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ABSTRACT

It is shown that the fermionic sector of N=1, D=11 supergravity can be rearranged in such a way that a minimal number of auxiliary fields are necessary to be introduced so that we can eliminate the quartic gravitino interaction terms. After functional integration over the gravitino we obtain a effective action in terms of these auxiliary fields and the bosonic fields of N=1, D=11 supergravity. The non-vanishing expectation value of these auxiliary fields can lead to mass-like terms for the gravitino and contributions to the cosmological constant.

INTRODUCTION

N=1 supergravity in d=11 dimensions[1] has recently received much attention due to two major aspects: a) obtention of N=8 supergravity in d=4 dimensions through the dimensional reduction scheme thus leading to the Cremmer-Julia[2] and de Wit-Nicolai[3] models; b) spontaneous compactification of the 11-dimensional space-time into a product of a 4-dimensional space-time with a compact 7-dimensional one. This property arises as a consequence of the fact that in 11-dimensional supergravity a three index totally anti-symmetric boson field is obligatory. In contrast to other higher dimensional models the 4-dimensional space-time comes out as an output[4] of the classical equation of motion in the 11-dimensional supergravity, as a result of the dimensional reduction scheme.

This scheme was, until now, almost only used to obtain 4-dimensional model starting with a classical d-dimensional one. Very little is known, on the other hand, if the dimensional reduction is performed <u>after</u> the d-dimensionally model is quantized, or at least partially quantized[5]. The quantization of higher dimensional models can present great simplifications because they are described by a simpler structure, as compared to 4-dimensional models. On the other hand, this d-dimensional quantized model need to be adequatelly analysed in terms of its 4-dimensional physical content. We are particularly interested in studying the quantization of the N=1, d=11 supergravity fermionic sector. Two major aspects are important through this procedure: a) dynamical mass generation and b) cosmological constant generation. The fermionic sector (which includes its interaction terms with the other 11-dimensional supergravity

bosonic fields) is uniquely described by a (128 degrees of freedom, on mass shell) Majorana Rarita-Schwinger field (gravitino). Its pure part has a rather peculiar structure because it is only described by quadratic and quartic terms, thus bearing a close analogy with the Jona-Lasinio-Nambu Model[6] (J-L-N). The Cooper pair formation in superconductivity[7], described in terms of a phonon mediated attractive force between the superconducting electrons (leading to an energy gap) was used as the starting point in the J-L-N model in order to understand the dynamical mass generation process in field theory. The quantization of the J-L-N model has been carried out by some authors[8,9] with the use of the functional path integral formalism, leading to the introduction of auxiliary fields in order to eliminate the quartic fermion self-interaction terms. Although this kind of model is unrenormalizable (from a perturbative power counting argument point of view) which forces us to introduce a ultraviolet cut-off representing an independent parameter, interesting informations can be obtained if some of the auxiliary fields develop a non-vanishing vacuum expectation value (v.e.v.).

The pure part of the fermionic sector in the 11-dimensional supergravity model presents similar kind of results as the J-L-N model. A closer inspection of this part shows us that the quartic gravitino interaction terms (q.g.i.t.) appear due to the presence of a quadratic contorsion product, since the spin connection is used as a dependent field, in the 1.5 formalism. Using a specific gauge for the gravitino, in addition to the Fierz identity, made it possible for us to re-arrange the q.g.i.t. in such a way that a <u>minimum</u> number of auxiliary fields where necessary to eliminate them, when fermionic quantization is performed. These auxiliary fields are of three types: a scalar, (two)three index totally anti-symmetric and (two)four index also totally anti-symmetric ones. Classically they represent only Lagrange multipliers, whose equation of motion consist of gravitino bi-linears in the various channels (scalar, three index totally anti-symmetric and four index totally anti-symmetric). But at the quantum level they acquire kinetic terms, which represents a change in their nature. Anyway, they consist of a new kind of entity thus leading, after the quantization of the fermionic sector, to a clear departure from the classical scenario through the development of v.e.v..

In the quantization procedure, the bosonic fields are classical background fields. Introducing the auxiliary fields, after the re-arrangement of the q.g.i.t. and performing functional quantization of the gravitino leads us to an effective action. This effective action is a very complicated (non-local) functional of the bosonic 11-dimensional supergravity fields and of the auxiliary fields. As mentioned before, new informations will be obtained when these auxiliary fields develop a v.e.v.. The quantities thus generated are a mass-like term for the 11-dimensional gravitino and a cosmological constant. We have to be carefull in interpreting this new generated quantities as physical ones. The mass term is called "mass-like" because a physical mass can only be understood by this if it appears in a 4-dimensional space-time with a vanishing cosmological constant. In this sense, we can point out that, after dimensional reduction the spin 1/2 and 3/2 fermions in 4 dimensions acquire mass terms and the 4 and 7 dimensional spaces receive a contribution to the cosmological constant.

The classical field equations of the 11-dimensional

supergravity, in addition to furnishing various kinds of spontaneous compactifications, gave a new insight to the cosmological constant problem. The three index totally anti-symmetric field, through its field strengh, leads (after dimensional reduction to 4-dimensions) to a different kind of contribution to the cosmological constant(10,11,12). Instead of being a fundamental element in the 4-dimensional equations of motion it consists of a undetermined equation of motion constant. This fact, although it does not furnish any deeper explanation of the almost vanishing 4-dimensional cosmological constant, demonstrates us that higher dimensional models containing world index tensor fields can furnish a contribution to the 4-dimensional cosmological constant. We will show that, in the case of 11-dimensional supergravity, the three index totally anti-symmetric field, <u>is not</u> the only ingredient to the obtention of a 4-dimensional cosmological constant because v.e.v. of the scalar and the (two)four index totally anti-symmetric auxiliary fields do also contribute.

This paper is organized as follows: in part 2 we introduce the model and re-arrange the q.g.i.t.. Next, introducing the various auxiliary fields we eliminate the q.g.i.t. and perform functional quantization on the gravitino field thus obtaining an effective action, in part 3. In part 4 we draw some comments on the 4-dimensional spin 1/2 and 3/2 fields mass terms, in addition to the generation of contributions to the cosmological constant in 4 and 7 dimensions. Finally we make final comments concerning open questions and future prospects.

