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SPONTANEOUS BREAKDOWN OF SYMMETRY, MACH'S PRINCIPLE AND CONFORMAL INVARIANCE IN SCHWINGER'S GRAVITY

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ABSTRACT

Schwinger's scalar-tensor theory of gravity is shown to have spontaneous breakdown of symmetry as an important feature of cosmological solutions. Machian features are also examined.

1. INTRODUCTION

In an attempt to solve the vexing problems connected with the formation of domains in the cosmological 3-space, which should, instead, be homogeneous[1], I proposed recently[2] a way out by exhibiting a pattern of spontaneous breakdown of symmetry[3] in which order-disorder phase transitions due to the thermal evolution of the universe[4] do not occur. It depends crucially on the presence, in the Higgs Lagrangian[5], of a term $\frac{R}{6}\phi^2$ which couples the zero-spin meson to gravity in a conformal way[6]. This microscopic breaking of the equivalence principle is not entirely "ad hoc", as it allows for a natural extension to arbitrary space-times of the 15-parameter conformal symmetry of a massless particle in Minkowski space-time[6]. This notwithstanding, it would be conceptually better to have the presence of such a term as a built in requisite of the coupling of gravity to matter in some (well motivated) modification of General Relativity.

Schwinger proposed such a theory a few years ago in the last pages of a wonderful book on relativistic quantum mechanics[7]. In this paper I review his work from a slightly different viewpoint, discuss some features of it which might be called Machian, and show that it contains my mechanism of spontaneous breakdown of symmetry in its very structure: this is possibly the most important property of the model, which singles it out from similar models of scalar-tensor gravity.

2. THE MODEL

The Lagrangian

$$L(\phi,g) = -\sqrt{-g}\,\frac{1}{2}\left\{g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \left(m^2+\frac{R}{6}\right)\phi^2\right\} \qquad (1)$$

(ϕ is a zero-spin field; R is the scalar curvature) is, for $m=0$, invariant under conformal transformations. More precisely, if ϕ and $g^{\mu\nu}$ are transformed into

$$\begin{aligned} \phi'(x) &= (\lambda(x))^{-\frac{1}{2}} \phi(x) \\ g'_{\mu\nu}(x) &= \lambda(x)\, g_{\mu\nu} \end{aligned} \tag{2}$$

and $m=0$, the action constructed with (1) remains invariant. It is sufficient to check for the infinitesimal version of (2), which reads

$$\begin{aligned} \delta\phi(x) &= -\frac{1}{2}\,\delta\lambda(x)\,\phi(x) \\ \delta g^{\mu\nu}(x) &= -\,\delta\lambda(x)\, g^{\mu\nu}(x) \end{aligned} \tag{3}$$

The response of (1) to these transformations is, in fact,

$$\delta_\lambda L = \partial_\nu \left\{\frac{1}{4}\,\phi^2 \sqrt{-g}\, g^{\mu\nu}\, \partial_\mu\, \delta\lambda\right\} \tag{4}$$

We refer the reader to Ref. (7), chapter 3-17, for details. The introduction of an additional, dimensionless, scalar field $\sigma(x)$ in such a way that the term m^2 in (1) is replaced by $m^2\phi^2(x)$, is sufficient to get a Lagrangian that maintains conformal invariance even for $m\neq 0$, provided $\sigma(x)$ transforms like $\phi(x)$, that is

$$\delta\sigma(x) = -\frac{1}{2}\,\delta\lambda(x)\,\sigma(x) \tag{5}$$

A slightly more complex Lagrangian still having this invariance is

$$\begin{aligned} L(g,\sigma,\phi) &= \frac{1}{2\kappa}\sqrt{-g}\left\{R\sigma^2 + 6g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma\right\} - \\ &\quad - \sqrt{-g}\;\frac{1}{2}\left\{g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + (m^2\sigma^2 + \frac{R}{6})\,\phi^2\right\} \end{aligned} \tag{6}$$

This is, however, too symetric: $\sigma(x)$, whose kinetic energy term has, by the way, the wrong sign, can be eliminated by a conformal transformation. Eq. (6) is, however, a good source of inspiration. Keeping the "matter Lagrangian" (the ϕ-dependent part) as it is, we modify the "gravitation Lagrangian" to get

$$\begin{aligned} L(g,\sigma,\phi) &= \frac{1+\alpha}{2\kappa}\sqrt{-g}\left\{R - \frac{2}{\alpha}\, g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma\right\} - \\ &\quad - \sqrt{-g}\;\frac{1}{2}\left\{g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + (m^2\sigma^2 + \frac{R}{6})\,\phi^2\right\} \end{aligned} \tag{7}$$

