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EFFECT OF COLLISIONS ON THE MECHANISM OF ISOTOPE SEPARATION BY ION-CYCLOTRON WAVES

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ABSTRACT

An expression for the ponderomotive force due to ion-cyclotron waves propagating in a collisional plasma is derived. Using this expression, the effect of collisions on the isotope separation induced by large-amplitude cyclotron waves in a two-ion-species plasma is calculated. It is shown that efficient isotope separation can still be achieved even in the evanescent propagation region.

1. INTRODUCTION

Due to a wide range of possible applications, the ponderomotive force (Pitaevskii, 1961) caused by ion-cyclotron waves propagating in magnetized plasmas has received renewed attention in recent years. Among the interesting applications are radio-frequency plugging of cusps [Hidekuma et al. (1974); Hiroe et al. (1975)], radio-frequency stabilization of magneto-hydrodynamic modes [Sanuki (1983); Myra & D'Ippolito (1984); Similon & Kaufman (1984)], and isotope separation [Weibel (1980); Festeau-Barrioz & Weibel (1980); Weibel & Festeau-Barrioz (1982)]. Isotope separation occurs in a magnetized plasma because the sign of the ponderomotive pseudo-potential depends on whether the frequency of the wave is larger or smaller than the cyclotron frequency (Motz & Watson, 1967). By properly choosing the frequency of a left-handed cyclotron wave between the resonance frequencies of two ion species in a plasma, it is possible to separate them along the direction parallel to the externally imposed magnetostatic field (Weibel, 1980). Hidekuma et al. (1974) have already experimentally demonstrated that essentially the same mechanism can efficiently be used for preferential plugging of a multi-ion-species plasma. This scheme of isotope separation is being actively pursued in Lausanne (Tran et al., 1982).

To be competitive with other modern schemes, in particular laser isotope separation of uranium, it is necessary to achieve effective separation in high-density plasmas with cold ions (ion temperature, $T_i < 1$ eV). Under these conditions, the ion-neutral collision frequency can be very large and affect the separation process. One deleterious effect of a large collision frequency, namely, ion-neutral charge exchange, can only be avoided by a fast extraction scheme. However, in this paper we concentrate on another effect of collisions, i.e., the modification of the ponderomotive force associated with the transfer of wave momentum during absorption. Preliminary calculations by the Lausanne Group [Tran et al. (1982); Sawley & Tran (1982); Sawley (1984)] have indicated that the effect of collisions on the ponderomotive force can be deleterious for isotope separation. However, by a self-consistent solution of the wave and moment equations including collisions, we show that efficient separation can still be achieved for relatively large values of the collision frequency.

The effect of collisions on the ponderomotive force in laser-produced unmagnetized plasmas was initially studied by Stamper (1976) and Miller & Hora (1979). They have clearly shown that collisions introduce a new term in the expression for the ponderomotive force that cannot be written as the gradient of a pseudo-potential. Dimonte, Lamb and Morales (1983) have studied another non-adiabatic effect on the ponderomotive force in the ion-cyclotron range of frequencies. They have shown that due to the finite transit time of a charged particle through the region of strong field variation, the ponderomotive potential becomes very small near the gyroresonance, instead of becoming infinite as predicted by the adiabatic theory (Motz & Watson, 1967). Actually, the condition for the adiabatic theory to be valid is that

$$|\omega - \Omega| >> \frac{v}{L} \quad ,$$

where ω is the wave frequency, Ω is the cyclotron frequency,

v is the characteristic particle velocity, and L is the characteristic scale length of the wave field variation. The parameter v/L can be considered as an effective collision frequency that limits the duration of the wave-particle resonant interaction. Thus, we expect the effect of collisions to be somewhat equivalent to the one described by Dimonte et al. (1983). Because the non-adiabatic effects decrease the pondero-motive pseudo-potential near the gyroresonance, Dimonte et al. (1983) conclude that the effectiveness of isotope separation by ion-cyclotron waves can be drastically reduced in comparison with the values predicted by the adiabatic theory. However, their conclusion is based upon a single-particle model (see also Lamb et al., 1984). Actually, in a real experiment there are strong space variations of the particle densities that are nonlinearly coupled to the field variations through the dispersion relation. In this case, the ponderomotive force on each species has to be self-consistently calculated with the wave and moment equations.

Fiedler Ferrari and Galvão (1984) and Sawley (1984) have derived similar expressions for the ponderomotive pseudo-potential including the effect of collisions. In the derivation of Fiedler-Ferrari & Galvão (1984), an important piece of the ponderomotive force has not been properly taken into account. In this paper, we derive a correct expression for the pondero-motive force in the presence of collisions and show that it cannot be written as the gradient of a pseudo-potential, in agreement with Stamper (1976) and Milley & Hora (1979). An equivalent expression for the force has also been recently obtained by Sawley (1984). Using a WKB perturbative calculation, he is able to write the ponderomotive force as the gradient of a pseudo-potential. Based upon the behaviour of this potential near the cyclotron resonances, he concludes that collisions decrease the effectiveness of isotope separation. However, because the scale length of field variations is comparable with the scale length of density variations, a WKB approximation is not applicable and his expression for the pseudo-potential is not valid for relevant values of the collision frequency. By numerically integrating the full set of nonlinear equations, we show that effective isotope separation can still be achieved in the presence of collisions.

