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MULTI-STEP COMPOUND MODEL OF HEAVY-ION FUSION

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ABSTRACT

Heavy ion fusion reactions have been analyzed within a multistep compound model composed of a di-nucleus configuration, coupled to particle and break-up channels as well as to an equilibrated compound nucleus configuration. The resulting fusion cross sections, defined as the summed particle emission cross sections from the equilibrated compound nucleus, are in reasonable agreement with the data for several systems. The resulting angular distributions as well as the time evolution of the system are also discussed.

I. INTRODUCTION

In the last several years, heavy ion fusion reaction have attracted great interest both theoretical and experimental[1). More than half a dozen models have been proposed, ranging from the more sophisticated microscopic TDHF to simple geometrical parametrizations. Several facts have emerged from these studies, the most important one of which is that the simple one-degree of freedom description, usually called the entrance channel model, is not fully adequate. For a recent review we refer the reader to Ref. 1).

In this paper, we develop a model which incorporates both the entrance channel effects and the compound nuclear characteristic in a consistent way. We feel that a model realistic enough to deal with fusion, must contain at least these effects.

We emphasize at this point that by entrance channel effects we do not mean just the restriction imposed through transmission factors calculated with a given entrance channel potential, rather, we also incorporate the effects arising from the formation of a di-nuclear configuration that preceeds the final equilibrated compound nucleus We allow the HI system to emit particles both from the intermediate stage and from the compound nucleus. The di-nuclear system is also allowed to break-up into two fragments.

The need for such a multi-step compound description of heavy ion fusion has already been pointed out in our previous

publication[2)], as well as in Ref. 3). The two-compound class model for fusion we develop here is based on the formulation of Agassi et al[4)].

The paper is organized as follows. In Section II we present the details of our multi-step compound model and discuss the formal consequences on heavy ion fusion, defined as the summed inclusive cross section for particle emission from the equilibrated compound system. In Section III we apply the model to a large variety of heavy ion systems ranging from light $^{12}C + ^{12}C$ to intermediate, $^{40}Ca + ^{40}Ca$. In Section IV we discuss another feature of our model, namely the characteristic angular momentum localization of the compound and di-nucleus statistical windows, and the corresponding angular distributions of emitted particles. The temporal aspect of the model is then considered in Section V, in connection with the life times of the two classes of compound configurations, and finally, in Section VI we present several concluding remarks.

II. MOTIVATION AND FORMALISM

It is a well known fact that heavy-ion systems such as $^{12}C + ^{12}C$, $^{16}O + ^{12}C$ etc, exhibit, in the elastic and compound nucleus (fusion) excitation functions intermediate structure, which is commonly related to the formation of isolated quasi-molecular resonances. It is also a common knowledge that heavier, or structurally more complex systems, do not show this behaviour. One is therefore tempted to suggest that these resonances, which may be isolated in $^{12}C + ^{12}C$ etc, at the energies considered, $\frac{E_{CM}}{A} \sim 2 - 3$ MeV, become overlapping at higher energies and/or in other systems.

In fact the experience one has gained from studying the dynamics of nuclear reactions over the last thirty years indicates clearly a gradual evolution of these "doorway" resonances as the energy is increased from isolated rather widely spaced structures to the overlapping regime, which requires a statistical treatment. Further, quite recently, several authors have suggested that the energy structure seem in the elastic scattering of C + C and O + C may be due to these evolved, quasimolecular resonances. In the heavy-ion case one may visualize these reasonances geometrically as two sticking nuclei (with a moment of inertia larger than that of the compound nucleus).

It is the aim of this paper to incorporate the overlapping quasimolecular resonances, in the description of heavy-ion fusion processes, commonly discussed within simple models[2]. We visualize the fusion process as in Fig. 1. The two approaching nuclei, first form a di-nucleus, which represents, geometrically, the overlapping quasimolecular resonances. This intermediate composite system is then allowed to emit particles and to break-up as well as couple to the equilibrated compound nucleus. There is no direct coupling between the entrance channel and the compound nucleus. The di-nucleus acts as a "doorway", and we shall call it such throughout this paper.

