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THE SYMMETRIES OF THE VACUUM

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ABSTRACT

The vacuum equation of state required by cosmological inflation is taken seriously as a general property of the cosmological vacuum. This correctly restricts the class of theories which admit inflation. A model of such a vacuum is presented that leads naturally to the cosmological principle.

1. INTRODUCTION

Modern theoretical physics has found many uses for the vacuum, the most sensational of which is probably that of source of asymmetry (spontaneous breakdown of symmetry). More recently it has been proposed that the vacuum itself (though an unstable, excited one) is the dominant source of the cosmological gravitational field for a certain period in the very early universe, allowing it to expand exponentially. This is the inflationary cosmology[1,2]. Exponential inflation is neatly described, in terms of a Robertson-Walker metric, by General Relativity, and proposes elegant solutions to the so called horizon, flatness and monopole problems, provided that the vacuum has a pressure p and an energy density ρ connected by the equation of state $p=-\rho$.

It is important to understand to what extent this equation of state is demanded by the vacuum properties, as opposed to being an "ad hoc" assumption.

In this paper the origin of the equation of state of the vacuum is traced back to its symmetries, which are characterized in a simple geometric way. Section 2 describes this characterization. In Section 3 it is used to show that the most reasonable scalar-tensor theories of gravity cannot accomodate inflation. In Section 4 a model of the vacuum is presented that provides some justification for the cosmological principles.

2. THE VACUUM STATE OF INFLATIONARY COSMOLOGY

Inflation[1,2] assumes that for some time during the first second the universe underwent exponential expansion.

In terms of a Robertson-Walker metric[3] one has (Einstein equations):

$$\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}a(\rho+3p) \tag{1}$$

$$\frac{1}{2}\left(\frac{da}{dt}\right)^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{k}{2} \tag{2}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p+\rho) = 0 \tag{3}$$

where a is the scale factor of the universe ("radius of the universe"), p is the pressure, ρ the energy density. A dot stands for differentiation in the time.

For very early times, $k \approx 0$. If we take as equation of state

$$\begin{aligned} p+\rho &= 0 \\ \rho &> 0 \end{aligned} \tag{4}$$

then it easily follows from the above equations that

$$a(t) = \exp\left(\sqrt{\frac{8\pi G\rho}{3}}\,t\right) = \exp(Ht) \tag{5}$$

H being Hubble's constant.

What is the physical interpretation of inflation? Of course $p=-\rho$ is not a usual equation of state, so that the medium must be peculiar. In the elegant scheme of Guth[1] this medium is the vacuum, though an excited, unstable one.

The argument is the following: consider a field theory defined in a curved space-time whose metric tensor is $g_{\mu\nu}$. Let $T_{\mu\nu}(x)$ denote the energy-momentum tensor of the field. At the vacuum configuration one has, if the vacuum has

(global) Lorentz invariance,

$$\langle 0|T_{\mu\nu}(x)|0\rangle = \Lambda g_{\mu\nu} , \tag{6}$$

Λ being a constant[1]. Now, the general expression for the energy-momentum tensor of a relativistic fluid is

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p\,g_{\mu\nu} \tag{7}$$

so that (6) leads to $\Lambda=-p=\text{constant}$, and also to $p=-\rho$. This is Eq. (4). However, the vacuum configuration of quantum field theories defined in a curved space-time does not have, in general, global Lorentz invariance[7], as this would imply a Minkowskian vacuum. For homogeneous space-times one can proceed as follows[4]: invoking local Lorentz invariance one can write (6) for every single point. Homogeneity then states that Λ is point independent, that is, a constant. This is, however, somewhat narrow, as one would like to characterize, through the quantity $\langle 0|T_{\mu\nu}(x)|0\rangle$, vacua with a wider class of symmetries.

In what follows we offer a more general derivation of (6) by separating the consequences of the vacuum symmetries from the direct consequences of Einstein's equations. In so doing we will be able to extend the discussion also to alternative theories of gravitation.