This is Schwinger's model ($\alpha > 0$ is a new empirical, dimensionless constant, and $\kappa = 8\pi G$): a scalar-tensor theory in which the matter Lagrangian is made conformally invariant by adequate use of both R and σ. This has several nice consequences, the first of which is the following: if we specialize the minimum-action principle to conformal variations $\delta\lambda(x)$, a relation between σ and R is immediately obtained, as the matter Lagrangian remains invariant. A similar thing happens in the theories of induced gravity propugnated by Adler and Zee[19,20]: there, an effective Lagrangian resembling Einstein's is obtained when the conformal symmetry of the gauge-symmetric, scalarless matter Lagrangian is broken by radiative corrections. Relations among the induced terms then follow by utilization of the technique we just described.

In our case the relation which follows from the conformal invariance of the matter Lagrangian is

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\sigma^2(x)\right) = \alpha R \tag{8}$$

Other equations of motion are

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}\,R = \frac{\kappa}{1+\alpha}\,t^{\mu\nu} \tag{9}$$

where $t^{\mu\nu}$ is the stress tensor corresponding to all pieces of the Lagrangian (7) except for the first one. It is written

$$t_{\mu\nu} = t_{m\mu\nu} + \frac{2}{\alpha}\,\frac{1+\alpha}{K}\,\left(\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}\,g_{\mu\nu}\,g^{\beta\gamma}\,\partial_\beta\sigma\,\partial_\gamma\sigma\right) \tag{10}$$

$$\begin{aligned} t_{m\mu\nu} = {} & \partial_\mu\phi\,\partial_\nu\phi + \frac{1}{6}\left\{R_{\mu\nu} - \frac{1}{2}\,(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu) + g_{\mu\nu}\Box\right\}\phi^2 - \\ & - g_{\mu\nu}\left\{\frac{m^2}{2}\,\sigma^2\phi^2 + \frac{1}{2}\,g_{\mu\nu}\,g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi\right\} \end{aligned} \tag{11}$$

$$t = t_m - \frac{2}{\alpha}\,\frac{1+\alpha}{K}\,g^{\beta\gamma}\,\partial_\beta\sigma\,\partial_\gamma\sigma \tag{12}$$

where $t = t^\mu_\mu$. The equation for $\sigma(x)$ can be written, using (9), as

$$-\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\sigma^2\right) = \frac{\alpha\kappa}{1+\alpha}\,t \tag{13}$$

Equations (9) and (13) govern the behaviour of the gravitational fields. It is seen that all gravitational fields have the same source, the stress tensor.

Schwinger's Lagrangian, Eq. (7), contains no strict mass terms. If $\sigma(x)$ is slowly varying the field ϕ has a slowly varying "mass" given by $m\sigma(x)$. In our interpretation (see below) this is a Machian feature.

To clarify the rules of the model a little more,

consider the coupling of a vector field of mass m to the gravitational fields. The last term of the matter Lagrangian

$$L = -\frac{1}{2}\,F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{4}\,F^{\mu\nu}\,g_{\mu\kappa}\,g_{\nu\lambda}\,F^{\kappa\lambda} - \frac{1}{2}\,m^2\,g^{\mu\nu}\,A_\mu A_\nu \tag{15}$$

breaks conformal invariance. The required modification consists in replacing it by

$$-\frac{1}{2}\,\frac{m^2}{\sigma(x)}\,g^{\mu\nu}\,A_\mu\,A_\nu \tag{16}$$

which depends on σ in a non-analytical way. To avoid this, one must take $m=0$. Electrodynamics comes out naturally.

3. MACH'S PRINCIPLE

Inertia is perhaps the fundamental property of matter[16]. Mach's principle addresses the problem of understanding it and asserts, "grosso modo", that inertia is determined by the interactions of a body with remote sources of gravity (see Refs. (8,9,16,17)). Mass, one of the parameters characterizing irreducible representations of the Poincaré group, may have the same origin as Poincaré invariance. In the presence of gravity the latter is valid only locally; mass can therefore also acquire a dependence on position and time. Schwinger's model has this feature embodied in its structure in a very elegant way. Let us consider the case of a scalar field, given by Eq. (7), and look for solutions with constant $\sigma(x)$. From Eq. (13), $t=0$. As, then, $t=t_m$ (Eq. (12)), $t_m=0$. From (11) it is straightforward to show that

$$t_m = - m^2 \sigma^2 \phi^2 + \frac{R}{3} \phi^2 \tag{17}$$

and this implies $m=0$. So, a constant σ is only consistent with a vanishing mass.