The fusion cross section is calculated as the summed "inclusive" cross section for particle emission from the equilibrated compound nucleus. The model we develop below is based on a generalization, to the heavy ion case, of the statistical multiclass compound model of Agassi, Weidenmüller and Mantzouranis[4]. The coupling between the di-nucleus and the compound nucleus is treated statistically. We do not attempt here to justify the model from first principles, leaving, this for a future work.

If we start from the traditional model of the compound nucleus, we can write the cross section for a given partial wave J as

$$\sigma_{fi} = \frac{\pi}{k^2}(2J+1)\frac{T_f\,T_i}{\sum_c T_c} \tag{1}$$

where T_i is the transmission coefficient which describes the probability to form the compound nucleus and the $T_f/\sum_c T_c$ can be interpreted as the branching ratio for decaying into the channel f.

If we sum over all channels, we find

$$\sum_f \sigma_{fi} = \frac{\pi}{k^2}(2J+1)\,T_i \tag{2}$$

which is the partial (J) reaction cross section for the channel i. At low energies, this quantity is very close to the total fusion cross section for the partial wave J. We know, however, that the fusion cross section at higher energies differs significantly from the reaction cross section. This is due to other competing processes such as deeply inelastic collisions. Motivated by the picture of fusion shown in Fig. 1, we construct below a two-step compound model, consistent with the picture of Ref. 2, based on

the formulation of Agassi et al. (AWM)[4]. In contrast with the one-class cross section of Eq. (1), AWM write the cross section for the final state f as

$$\sigma_{fi} = \frac{\pi}{k^2}(2J+1)\sum_{a,b} T_f^{a}\,\Pi_{ab}\,T_i^{b} \tag{3}$$

where the transmission coefficient, T_c^b, now describe the probability of the channel c to form states of class b in the composite system. The factor Π_{ab} describes the transitions among the classes of states of the composite system and can be defined by

$$\Pi_{ab}^{-1} = \delta_{ab}\,2\pi\rho_a(\Gamma_a^{\uparrow}+\Gamma_a^{\downarrow}) - T_{ab}^{\uparrow} - T_{ab}^{\downarrow} \tag{4}$$

where

$$2\pi\rho_a\Gamma_a^{\downarrow} = \sum_b T_{ab}^{\downarrow} \tag{5}$$

and

$$2\pi\rho_a\Gamma_a^{\uparrow} = \sum_c T_c^{a} \tag{6}$$

The factor $T_{ab}^{\downarrow}$ describes the internal mixing among classes a and b and is defined to be

$$T_{ab}^{\downarrow} = 2\pi\rho_a\,\overline{V_{ab}^2}\,2\pi\rho_b \tag{7}$$

The external mixing among the classes a and b due to open channels is described by $T^{\uparrow}_{ab}$. We neglect this, taking $T^{\uparrow}_{ab}=0$.

We can also define a partial cross section

$$\sigma_{fi,ab} = \frac{\pi}{k^2}(2J+1)\, T_f^{a}\, \Pi_{ab}\, T_i^{b} \tag{9}$$

which can be interpreted as the cross section for channel i to form states of class b and later decay from class a to channel f. Note that if we take $T^{\downarrow}_{ab} = 0$ as well as $T^{\uparrow}_{ab} = 0$, π_{ab} is diagonal and the corresponding cross section separates into a sum of independent contributions from each of the classes.