2. REARRANGEMENT OF THE GRAVITINO QUARTIC INTERACTION TERMS IN N=1, d=11 SUPERGRAVITY

The N=1, d=11 supergravity lagrangian is described by a "elf-bein" e_M^A, a Rarita-Schwinger field (gravitino) ψ_M^α and a totally anti-symmetric three index tensor field A_{MNP}. World indices M,N,... (middle part of the alphabet) and tangent space indices A,B,... (early part of the alphabet) range from 1 to 11, while spinor index α goes from 1 to 32. Its complete lagrangian is(*)

$$
\begin{aligned}
L = & -\frac{1}{2} e R(e,\omega) - \frac{e}{2} \bar{\psi}_M \Gamma^{MNP} D_N\left(\frac{\omega+\hat{\omega}}{2}\right) \psi_P - \\
& - \frac{e}{48} F^2_{MNPQ} - \frac{3}{4} De \left[\bar{\psi}_R \Gamma^{MNPQRS} \psi_S + 12 \bar{\psi}^M \Gamma^{NP} \psi^Q\right] \times \\
& \times \left[F_{MNPQ} + \hat{F}_{MNPQ}\right] + C \varepsilon^{M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11}} \times \\
& \times F_{M_1 M_2 M_3 M_4} F_{M_5 M_6 M_7 M_8} \times A_{M_9 M_{10} M_{11}}
\end{aligned}
\tag{2.1}
$$

where(**)

$$F_{MNPQ} = \partial_{[M} A_{NPQ]} \tag{2.2a}$$

$$\hat{F}_{MNPQ} = F_{MNPQ} - 3 \bar{\psi}_{[M} \Gamma_{ND} \psi_{Q]} \tag{2.2b}$$

and

$$C = -\sqrt{2}\, i / 3456 \quad , \quad D = \sqrt{2}/144 \tag{2.3}$$

(*) We use conventions of refs. (13) and (14).

(**) The symbol [ ] means total anti-symmetrization.

The spin connection used as a dependent field (in the 1.5 formalism), is written in terms of e_M^A and ψ_N as:

$$\omega_M^{AB} = \omega_M^{AB}(e) + \kappa_M^{AB} \tag{2.4}$$

with the contorsion term κ_M^{AB} given by

$$\kappa_M^{AB} = \frac{1}{8} \bar{\psi}_N \Gamma_M^{ABNP} \psi_P + \frac{1}{4} \left[\bar{\psi}_M \Gamma^A \psi^B - \bar{\psi}_M \Gamma^B \psi^A + \bar{\psi}^A \Gamma_M \psi^B\right] . \tag{2.5}$$

Also

$$D_M = \partial_M + \frac{1}{4} \omega_M^{AB} \Gamma_{AB} \tag{2.6}$$

and

$$\hat{\omega}_M^{AB} = \omega_M^{AB} - \frac{1}{8} \bar{\psi}_N \Gamma_M^{ABNP} \psi_P . \tag{2.7}$$

The scalar curvature and the curvature tensor are respectively represented as

$$R(e,\omega) = e_A^M e_B^N R_{MN}^{AB} \tag{2.8}$$

and

$$R_{MN}^{AB} = \partial_M \omega_N^{AB} + \omega_M^{AC} \omega_{NC}^{\ \ B} - (M \to N) . \tag{2.9}$$

The gamma matrices with N indices are defined by

$$\Gamma_{A_1 \ldots A_N} = \frac{1}{N!} \Gamma_{[A_1 \ldots A_N]} . \tag{2.10}$$

Lagrangian (2.1) is invariant, on shell, under the following supersymmetry transformations

$$\delta e_M^A = \frac{1}{2}\,\bar{\varepsilon}\,\Gamma^A\,\psi_M \qquad (2.11)$$

$$\delta\psi_M = D_M(\hat{\omega})\,\varepsilon + \frac{\sqrt{2}}{288}\left[\Gamma_M^{\ NPQR} - 8\,\delta_M^N\,\Gamma^{PQR}\right]\varepsilon\,\hat{F}_{NPQR} \qquad (2.12)$$

and

$$\delta A_{MNP} = -\frac{\sqrt{2}}{8}\,\bar{\varepsilon}\,\Gamma_{[MN}\,\psi_{P]} \qquad (2.13)$$

being ε an infinitesimal Majorana spinor.

We know that the quantization of any gauge model requires algebra closing, as can be deduced by the BRST invariance. In contrast to N=1 and N=2 4-dimensional supergravity N=1, d=11 supergravity does not close supersymmetry algebra on all fields due to the lack of auxiliary fields. But for the particular quantization of the gravitino it is possible to circumvent this problem, due to the existence of a set of auxiliary fields(13) which close the gravitino algebra, as will be discussed in more detail in the next section.

The quartic gravitino terms arise from the following parts, in expression (2.1)

$$-\frac{1}{2}\,e\,R(e,\omega) \qquad (2.14)$$

$$-\frac{1}{2}\,e\,\bar{\psi}_M\,\Gamma^{MNP}\,D_N\left(\frac{\omega+\hat{\omega}}{2}\right)\psi_P \qquad (2.15)$$

and

$$-\frac{\sqrt{3}}{4}\,D\,k\,e\left[\bar{\psi}_M\,\Gamma^{MNPQRS}\,\psi_S + 12\,\bar{\psi}^N\,\Gamma^{PQ}\,\psi^R\right]\hat{F}_{NPQR}\,. \qquad (2.16)$$

Starting with term (2.14), the quartic gravitino terms come from expression

$$e_A^M\,e_B^N\left[\kappa_M^{AC}\,\kappa_{NC}^{\ B} - \kappa_N^{AC}\,\kappa_{MC}^{\ B}\right] \qquad (2.17)$$

as can be easily checked through the use of (2.4), (2.5), (2.8) and (2.9). Using the gauge

$$\Gamma^M\,\psi_M = 0 \qquad (2.18)$$

and Fierz identity, so that

$$\bar{\psi}_M\,\Gamma^N\,\psi^M = 0 \qquad (2.19)$$

and

$$\Gamma_M^{\ MNPQ} = 0\,, \qquad (2.20)$$

we can conclude that

$$e_A^M\,e_B^N\,\kappa_M^{AC}\,\kappa_{NC}^{\ B} = 0\,. \qquad (2.21)$$

Thus (2.17) reduces to

$$e_A^M\,e_B^N\,\kappa_N^{AC}\,\kappa_{NC}^{\ B} = \underbrace{\frac{1}{64}\,(\bar{\psi}_P\,\Gamma^{NQRPS}\,\psi_S)^2}_{(a)} +$$

$$+\underbrace{\frac{1}{16}\,(\bar{\psi}_R\,\Gamma^{NPQRS}\,\psi_S)(\bar{\psi}_{[N}\,\Gamma_P\,\psi_{Q]})}_{(b)} + \underbrace{\frac{1}{8}\,(\bar{\psi}_M\,\Gamma_N\,\psi_P)(\bar{\psi}^P\,\Gamma^M\,\psi^N)}_{(c)} -$$

$$-\underbrace{\frac{1}{16}\,(\bar{\psi}_M\,\Gamma_N\,\psi_P)^2}_{(d)}\,. \qquad (2.22)$$