The expression for the ponderomotive force is derived in Section 2. The coupled system of wave equations and equations for the particle densities is derived and numerically integrated in Section 3. The conclusions are presented in Section 4.

2. PONDEROMOTIVE FORCE

We consider a left-handed circularly polarized wave, $\vec{E}(z,t) = E(z)(\hat{x}+i\hat{y})e^{i\omega t}$ + c.c. , where $E(z)$ is the complex amplitude of the electric field and ω is the frequency of the wave, propagating in an infinite plasma column immersed in an homogeneous magnetic field $B_0\hat{z}$. The wave phase velocity is assumed to be much larger than the thermal particle velocities. Accordingly, kinetic effects are not important and the wave propagation can be described by a fluid plasma model. The basic equations of the model are the moment balance equation

$$\left(\frac{\partial}{\partial t} + \vec{v}_\sigma\cdot\nabla\right)\vec{v}_\sigma = \frac{q_\sigma}{m_\sigma}\left[\vec{E} + \vec{v}_\sigma \times (\vec{B}+B_0\hat{z})\right] - \frac{\nabla p_\sigma}{n_\sigma} - \nu\vec{v}_\sigma \quad , \tag{1}$$

Faraday's law,

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad , \tag{2}$$

and Ampère's law,

$$\nabla \times \vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \quad . \tag{3}$$

In Eq.(1), m_σ is the mass, q_σ is the charge, $\vec{v}_\sigma$ is the macroscopic fluid velocity, p_σ is the kinetic pressure, and n_σ is the particle density of the species labelled by the subscript σ. We assume that the collision frequency ν is constant and equal for both ion species. Actually, for the parameters of interest, the dominant collisional momentum transfer is due to ion-neutral collisions (Tran et al., 1982). Electron collisions are not important. The collision frequency can also be considered in a phenomenological way to simulate the effect of other non-adiabatic processes, as finite transit time (Dimonte et al., 1983). The temperature T_σ of the different species is assumed constant and uniform.

For cyclotron waves, the ponderomotive force comes only from the Lorentz term $\vec{v}_\sigma \times \vec{B}$ in Eq.(1) (Tajima, 1977; Fasteau-Barrioz & Weibel, 1982). In this case, an exact solution of the momentum balance equation with $v_{\sigma z} = 0$ is obtained by separating the components parallel and perpendicular to the external magnetic field,

$$n_\sigma q_\sigma \frac{\partial U}{\partial z} - n_\sigma F_\sigma + k_B T_\sigma \frac{\partial n_\sigma}{\partial z} = 0 \tag{4}$$

and

$$\frac{\partial\vec{v}_{\sigma\perp}}{\partial t} + \vec{\Omega}_\sigma \times \vec{v}_{\sigma\perp} = \frac{q_\sigma}{m_\sigma}\vec{E} - \nu\vec{v}_{\sigma\perp} \quad , \tag{5}$$

respectively. In these equations, U is the ambipolar electrostatic potential, $E_z = -\partial U/\partial z$, $\vec{F}_\sigma = F_\sigma \hat{z} = q_\sigma(\mathrm{Re}\vec{v}_{\sigma 1} \times \mathrm{Re}\vec{B})$ is the ponderomotive force, and $\vec{\Omega}_\sigma = (q_\sigma B_0/m_\sigma)\hat{z}$ is the cyclotron angular velocity. The expression for the ponderomotive force can be readily obtained by solving Eq.(2) for $\vec{B}$ and Eq.(5) for $\vec{v}_{\sigma 1}$. Writing the complex amplitude of the electric field as $E(z) = |E(z)| \exp[i\phi(z)]$ and neglecting terms of $O(\nu^2/\omega^2)$, we obtain

$$\vec{F}_\sigma(z) = -\hat{z}\frac{q_\sigma^2}{2m_\sigma\omega}\frac{(\omega^2-\Omega_\sigma^2)(\omega+\Omega_\sigma)}{(\omega^2-\Omega_\sigma^2)^2 + 2\nu^2(\omega^2+\Omega_\sigma^2)}\left[\frac{dE^2}{dz} + 2\frac{\nu}{\omega-\Omega_\sigma}E^2\frac{d\phi}{dz}\right]. \quad (6)$$

It follows from this equation that, in the absence of collisions, the ponderomotive force can be written as the gradient of a pseudo-potential that diverges at the resonance, $\omega=\Omega_\sigma$, and has opposite signs for $\omega < \Omega_\sigma$ and $\omega > \Omega_\sigma$. The new term proportional to $\nu(d\phi/dz)$ that appears in the presence of collisions comes from the transfer of wave momentum to the particles during absorption. This transfer is equivalent to a collisional drag that acts on the same direction for both ion species (the term proportional to $d\phi/dz$ does not change sign at $\omega=\Omega_\sigma$). With the collisional term included, instead of diverging, the ponderomotive forces assumes a finite value $F_\sigma = -q_\sigma^2 E^2(d\phi/dz)/m_\sigma \nu \Omega_\sigma$ at $\omega=\Omega_\sigma$ and crosses zero somewhere away from the resonance. In Fig. 1 we show a sketch of F_σ as a function of ω for a fixed position inside the plasma. We note the similarity between the behaviour of F_σ and the behaviour of the ponderomotive potential derived by Lamb et al. (1984) including the finite transit-time effect. For the application in isotope separation, ω is chosen somewhere between the values of the gyrofrequencies of the two species.

