is possible to rewrite the various parts of W as follows

$$W_S = \frac{C_S}{3} \vec{\tau}^{(1)}.\vec{\tau}^{(2)} \left\{ U_1(r_{31}) U_1(r_{23}) - \frac{1}{\mu} \frac{\partial G(r_{31})}{\partial r_{31}} U_1(r_{23}) - U_1(r_{31}) \frac{1}{\mu} \frac{\partial G(r_{23})}{\partial r_{23}} \right\} \times$$

$$\times \left[S_{12}(\hat{r}_{31}, \hat{r}_{23}) + \cos\theta_3 \, \vec{\sigma}^{(1)}.\vec{\sigma}^{(2)} \right] + \text{c.p.} \qquad (60)$$

$$W_P = \frac{C_P}{9} \vec{\tau}^{(1)}.\vec{\tau}^{(2)} \left\{ \left[\left(U(r_{31}) - G(r_{31})\right) \left(U(r_{23}) - G(r_{23})\right) \right.\right.$$

$$\left. + 2 P_2(\cos\theta_3) U_2(r_{31}) U_2(r_{23}) \right] \vec{\sigma}^{(1)}.\vec{\sigma}^{(2)}$$

$$+ \left[U(r_{31}) - G(r_{31}) - U_2(r_{31}) \right] U_2(r_{23}) S_{12}(\hat{r}_{23}, \hat{r}_{23})$$

$$+ U_2(r_{31}) \left[U(r_{23}) - G(r_{23}) - U_2(r_{23}) \right] S_{12}(\hat{r}_{31}, \hat{r}_{31})$$

$$\left. + 3 \cos\theta_3 \, U_2(r_{31}) U_2(r_{23}) S_{12}(\hat{r}_{31}, \hat{r}_{23}) \right\} + \text{c.p.} \qquad (61)$$

$$W'_p = \frac{C'_p}{9}\,(i\,\vec{\tau}^{(1)} \times \vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) \Big\{ \Big[\big(U(r_{31}) - G(r_{31})\big)\big(U(r_{23}) - G(r_{23})\big) \times$$

$$\times\,(i\,\vec{\sigma}^{(1)} \times \vec{\sigma}^{(2)} \cdot \vec{\sigma}^{(3)}) \Big]$$

$$+ \Big[U(r_{31}) - G(r_{31})\Big] U_2(r_{23})\,\frac{1}{2}\Big[S_{12}(\hat{r}_{23},\hat{r}_{23})\vec{\sigma}^{(3)}\cdot\vec{\sigma}^{(1)} - \vec{\sigma}^{(3)}\cdot\vec{\sigma}^{(1)} S_{12}(\hat{r}_{23},\hat{r}_{23})\Big]$$

$$+ U_2(r_{31})\Big[U(r_{23}) - G(r_{23})\Big]\frac{1}{2}\Big[-S_{12}(\hat{r}_{31},\hat{r}_{31})\vec{\sigma}^{(2)}\cdot\vec{\sigma}^{(3)} + \vec{\sigma}^{(2)}\cdot\vec{\sigma}^{(3)} S_{12}(\hat{r}_{31},\hat{r}_{31})\Big]$$

$$+ U_2(r_{31})U_2(r_{23})\,\frac{1}{2}\Big[S_{31}(\hat{r}_{31},\hat{r}_{31})S_{23}(\hat{r}_{23},\hat{r}_{23}) - S_{23}(\hat{r}_{23},\hat{r}_{23})S_{31}(\hat{r}_{31},\hat{r}_{31})\Big]\Big\}$$

$$+ \text{c.p.} \tag{62}$$

where

$$\cos\theta_3 \equiv \frac{\vec{r}_{31}\cdot\vec{r}_{23}}{r_{31}\,r_{23}} \quad . \tag{63}$$

The expressions for W_p and W'_p are the same as those of eq. (67) of ref. (5), except for an unfortunate misprint in the sign of the term proportional to C'_p in that equation. The expression for W_s, on the other hand, includes now the derivatives of the function $G(r)$, originating from the factors $(\vec{\nabla}^2 - \mu^2)$.

The final form of the $\pi\pi$E-3NP is obtained by eliminating all occurences of G(r) and its derivatives from the above equations. The motivation for this redefinition of the potential is discussed at length in ref. (18) and can be summarized as follows. In the absence of form-factors U(r) becomes the Green's function for eq. (56) and G(r) is

proportional to a δ-function, describing a contact interaction between two of the nucleons. When form factors are present, G(r) describes a "contact" interaction between two extended nucleons. Hence the terms of eqs. (60-62) containing this function cannot be associated with the propagation of pions between different points in space and do not correspond to a proper $\pi\pi$E-3NP. Rather, they could be called π?E-3NP, because the parameter of the form factor does not let us know the type of particles being exchanged. Another reason for dropping G(r) and its derivatives is that, when they are present, the form factors do not just correct the potential for small internucleon distances. Instead, they determine its form in a much larger region. Form factors are relativistic corrections and therefore this behaviour is inconsistent with the non-relativistic hypothesis that has led to the final form of the potential. These problems are not present in the redefined version of the potential, that is denoted by $\hat{W}$.