Now, using that(*)

$$(\overline{\psi}_R\,\Gamma^{NPQRS}\,\psi_S) = 3!\,(\overline{\psi}^{[N}\,\Gamma^P\,\psi^{Q]}) - (\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)\quad, \tag{2.23}$$

we obtain for expression (a) in (2.22)

$$\frac{1}{64}\,(\overline{\psi}_R\,\Gamma^{NPQRS}\,\psi_S)^2 = \frac{9}{16}\,(\overline{\psi}^{[N}\,\Gamma^P\,\psi^{Q]})^2 + \frac{1}{64}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)^2 -$$

$$-\frac{9}{32}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)\,\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}\quad, \tag{2.24}$$

and the same for (b), leads to

$$\frac{1}{16}\,(\overline{\psi}_R\,\Gamma^{NPQRS}\,\psi_S)\,(\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}) = \frac{3}{8}\,(\overline{\psi}^{[N}\,\Gamma^P\,\psi^{Q]})^2 -$$

$$-\frac{1}{16}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)\,\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}\quad. \tag{2.25}$$

Further, using Fierz identity in expression (c) gives us that

$$\frac{1}{8}\,(\overline{\psi}_M\,\Gamma_N\,\psi_P)(\overline{\psi}^P\,\Gamma^M\,\psi^N) = -\frac{1}{128}\Big\{(\overline{\psi}_M\,\psi^M)^2 + \frac{1}{6}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi_M)^2 +$$

$$+\,4(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)(\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}) + \frac{1}{24}\,(\overline{\psi}_M\,\Gamma^{NPQR}\,\psi^M)^2 -$$

$$-\,(\overline{\psi}_M\,\Gamma^{NPQR}\,\psi^M)\,\overline{\psi}_{[N}\,\Gamma_{PQ}\,\psi_{R]}\Big\}\quad. \tag{2.26}$$

Finally, for term (d) we have

$$-\frac{1}{16}\,(\overline{\psi}_M\,\Gamma_N\,\psi_P)^2 = -\frac{1}{512}\Big\{11(\overline{\psi}_M\,\psi^M)^2 + \frac{7}{3}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)^2 +$$

$$-\frac{181}{24}\,(\overline{\psi}_M\,\Gamma^{NPQR}\,\psi^M)^2\Big\}\quad. \tag{2.27}$$

(*) Subsequently we will use always gauge (2.15) without mention.

Now we insert (2.24), (2.25), (2.26) and (2.27) into (2.22), so that

$$e^M_A\,e^N_B\,\kappa^{AC}_N\,\kappa^B_{MC} = -\frac{15}{512}\,(\overline{\psi}_M\,\psi^M)^2 + \frac{15}{16}\,(\overline{\psi}^{[N}\,\Gamma^P\,\psi^{Q]})^2 +$$

$$-\frac{15}{1536}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)^2 - \frac{3}{8}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)(\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}) +$$

$$+\frac{23167}{3072}\,(\overline{\psi}_M\,\Gamma^{NPQR}\,\psi^M)\,\overline{\psi}_{[N}\,\Gamma_{PQ}\,\psi_{R]}\quad. \tag{2.28}$$

For expression (2.14) we write

$$-\frac{1}{2}e\,R(e,\omega) = -\frac{1}{2}e\,R(e) - \frac{e}{2}\Big\{e^M_A\,e^N_B\Big[\partial_M\Big[\frac{1}{8}\,(\overline{\psi}_P\,\Gamma_N^{ABPQ}\,\psi_Q) +$$

$$+\frac{1}{4}\Big[\overline{\psi}_N\,\Gamma^A\,\psi^B - \overline{\psi}_N\,\Gamma^B\,\psi^A + \overline{\psi}^A\,\Gamma_N\,\psi^B\Big]\Big] + \omega^A_{MC}(e)\Big[\frac{1}{8}\,(\overline{\psi}_P\,\Gamma_N^{BCPQ}\,\psi_Q) +$$

$$+\frac{1}{4}\Big[\overline{\psi}_N\,\Gamma^B\,\psi^C - \overline{\psi}_N\,\Gamma^C\,\psi^B + \overline{\psi}^B\,\Gamma_N\,\psi^C\Big] + \omega^B_{NC}(e)\Big[\frac{1}{8}\,(\overline{\psi}_P\,\Gamma_M^{ACPQ}\,\psi_Q) +$$

$$+\frac{1}{4}\Big[\overline{\psi}_M\,\Gamma^A\,\psi^C - \overline{\psi}_M\,\Gamma^C\,\psi^A + \overline{\psi}^A\,\Gamma_M\,\psi^C\Big]\Big] - (M\to N)\Big\} + \frac{15e}{1024}\,(\overline{\psi}_M\,\psi^M)^2 -$$

$$-\frac{15e}{32}\,(\overline{\psi}^{[N}\,\Gamma^P\,\psi^{Q]})^2 - \frac{15e}{3072}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)^2 + \frac{3e}{16}\,(\overline{\psi}_M\,\Gamma^{NPQ}\,\psi^M)\,\times$$

$$\times\,(\overline{\psi}_{[N}\,\Gamma_P\,\psi_{Q]}) - \frac{23167e}{6144}\,(\overline{\psi}_M\,\Gamma^{NPQR}\,\psi^M)(\overline{\psi}_{[N}\,\Gamma_{PQ}\,\psi_{R]}) \tag{2.29}$$

where $R(e)$ is the scalar curvature without torsion.