In this paper, we consider only the case $\omega = (\Omega_1 + \Omega_2)/2$ studied by Weibel (1980). As shown in Fig. 1, the discrepancy between the values of F_σ with and without collisions is not very large in this case. However, this is valid only if $|(dE^2/dz)|/E^2 \sim |d\phi/dz|$. During the wave propagation, there can occur points in the plasma where $|(dE^2/dz)|/E^2 << |d\phi/dz|$ and the term due to collisions becomes locally dominant. Finally, we remark that in the presence of collisions the ponderomotive force cannot be written as the gradient of a pseudo-potential, as it follows from Eq.(6).

3. WAVE AND DENSITY EQUATIONS

Computing the current density, $\vec{J} = \sum_\sigma n_\sigma q_\sigma \vec{v}_\sigma$, substituting into Eq.(3), and using Eq.(2), we derive the well-known wave equation for cyclotron waves propagating along the magnetic field. In the presence of collisions, this is a complex equation for the complex amplitude $E(z)$. We split it into two real equations for $|E(z)|$ and $\phi(z)$, i.e.,

$$\frac{d^2|E|}{dz} - |E|\left(\frac{d\phi}{dz}\right)^2 + \frac{\omega^2}{c^2}\left[1 - \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega}\frac{(\omega-\Omega_\sigma)}{(\omega-\Omega_\sigma)^2+\nu^2}\right]|E| = 0 \quad (7)$$

and

$$\frac{d^2\phi}{dz^2} + 2\left[\frac{1}{|E|}\frac{d|E|}{dz}\right]\frac{d\phi}{dz} - \frac{\omega^2}{c^2}\left[\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega}\frac{\nu}{(\omega-\Omega_\sigma)^2+\nu^2}\right] = 0 \quad , \quad (8)$$

where $\omega_{p\sigma} = (n_\sigma q_\sigma/m_\sigma \varepsilon_0)^{1/2}$ is the plasma frequency of species σ. The wave equations (7) and (8) are nonlinear because the

the particle densities that appear in the plasma frequencies are coupled to $|E|$ and ϕ through Eqs.(4) and (6).

Because the scale lengths of interest are much larger than the Debye length, the ambipolar potential can be eliminated from the problem by assuming charge neutrality, i.e., $\sum_\sigma n_\sigma q_\sigma = 0$ (Weibel, 1980 and Festeau-Barrioz & Weibel, 1982). Then, assuming that we have two ion species of equal temperatures, $T_1 = T_2 = T_i$, and summing Eq.(4) over all species, we obtain

$$k_B(T_i + T_e)\frac{d}{dz}(n_1 + n_2) - (n_1F_1 + n_2F_2 + n_eF_e) = 0 \quad , \tag{9}$$

where $n_e = n_1 + n_2$. Finally, subtracting Eq.(4) for the ion species 2 from the corresponding one for the ion species 1 $(q_1 = q_2)$, we obtain a second equation for the densities

$$k_B T_i\left(\frac{1}{n_1}\frac{dn_1}{dz} - \frac{1}{n_2}\frac{dn_2}{dz}\right) - (F_1 - F_2) = 0 \quad . \tag{10}$$

Equations (7) to (10) form a complete set of nonlinear equations for $|E|$, ϕ, n_1, and n_2, with the expression for the ponderomotive forces given by Eq.(6).

We consider the ion species 1 to be the naturally more abundant one. The wave frequency is chosen as

$$\omega = \frac{\Omega_1 + \Omega_2}{2} = (\alpha+1)\Omega_1 \quad , \tag{11}$$

where the parameter α is given by (Weibel, 1980)

$$\alpha = \frac{\Omega_2 - \Omega_1}{2\Omega_1} = \frac{m_1 - m_2}{2m_2} \quad . \tag{12}$$

We note that in general α can be positive or negative; however,

for the cases that we are interested, we have $0 < \alpha << 1$. Based upon the smallness of the parameter α and of the ratios ω/Ω_e, ν/ω, and m_e/m_i, a number of simplifying assumptions can be made in Eqs.(7) to (10). In Eq.(9), the ponderomotive force on the electrons can be neglected because

$$\left|\frac{F_e}{F_1}\right| \approx \alpha << 1 \quad . \tag{13}$$

In Eq.(7), the plasma contribution to the dispersion relation is approximately given by

$$\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega}\frac{\omega - \Omega_\sigma}{(\omega-\Omega_\sigma)^2 + \nu^2} \approx -\frac{\omega_{pe}^2}{\Omega_e\Omega_1} + \frac{\omega_{p1}^2}{\Omega_1^2}\left[\frac{1 - \frac{n_2}{n_1}(1+2\alpha)}{\alpha\left(1 + \frac{\nu^2}{\alpha^2\Omega_1^2}\right)}\right] \quad . \tag{14}$$