Let us take as a model the Minkowski vacuum. If we construct idealized particle detectors[5,6] in different inertial motions in Minskowski space-time, they will all agree as to the absence or presence of particles[7]. This shows that the Minkowski vacuum state is invariant under transformations of the Poincaré group. These transformations also happen to be those generated by the Killing fields[3] of the Minkowski metric tensor. This observation is the key to understanding the symmetries of the vacuum state. Consider a field theory defined in a curved space-time whose metric tensor is $g_{\mu\nu}$. The vacuum state should be invariant under the transformations generated by the Killing fields of $g_{\mu\nu}$. Given coordinate transformations

$$x'^{\mu} = x^{\mu} + \xi^{\mu}(x) \tag{8}$$

where ξ^{μ} is an infinitesimal 4-vector field, the Killing fields of the metric are those ξ^{μ} which satisfy the equations

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0 \quad . \tag{9}$$

It easily follows[3] that the transformed metric $g'_{\mu\nu}$ has the same form as $g_{\mu\nu}$, that is

$$g'_{\mu\nu}(x) = g_{\mu\nu}(x) \tag{10}$$

or, in a more suggestive way, equivalent obervers (connected by (8) and (9)) experience the same gravitational field. Formally, the Lie derivative[8], with respect to ξ^{μ}, of $g_{\mu\nu}$, vanishes:

$$L_{\xi}\, g_{\mu\nu} = 0 \quad . \tag{11}$$

In an analogous way, two equivalent observers which probe into the vacuum should observe the same energy-momentum tensor.

That is to say,

$$L_{\xi}\langle 0|T_{\mu\nu}(x)|0\rangle = 0 \quad . \tag{12}$$

Comparing (11) and (12) one is led to

$$\langle 0|T_{\mu\nu}(x)|0\rangle = \Lambda(x)g_{\mu\nu}(x) \tag{13}$$

where $\Lambda(x)$ is a scalar field such that

$$L_{\xi}\Lambda(x) \equiv \xi^{\mu}(x)\partial_{\mu}\Lambda(x) \equiv 0 \quad . \tag{14}$$

Form invariance cannot be restricted to $g_{\mu\nu}$ and $T_{\mu\nu}$. We therefore require that all fields present in the vacuum be form invariant under the transformations generated by the Killing fields of $g_{\mu\nu}$. This ends our characterization of the vacuum.

Remark that, to this point, we have made no use of General Relativity: we just assumed the gravitational theory to be a metrical one. If, now, the space-time is one of maximum symmetry, then it follows, without more ado[3], that $\Lambda(x)$ is a constant. This covers the case of Minkowski, and shows that (12) is a generalization of (6). Remark that the Robertson-Walker metrics do not correspond to space-times of maximum symmetry: in this case we have

$$\langle 0|T_{\mu\nu}(x)|0\rangle = \Lambda(t)g_{\mu\nu}(x) \tag{15}$$

as there are no Killing fields in the t-direction.

Let us now, for the first time, use a direct consequence of Einstein equations, namely,

$$T^{\mu\nu}{}_{;\mu} = 0 \quad ,$$

to get, from (13),

$$\langle 0|T^{\mu\nu}{}_{;\nu}|0\rangle = g^{\mu\nu}(x)\partial_{\nu}\Lambda = 0$$

so that

$$\Lambda = \text{constant} \tag{16}$$

in General Relativity.

Taking now

$$T^{vac}_{\mu\nu} \equiv <0|T_{\mu\nu}(x)|0> = (p+\rho)u_\mu u_\nu - p\, g_{\mu\nu} \tag{17}$$

it follows that

$$\begin{aligned} p &= -\Lambda = \text{constant} \\ p + \rho &= 0 \end{aligned} \tag{18}$$

so that also ρ is a constant. Notice that in alternative theories in which $T^{\mu\nu}{}_{;\nu} \neq 0$ [9], p would not necessarily be a constant, although one would still have $p+\rho = 0$. So, the symmetries of the vacuum imply the equation of state, but not the constancy of p and ρ. Notice that the constancy of ρ is needed for exponential expansion.