Now, for term (2.15) we have:

$$-\frac{1}{2}\,e\,\overline{\psi}_M\,\Gamma^{MNP}\,D_N\Big(\frac{\omega+\hat{\omega}}{2}\Big)\psi_P = -\frac{1}{2}\,e\,\overline{\psi}_M\,\Gamma^{MNP}\,\partial_N\,\psi_P -$$

$$-\frac{1}{8}\,e\,\overline{\psi}_M\,\Gamma^{MPN}\,\Gamma_{AB}\,\psi_P\,\omega_N^{AB}(e) - \frac{e}{128}\,\overline{\psi}_M\,\Gamma^{MNP}\,\psi_{AB}\,\psi_P\Big[(\overline{\psi}_R\,\Gamma_N^{ABRS}\,\psi_S) +$$

$$+\,4\Big[\overline{\psi}_N\,\Gamma^A\,\psi^B - \overline{\psi}_N\,\Gamma^B\,\psi^A + \overline{\psi}^A\,\Gamma_N\,\psi^B\Big]\Big]\quad. \tag{2.30}$$

Since

$$\overline{\psi}_N \Gamma^{MNP} \Gamma_{AB} \psi_P = 2\left[(\overline{\psi}^M \Gamma_A \psi_B) - (\overline{\psi}^M \Gamma_B \psi_A)\right] + (\overline{\psi}_P \Gamma^M \Gamma_{AB} \psi^P) \qquad (2.31)$$

and

$$(\overline{\psi}_R \Gamma^{RS}_{NAB} \psi_S) = 3!\ (\overline{\psi}_{[N} \Gamma_A \psi_{B]}) - (\overline{\psi}_S \Gamma_{NAB} \psi^S) \ , \qquad (2.32)$$

expression (2.30) reduces to:

$$\begin{aligned}
&- \frac{1}{2}\, e\, \overline{\psi}_M \Gamma^{MNP} D_N\left(\frac{\omega+\hat{\omega}}{2}\right)\psi_P = - \frac{1}{2}\, e\, \overline{\psi}_M \Gamma^{MNP} \partial_N \psi_P - \\
&- \frac{1}{8}\, e\, \overline{\psi}_M \Gamma^{MNP} \Gamma_{AB} \psi_P\, \omega_N^{AB}(e) - \frac{11e}{256} (\overline{\psi}_M \psi^M)^2 - \\
&- \frac{7e}{64} (\overline{\psi}_M \Gamma^{NAB} \psi^M)\ \overline{\psi}_{[N} \Gamma_A \psi_{B]} - \frac{e}{768} (\overline{\psi}_M \Gamma^{NAB} \psi^M)^2 - \\
&- \frac{3e}{16} (\overline{\psi}_{[N} \Gamma_A \psi_{B]})^2 + \frac{181e}{6144} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)^2 \qquad (2.33)
\end{aligned}$$

Finally expression (2.16) gives us:

$$\begin{aligned}
&- \frac{\sqrt{2}\, e}{384} \left[\overline{\psi}_M \Gamma^{MNPQRS} \psi_S + 12\ \overline{\psi}^N \Gamma^{PQ} \psi^R\right] \hat{F}_{NPQR} = \\
&= - \frac{\sqrt{2}\, e}{384} \left[\overline{\psi}_M \Gamma^{MNPQRS} \psi_S + 12\ \overline{\psi}^N \Gamma^{PQ} \psi^R\right] F_{NPQR} + \\
&+ \frac{\sqrt{2}\, e}{128} \left[\overline{\psi}_M \Gamma^{MNPQRS} \psi_S + 12\ \overline{\psi}^N \Gamma^{PQ} \psi^R\right] \overline{\psi}_{[N} \Gamma_{PQ} \psi_{R]} \ , \qquad (2.34)
\end{aligned}$$

which after the use of relation

$$\overline{\psi}_M \Gamma^{MNPQRS} \psi_S = 4!\ \overline{\psi}^{[N} \Gamma^P \Gamma^Q \psi^{R]} - (\overline{\psi}_M \Gamma^{NPQR} \psi^M) \qquad (2.35)$$

is:

$$\begin{aligned}
&- \frac{\sqrt{2}\, e}{384} \left[\overline{\psi}_M \Gamma^{MNPQRS} \psi_S + 12\ \overline{\psi}^N \Gamma^{PQ} \psi^R\right] \hat{F}_{NPQR} = \\
&= - \frac{\sqrt{2}\, e}{32}\ \overline{\psi}^{[N} \Gamma^P \Gamma^Q \psi^{R]}\ F_{NPQR} + \frac{\sqrt{2}\, e}{384} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)\ F_{NPQR} + \\
&+ \frac{9\sqrt{2}}{32} (\overline{\psi}^{[N} \Gamma^P \Gamma^Q \psi^{R]})^2 - \frac{\sqrt{2}\, e}{128} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)\ \overline{\psi}_{[N} \Gamma_{PQ} \psi_{R]} \ . \qquad (2.36)
\end{aligned}$$

Inserting expressions (2.29), (2.33) and (2.36) into (2.1) results in:

$$\begin{aligned}
L &= - \frac{1}{2}\, e\, R(e) - \frac{e}{2}\ e_A^M e_B^N \left[\partial_M \left[\frac{1}{8} (\overline{\psi}_P \Gamma_N^{ABPQ} \psi_Q + \right.\right. \\
&+ \frac{1}{4} \left.\left.\left[\overline{\psi}_N \Gamma^A \psi^B - \overline{\psi}_N \Gamma^B \psi^A + \overline{\psi}^A \Gamma_N \psi^B\right]\right]\right] + \omega_{MC}^{A}\ (e) \left[\frac{1}{8}\ \overline{\psi}_P \Gamma_N^{BCPQ} \psi_Q + \right. \\
&+ \frac{1}{4} \left[\overline{\psi}_N \Gamma^B \psi^C - \overline{\psi}_N \Gamma^C \psi^B + \overline{\psi}^B \Gamma_N \psi^C\right] + \omega_{NC}^{B}(e) \left[\frac{1}{8}\ \overline{\psi}_P \Gamma_M^{ACPQ} \psi_Q + \right. \\
&+ \frac{1}{4} \left.\left.\left[\overline{\psi}_M \Gamma^A \psi^C - \overline{\psi}_M \Gamma^C \psi^A + \overline{\psi}^A \Gamma_M \psi^C\right]\right]\right] - (M \to N) - \frac{29e}{1024} (\overline{\psi}_M \psi^M)^2 + \\
&- \frac{21}{32}\, e (\overline{\psi}_{[N} \Gamma_P \psi_{q]})^2 - \frac{19e}{3072} (\overline{\psi}_M \Gamma_{NPQ} \psi^M)^2 + \frac{5e}{64} (\overline{\psi}_M \Gamma^{NPQ} \psi^M) \overline{\psi}_{[N} \Gamma_P \psi_{Q]} + \\
&+ \frac{181e}{6144} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)^2 + \frac{(23167 - 24\sqrt{2})e}{3072} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)\ \overline{\psi}_{[N} \Gamma_{PQ} \psi_{R]} + \\
&+ \frac{9\sqrt{2}\, e}{32} (\overline{\psi}^{[N} \Gamma^{PQ} \psi^{R]})^2 - \frac{1}{2}\, e\, \overline{\psi}_M \Gamma^{MNP} \partial_N \psi_P - \frac{1}{8}\, e (\overline{\psi}_M \Gamma^{MNP} \Gamma_{AB} \psi_P)\, \omega_N^{AB}(e) - \\
&- \frac{e}{48} F^2_{MNPQ} - \frac{\sqrt{2}\, e}{32}\ \overline{\psi}^{[N} \Gamma^{PQ} \psi^{R]}\ F_{NPQR} + \frac{\sqrt{2}\, e}{384} (\overline{\psi}_M \Gamma^{NPQR} \psi^M)\ F_{NPQR} - \\
&- \frac{\sqrt{2}\, i}{3456}\ \varepsilon^{M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11}}\ F_{M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11}} \ . \qquad (2.37)
\end{aligned}$$