The first term in the right-hand side of Eq.(14) is usually neglected (Festeau-Barrioz & Weibel, 1982) because the ratio of the first to the second term is of order α. However, if $n_1 = n_2$, the two terms become of the same order. Since in the process of separation there can occur points inside the plasma where $n_1 = n_2$, we keep the electron contribution to the dispersion relation. The third term in Eq.(8) is given approximately by

$$\frac{\omega^2}{c^2}\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega}\frac{\nu}{(\omega-\Omega_\sigma)^2 + \nu^2} \approx \frac{\omega^2}{\alpha c^2}\left(\frac{\omega_{p1}^2}{\Omega_1^2}\right)\left(\frac{\nu}{\alpha\Omega_1}\right)\frac{1 + \frac{n_2}{n_1}}{1 + \frac{\nu^2}{\alpha^2\Omega_1^2}} \quad . \tag{15}$$

Finally, Eqs.(7) to (10) can be written in dimensionless form by introducing convenient normalization parameters. Densities are normalized to the density n_{10} of species 1 at $z=0$. Lengths are normalized to the characteristic length for isotopic separation given by

$$L = \frac{\sqrt{\alpha}\, c}{(1+\alpha)\omega_{p10}} , \tag{16}$$

where $\omega_{p10} = \omega_{p1}$ (z=0). The normalized collision frequency is defined as

$$\xi = \frac{\nu}{\omega - \Omega_1} = \frac{\nu}{\alpha\Omega_1} . \tag{17}$$

We see that, although $\nu/\Omega_1 \ll 1$ for the conditions of interest, the collisionality parameter ξ can be of order one because $\alpha \ll 1$. The ratio of the Alfven velocity, $V_A = B_0/\sqrt{\mu_0 n_{10} m_1}$, to the speed of light is characterized by the parameter $\eta = V_A^2/c^2 \ll 1$. The normalized electric field is given by

$$\varepsilon = \frac{|E|}{\sqrt{2\alpha k_B T_i B^2/m_i}} . \tag{18}$$

Using Eqs.(11) to (18) and the expression for the ponderomotive force, Eq.(6), the basic set of equations (7) to (10) can be written in normalized form as

$$\frac{d^2\varepsilon}{dz^2} - \varepsilon\left(\frac{d\phi}{dz}\right)^2 + \left[\eta - \frac{n_1 - (1+2\alpha)n_2}{1+\xi^2} + \alpha(n_1+n_2)\right]\varepsilon = 0 , \tag{19}$$

$$\frac{d^2\phi}{dz^2} + 2\left[\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\right]\frac{d\phi}{dz} - \frac{\xi}{1+\xi^2}(n_1+n_2) = 0 , \tag{20}$$

$$\frac{dn_1}{dz} + \frac{n_2}{(1+\xi^2)(n_1+n_2)}\left[\frac{n_1(n_1-n_2)}{n_2\left(1+\frac{T_e}{T_i}\right)} + 2(1+\alpha)n_1\right]\frac{d\varepsilon^2}{dz} + \frac{2\xi}{1+\xi^2}\,\frac{n_1}{1+\frac{T_e}{T_i}}\,\varepsilon^2\frac{d\phi}{dz} = 0 , \tag{21}$$

and

$$\frac{dn_2}{dz} + \frac{n_2}{(1+\xi^2)(n_1+n_2)}\left[\frac{n_1-n_2}{1+\frac{T_e}{T_i}} - 2(1+\alpha)n_1\right]\frac{d\varepsilon^2}{dz} + \frac{2\xi}{1+\xi^2}\,\frac{n_2}{1+\frac{T_e}{T_i}}\,\varepsilon^2\frac{d\phi}{dz} = 0 . \tag{22}$$

We note from Eq.(19) that, for $\eta \ll 1$ and $\alpha \ll 1$, the wave is evanescent for $n_1 \gg n_2$. This is because $\omega > \Omega_1$ and cyclotron waves are evanescent above the cyclotron frequency. For $n_2 \gg n_1$, the wave is propagatory with a damping due to collisions. To solve Eqs.(19) to (22) for the case that we are interested in, namely, the separation of uranium isotopes, we assume that the two isotopes are at their natural abundance at z=0. The equations are integrated in the direction of positive z with initial conditions $\varepsilon \ll 1$ and $d\varepsilon/dz = 0$ at $z=0$. Thus, the solutions presented below correspond to an idealized situation where an ionized plasma of natural uranium is continuously introduced into a separation chamber from the left to the right and the wave is excited to propagate from the right to the left.