IV. MATRIX ELEMENTS

The two-body potential V (eq.(45)) acting on the principal S state (eq. (18)) yields

$$V|S\rangle = S(\vec{x},\vec{y})\,\Big\{\frac{1}{2}\Big[V^C_{13}(r_{12}) + V^C_{31}(r_{12})\Big]\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a)$$

$$+ \frac{1}{2}\Big[V^C_{13}(r_{12}) - V^C_{31}(r_{12})\Big]\Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(m^-) - 2\,V^T_{31}(r_{12})\Omega^{\mu}_{12}|m^- i\rangle_I\Big\} + \text{c.p.} \tag{64}$$

In this evaluation we have used the following results

$$\begin{aligned} S_{12}(\hat{r}_{ij},\hat{r}_{ij}), |m^-\mu\rangle_S &= 0 \\ S_{12}(\hat{r}_{ij},\hat{r}_{ij}) \; |m^+\mu\rangle_S &= 2\sqrt{2}\;\Omega^{\mu}_{ij} \;. \end{aligned} \qquad (65)$$

The contribution of W_s is given by

$$\hat{W}_s|S\rangle = S(\vec{x},\vec{y})\, C_s \Big\{ -\cos\theta_3 \; U_1(r_{23})U_1(r_{31}) \; \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) + 2\, U_1(r_{23})U_1(r_{31})\, \Omega^{\mu}_3 \, |m^- i\rangle_I \Big\} + \text{c.p.} \qquad (66)$$

where we have employed

$$\begin{aligned} S_{12}(\hat{r}_{ki},\hat{r}_{jk}) \; |m^-\mu\rangle_S &= \text{terms with } \ell = 1 \\ S_{12}(\hat{r}_{ki},\hat{r}_{jk}) \; |m^+\mu\rangle_S &= 2\sqrt{2}\; \Omega^{\mu}_{k} + \text{terms with } \ell = 1 \;. \end{aligned} \qquad (67)$$

The action of W_p yields

$$\begin{aligned} \hat{W}_p|S\rangle = S(\vec{x},\vec{y})\, \frac{C_p}{3} \Big\{ &-\Big[U(r_{23})U(r_{31}) + 2\, P_2(\cos\theta_3) U_2(r_{23})U_2(r_{31}) \Big] \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) \\ &+ 2 \Big[U_2(r_{23}) \big(U(r_{31}) - U_2(r_{31})\big)\, \Omega^{\mu}_{23} + \big(U(r_{23}) - U_2(r_{23})\big) U_2(r_{31})\, \Omega^{\mu}_{31} \\ &+ 3\cos\theta_3\, U_2(r_{23})U_2(r_{31})\, \Omega^{\mu}_3 \Big] \; |m^- i\rangle_I \Big\} \end{aligned} \qquad (68)$$

Finally, W'_p produces

$$\begin{aligned} \hat{W}'_p|S\rangle = S(\vec{x},\vec{y})\, \frac{4C'_p}{3} \Big\{ &\Big[U(r_{23})U(r_{31}) - P_2(\cos\theta_3) U_2(r_{23})U_2(r_{31}) \Big] \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) \\ &+ U_2(r_{23})U(r_{31})\, \Omega^{\mu}_{23} \Big(-\frac{\sqrt{3}}{2} \; |m^+ i\rangle_I - \frac{1}{2} \; |m^- i\rangle_I \Big) \\ &- U(r_{23})U_2(r_{31})\, \Omega^{\mu}_{31} \Big(\frac{\sqrt{3}}{2} \; |m^+ i\rangle_I - \frac{1}{2} \; |m^- i\rangle_I \Big) \\ &+ U_2(r_{23})U_2(r_{31}) \Big[3\cos\theta_3\, \Omega^{\mu}_3 - (\Omega^{\mu}_{23} + \Omega^{\mu}_{31}) \Big] |m^- i\rangle_I \Big\} + \text{c.p.} \end{aligned} \qquad (69)$$

where we have used

$$\begin{aligned} &S_{31}(\hat{r}_{31},\hat{r}_{31}) S_{23}(\hat{r}_{23},\hat{r}_{23}) \; |m^-\mu\rangle_S \\ &= S_{31}(\hat{r}_{31},\hat{r}_{31}) S_{23}(\hat{r}_{23},\hat{r}_{23}) \; \sqrt{3} \; |m^+\mu\rangle_S \\ &= 2\sqrt{6} \Big[\cos\theta_3 \Omega^{\mu}_3 - (\Omega^{\mu}_{23} + \Omega^{\mu}_{31}) \Big] - 4\sqrt{3} \; P_2(\cos\theta_3) \\ &\qquad \times \Big(-\frac{1}{2} \, |m^+\mu\rangle_S + \frac{\sqrt{3}}{2} \; |m^-\mu\rangle_S \Big) + \text{terms with } \ell=1 \;. \end{aligned} \qquad (70)$$

All the preceding results are summarized in the following expression

$$\begin{aligned} \hat{W}|S\rangle = S(\vec{x},\vec{y}) \Big\{ \Big[&-\frac{1}{3} (C_p - 4C'_p) U(r_{23})U(r_{31}) - C_s \cos\theta_3 \; U_1(r_{23})U_1(r_{31}) \\ &- \frac{2}{3} \; (C_p + 2C'_p) \; P_2(\cos\theta_3) \; U_2(r_{23})U_2(r_{31}) \Big] \; \Gamma^{\frac{1}{2}\mu}_{\frac{1}{2}i}(a) \\ &+ \Big[2\, C_s \; U_1(r_{23})U_1(r_{31}) + 2\, (C_p + 2C'_p) \cos\theta_3 \; U_2(r_{23})U_2(r_{31}) \Big] \Omega^{\mu}_3 |m^- i\rangle_I \end{aligned}$$

$$+\frac{2}{3}\,(C_p+2C_p')\Big[U_2(r_{23})\big(U(r_{31})-U_2(r_{31})\big)\Omega^{\mu}_{23}+\big(U(r_{23})-U_2(r_{23})\big)U_2(r_{31})\Omega^{\mu}_{31}\Big]|m^-i\rangle_I$$

$$+\frac{4C_p'}{\sqrt{3}}\Big[U_2(r_{23})U(r_{31})\,\Omega^{\mu}_{23}\Big(-\frac{1}{2}\,|m^+i\rangle_I-\frac{\sqrt{3}}{2}\,|m^-i\rangle_I\Big)$$

$$+U(r_{23})U_2(r_{31})\,\Omega^{\mu}_{31}\Big(\frac{1}{2}\,|m^+i\rangle_I-\frac{\sqrt{3}}{2}\,|m^-i\rangle_I\Big)\Big]\Big]\Big\}+\text{c.p.}\qquad(71)$$