Lagrangian (2.37) is totally equivalent to (2.1) if we use

Fierz identity and gauge (2.18) but presents a more simple structure concerning the q.g.i.t. thus permiting the use of a minimal number of auxiliary fields to eliminate them in the quantization procedure.

3. QUANTIZATION OF THE GRAVITINO

We quantize the gravitino by using functional path integral formalism, with e_M^A and A_{MNP} represented as background fields. Supergravity quantization(15) (in the case of algebra closing models) reveals some novel features if compared to the quantization of abelian or non-abelian theories. For example in the case of N=1, d=4 supergravity four-ghost coupling terms have to appear as a direct consequence of the BRST invariance. This property implies that the supersymmetry algebra is closed, using a set of auxiliary fields, in its minimal version given by S (scalar), P (pseudo-scalar) and A_μ (axial). On the other hand, in the case of N=1, d=11 supergravity there does not exist (at least until now) a complete set of algebra closure auxiliary fields. But for the gravitino it was discovered(13) that its algebra can be closed (also for the elf-bein) if we use a specific number of auxiliary fields: a scalar, two totally anti-symmetric three index fields and two totally anti-symmetric four index fields. This information is sufficient to ensure the quantization of the gravitino.

The Green function functional generator is

$$Z(J,T,\zeta) = N^{-1} \int [d e_M^A]\,[d\psi_M]\,[dA_{MNP}]\,[dC^i]\ \delta(\text{gauge fixing}) \times$$

$$\times \exp i \Big\{ S_{c\ell} + S_{gh} + \int d_x^{11} e \Big\{ \Big[J_A^M e_M^A + \frac{1}{2} \bar{\zeta}^M \psi_M + \frac{1}{2} \bar{\psi}^M \zeta_M + T^{MNP} A_{MNP} \Big] + \begin{bmatrix} \text{supergravity algebra closing} \\ \text{auxiliary fields quadratic terms} \end{bmatrix} \Big\}\Big\} \tag{3.1}$$

where $S_{c\ell}$ is the classical action, C^i represents the various ghosts and the ghost action, for the case of the gravitino quantization, is given by

$$S_{gh}^{\psi_M} = \int d_x^{11}\, e \left[\bar{C}^\alpha\, \bar{\mathcal{D}}_{\alpha\beta}\, C^\beta \right] \tag{3.2}$$

with

$$\bar{D}_M = D_M(\hat{\omega}) + \frac{\sqrt{2}}{288} \left[\Gamma_M^{NPQR} - 8\, \delta_M^N\, \Gamma^{PQR} \right] \hat{F}_{NPQR} + \begin{bmatrix} \text{supergravity algebra closing} \\ \text{auxiliary fields} \end{bmatrix} . \tag{3.3}$$

As we mentioned in the last section, the re-arranged lagrangian (2.37) is suitable for quantization, since the set of auxiliary fields necessary to eliminate the quartic terms are minimal. Now we start introducing these auxiliary fields.

In the scalar channel we write

$$\exp - \frac{29}{1024}\, i \int d_x^{11}\, e\, (\bar{\psi}_M \psi^M)^2 = N^{-1} \int [d\sigma] \exp i \int d_x^{11}\, e \left\{ \sigma^2 + \frac{\sqrt{29}}{16} (\bar{\psi}_M \psi^M) \right\} . \tag{3.4}$$

Next, in the three index channel we introduce two different fields, α_{MNP} and β_{MNP}.

First,

$$\exp - \frac{21}{32}\, i \int d_x^{11}\, e(\bar\psi_{[N}\Gamma_P\psi_{Q]})^2 + \frac{5i}{64}\int d_x^{11}\, e(\bar\psi_M\Gamma^{NPQ}\psi_M)\bar\psi_{[N}\Gamma_P\psi_{Q]} =$$

$$= N^{-1}\int [d\alpha_{MNP}]\,\exp i\int d_x^{11}\, e\left\{\alpha^2_{MNP} + \alpha^{MNP}\left[\sqrt{\frac{21}{8}}\,\bar\psi_{[M}\Gamma_N\psi_{P]} + \right.\right.$$

$$\left.\left. + \frac{9\sqrt{\frac{21}{32}}}{42}\,(\bar\psi_R\Gamma_{MNP}\psi^R)\right] + \frac{1701}{225792}\,(\bar\psi_R\Gamma_{MNP}\psi^R)^2\right\} \quad . \tag{3.5}$$

Second, summing the last term in (3.5) with the same kind of quartic term appearing in (2.38) leads us to:

$$\frac{5225453\,i}{693733024}\int d_x^{11}\; e(\bar\psi_R\Gamma_{MNP}\psi^R)^2 =$$

$$= N^{-1}\int [d\beta_{MNP}]\,\exp i\int d_x^{11}\left\{-\beta^2_{MNP} + \frac{1}{2}\sqrt{\frac{5225453}{693733024}}\; \times\right.$$

$$\left.\times\, \beta^{MNP}\,(\bar\psi_R\Gamma_{MNP}\psi^R)\right\} \quad . \tag{3.6}$$