We consider a uranium plasma immersed in a magnetic field of 5 kG. The cross-section for ion-neutral collisions is approximately $\sigma = 5 \times 10^{-15}$ cm^2. Considering that for the cases of interest the neutral density can vary from 10^{12} to 10^{13} atoms/cm^3 and assuming $T_i \approx 0.1$ eV ($T_e = 1$ eV), we obtain that the parameter ξ can vary approximately from 0.078 to 0.8. In Fig. 2 we show the solution of Eqs.(19) to (22) for $\xi=0$ (Fig. 2a), 0.078 (Fig. 2b), 0.2 (Fig. 2c), and 0.8 (Fig. 2d).

The initial densities at $z=0$ are $n_1 = 1$ and $n_2 = \rho/(1-\rho)$, where ρ is the natural percentual abundance of the second isotope. The solution for $\xi=0$ corresponds to the one already obtained by Festeau-Barrioz & Weibel (1980). However, they have imposed $n_1+n_2 = 1$ throughout the calculation, i.e., the electron density is assumed uniform. We do not impose the same condition because the collisional term in the ponderomotive force acts on the same direction on both isotopes, producing a spatial variation in the total density of the ions that has to be followed by the electrons to keep charge neutrality. We use the condition $n_1+n_2 = 1$ as an accuracy test of the numerical method in the case $\xi=0$.

The solution for $\xi=0$ produces a region of high enrichment of width approximately equal to $2L$. Equal regions of high enrichment repeat periodically because the solution has no damping. Naturally, for practical applications, the system would have to be just long enough to accomodate one such a region. For moderate values of the collisionality parameter, such as $\xi = 0.078$ (Fig. 2b), regions of high enrichment are still obtained. The width of these regions are also of the order of $2L$ but the separation between two such regions decreases considerably in comparison with the case $\xi=0$. This can be somewhat troublesome when practical schemes for extraction are considered. For large values of the collisionality parameter, on the other hand, the separation can be favored. This is shown in Fig. 2c for $\xi=0.2$. After the first region of high enrichment, there occurs a region of low enrichment and then a continuous region of high enrichment where the density of the second isotope damps out. This is even more drastically shown in Fig. 2d for $\xi=0.8$. Such situation produces a clear

separation between the two isotopes although the value of the wave intensity at the antenna (assumed to be located at the right of Figs. 2) substantially increases in the presence of collisions because of wave damping. However, for realistic calculation of the required power densities, one has to solve the more involved problem of a bounded plasma column (Weibel & Festeau-Barrioz, 1982, and Sawley & Tran, 1982) in the presence of collisions.

4. CONCLUSIONS

We have derived an expression for the ponderomotive force due to ion-cyclotron waves in a magnetized plasma. This expression shows that the momentum transfer to the plasma due to wave absorption produces an extra term in the ponderomotive force that remains finite at the gyroresonances. Applying our results to the problem of isotope separation in a magnetized plasma (Weibel, 1980), we conclude that effective separation can still be obtained in the presence of collisions. The required power levels with and without collisions are of the same order at the place of separation inside the plasma. However, because of wave damping, the power level at the location of the exciting antenna is much larger in the presence of strong collisions. The expression for the ponderomotive force in the presence of collisions may also be important in other applications, such as stabilization of magnetohydrodynamic modes (Sanuki, 1983).

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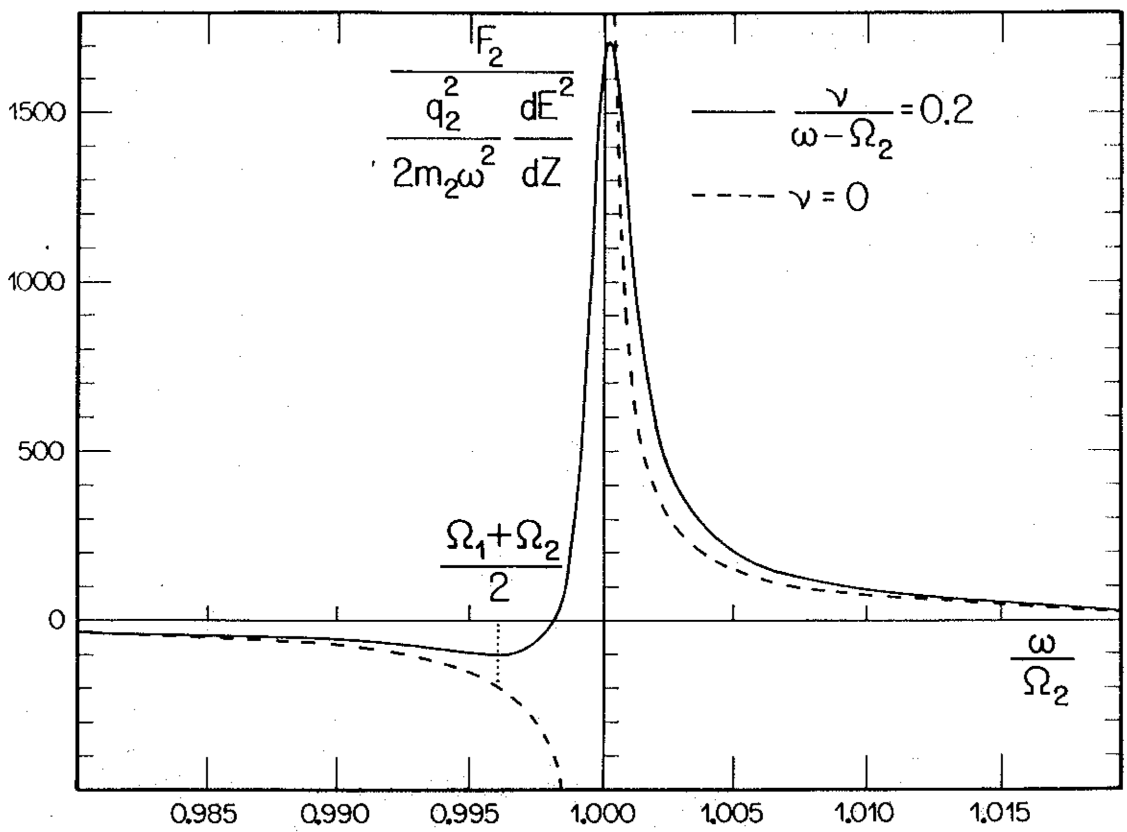
FIGURE CAPTIONS