In our model, we will assume the existence of a class of doorway states and a class of compound nucleus states. We will assume that the doorway states can decay by breakup or particle emission. We write the corresponding transmission coefficients as T_b^d and T_p^d, respectively. We will assume that the compound nucleus can decay by particle emission only, so that $T_p^c \neq 0$ while $T_b^c = 0$. We can then write the escape width for the compound nucleus as

$$2\pi \rho_c \Gamma_c^{\uparrow} = \sum_p T_p^c \tag{10}$$

while the escape width for the doorway states is

$$\begin{aligned} 2\pi \rho_d \Gamma_d^{\uparrow} &= 2\pi \rho_d \Gamma_{d,p}^{\uparrow} + 2\pi \rho_d \Gamma_{d,b}^{\uparrow} \\ &= \sum_p T_p^d + \sum_b T_b^d \end{aligned} \tag{11}$$

For the mean square matrix element in the internal mixing factor, we take an extremely simplified form of that used by Agassi et al[4)]

$$\overline{V_{dc}^2} = \frac{\overline{V_o^2}}{\sqrt{\rho_d \rho_c}} \tag{12}$$

The inverse dependence on the densities of states of the mean square matrix element is consistent with the increasing complexity of the states and their diminishing overlap with increasing excitation energy. It is also consistent with the smooth energy dependence expected of its sum over final states.

As we are not dealing with the particle-hole excitations treatment by Agassi et al, we cannot give an order of magnitude estimate of the constant $\overline{V_o^2}$. We will treat it as a parameter to be adjusted.

To describe the doorway states, we rely on recent observations which have suggested that the composite system behaves like a dinuclear system living long enough to damp all of its energy and angular momentum but for which the decay occurs before the compound nucleus is formed. Such dinuclear systems, and their description in terms of the sticking configuration, are well-known in DIC.

We will assume that to continue beyond a DIC and form the compound nucleus, the two fragments must attain the sticking limit. Since the sticking configuration can be obtained as the most probable one by summing over all possible states of the two fragments, we take here the point of view that in the initial stage of the reaction, all states consistent with the conservation laws are excited with equal probability. We thus write the doorway level density as

$$\rho_d(\varepsilon, J, R) = \int d^3I_1\, d^3I_2\, d^3L\, d\varepsilon_1\, d\varepsilon_2\, \rho_1(\varepsilon_1, I_1)\, \rho_2(\varepsilon_2, I_2) \times \delta\Big(E - \varepsilon_1 - \varepsilon_2 - V(R) - \frac{\hbar^2 L(L+1)}{2\mu R^2} - \frac{\hbar^2 I_1(I_1+1)}{2\mathcal{J}_1} - \frac{\hbar^2 I_2(I_2+1)}{2\mathcal{J}_2}\Big)\, \delta(J - L - I_1 - I_2) \tag{13}$$

Here, ε is the total excitation energy

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{14}$$

with ε_1 and ε_2 the excitation energies of the fragments.

$\vec{J}$ is the total angular momentum

$$\vec{J} = \vec{L} + \vec{I}_1 + \vec{I}_2 \tag{15}$$

with $\vec{L}$ the orbital angular momentum and $\vec{I}_1$ and $\vec{I}_2$ the spins of the two fragments, E is the total energy, R is the distance between the centers of the two fragments and μ is the deduced mass.

For the level densities of the fragments, we have taken a modified Fermi-gas form,

$$\rho_i(\varepsilon_i, I_i) = \frac{1}{12}\left(\frac{\hbar^2 c A_i}{2\mathcal{J}_i}\right)^{3/2} \frac{(cA_i)(cA_i\varepsilon_i)}{(cA_i\varepsilon_i + 2)^3}\,(2I_i+1)\exp\left[2\sqrt{cA_i\varepsilon_i}\right], \quad i = 1,2 \tag{16}$$

where A_1 and A_2 are the mass numbers of the two fragments, Q_1 and Q_2 are their moments of inertia and C is a constant expected to be of the order of $\frac{1}{7}$ to $\frac{1}{8}$ for light to medium systems.