Schwinger decomposes

$$\sigma(x) = 1 + \psi(x) \tag{18}$$

and interprets $\psi(x)$ as the field due to nearby sources, whereas 1 is the field of the very distant sources, connected to Mach's principle. I think, instead, that the decomposition (18) has no physical meaning, for $\sigma(x)$, like $g_{\mu\nu}$, is not a field strength, but a potential. A constant $\sigma(x)$ means zero (scalar gravity) field strength. No wonder that a constant $\sigma(x)$ requires a vanishing mass: it represents the absence of (scalar) gravity. It is this very fact that makes of Schwinger's model a Machian one: inertia (as measured by mass) is an effect of gravity, scalar gravity, to be precise.

We may take profit of the fact that the model has simple transformation properties under dilatations to write it[13] in a form resembling the Brans-Dicke theory[14]. Essentially, $\sigma(x)$ is absorbed by $\phi(x)$ and $R(x)$, the "mass term" of $\phi(x)$ being transformed into $(m^2 + \frac{\bar{R}}{6})\bar{\phi}^2$, which has a more familiar look (the mass is constant). The metric is, of course, changed, and the price to pay is that the gravitational "constant" is now a field. The two formulations are physically equivalent if considered as classical theories. We prefer the original formulation of Schwinger, with constant G, because of the possibility of borrowing, in many cases, the solutions of Einstein's equations of General Relativity. To argue further for our contention, that the relevant field for inertia effects

is not $\sigma(x)$ but $\partial_\mu \sigma(x)$, we consider now a simple situation in which the physical role of the scalar field is made transparent. The geometry will be that of the Robertson-Walker metric for $k=0$, that is, the flat case; in agreement with the spatial symmetries we assume $\sigma(x)$ to depend only on time. Writing the fundamental form as

$$- ds^2 = dt^2 - S(t^2) \sum_k (dx_k)^2 \tag{19}$$

the field equations read

$$3 \frac{\dot{S}^2}{S^2} + 3 \frac{k}{S^2} = \frac{1}{\alpha} \sigma^2 + \frac{\kappa}{1+\alpha} \rho \tag{20}$$

$$2 \frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = - \frac{\kappa}{1+\alpha} p - \frac{1}{\alpha} \dot{\sigma}^2 \tag{21}$$

$$\frac{\sigma}{S^3} \frac{d}{dt} (S^3 \dot{\sigma}) = \frac{1}{2} \frac{\alpha\kappa}{1+\alpha} (3p - \rho) \tag{22}$$

where ρ is the energy density and p is the pressure. We now take $p=0$ and combine (21) and (22), obtaining

$$\frac{\ddot{S}}{S} = - \frac{\kappa}{6(1+\alpha)} \rho - \frac{2}{3\alpha} \dot{\sigma}^2 \tag{23}$$

The spherical symmetry of the problem, allied to the vanishing pressure, should allow for a Newtonian interpretation of these equations, in analogy with the case of General Relativity[15] provided that we recover Newton's law in the limit of weak fields. This is most easily done in the source formalism developed by Schwinger[7], where the interaction energy of a fixed body of mass M is

$$E_{int}(x^0) = -\frac{\kappa M}{8\pi}\int d^3x\,\frac{1}{|\vec{x}|}\left[t^{00}(\vec{x},x^0) + \frac{1-\alpha}{1+\alpha}\,t_{kk}(\vec{x},x^0)\right] \tag{24}$$

for weak fields in the scalar-tensor theory, whereas General Relativity gives

$$E_{int}(x^0) = -\frac{\kappa M}{8\pi}\int d^3x\,\frac{1}{|\vec{x}|}\left[t^{00}(\vec{x},x^0) + t_{kk}(\vec{x},x^0)\right] . \tag{25}$$

When $t^{00} >> t_{kk}$ (vanishing pressure) the limits coincide, and reproduce Newton's law. By considering the Newtonian equation of motion of a point at the surface of a sphere of radius S containing mass M, one has

$$\ddot{S} = -\frac{GM}{S^2} = -\frac{4}{3}\pi G\rho S \tag{26}$$

which is Eq. (23) with the last term, given essentially by $\dot{\sigma}^2 S$, lacking. This term, which looks like a centripetal force, is showing that the disagreement between Eqs. (26) and (23) (they agree in General Relativity!) can be interpreted as meaning that the chosen reference system is not inertial, or, equivalently, that particles do not move along geodesics. It vanishes for constant σ, which is, then, a condition for the frame to be inertial. Here it is particularly clear that inertia is connected directly to $\dot{\sigma}$, and not to σ.