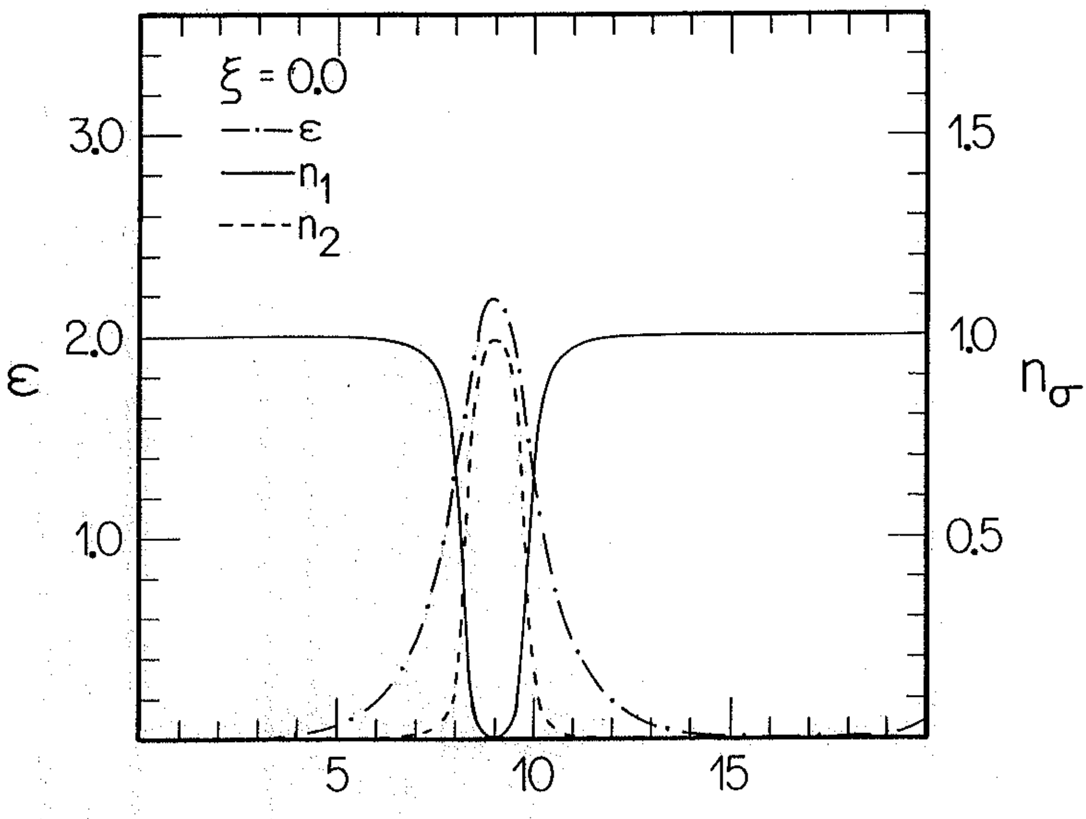
Fig. 1 – Sketch of $F\sigma/(\frac{q_\sigma^2}{2m_\sigma\omega^2}\frac{dE^2}{dz})$ as a function of ω/Ω_σ for a fixed position inside the plasma where $dE^2/dz \approx E^2\, d\phi/dz$ (plot for U^{235} with $\nu/\omega-\Omega_2 = 0.2$). The case with (without) collisions is given in continuous (dashed) line.

Fig. 2 – Solution of Eqs. (19) to (22). The normalized electric field is ε and the normalized densities are n_σ (n_1 for U^{238} and n_2 for U^{235}). (a) $\xi = \nu/\omega-\Omega_1 = 0$ (without collisions); (b) $\xi = 0.078$; (c) $\xi = 0.2$; (d) $\xi = 0.8$.