The results given by eqs. (64) and (71) produce the following expectation values for the total potential between $|S\rangle$ and the S and D states

$$\langle S|V+\hat{W}|S\rangle=\Big[S(\vec{x},\vec{y})\Big]^2\Big\{\frac{1}{2}\Big[V^C_{13}(r_{12})+V^C_{31}(r_{12})\Big]$$

$$+\Big[-\frac{1}{3}\,(C_p-4C_p')\;U(r_{23})\,U(r_{31})-C_s\cos\theta_3\,U_1(r_{23})\,U_1(r_{31})$$

$$-\frac{2}{3}\,(C_p+2C_p')\;P_2(\cos\theta_3)\,U_2(r_{23})\,U_2(r_{31})\Big]\Big\}+\text{c.p.}\qquad(72)$$

$$\langle S'|V+\hat{W}|S\rangle=S_+'(\vec{x},\vec{y})\;S(\vec{x},\vec{y})\;\Big\{\frac{1}{2}\Big[V^C_{13}(r_{12})-V^C_{31}(r_{12})\Big]\Big\}+\text{c.p.}\qquad(73)$$

$$\langle D|V+\hat{W}|S\rangle=D(\vec{x},\vec{y})\;S(\vec{x},\vec{y})\Big\{-2\,V^T_{31}(r_{12})\Big[2-P_2(\cos\theta_1)-P_2(\cos\theta_2)\Big]$$

$$+\Big[2C_s\,U_1(r_{23})U_1(r_{31})+2(C_p+2C_p')\cos\theta_3\,U_2(r_{23})U_2(r_{31})\Big]3(\cos\theta_1\cos\theta_2-\cos\theta_3)$$

$$+\frac{2}{3}\,(C_p+2C_p')\Big[U_2(r_{23})\big(U(r_{31})-U_2(r_{31})\big)\big(2P_2(\cos\theta_2)-1-P_2(\cos\theta_3)\big)$$

$$+\big(U(r_{23})-U_2(r_{23})\big)U_2(r_{31})\big(2P_2(\cos\theta_1)-1-P_2(\cos\theta_3)\big)\Big]+$$

$$+4C_p'\Big[U_2(r_{23})U(r_{31})\big(P_2(\cos\theta_3)-P_2(\cos\theta_2)\big)$$

$$+U(r_{23})U_2(r_{31})\big(P_2(\cos\theta_3)-P_2(\cos\theta_1)\big)\Big]\Big\}+\text{c.p.}\qquad(74)$$

$$\langle D'|V+\hat{W}|S\rangle=D_+'(\vec{x},\vec{y})S(\vec{x},\vec{y})\Big\{-2\,V^T_{31}(r_{12})\Big[1+P_2(\cos\theta_1)+P_2(\cos\theta_2)\Big]$$

$$+\Big[2C_s\,U_1(r_{23})U_1(r_{31})+2(C_p+2\;C_p')\cos\theta_3\,U_2(r_{23})\,U_2(r_{31})\Big]\frac{3}{2}(\cos\theta_1\cos\theta_2+\cos\theta_3)$$

$$+\frac{2}{3}\,(C_p+2C_p')\Big[U_2(r_{23})\;\big(U(r_{31})-U_2(r_{31})\big)\big(P_2(\cos\theta_2)+1+P_2(\cos\theta_3)\big)$$

$$+\big(U(r_{23})-U_2(r_{23})\big)\;U_2(r_{31})\;\big(P_2(\cos\theta_1)+1+P_2(\cos\theta_3)\big)\Big]\Big\}$$

$$-\frac{4C_p'}{\sqrt{3}}\Big\{U_2(r_{23})U(r_{31})\big(P_2(\cos\theta_3)+1+P_2(\cos\theta_2)\big)\Big[\frac{\sqrt{3}}{2}\,D_+'(\vec{x},\vec{y})-\frac{1}{2}\,D_-'(\vec{x},\vec{y})\Big]S(\vec{x},\vec{y})$$

$$+U(r_{23})U_2(r_{31})\big(P_2(\cos\theta_3)+P_2(\cos\theta_1)+1\big)\Big[\frac{\sqrt{3}}{2}\,D_+'(\vec{x},\vec{y})+\frac{1}{2}\,D_-'(\vec{x},\vec{y})\Big]\;S(\vec{x},\vec{y})\Big\}$$

$$+\text{c.p.}\qquad(75)$$

$$\langle D''|V+\hat{W}|S\rangle=\Big\{-2\,V^T_{31}(r_{12})\Big[D_+''(\vec{x},\vec{y})\,\big(2-P_2(\cos\theta_1)-P_2(\cos\theta_2)\big)$$

$$-\sqrt{3}\;D_-''(\vec{x},\vec{y})\;\big(P_2(\cos\theta_1)-P_2(\cos\theta_2)\big)\Big]\Big\}\;S(\vec{x},\vec{y})$$