Evaluating the doorway level density by the method of steepest descent (taking the terms outside the exponentials in the fragment level densities as slowly varying), we obtain the sticking conditions

$$\vec{L} = \frac{\mu R^2}{\mathcal{J}_T(R)}\,\vec{J}, \qquad \vec{I}_1 = \frac{\mathcal{J}_1}{\mathcal{J}_T(R)}\,\vec{J}, \qquad \vec{I}_2 = \frac{\mathcal{J}_2}{\mathcal{J}_T(R)}\,\vec{J} \tag{17}$$

where $\mathcal{J}_T(R)$ is the total moment of inertia

$$\mathcal{J}_T(R) = \mu R^2 + \mathcal{J}_1 + \mathcal{J}_2 \quad ,$$

as well as the condition of equilibration in excitation energy,

$$\varepsilon_1 = \frac{A_1}{A}\varepsilon \quad , \quad \varepsilon_2 = \frac{A_2}{A}\varepsilon \tag{18}$$

with

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = E - V(R) - \frac{\hbar^2 J(J+1)}{2\mathcal{J}_T(R)}$$

Note that the sticking conditions are consistent with the picture of two fragments stuck and rotating together while the equilibration condition implies a common temperature for the two fragments.

The resulting doorway level density has the form

$$\begin{aligned}\rho_d(\varepsilon, J, R) = {} & \frac{\pi^{7/2}}{72}\left(\frac{\mu R^2}{\mathcal{J}_T(R)}\right)^{3/2}\left(\frac{A^2}{A_1A_2}\right)(cA)\left(2\frac{\mathcal{J}_1}{\mathcal{J}_T(R)}J+1\right) \\ & \times\left(2\frac{\mathcal{J}_2}{\mathcal{J}_T(R)}J+1\right)\frac{(cA\varepsilon)^{17/4}}{\left(cA\varepsilon+2(A/A_2)^2\right)^3\left(cA\varepsilon+2(A/A_2)^2\right)^3} \\ & \times \exp\left[2\sqrt{cA\varepsilon}\right]\end{aligned} \tag{19}$$

where A is the total mass number.

To eliminate the radial dependence of the doorway level density we assume that the system will prefer the radius that maximizes the density of states. We thus choose R_J by maximizing the excitation energy and minimizing the effective potential energy.

We take the effective potential for partial wave J to then be

$$V(J) = \left[V(R_J) + \frac{\hbar^2 J(J+1)}{2\mathcal{J}_T(R_J)}\right]\Bigg|_{min} + \frac{\hbar\omega(R_J)}{2} \tag{20}$$

The term $\frac{h\omega}{2}$, proportional to the curvature of the potential, is added to take into account the minimum energy of the fragments trapped in the potential well. The final form of the doorway level density is then given by Eq. (19) evaluated at R_J with

$$\varepsilon = E - V_{min}(J) \tag{21}$$

For the potential $V(R)$, we use use a Wood-Saxon + Coulomb potential given by

$$\begin{aligned} V(R) &= \frac{Z_1Z_2e^2}{R} + 20.11\frac{R_1R_2}{R_1+R_2}\left(1+1.0414\left(\frac{N-Z}{A}\right)^2\right) \\ &\qquad \times\left[1+\exp\left((R-R_0)/0.4454\right)\right]^{-1} \\ R_0 &= R_1 + R_2 + 0.29 \ [fm] \\ R_{1,2} &= 1.2998\,A_{1,2}^{1/3} - 0.4286\,A_{1,2}^{-1/3} \ [fm]\end{aligned} \tag{22}$$

The potential parameters were obtained from a simultaneous least-squares fit to the Region I fusion data for 32 systems ranging from $^{12}C + ^{12}C$ to $^{40}Ca + ^{40}Ca$ (see next Section).

We write the breakup width as the sum over all possible final fragment states of the corresponding Hill-Wheeler transmission coefficients,

$$
\begin{aligned}
2\pi \rho_d \Gamma^{\uparrow}_{d,p} &= \sum_b T_b^d \\
&= \int d^3I_1\, d^3I_2\, d^3L\, d\varepsilon_1 d\varepsilon_2\, \rho_1^f(\varepsilon_1, I_1)\, \rho_2^f(\varepsilon_2, I_2) \\
&\quad \times \left[1 + \exp\left[\left(V_{max}(J) + \varepsilon_1 + \varepsilon_2 - E\right) \Big/ \frac{\hbar\omega_{max}(J)}{2\pi}\right]\right]^{-1} \\
&\quad \times \delta(\vec{J} - \vec{L} - \vec{I}_1 - \vec{I}_2)
\end{aligned} \tag{23}
$$

We characterize the density of final fragment states by again assuming all possible states (consistent with the conservation laws) to be equally probable.