Just to close the argument we analyse a simple and curious situation that once more stresses the role of $\dot{\sigma}$. Consider Eqs. (20), (21) and (22) with $\dot{\sigma} = \text{constant} \neq 0$ and $3p=\rho$. From (22) it immediately follows that S is constant, that is, the universe is stationary. Then Eq. (20) gives

$$\rho = \left(\frac{3k}{S^2} - \frac{1}{2}\dot{\sigma}^2\right)\frac{1+\alpha}{\kappa} = \text{constant} \tag{27}$$

and using (21) and the equation of state, it follows that

$$k = \frac{\kappa - 3(1+\alpha)}{(1+\alpha+\kappa)\alpha}\,S^2\dot{\sigma}^2 \tag{28}$$

and

$$\rho = A\dot{\sigma}^2$$

where A is a constant. It is $\dot{\sigma}$ which is directly connected to physics.

4. SPONTANEOUS BREAKDOWN OF SYMMETRY

We start by enriching the physics of the model, adding a self-interaction term to the ϕ-Lagrangian. Let it read

$$L = \frac{1+\alpha}{2\kappa}\sqrt{-g}\left\{R - \frac{2}{\alpha}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right\} - \sqrt{-g}\,\frac{1}{2}\left\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + (m^2\sigma^2 + \frac{R}{6})\phi^2 + \frac{\lambda}{6}\phi^4\right\} \tag{29}$$

and notice that the new term does not break the conformal invariance required of the matter Lagrangian. In the case $m=0$ we are allowed to look for solutions with constant σ: the gravitational problem reduces, therefore, to a problem of General Relativity, except for the conformal coupling. The ϕ-Lagrangian

$$-\sqrt{-g}\,\frac{1}{2}\left\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{6}R\phi^2 + \frac{\lambda}{6}\phi^4\right\} \tag{30}$$

can, under certain circumstances, reproduce Goldstone's Lagrangian[3], the curvature term playing the role of wrong-signed mass term. In references (10) and (11) this has been done for complex scalar fields in the context of General Relativity modified by the addition of the conformal coupling term. In Ref. (2) the case of a real pseudo-scalar field is considered, and connected to the CP violation of the K^0 system. In the present context the R-proportional term comes out of the theory itself, and the spontaneous breakdown of symmetry is an unavoidable consequence of its rules. It is also seen that the problem has a different nature when ϕ is massive, for then it is no longer possible to assume constancy of σ. This very interesting case is presently under development.

The following situation is potentially relevant for particle physics and simple enough for computation: ϕ propagates in an open Friedman universe considered as a background metric. This corresponds, so to say, to studying ϕ in the laboratory but taking into account the cosmological field. The curvature of the universe is not due to that sample of ϕ we are observing, but, mainly, to the cosmic matter. In these circumstances the existence of Friedman's solutions is ensured by the fact that σ is being assumed constant. Then, according to the Machian interpretation of the precedent section, one would expect the mass to be zero. Because of Eqs. (8) and (13), a constant σ is only possible if $R=t=0$, conditions that are satisfied for matter with the equation of state

$$p = \frac{\rho}{3} \tag{31}$$

The metric is[12]

$$-ds^2 = S^2(\eta)\left\{d\eta^2 - d\chi^2 - \sinh^2\chi\,(d\theta^2 + \sin^2\theta\, d\varphi^2)\right\} \tag{32}$$

The equation of motion being

$$\Box\phi - \frac{R}{6}\phi - \frac{\lambda}{3}\phi^3 = 0 \tag{33}$$

one gets for the vacuum expectation value

$$g(\eta) \equiv \langle 0|\phi(x)|0\rangle \tag{34}$$

the following equation, valid in the tree approximation (see Ref. (11) for details):

$$\ddot{g} + 2\frac{\dot{S}}{S}\dot{g} + \left(\frac{\ddot{S}}{S} - 1\right)g + \frac{\lambda S^2}{3}g^3 = 0 \tag{35}$$