$$+D_+''(\vec{x},\vec{y})S(\vec{x},\vec{y})\Big[2C_s\,U_1(r_{23})U_1(r_{31})+2(C_p+2C_p')\;\cos\theta_3\;U_2(r_{23})U_2(r_{31})\Big]\;\times$$

$$\times\;3\;(\cos\theta_1\;\cos\theta_2-\cos\theta_3)$$

$$+\frac{2}{3}\,(C_p+2C_p')\;\Big\{U_2(r_{23})\Big[U(r_{31})-U_2(r_{31})\Big]\Big[D_+''(\vec{x},\vec{y})\big(2P_2\,(\cos\theta_2)-1-P_2(\cos\theta_3)\big)+$$

$$+ \sqrt{3}\, D''_{-}(\vec{x},\vec{y}) \left(1 - P_2(\cos\theta_3)\right)\Big] + \Big[U(r_{23}) - U_2(r_{23})\Big]\, U_2(r_{31}) \times$$

$$\times \Big[D''_{+}(\vec{x},\vec{y}) \left(2P_2(\cos\theta_1) - P_2(\cos\theta_3) - 1\right) + \sqrt{3}\, D''_{-}(\vec{x},\vec{y}) \left(P_2(\cos\theta_3) - 1\right)\Big]\Big\}\, S(\vec{x},\vec{y})$$

$$+ \frac{4C'_p}{\sqrt{3}} \Big\{U_2(r_{23})U(r_{31}) \Big[\sqrt{3}\, D''_{+}(\vec{x},\vec{y}) \left(1 - P_2(\cos\theta_2)\right) + D''_{-}(\vec{x},\vec{y}) \left(2P_2(\cos\theta_3)\right.$$

$$\left. - 1 - P_2(\cos\theta_2)\right)\Big] + U(r_{23})U_2(r_{31}) \Big[\sqrt{3}\, D''_{+}(\vec{x},\vec{y}) \left(1 - P_2(\cos\theta_1)\right) +$$

$$- D''_{-}(\vec{x},\vec{y}) \left(2P_2(\cos\theta_3) - 1 - P_2(\cos\theta_1)\right)\Big]\Big\}\, S(\vec{x},\vec{y}) + c.p. \qquad (76)$$

In the next section the matrix elements displayed above are used in the construction of the equipotential plots.

V. EQUIPOTENTIAL PLOTS AND CONCLUSIONS

We study here the qualitative features of the coupling between S and D states in trinucleon systems, due to two and three-nucleon potentials. This study is performed with the help of the energy diagrams already employed by Brandenburg and Glöckle[21] and by ourselves[8,18] and that are very useful because of their relative simplicity.

The equipotential plots are constructed by fixing the positions of two of the nucleons and taking the third one as a probe. The coordinate system used to describe the tri-nucleon is shown in fig. 1 . All diagrams are symmetric under rotations around the X direction and under reflections about the

Y direction. Hence the specification of a single quadrant is enough to determine uniquely the complete spatial diagram. We adopt the value $r_{12} = 0.88$ fm for the fixed internucleon distance, corresponding to the minimum of the Reid two-body potential[16].

The principal S state expectation value of the two-body potential is the basic element in the construction of the ground state wave function of the trinucleon. The corresponding energy diagram has already been studied in refs. (21) and (8) and is shown again in fig.2 for the sake of completeness. The present diagram is slightly different from that of ref. (8) because here we have used an alternate form of Reid's potential. The examination of the figure allows us to conclude that the triangular configuration is favoured, since it corresponds to the minimum potential energy.

In figures 3a and 3b we show the plots for two versions of the matrix element $\langle S|W|S\rangle$, where W is the $\pi\pi$E-3NP given by eqs. (48,60-62) and we have adopted the value $\Lambda = 5\ fm^{-1}$ for the parameter of the form factor. In the former figure we used the fully redefined version of the potential, denoted by $\hat{W}$ and obtained by neglecting G(r) everywhere. In fig. 3b, on the other hand, we have eliminated G(r) in $\hat{W}_s$, keeping it in W_p and W'_p . The motivation for this choice is that the role of G(r) in $\langle S|W_s|S\rangle$ has already been studied in ref. (18). Comparing both figures we find that, in the case of the principal S state, the neglect of G(r) in $\hat{W}_p$ and $\hat{W}'_p$ does not produce the same dramatic changes as observed in $\hat{W}_s$[18]. The reason for this behaviour is that the strength parameter $\frac{1}{3}(C_p - 4C_p')$ entering eq. (72) is smaller than the others, minimizing the effects of the replacement $[U(r) - G(r)] \to U(r)$. In fig. 3a we note that $\langle S|\hat{W}|S\rangle$ is attractive in the triangular configuration, adding constructively to the two-nucleon potential. The same conclusion can be drawn from fig. 4, where the joint effect of V and $\hat{W}$ is displayed.

The energy diagram describing the coupling between the wave S' and the principal S state is shown in fig. 5. This coupling is due only to the two-nucleon force, as indicated in eq. (73), and involves the unknown functions $S_+'(\vec{x},\vec{y})$ and $S_-'(\vec{x},\vec{y})$. So, in the evaluation of the diagram we have approximated them by the first term of eq. (21). One important feature of the coupling between S' and S is that the transformation properties of the former under the permutation group force it to vanish at the triangular configuration. This means that the wave S' is constrained to be small at the points where the principal S state is large and hence the coupling between them is relatively weak.