3. NON STANDARD THEORIES

We here study situations in which the vacuum states do not necessarily have constant p and ρ. This happens either when $T^{\mu\nu}{}_{;\mu} \neq 0$ or when $g^{\mu\nu}{}_{;\mu} \neq 0$. Typical of the former case are field theories defined in a background metric: the energy-momentum tensor of the field does not satisfy the Einstein equations, so that $T^{\mu\nu}{}_{;\mu} \neq 0$ for the total energy-momentum tensor. General Relativity is, therefore, truncated. We give an example. Consider a scalar field ϕ with Lagrangian

density

$$L = \frac{1}{2} g^{\mu\nu} \partial_\mu\phi\, \partial_\nu\phi + \frac{R}{12}\phi^2 - \frac{\lambda}{12}\phi^4 \tag{19}$$

defined in Friedmann's open universe, with metric

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta\, d\theta^2)\right\} \quad . \tag{20}$$

The space-time is fixed, in the sense that the energy-momentum tensor of ϕ,

$$t^\mu{}_\nu = \partial^\mu\phi\, \partial_\nu\phi + \left[R^\mu{}_\nu - \nabla^\mu\nabla_\nu + \delta^\mu{}_\nu \Box\right]\frac{\phi^2}{6} - \delta^\mu{}_\nu L \tag{21}$$

is such that

$$t^\mu{}_{\nu;\mu} \neq 0 \quad .$$

It is not difficult [10] to show that

$$<0|t^\mu{}_\nu(x)|0> = \frac{3C}{2\lambda a^4(t)}\,\delta^\mu{}_\nu \tag{22}$$

where the minimum value of $C = -\frac{1}{2}$. Then, redefining the density of the true vacuum to be zero, the false vacuum with the lowest energy will have

$$p(t) = \frac{3}{4}\,\frac{1}{\lambda a^4(t)} = -\rho(t) \quad . \tag{23}$$

As a second example we consider scalar-tensor theories of gravity. In order that the scalar field ϕ be a legitimate gravitational field, its source must be the trace of the energy-momentum tensor [11]. This condition turns out to be too restrictive to allow for a "bona fide" inflationary stage. We demonstrate this in the context of the scalar-tensor theory due to Schwinger[12]. The result extends trivially to Brans-Dicke[13]. The gravitational fields, $g_{\mu\nu}$ and $\sigma(x)$, are coupled to a "matter" scalar field ϕ as expressed by the Lagrangian

$$L(g,\sigma,\phi) = \frac{1+\alpha}{2\kappa}\sqrt{-g}\left\{R - \frac{2}{\alpha} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right\} - $$
$$- \sqrt{-g}\,\frac{1}{2}\left\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + (m^2\sigma^2 + \frac{R}{6})\phi^2\right\} \quad . \tag{24}$$

The "matter" Lagrangian is made invariant under local dilatations by adequate use of σ and R. α is a new dimensionless coupling constant. κ is Newton's constant. The scalar gravity field obeys

$$- \frac{1}{\sqrt{-g}}\partial_\mu\left\{\sqrt{-g}\, g^{\mu\nu}\partial_\nu\sigma^2\right\} = \frac{\alpha\kappa}{1+\alpha}\, t \tag{25}$$

where $t \equiv t^\mu{}_\mu$; $t^\mu{}_\nu$ is the energy-momentum tensor associated to the Lagrangian of Eq. (24). According to Section 2 we have, for the false vacuum,

$$<0|t^\mu{}_\nu(x)|0> = \Lambda\,\delta^\mu{}_\nu \tag{26}$$

and

$$<0|t|0> = 4\Lambda \quad . \tag{27}$$

This gives, using (25),

$$- \frac{1+\alpha}{\alpha\kappa}\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu <0|\sigma^2(x)|0> = 4\Lambda \quad . \tag{28}$$

In the tree approximation, consistent with our classical approach,

$$<0|\sigma^2(x)|0> = <0|\sigma(x)|0><0|\sigma(x)|0>$$

and, in the absence of a self-interaction for $\sigma(x)$,

$$<0|\sigma(x)|0> = 0 \quad . \tag{29}$$

So (28) gives $\Lambda=0$, implying that $\rho=0$. This gives no expansion, hence no inflation. The same is true for Brans-Dicke[13]. Scalar-tensor theories like Zee's[9] escape this mechanism, as a self-interaction term for the scalar field $\sigma(x)$ allows for a nonvanishing vacuum expectation value and introduces a new term in Eq. (28). Inflation is then allowed, but at a price: the scalar field is not a clean-cut gravitational field.