To take into account the observation that the angular degrees of freedom in DIC equilibrate much more slowly than the radial one, we take the distribution of angular momenta to be that determined by the sticking conditions, Eq. (17), at the separation distance, R_J, for which the doorway level density attains its maximum. In the radial potential through which we obtain the maximum, $V_{max}(J)$, we decouple the angular degrees of freedom, taking

$$
V_{max}(J) = \left[V(R) + \frac{\hbar^2 L(L+1)}{2\mu R^2} + \frac{\hbar^2 I_1(I_1+1)}{2\mathcal{J}_1} + \frac{\hbar^2 I_2(I_2+1)}{2\mathcal{J}_2}\right] \tag{24}
$$

with the angular momenta determined by the sticking conditions at the separation distance, R_J. We take $h\omega_{max}(J)$ proportional to the curvature of this potential at the maximum. We point out that these assumptions are consistent with the picture of a dinuclear system which has attained equilibrium in energy and angular momentum but which separates too rapidly for the angular momenta to follow, freezing them at their equilibrium values.

Using the steepest descent method, we can now reduce the expression for the breakup width, Eq. (23), to a single integral

$$
2\pi \rho_d \Gamma^{\uparrow}_{d,p} = \int d\varepsilon\, \rho_d(\varepsilon, J) \left[1 + \exp\left[\frac{V_{max}(J) + \varepsilon - E}{\hbar\omega_{max}(J)/2\pi}\right]\right]^{-1} \tag{25}
$$

where ρ_d is given by Eq. (19) evaluated at the separation R_J but without the energy conservation condition of Eq. (18).

This last integral can also be evaluated by the method of steepest descent. The condition for the maximum, which must be solved numerically, is

$$\sqrt{\frac{\varepsilon}{cA}} = \frac{\hbar\,\omega_{max}(J)}{2\pi}\left[1 + \exp\left[\frac{E - V_{max}(J) - \varepsilon}{\hbar\,\omega_{max}(J)}\right]\right] \tag{26}$$

We note that for sufficiently high energy, this condition results in an asymptotic kinetic energy that is smaller than the barrier energy, again consistent with DIC.

For the decay width due to particle emission of a pair of overlapping fragments at separation R_J with excitation energies ε_1 and ε_2 and spins I_1 and I_2, we take

$$2\pi\left(\rho_1(\varepsilon_1, I_1)\,\Gamma_1(\varepsilon_1)\,g_1(J) + \rho_2(\varepsilon_2, I_2)\,\Gamma_2(\varepsilon_2)\,g_2(J)\right) \tag{27}$$

where the $\Gamma_i(\varepsilon_i)$ are correlation widths, parametrized by

$$\Gamma_i(\varepsilon_i) = 14.0\,\exp\left[-4.69\sqrt{A_i/\varepsilon_i}\,\right]\,[\mathrm{Mev}] \tag{28}$$

and the g_i are geometrical factors which discount emission from the overlapping surface regions of the two nuclei

$$\vec{g}_i(J) = \frac{(R_i + R_J)^2 - R_{3-i}}{4R_i R_J} \qquad i = 1,2 \tag{29}$$