The couplings of D waves to the principal S state are also strongly influenced by structural constraints. As discussed at the end of section II, the D states are such that $|D\rangle$ vanishes in the colinear configuration, $|D'\rangle$ in the triangular configuration and $|D''\rangle$ in both of them. These constraints determine a hierarchy among the states, since figs. 2 and 4 allow us to expect the principal S wave function to be large when the nucleous form a triangle and small when they are aligned. This means that this state will couple

strongly to those not constrained to vanish in the triangular configuration, as is the case of $|D\rangle$, which can be called the principal D state. The state $|D'\rangle$, on the other hand, is more important than $|D''\rangle$ because the latter is doubly constrained.

The equipotential plot describing the coupling of the principal D and S states due to the two-nucleon force is given in fig. 6. This is usually assumed to be the basic coupling for D waves and is the point of departure for the inclusion of the effects of three-body forces. It is worth pointing out that the absolute values of the energies appearing in this figure do not mean much and, in particular, should not be directly compared to those of fig. 2, because they are multiplied by spatial wave-functions with different normalizations.

The energy diagrams obtained from $\hat{W}_s$, $\hat{W}_p$ and $\hat{W}_p'$, the components of the fully redefined version of the ππE-3NP, are shown in figs. 7a-c. Examining them we note that only equipotentials with positive sign are present, meaning that at any given point all contributions add constructively. Moreover, the maxima of these figures occur roughly at the same region, namely that corresponding to the triangular configuration, confirming that the coupling between $|D\rangle$ and $|S\rangle$ is strong. As far as the magnitude of the various terms is concerned, we can conclude that $\hat{W}_p$ and $\hat{W}_p'$ are much more important than $\hat{W}_s$.

When dealing exclusively with the principal S state we have noted that the elimination of the function G(r) from W_p and W_p' in eqs. (61) and (62) has very little influence on the energy diagram. In the case of the coupling of the principal D and S states, on the other hand, the situation is quite different, as it is shown by figs. 8a and b. The first of them describes the matrix element $\langle D|\left[\hat{W}_s + \hat{W}_p + \hat{W}'_p\right]|S\rangle$ where, as before, the symbol $(\wedge)$ indicates the neglect of $G(r)$. Thus fig. 8a corresponds to the superposition of figs. 7a-c, whereas fig. 8b represents $\langle D|\left[\hat{W}_s + W_p + W_p\right]|S\rangle$. Inspecting these diagrams we conclude that the presence of $G(r)$ is noticeable and determines the short distance behaviour of the potential. The close association of this function with "contact" interactions having a rather loose dynamical meaning suggests that it should be neglected everywhere in order to allow the ππE-3NP to be interpreted as being due to pions propagating between different points in space[18].

The joint effect of the two and three-nucleon potentials is displayed in fig. 9. Comparing it with fig. 6 we note that the inclusion of the ππE-3NP produces significant modifications in that equipotential plot. The extent of this modification may be better grasped by inspecting fig. 10, where we show the curves describing the potential energies along the Y axis of fig. 1, representing the intersection of the energy surfaces of figs. 6 and 9 with a plane perpendicular to X and containing Y. The continuous line is due to the two-nucleon force, whereas the broken one is the combined contribution of two and three-nucleon forces. It is possible to see that in the regions around the triangular configuration the effects of the ππE-3NP are of the same order of magnitude as those of the two-body force. This means that the usual

treatment of the former by means of perturbation theory is not justified. As an illustration we also include in fig. 10 the dotted curve, corresponding to $\langle D|\left[V + \hat{W}_s + W_p + W'_p\right]|S\rangle$.

The coupling between the states $|D'\rangle$ and $|S\rangle$ due to the two-body force is displayed in fig. 11a, whereas that due to the combined two and three-nucleon potentials is given in fig. 11b. Similarly to the case of $|S'\rangle$, we have approximated the unknown functions $D'_+(\vec{x},\vec{y})$ and $D'_-(\vec{x},\vec{y})$ of eq. (75) by the first term of eq. (43). Comparing both diagrams we note that the influence of the ππE-3NP is strong. Nevertheless, the energies around the triangular configuration are small, meaning that the coupling is weak. The same qualitative conclusions are valid for the case of the state $|D''\rangle$, corresponding to the equipotential plots given in figs. 12a and b.

In this work we have studied the couplings of the D-waves to the principal S state of trinucleon systems. Our two main general conclusions are the following: first, the redefinition of the ππE-3NP by neglecting "contact" interactions affect significantly the results; second, the importance of two and three-nucleon potentials are comparable and hence the use of perturbation theory is not advisable in this case. As a final remark, we would like to mention that the test in realistic calculations of the conclusions presented in this work would be very helpful in improving our understanding of the problem.

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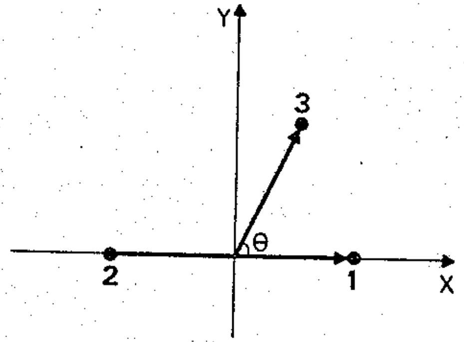

Fig. 1 - Coordinates of the trinucleon system.