Remark that though $R=0$, the term $\frac{R}{6}\phi^2$ in Eq. (30) cannot be simply ignored, as it gives origin, in the stress tensor of Eq. (11), to terms which do not vanish with R. So, it is wiser to keep all R terms to the end. To determine which of the solutions of (35) is the real vacuum expectation value, one has to select those which correspond to the minimum value of the energy density. Introducing the new variable defined by

$$g = \sqrt{\frac{3}{\lambda}}\,\frac{f}{S} \tag{36}$$

in Eq. (35), it simplifies to

$$\ddot{f} - f + f^3 = 0 \tag{37}$$

whose general solution is

$$\frac{f^2}{1+B} = 1 - 2B\,\mathrm{sn}^2\left\{\sqrt{\frac{1+B}{2}}\,\eta\,,\ \sqrt{\frac{2B}{1+B}}\right\} \quad ; \quad B = \sqrt{1+2C} \tag{38}$$

with $C \geq -\frac{1}{2}$. Here sn denotes the Jacobian elliptic function called sine amplitude.

From

$$t_{m\nu}^{\ \mu} = \partial^\mu\phi\,\partial_\nu\phi + \frac{1}{6}\left\{R^\mu_\nu - \frac{1}{2}(\nabla^\mu\nabla_\nu + \nabla_\nu\nabla^\mu) + \delta^\mu_\nu\,\Box\right\}\phi^2 \qquad (39)$$

(see Eq. (11)) it follows, for the vacuum energy density

$$\varepsilon(\eta) \equiv \langle 0|t_{m\ 0}^{\ 0}|0\rangle = \frac{3}{2\lambda S^4}\left\{\dot{f}^2 - f^2 + \frac{f^4}{2}\right\} \qquad (40)$$

or, in terms of the solutions of Eq. (37),

$$\varepsilon(\eta) = \frac{3C}{2\lambda S^4} \qquad (41)$$

So, the minimum $\varepsilon(\eta)$ corresponds to the smallest C. This is, from Eq. (38),

$$f = \pm 1 \qquad (42)$$

or

$$\langle 0|\phi(x)|0\rangle = g(\eta) = \pm\sqrt{\frac{3}{\lambda}}\ \frac{1}{S(\eta)} \qquad (43)$$

and

$$\varepsilon(\eta) = -\frac{3}{4\lambda S^4} \qquad (44)$$

The vacuum expectation value of $\phi(x)$ is nonvanishing and degenerate. The symmetry $\phi \leftrightarrow -\phi$ is therefore spontaneously broken, in the stated conditions. A host of phenomena follow from this consequence of conformal coupling. For instance, if ϕ is pseudoscalar, parity symmetry is violated in the solutions, though maintained in the Lagrangian. Potential applications to particle physics are indicated elsewhere[2]. More important

to the present analysis is the fact that mass generation for the field ϕ is found to happen. In fact, rewriting the theory in terms of fields with vanishing vacuum expectation values,

$$\tilde{\phi}(x) = \phi(x) - g(\eta) \quad ,$$

a quadratic term in $\tilde{\phi}^2$ will appear in (29), to be interpreted as a mass, given by

$$m = \frac{\sqrt{3}}{S}$$

This is the only mass term, as, for the adopted equation of state, $R=0$.

The phenomenon just described is connected to the very foundations of Schwinger's gravity, namely, to the requirement that the matter Lagrangian be conformally invariant. It has been shown, therefore, that not all mass is connected to the field $\sigma(x)$. However, as the generated mass has geometrical character, it can still be said that its origin is gravitational. So, no harm is done to the Machian features of the theory.

5. CONCLUSIONS

In this paper we reviewed the theory of gravity proposed by Schwinger, in which conformal symmetry plays an important role. For weak fields it gives the same results as the theory of Brans and Dicke[13], so that the differences should be looked for elsewhere, as in cosmology. In fact, as demonstrated in the case of scalar fields, the coupling $\frac{R}{6}\phi^2$ can give rise, through spontaneous breakdown of symmetry, to unexpected phenomena which are particularly important in the early universe. We also analyse a Machian property of the model, the connection between the σ field and the masses, in a more complete way, including the mass generation connected to the just mentioned symmetry breakdown.

This theory, though endowed with an extra field, has a manifold of solutions severely restricted by the conformal invariance of the matter Lagrangian. In particular, it is not hard to show, through the use of Eqs. (20), (21) and (22), that inflationary solutions[18] with a Friedman metric do not exist.

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