To calculate the contribution of particle emission to the doorway decay width, we average (27) over all configurations consistent with the conservation laws

$$\begin{aligned}
2\pi\rho_d\,\Gamma_{d,p}^{\uparrow} &= \sum_p T_p^d \\
&= \frac{1}{\rho_d(\varepsilon, J)}\int d^3L\, d^3I_1\, d^3I_2\, \rho_1(\varepsilon_1, I_1)\,\rho_2(\varepsilon_2, I_2) \\
&\quad \times \delta(\vec{J} - \vec{L} - \vec{I}_1 - \vec{I}_2)\;\delta\Big(E - \varepsilon_1 - \varepsilon_2 - V(R) - \\
&\quad - \frac{\hbar^2 L(L+1)}{2\mu R^2} - \frac{\hbar^2 I_1(I_1+1)}{2\mathcal{J}_1} - \frac{\hbar^2 I_2(I_2+1)}{2\mathcal{J}_2}\Big) \\
&\quad \times 2\pi\left[\rho_1(\varepsilon_1, I_1)\,\Gamma_1(\varepsilon_1)\,g_1(J) + \rho_2(\varepsilon_2, I_2)\,\Gamma_2(\varepsilon_2)\,g_2(J)\right]
\end{aligned} \tag{30}$$

Evaluating this integral by the method of steepest descent, we again obtain the sticking conditions of (17). Assuming that the correlation widths vary slowly with energy, we find the maximum of the integrand describing emission from nucleus i to occur at

$$\varepsilon_i = \frac{4A_i}{4A_i + A_{3-i}}\,\varepsilon \qquad i = 1,2 \tag{31}$$

with

$$\varepsilon = E - V(R) - \frac{\hbar^2 J(J+1)}{2\mathcal{J}_T(R)}$$

The dominant contribution to the doorway decay width of particle emission from fragment i thus comes from those configurations in which the excitation energy is concentrated in that fragment. We note that the partial decay width due to particle emission attains its maximum at the separation distance R_J , consistent with our description of the doorway states. We evaluate it there.

Finally, we assume that the elastic channel does not couple directly to the compound nucleus states. This implies that

$$T_o^c = 0 \tag{32}$$

For the coupling to the doorway states, we use a Hill-Wheeler transmission coefficient

$$T_o^d = \left[1 + \exp\left[\frac{V_o(J) - E}{\frac{\hbar\,\omega_o(J)}{2\pi}}\right]\right]^{-1} \tag{33}$$

where $V_o(J)$ is the barrier height of the effective potential in the elastic channel

$$V_o(J) = \left[V_o(R) + \frac{\hbar^2 J(J+1)}{2\mu R^2}\right]\Bigg|_{max.} \tag{34}$$

and $\hbar\omega_o(J)$ is proportional to the curvature of the potential at that point.

With the definition of these elements, we can now

readily perform the evaluation of the fusion cross section within our model. As in Eq. (9), we first construct the cross section for emission from the doorway into particle channel i,

$$\sigma_{p_i,d} = \frac{\pi}{k^2}(2J+1)\, T_{p_i}^d\, \Pi_{dd}\, T_o^d \tag{35a}$$

the cross section for emission from the compound nucleus into particle channel j

$$\sigma_{p_1,c} = \frac{\pi}{k^2}(2J+1)\, T_{p_1}^c\, \Pi_{cd}\, T_o^d \tag{35b}$$

and the cross section for emission from the doorway into breakup channel k ,

$$\sigma_{b_k,d} = \frac{\pi}{k^2}(2J+1)\, T_{b_k}^d\, \Pi_{dd}\, T_o^d \tag{35c}$$

Following Eq. (4), we write the inverse of the matrix π as

$$\Pi^{-1} = \begin{pmatrix} 2\pi\rho_d\Gamma_d^{\uparrow} + T^{\downarrow} & -T^{\downarrow} \\ -T^{\downarrow} & 2\pi\rho_c\Gamma_c^{\uparrow} + T^{\downarrow} \end{pmatrix} \tag{36}$$

where, by (7) and (12), we have for the internal mixing factor

$$T^{\downarrow} = (2\pi)^2\sqrt{\rho_c\,\rho_d}\;\overline{V_o^2} \tag{37}$$

Recall that

$$2\pi \rho_d \Gamma_d^{\uparrow} = 2\pi \rho_d (\Gamma_{d,p}^{\uparrow} + \Gamma_{d,b}^{\uparrow})$$