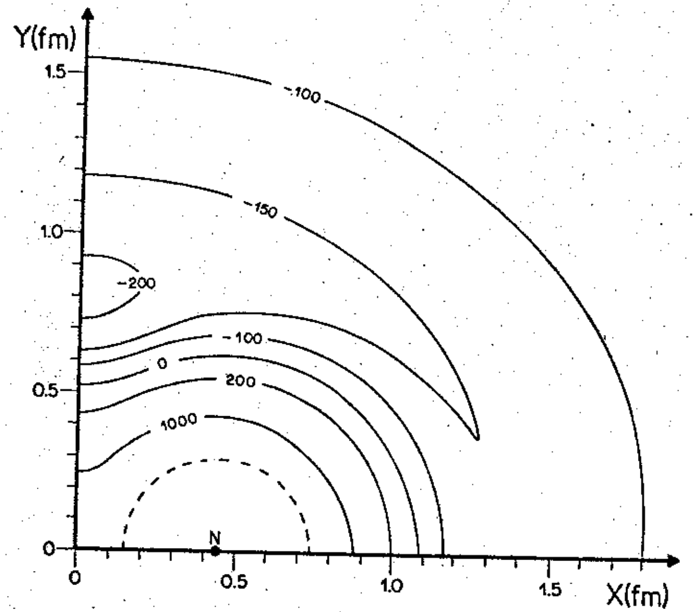

Fig. 2 - Equipotentials for $\langle S|V|S\rangle$. All energies are in MeV and N indicates the position of one of the fixed nucleons.

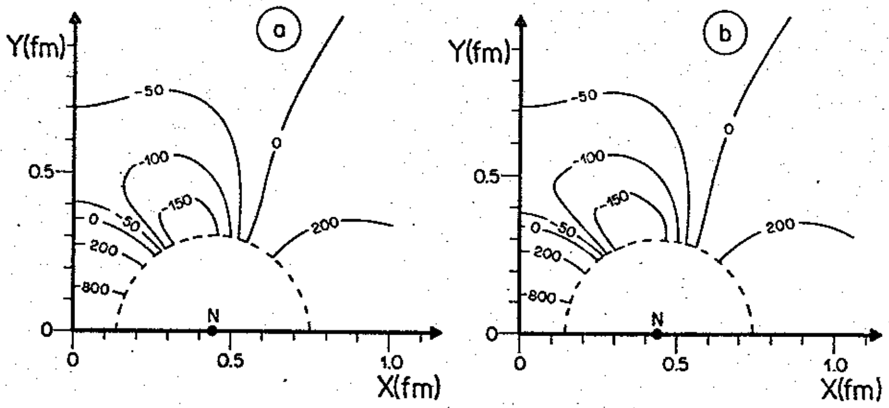

Fig. 3 - Equipotentials for (a) $\langle S|[\hat{W}_s + \hat{W}_p + \hat{W}'_p]|S\rangle$ and (b) $\langle S|[\hat{W}_s + W_p + W'_p]|S\rangle$. Conventions as in fig. 2.

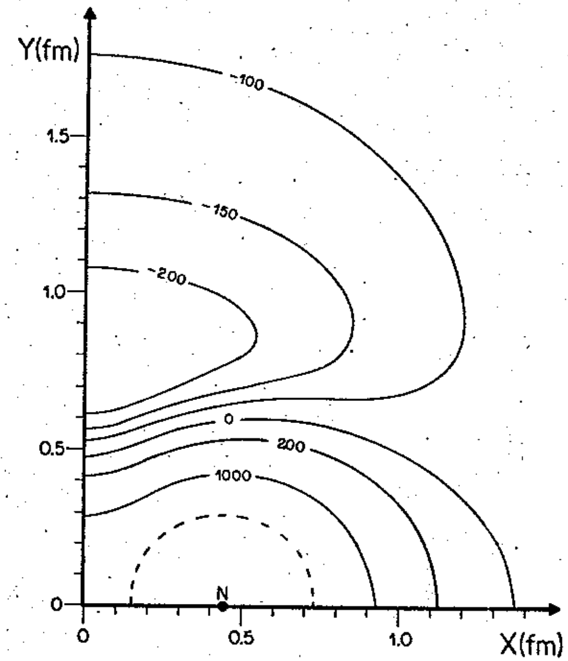

Fig. 4 - Equipotentials for $\langle S|[V + \hat{W}_s + \hat{W}_p + \hat{W}'_p]|S\rangle$. Conventions as in fig. 2.

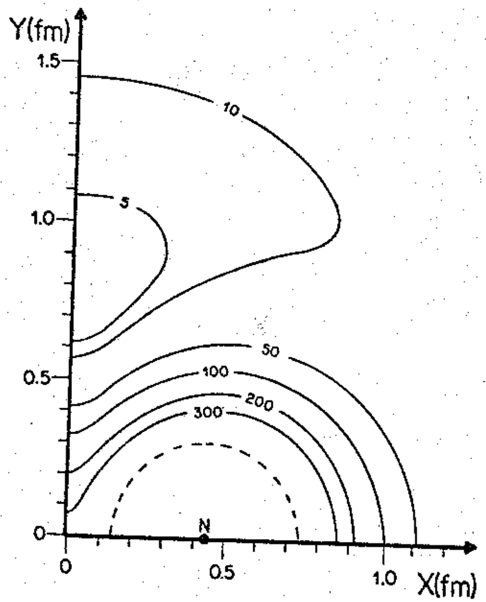

Fig. 5 - Equipotentials for $\langle S'|V|S\rangle$. Conventions as in fig. 2.