4. A MODEL OF THE VACUUM

In this section we will exploit our characterization of a vacuum state in a more ambitious way. It is conceivable that matter determines not only the particular Riemannian geometry of space-time, but the very nature (Riemannian or other) of that geometry. This is because matter provides standards of length and time[14]. In the absence of matter one is free to liberally change all standards, from point to point, without any consequences. Weyl[15] discovered a generalization of Riemannian geometry which allows for a natural formulation of this possibility, of changing standards in an arbitrary manner. We propose that the vacuum state is naturally described by Weyl's geometry. This is inconsistent with the presence of matter unless $F_{\mu\nu}$, the "length curvature"(16) vanishes at the points where matter is located. The physics of vacuum is then described by the action

$$S = \int \sqrt{-g}\, d^4x \left\{-\frac{1}{16\pi k}\left(\dot{R} - \frac{\lambda}{2}\right) - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}\varphi_\alpha \varphi^\alpha\right\} . \qquad (30)$$

We are adopting the notation and conventions of the excellent review of Weyl's theory contained in Ref. (16). Eq. (30) is equivalent to Eq. (15.82) of this reference. Formally we have the coupling of gravity, with a cosmological constant, to massive electrodynamics ($F_{\mu\nu} \equiv \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu$). The equation of state of vacuum is a relation between $F_{\mu\nu}$ and φ_α which is not difficult to find out. In fact, from the general expression of $T_{\mu\nu}$,

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p\, g_{\mu\nu}$$

we see that $p+\rho = 0$ implies that T_{i0}, the energy-flux density ("Poynting vector"), is vanishing:

$$T_{i0} = -\frac{(p+\rho)v_i}{1-v^2} = 0 . \qquad (31)$$

In terms of the electromagnetic analogy of the action of Eq. (30), therefore, the vacuum equation of state is

$$\vec{S} = 0 ,$$

$\vec{S}$ being the Poyinting vector. This means that

$$\mathbb{E} \times \mathbb{B} + m^2 \varphi_0 \vec{\varphi} = 0 . \qquad (32)$$

As is always the case in field theories in curved space, the vacuum geometry is the same as that of the state containing matter. The following question is, therefore, relevant: are there situations in which the Weyl geometry of the vacuum degenerates into a Riemann geometry in a natural way (that is, without introducing more dynamics)? Weyl geometry is Riemann geometry if $F_{\mu\nu} = 0$. We will show that if the vacuum has a certain amount of symmetry, this turns out to be the case.

As a first solution, suppose the vacuum is a maximally symmetric space-time. Then, as shown in Ref. (3), Chapter 13, a form invariant antisymmetric tensor must vanish. Therefore $F_{\mu\nu} = 0$, Eq. (32) is satisfied and the geometry is Riemannian. Adding matter in such a way that vacuum symmetry is respected leads to the so-called perfect cosmological principle. Notice that this is the real vacuum, as $\rho = p = 0$.

As a second solution, suppose the vacuum is of the Robertson-Walker type, that is, that the spatial part of its space-time is maximally symmetric. Then (Ref. (3), Chapter 13) the form invariance of $\mathbb{E}$ and $\mathbb{B}$ implies that $\mathbb{E} = 0$ and $\mathbb{B} = 0$. This also satisfies (32) and, as $F_{\mu\nu} = 0$, the geometry is Riemannian. Adding matter in such a way that vacuum symmetry is respected leads to the standard cosmological principle.

So, in our model of the vacuum, the cosmological principle comes out in a natural way.

A less symmetric space-time is not able to enforce, by sole symmetry arguments, $F_{\mu\nu} = 0$. Therefore, addition of matter would require the unnatural, ad hoc assumption of the vanishing of $F_{\mu\nu}$. Consider, for instance, the solution of Eq. (32) given by $\varphi_0 = 0$ and $\frac{d\vec{\psi}}{dt} = 0$. This implies $\mathbb{E} = 0$, but, if the space-time has no particular symmetry, $\mathbb{B} \neq 0$. Then, of course, $F_{\mu\nu} \neq 0$.

To summarize: the vacuum configuration of a quantum field in a curved space-time is supposed, as usually, to have the same symmetry as the nonvoid configuration. If it is described by Weyl's geometry with the requirement that the sole symmetry lead to $F_{\mu\nu} = 0$, that is, to a (compatible with matter) Riemannian geometry, then the space-time satisfies the perfect cosmological principle or the standard cosmological principle.

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