Inverting the matrix, we obtain

$$\Pi = \left[2\pi(\rho_d \Gamma_d^{\uparrow} + \rho_c \Gamma_c^{\uparrow}) T^{\downarrow} + (2\pi\rho_c \Gamma_c^{\uparrow})(2\pi\rho_d \Gamma_d^{\uparrow}) \right]^{-1}$$
$$\times \begin{pmatrix} 2\pi\rho_c \Gamma_c^{\uparrow} + T^{\downarrow} & T^{\downarrow} \\ T^{\downarrow} & 2\pi\rho_d \Gamma_d^{\uparrow} + T^{\downarrow} \end{pmatrix} \quad (38)$$

Since we are only interested in the inclusive cross sections, we sum the partial cross of (35) over all particle and breakup channels. Using (10) and (11), we thus obtain the cross section for particle emission from the doorway,

$$\sigma_{p,d} = \frac{\pi}{k^2}(2J+1)\, 2\pi \rho_d \Gamma_{d,p}^{\uparrow} \Pi_{dd} T_o^d \quad (39a)$$

the cross section for particle emission from the compound nucleus

$$\sigma_{p,c} = \frac{\pi}{k^2}(2J+1)\, 2\pi \rho_c \Gamma_c^{\uparrow} \Pi_{cd} T_o^d \quad (39b)$$

and the cross section for breakup of the doorway,

$$\sigma_{b,d} = \frac{\pi}{k^2}(2J+1)\, 2\pi \rho_d \Gamma_{d,b}^{\uparrow} \Pi_{dd} T_o^d \quad (39c)$$

We interpret the emission of particles from the doorway as pre-equilibrium particle emission. Its cross section is usually a very small fraction of the total. However, at sufficiently high energies, we find it can make an important contribution to the cross section for partial waves in a small region between those dominated by emission from the compound nucleus and those dominated by breakup.

We interpret the breakup cross section as a DIC cross section. We cannot interpret it as a fast fission or equilibrated DIC cross section. At high energies and high angular momenta, the doorway states allowed by the entrance channel transmission coefficient, T_o^d , do not live long enough to attain the equilibrium we have assumed.

This breakdown in the model should not effect the particle emission cross sections, however. These relatively slower processes dominate at lower energies and lower angular momenta. The fact that they do dominate implies a lifetime of the composite system sufficient to attain the type of equilibrium we have assumed.

Finally, we interpret the summed cross section for particle emission from the compound nucleus as the fusion cross section. Writing it out in full for a given partial wave J , we have

$$\sigma_F^{J} = \frac{\pi}{k^2}(2J+1)\frac{2\pi\rho_c\Gamma_c^{\uparrow}T^{\downarrow}T_o^{d}}{2\pi(\rho_d\Gamma_d^{\uparrow}+\rho_c\Gamma_c^{\uparrow})T^{\downarrow}+(2\pi\rho_c\Gamma_c^{\uparrow})(2\pi\rho_d\Gamma_d^{\uparrow})} \tag{40}$$

We find that this expression reproduces well the observed behavior of the fusion cross section. At low energies, it is close to the reaction cross section. At higher energies, it is limited to the lower partial waves due to the competition with the doorway decay modes. The details of our results are discussed in the next section.

III. RESULTS AND DISCUSSION

In this section, we present the results of the calculations which we perform within the formalism presented in the previous section.