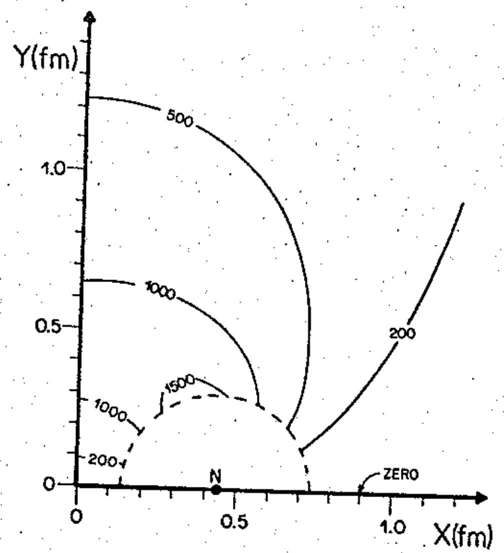

Fig. 6 - Equipotentials for $\langle D|V|S\rangle$. Conventions as in fig. 2.

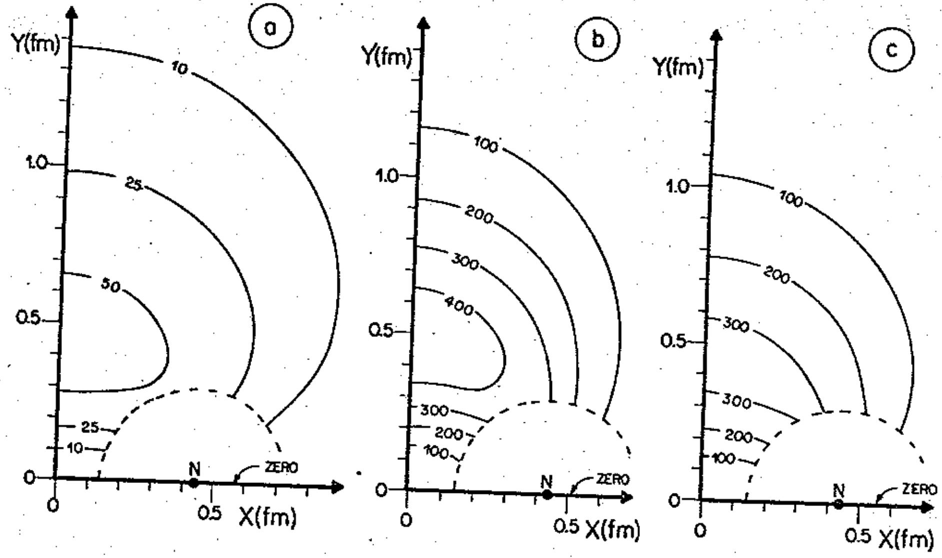

Fig. 7 - Equipotentials for (a) $\langle D|\hat{W}_s|S\rangle$, (b) $\langle D|\hat{W}_p|S\rangle$ and (c) $\langle D|W_p'|S\rangle$. Conventions as in fig. 2.

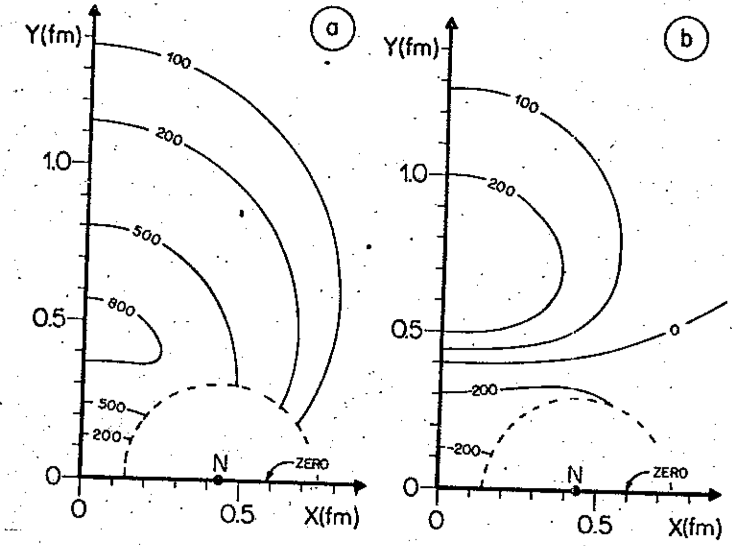

Fig. 8 - Equipotentials for (a) $\langle D|[\hat{W}_s + \hat{W}_p + \hat{W}_p']|S\rangle$ and (b) $\langle D|[\hat{W}_s + W_p + W_p']|S\rangle$. Conventions as in fig. 2.

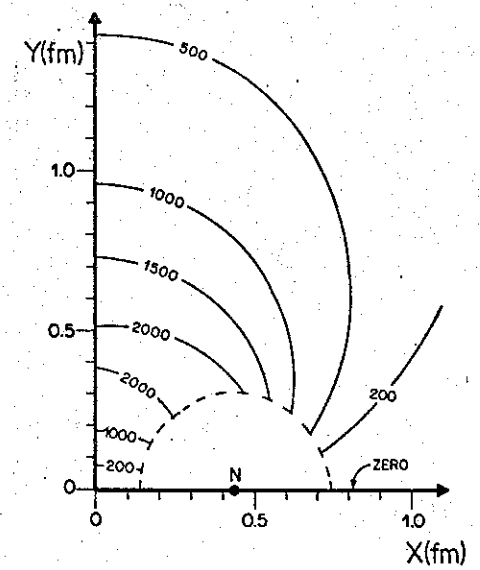

Fig. 9 - Equipotentials for $\langle D|[V+\hat{W}_s+\hat{W}_p+\hat{W}_p']|S\rangle$.

Conventions as in fig. 2.