Finally, in the four index channel, the two totally anti-symmetric auxiliary fields which have to be used are γ_{MNPQ} and δ_{MNPQ}. So

$$\exp\frac{181 i}{6144}\int d_x^{11}\; e(\bar\psi_M\Gamma^{NPQR}\psi^M)^2 + \frac{23167 - 24\sqrt{2}}{3072}\, i\, \times$$

$$\times \int d_x^{11}\; e(\bar\psi_M\Gamma^{NPQR}\psi^M)(\bar\psi_{[N}\Gamma_{PQ}\psi_{R]}) = N^{-1}\int [d\gamma_{MNPQ}]\; \times$$

$$\times\,\exp i\int d_x^{11}\; e\left\{-\gamma^2_{MNPQ} - \gamma^{MNPQ}\left[\sqrt{\frac{181}{1536}}\,(\bar\psi_R\Gamma_{MNPQ}\psi^R) + \right.\right.$$

$$\left.\left. + \frac{(23167 - 24\sqrt{2})}{181}\sqrt{\frac{181}{1536}}\,(\bar\psi_{[M}\Gamma_{NP}\psi_{Q]})\right] + \frac{536611041 - 1112016\sqrt{2}}{1112064}\,(\bar\psi^{[N}\Gamma^{PQ}\psi^{R]})^2\right]\right\} \tag{3.7}$$

and, summing the last term of (3.7) with the same type appearing in (2.38),

$$\exp\frac{536611041 - 799248\sqrt{2}}{1112064}\, i\int d_x^{4}\;(\bar\psi^{[N}\Gamma^{PQ}\psi^{R]})^2 =$$

$$= N^{-1}\int [d\Delta_{MNPQ}]\,\exp i\int d_x^{11}\; e\left\{-\Delta^2_{MNPQ} + \sqrt{\frac{536611041 - 799148\sqrt{2}}{278016}}\; \times\right.$$

$$\left.\times\,\Delta^{MNPQ}\,(\bar\psi_{[M}\Gamma_{NP}\psi_{Q]})\right\} \quad . \tag{3.8}$$

The four auxiliary fields α_{MNP}, β_{MNP}, γ_{MNPQ} and Δ_{MNPQ} are totally anti-symmetric in their indices.

Putting the expressions (3.4), (3.5), (3.6), (3.7) and (3.8) into (2.1) and using that

$$\delta(\Gamma^M\psi_M) = \int [d\xi]\,\exp\frac{i}{2}\int d_x^{11}\; e\left[\bar\psi_M\Gamma^M\xi - \bar\xi\,\Gamma^M\psi_M\right] \tag{3.9}$$

we have, after the functional integration over the gravitino field, the following result:

$$Z(J,T,\zeta) = N^{-1}\int [d e^A_M][dA_{MNP}][d\sigma][d\alpha_{MNP}][d\beta_{MNP}][d\gamma_{MNPQ}]\; \times$$

$$\times\,[d\Delta_{MNPQ}][dC^i]\;\delta\begin{pmatrix}\text{other fields}\\ \text{gauge fixing}\end{pmatrix}\,\exp i\left\{S_{eff} + \right.$$

$$+ \int d_x^{11}\; e\left\{-\frac{1}{2}\left[\left[-\bar\xi\,\Gamma^M\Delta^{-1}_{MN} + \zeta^M\Delta^{-1}_{MN}\right]\Delta^{NP}\left[\Delta^{-1}_{PQ}\Gamma^Q\xi + \Delta^{-1}_{PQ}\zeta^Q\right]\right] + \right.$$

$$+ \bar C^\alpha\left[\Gamma^M\left[\partial_M + \frac{1}{4}\omega^{AB}_M(e)\Gamma_{AB} + \frac{\sqrt{2}}{288}\left[\Gamma_M^{NPQR} - 8\,\delta^N_M\Gamma^{PQR}\right]F_{NPQR} + \right.\right.$$

$$\left.\left. + \left[\begin{matrix}\text{supergravity algebra closing}\\ \text{auxiliary fields}\end{matrix}\right]\right]\right]_{\alpha\beta} C^\beta + J^M_A\, e^A_M + T^{MNP}A_{MNP} +$$

$$+ \begin{bmatrix}\text{supergravity algebra closing} \\ \text{auxiliary fields quadratic terms}\end{bmatrix} + S_{\substack{\text{other} \\ \text{ghosts}}} \Big\} \tag{3.10}$$

where the effective action is given by

$$S_{eff} = \frac{i}{2} \operatorname{Tr} \ell n\, \Delta_{MN} + \int d^{11}_x\, e \Big\{ \sigma^2 + \alpha^2_{MNP} - \beta^2_{MNP} - \gamma^2_{MNPQ} -$$

$$- \Delta^2_{MNPQ} - \frac{1}{48} F^2_{MNPQ} - \frac{1}{2} R(e) - \frac{\sqrt{2}\,ki}{3456} \varepsilon^{M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11}} \times$$

$$\times F_{M_1 M_2 M_3 M_4} F_{M_5 M_6 M_7 M_8} A_{M_9 M_{10} M_{11}} \Big\} \quad , \tag{3.11}$$

and

$$\Delta_{MN} = i \Big\{ \Big\{ \frac{\partial_Q}{e} \left[e^Q_A\, e^P_B\, e \right] \Big[\frac{1}{8} \Gamma^{AB}_{PMN} + \frac{1}{4} \Big[\delta_{PM} \Gamma^A \delta^B_N - \delta_{PM} \Gamma^B \delta^A_N +$$

$$+ \delta^A_M \Gamma_P \delta^B_N \Big] + \omega^A_{QC}(e) \Big[\frac{1}{8} \Gamma^{BC}_{PMN} + \frac{1}{4} \Big[\delta_{MP} \Gamma^B \delta^C_N - \delta_{MP} \Gamma^C \delta^B_N +$$

$$+ \delta^B_M \Gamma_P \delta^C_N \Big]\Big] + \omega^B_{PC}(e) \Big[\frac{1}{8} \Gamma^{AC}_{QMN} + \frac{1}{4} \Big[\delta_{QM} \Gamma^A \delta^C_N - \delta_{QM} \Gamma^C \delta^A_N +$$