F_2
$\frac{q_2^2}{2m_2\omega^2} \frac{dE^2}{dZ}$
$\frac{\nu}{\omega-\Omega_2}=0.2$
$\nu = 0$
1500
1000
500
0
$\frac{\Omega_1+\Omega_2}{2}$
$\frac{\omega}{\Omega_2}$
0.985
0.990
0.995
1.000
1.005
1.010
1.015

Fig. 1



$\xi = 0.0$
ε
n_1
n_2
3.0
2.0
1.0
ε
1.5
1.0
0.5
n_σ
5
10
15
Z

Fig. 2a

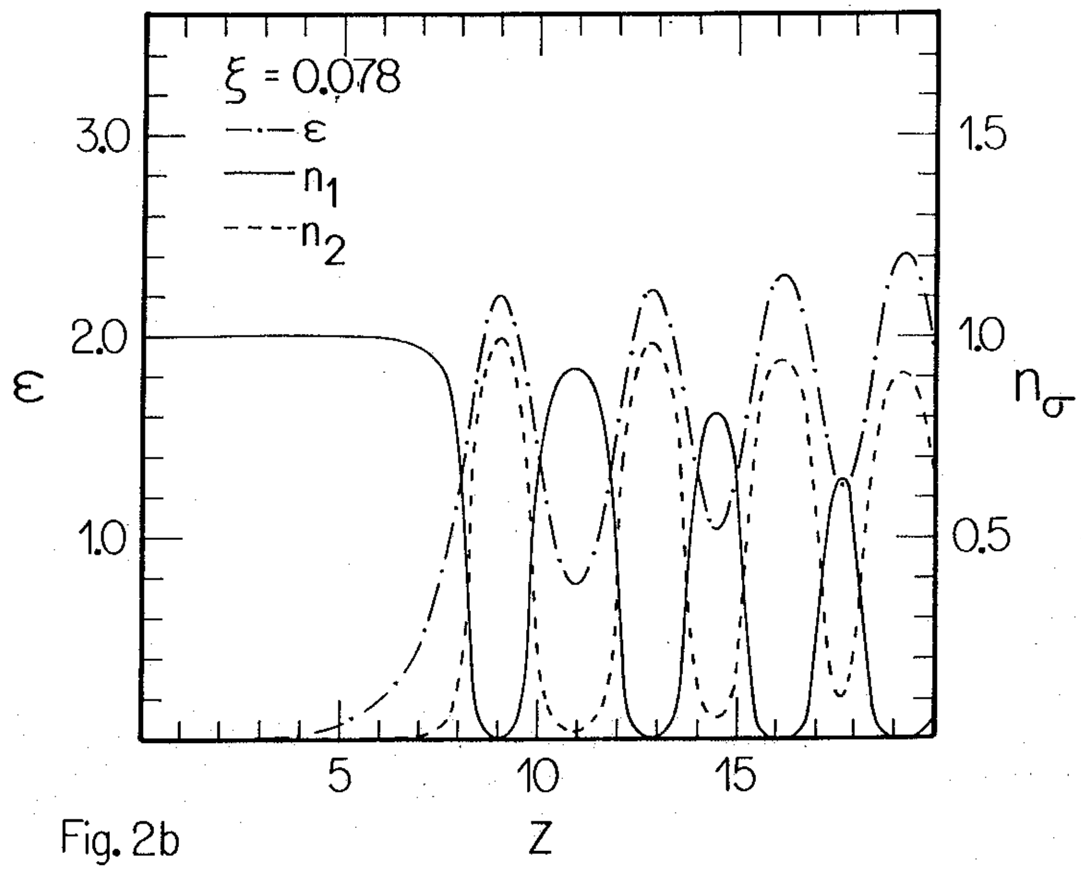

ξ = 0.078
ε
n1
n2
3.0
2.0
1.0
1.5
1.0
0.5
ε
nσ
5
10
15
Z

Fig. 2b

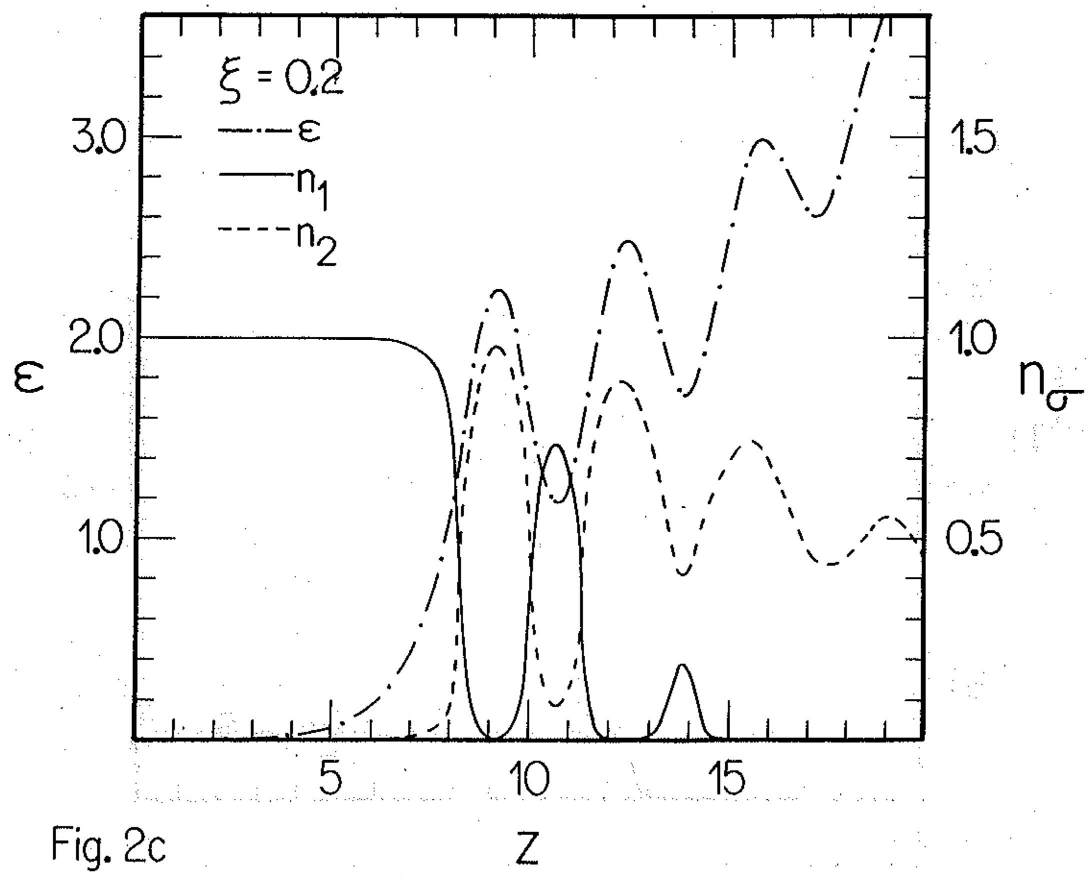

ξ = 0.2
ε
n1
n2
3.0
2.0
1.0
1.5
1.0
0.5
ε
nσ
5
10
15
Z

Fig. 2c

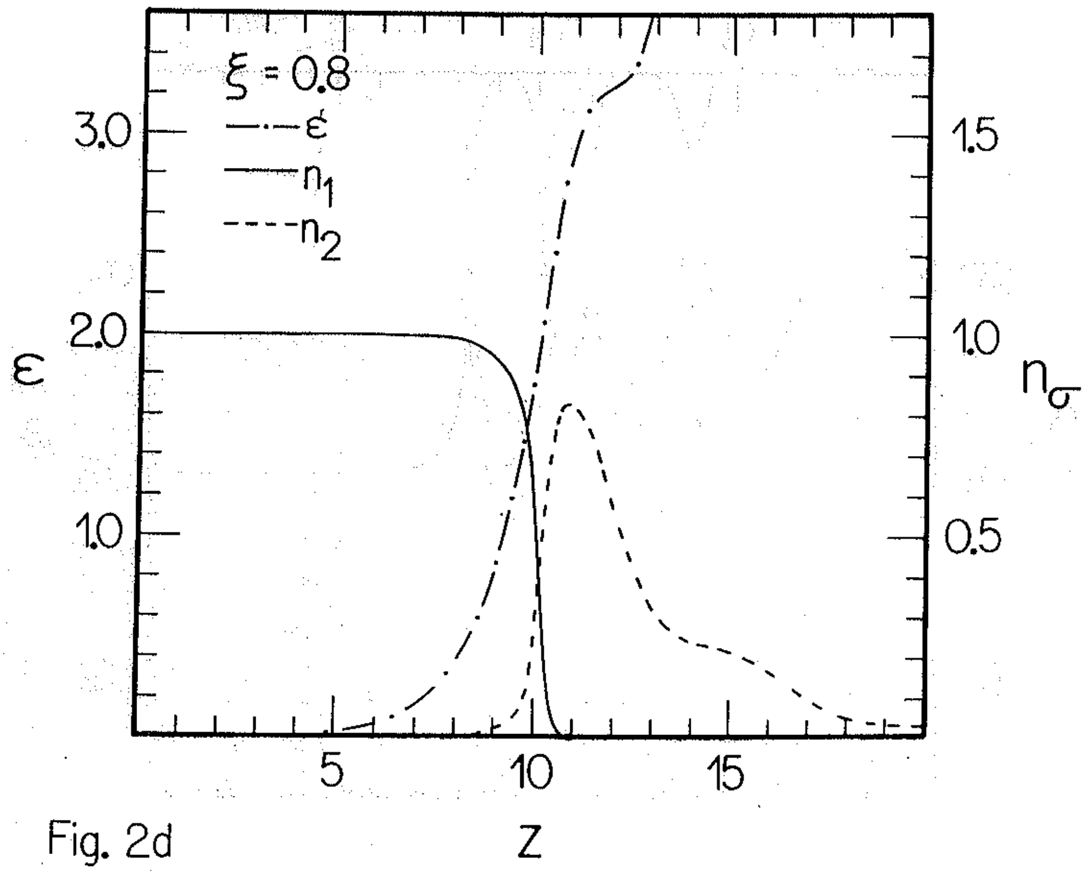


Fig. 2d