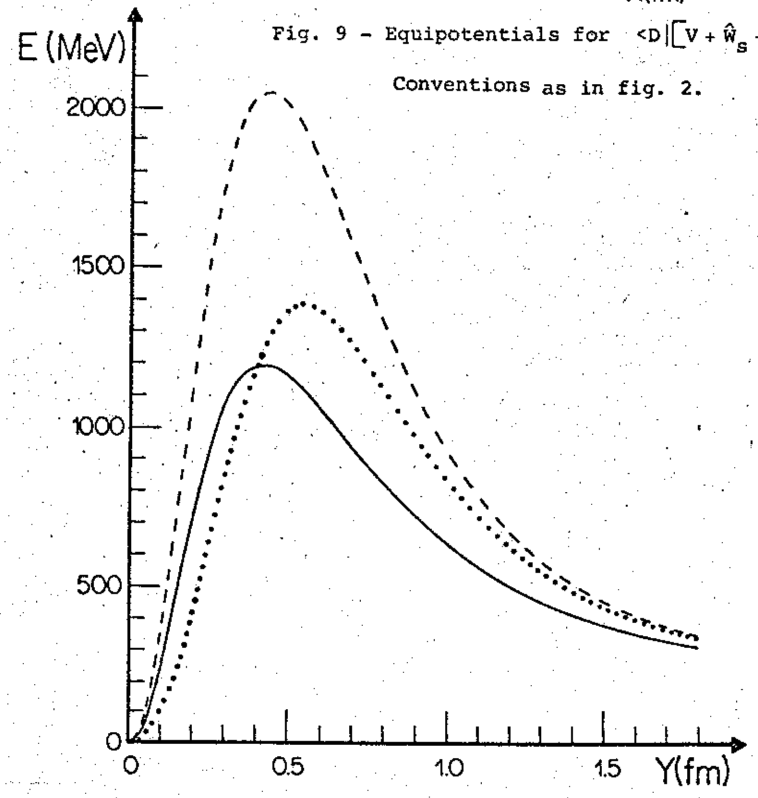

Fig. 10 - Energy curves along the Y axis for $\langle D|V|S\rangle$ (continuous line), $\langle D|[V+\hat{W}_s+\hat{W}_p+\hat{W}_p']|S\rangle$ (broken line) and $\langle D|[V+\hat{W}_s+W_p+W_p']|S\rangle$ (dotted line).

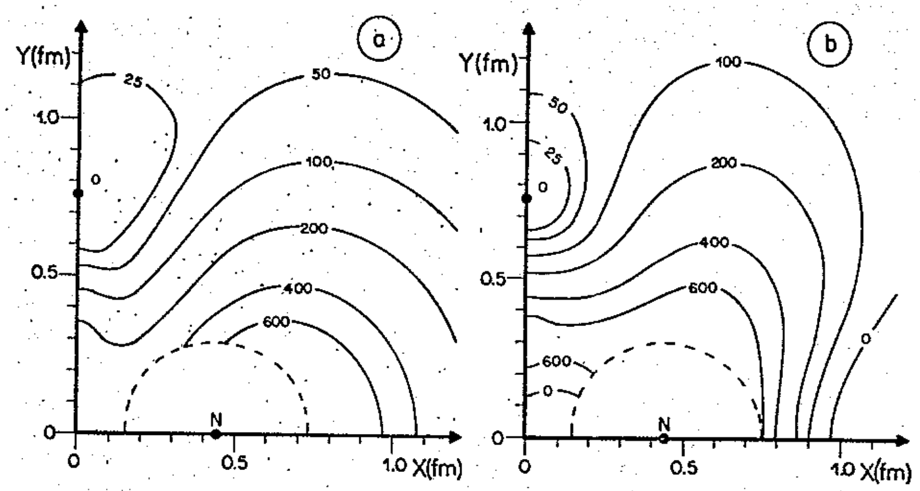

Fig. 11 - Equipotentials for (a) $\langle D'|V|S\rangle$ and (b) $\langle D'|[V+\hat{W}_s+\hat{W}_p+\hat{W}_p']|S\rangle$.

Conventions as in fig. 2.

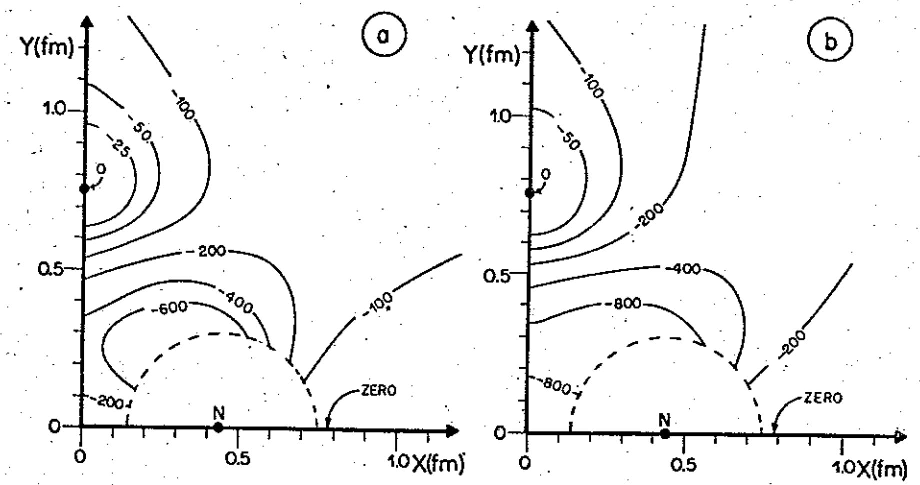

Fig. 12 - Equipotentials for (a) $\langle D''|V|S\rangle$ and (b) $\langle D''|[V+\hat{W}_s+\hat{W}_p+\hat{W}_p']|S\rangle$.

Conventions as in fig. 2.