$$+ \delta^A_M \Gamma_Q \delta^C_N \Big]\Big] - (M \to N) \Big\} - \frac{1}{4} \Gamma_{MPN} \Gamma_{AB}\, \omega^{PAB}(e) - \Gamma_{MPN} \partial^P - \frac{\sqrt{29}}{8} \delta_{MN}\, \sigma +$$

$$+ \alpha^{PQR} \Big[\sqrt{\frac{21}{2}}\, \delta_{M[P} \Gamma_Q \delta_{R]N} + \frac{9\sqrt{\frac{21}{8}}}{21} \delta_{MN} \Gamma_{PQR} +$$

$$+ \sqrt{\frac{5225453}{693733024}}\, \beta^{PQR} \delta_{MN} \Gamma_{PQR} - \gamma^{PQRS} \Big[\sqrt{\frac{181}{384}}\, \delta_{MN} \Gamma_{PQRS} +$$

$$+ \frac{23167 - 24\sqrt{2}}{181} \sqrt{\frac{181}{384}}\, \delta_{M[P} \Gamma_{QR} \delta_{S]N} \Big] + \sqrt{\frac{536\,611\,041 - 799\,248\sqrt{2}}{69504}} \times$$

$$\times \Delta^{PQRS} \delta_{M[P} \Gamma_{QR} \delta_{S]N} - \frac{\sqrt{2}}{16} F^{PQRS} \delta_{M[P} \Gamma_{QR} \delta_{S]N} +$$

$$+ \frac{\sqrt{2}}{192} \delta_{MN} \Gamma_{PQRS} F^{PQRS} + \overline{C}^{\alpha} \Big\{ \Gamma^J \Big\{ \Big\{ \frac{1}{8} \delta_{RM} \delta_{PN} \Gamma^{AB}_J +$$

$$+ \frac{1}{4} \Big[\delta_{JM} \Gamma^A \delta^B_N - \delta_{JM} \Gamma^B \delta^A_N + \delta^A_M \delta^B_N \Gamma_J \Big] \Big\} \Gamma_{AB} -$$

$$- \frac{\sqrt{2}}{48} \Big[\Gamma_{PQRSJ} - 8\, \delta_{PJ} \Gamma_{QRS} \Big] \delta^P_M \delta^S_N \Gamma^{QR} \Big\}_{\alpha\beta} C^{\beta} \quad . \tag{3.12}$$

4. <u>THE EFFECTIVE ACTION AND DIMENSIONAL REDUCTION</u>

The effective action (3.11), obtained by the quantization of the gravitino field in N=1, d=11 supergravity presents some novel aspects which induce a drastic departure from the classical scenario. In the first place, this effective action is a complicated functional of the elf-bein e^A_M, the A_{MNP} field and the auxiliary fields σ, α_{MNP}, β_{MNP}, γ_{MNPQ} and Δ_{MNPQ}. These set of auxiliary fields are not arbitrary in our oppinion. We think that they reveal a unexplored relation to the set of auxiliary fields necessary to close supersymmetry algebra on the gravitino field[13]. This connection was also noticed in N=1, d=4 supergravity[16]. There we discovered the existence of a condensate in the scalar, pseudo-scalar and axial channels in the same way as the auxiliary fields (in their minimal version) necessary to close the super-symmetry algebra are scalar, pseudo-scalar and axial in nature. Although it is premature to give any deeper interpretation to this fact, we used it as a guiding line to search for the minimal set of auxiliary fields necessary to eliminate the quartic gravitino self-interaction terms.

Second, in d=11 dimensions it is possible to have a bare mass term, in the Majorana representation of the Rarita-Schwinger fields, whose general structure is like

$$M_{3/2}^{(d=11)}\ \overline{\psi}_M\ \Gamma^{MN}\ \psi^N \quad , \tag{4.1}$$

since $\overline{\psi}_M \equiv \psi_M^T C$ and $C^T = -C$. This makes it possible to think in a formal mass generation mechanism, through the v.e.v. of the auxiliary fields, in a close analogy with the 4-dimensional theories(8). The candidates for this process are the expressions $\sqrt{29}/8\ \sigma$, $(24\sqrt{2} - 23,167)/\sqrt{69\,504}\ \gamma^{PQRS}\ \delta_{M[P}\ \Gamma_{QR}\ \delta_{S]N}$, $\sqrt{(536611041 - 799248\sqrt{2})/69\,504}\ \Delta^{PQRS}\ \delta_{M[P}\ \Gamma_{QR}\ \delta_{S]N}$ and $-\sqrt{2}/16\ F^{PQRS}\ \delta_{M[P}\ \Gamma_{QR}\ \delta_{S]N}$, appearing in the effective action (3.11). We call this process a formal mass generation mechanism, and the corresponding mass term "mass-like" because: a) we can only interpret physical masses in a 4-dimensional space-time, b) the background metric has to be Minkowskian so that this amounts to take the cosmological constant as being zero. Anyway, after a dimensional reduction, the auxiliary fields σ, γ^{PQRS} and δ^{PQRS} in addition to F^{PQRS} give their contributions to the physical spin 3/2 and 1/2 fields masses if conditions a) and b) are taken into account.

A third important aspect of the effective action (3.11) corresponds to the fact that v.e.v. of the auxiliary fields contribute to the cosmological constant in 4 and 7 dimensions.

It is known in literature(10,11,12) that the presence of a totally anti-symmetric third rank field yields through its field strengh, when dimensionally reduced to 4 dimensions a contribution to the cosmological constant in the form of a undetermined equation of motion constant. We deduce from (3.11) and (3.12) that the cosmological constant in 4 dimensions receives contribution from the v.e.v. of the auxiliary fields σ, $\gamma_{\mu\nu\lambda\rho}$ and $\Delta_{\mu\nu\lambda\rho}$, in addition to $F_{\mu\nu\lambda\rho}$(17).

At the classical level v.e.v. of fermion bilinears have been taken into account in order to obtain a (geometrical) mechanism by which the 4 dimensional cosmological constant is zero(16,17). Although we showed that the cosmological constant (in 4 dimensions) contains contributions from v.e.v. of auxiliary fields, we wonder if it is possible to maintain such kind of mechanism in order to make the cosmological constant vanishing, since the classical vacuum structure of the model might not survire quantization of the gravitino.

5. FINAL COMMENTS

We showed that, although not trivial, it was possible to quantize the gravitino field in the N=1, d=11 supergravity model, using the well known technic in 4 and 2 dimensional models(8) of introducing auxiliary fields so as to eliminate the quartic gravitino self-interaction terms. These auxiliary fields form condensates which, through its v.e.v., lead to a formal bare mass and cosmological constant generation. Many questions still wait for a definite answer. One of them concerns the vacuum structure of this (semi)quantized models since we know that at the classical level it exhibits a spontaneous compactification of the 11-dimensional space-time into a product of a 4 (or 7)-dimensional space-time with a 7(4)-dimensional compact space(4). Does this property survire quantization?(22) Another important question concerning quantization of higher-dimensional models concerns their physical interpretation after dimensional reduction has been executed. Since higher-dimensional models present a simpler structure, and as a consequence a higher unification, the more dimensions we take into account, questions concerning quantization of this models before dimensional reduction are of great importance. In this connection, a lot of work has been done concerning the anomaly structure of higher dimensional models(18,19,20), which sets a clear limitation in the set of models apt for quantization.

Finally, fermion quantum numbers(11) may perhaps gain a new insight using the procedure of quantizing higher dimensional supergravity models.

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APPENDIX A

We define the $d=11$ dimensional gamma matrices in the Majorana representation, where C is the charge conjugation matrix, satisfying(*)

$$\{\Gamma_M , \Gamma_N\} = 2\,\eta_{MN} \tag{A.1}$$

where

$$\eta_{MN} = \mathrm{diag}(-,+,\ldots,+) \tag{A.2}$$

and

$$C\,\Gamma_M\,C^{-1} = -\,\Gamma_M^T \quad . \tag{A.3}$$

Also

$$C^T = -\,C \tag{A.4}$$

and

$$\overline{\psi} \equiv \psi^T C \quad . \tag{A.5}$$

The gamma matrices 1, Γ_M, Γ_{MN}, Γ_{MNP}, Γ_{MNPQ} and Γ_{MNPQ} form an independent set $\Gamma^{(m)}$ in terms of which we define the following Fierz identity (being $\psi_1 , \ldots , \psi_4$ Majorana spinors):

$$(\overline{\psi}_1 O_1 \psi_2)(\overline{\psi}_3 O_2 \psi_4) = -\frac{1}{32}\sum_{n=0}^{5}\frac{1}{n!}(\overline{\psi}_1 \Gamma^{(n)} \psi_4)(\overline{\psi}_3 O_2 \Gamma^{(n)} O_1 \psi_2) \times$$

$$\times\ (-1)^{n=2,3} \tag{A.6}$$

(*) We adopt the same conventions as ref. (13).

APPENDIX B

Using Fierz identity (A.6) and gauge condition we will show how to obtain relation (2.24):

$$(\bar{\psi}_M \Gamma_N \psi_P)(\bar{\psi}^P \Gamma^M \psi^N) = -\frac{1}{32}\Big\{\underset{(a)}{(\bar{\psi}_M \psi^M)(\bar{\psi}_N \Gamma_P \Gamma^N \psi^P)} -$$

$$-\frac{1}{3!}\underset{(b)}{(\bar{\psi}_M \Gamma_{NPQ} \psi^M)(\bar{\psi}_R \Gamma_S \Gamma^{NPQ} \Gamma^R \psi^S)} + \frac{1}{4!}(\bar{\psi}_M \Gamma_{NPQR} \psi^M) \times$$

$$\times \underset{(c)}{(\bar{\psi}_S \Gamma_T \Gamma^{NPQR} \Gamma^S \psi^T)} \qquad (B.1)$$

since

$$(\bar{\psi}_M \Gamma_N \psi^M) = (\bar{\psi}_M \Gamma_{NP} \psi^M) = (\bar{\psi}_M \Gamma_{NPQRS} \psi^M) = 0 . \qquad (B.2)$$

For term (a) we have:

$$-\frac{1}{32}(\bar{\psi}_M \psi^M)(\bar{\psi}_N \Gamma_P \Gamma^N \psi^P) = -\frac{1}{16}(\bar{\psi}_M \psi^M)^2 \qquad (B.3)$$

using (A.1) and gauge (2.14).

Next, expression (b) reduces to:

$$\frac{1}{192}(\bar{\psi}_M \Gamma_{NPQ} \psi^M)(\bar{\psi}_R \Gamma_S \Gamma^{NPQ} \Gamma^R \psi^S) = -\frac{1}{4}(\bar{\psi}_M \Gamma_{NPQ} \psi^M)(\bar{\psi}^N \Gamma^P \psi^Q) -$$

$$-\frac{1}{96}(\bar{\psi}_M \Gamma_{NPQ} \psi^M)^2 . \qquad (B.4)$$

Finally, for term (c) we have to use that:

$$(\bar{\psi}_S \Gamma_T \Gamma^{NPQR} \Gamma^S \psi^T) = 2\left\{-4!(\bar{\psi}^{[N} \Gamma^P \Gamma^Q \psi^{R]}) + (\bar{\psi}_M \Gamma^{NPQR} \psi^M)\right\} . \qquad (B.5)$$

So

$$-\frac{1}{768}(\bar{\psi}_M \Gamma_{NPQR} \psi^M)(\bar{\psi}_S \Gamma_T \Gamma^{NPQR} \Gamma^S \psi^T) =$$

$$= -\frac{1}{384}(\bar{\psi}_M \Gamma_{NPQR} \psi^M)^2 + \frac{1}{16}(\bar{\psi}_M \Gamma_{NPQR} \psi^M)(\bar{\psi}^{[N} \Gamma^P \Gamma^Q \Gamma^{R]}) . \qquad (B.6)$$

Putting expressions (B.3), (B.4) and (B.6) into (B.1) gives us:

$$(\bar{\psi}_M \Gamma_N \psi_P)(\bar{\psi}^P \Gamma^M \psi^N) = -\frac{1}{16}\Big\{(\bar{\psi}_M \psi^M)^2 + \frac{1}{6}(\bar{\psi}_M \Gamma_{NPQ} \psi^M)^2 +$$

$$+ 4(\bar{\psi}_M \Gamma_{NPQ} \psi^M)(\bar{\psi}^{[N} \Gamma^P \psi^{Q]}) + \frac{1}{24}(\bar{\psi}_M \Gamma_{NPQR} \psi^M)^2 +$$

$$+ (\bar{\psi}_M \Gamma_{NPQR} \psi^M)(\bar{\psi}^{[N} \Gamma^P \Gamma^Q \psi^{R]})\Big\} . \qquad (B.7)$$

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