We describe the intermediate states as being associated with a two touching spheres configuration. Accordingly when the heavy ion system reaches this stage, we expect the energy and the angular momentum to be distributed among collective and intrinsic degrees of freedom. The collective degrees of freedom describe the rotational motion of the composite system while the intrinsic ones describe its internal excitation represented by a certain number of excitons. For the collective rotational motion, we assume the usual form for the associated energy

$$E_J = \frac{\hbar^2}{2\mathcal{J}}J(J+1) \tag{41}$$

where the moment of inertia of the composite system is given by

$$\mathcal{J}_o^{d} = \frac{2}{5}MR^2 + \mathcal{J}_1 + \mathcal{J}_2 \tag{42}$$

To simplify the calculations we consider only the collective degrees of freedom in constructing the level density of states for the composite system, Eq. (19). To take into account the intrinsic degrees of freedom in the composite (doorway) system we merely adjust the level density parameter $\frac{A}{8}$, to be $\frac{A}{8x}$, with x being a parameter. This is motivated by the fact that in ref. 2 it was found that the internal energy of the compound nucleus $\Delta Q = 0.27\ A_{CN}$ could be related to the average number of excitons $n \simeq 0.2\ A_{CN}$. This number is used to relate the level density parameter a_d to the corresponding one for the equilibrated compound nucleus a_c through

$$a_d = \frac{\bar{n}}{A_{CN}} a_c \simeq 0.2\ a_c \tag{42}$$

As for the density of state of the equilibrated compound nucleus we use the standard Fermi gas Formula with the parameter a_c given by the estimate $A_c = \frac{A}{7.1}$ and the corresponding moment of inertia $\mathcal{J}$ given by

$$\mathcal{J} = \frac{2}{5} M R^2 \tag{43}$$

The transmission coefficients which appear in the cross section formulae are evaluated by using a simple ion-ion potential as was discussed in Section II. We have chosen for such a potential the conveniently parametrized global interaction specified in Eq. (22). This potential was used to construct the effective barriers which are subsequently employed in the evaluation of the penetrabilities (T_ℓ), using the Hill-Wheeler expression:

$$T_J = \left[1 + \exp \frac{2\pi}{\hbar\omega}\left[E - V(R) - \frac{\hbar^2 J(J+1)}{2\mathcal{J}(R)}\right]\right]^{-1} \tag{44}$$

The results of our calculation of σ_f using Eq. (40) which contains the effect of the doorways (see previous section) are shown in Figs. 1-9. The value of the coupling strength was taken to be $\overline{V_0^2} = 21.5$ (MeV). The data points were taken from Refs. 1 and 5. Also shown in these figures (dashed lines) are the summed cross section $\sigma_F + \sigma_{p,D}$, where $\sigma_{p,D}$ is the summed particle emission cross sections from the "doorway", Eq. (39a).

The fusion excitation function in "region I" where the complete fusion process exhausts the reaction cross sections is well reproduced by our model. Moreover, the energy corresponding to maximum fusion cross-section is also systematically predicted by the calculations. The evaluation of the fusion cross section in region II where the competition between the fusion and more rapid processes is considerable, is also satisfactorily reproduced. In particular the feature of the σ_F vs. E_{CM}^{-1}, that depends on the entrance channel, and which is reflected by positive, null or negative values of $V_{critical}$, can be predicted by this model e.g. fusion of $^{12}C + {}^{16}O$ $^{16}O + {}^{27}Al$ or other light systems indicate $V_{cr} < 0$, whereas in $^{16}O + {}^{40}Ca$ and $^{40}Ca + {}^{40}Ca$, $V_{cr} \gtrsim 0$[1].

It is interesting to observe that our calculation reproduce well the recent $^{40}Ca + {}^{40}Ca$ of Aljuwair et al[5], being off the old data of Doubre et al[6].

The contribution of particle emission from the dinuclear (doorway) configuration is shown summed to σ_F is already mentioned earlier. We see clearly that this effect is mostly important in the maximum fusion, σ^F_{max} region. We also notice that there seems to be a clear connection between the value of σ^F_{max} and $\sigma^{max}_{p,D}$; the larger σ^F_{max}, the smaller $\sigma^{max}_{p,D}$ (see e.g. Fig. 2).

The fact that the general trends of the excitation functions are predicted by the model, independently of the system, reveals that the most important features of the dynamics of the collision are taken into account in the calculations.

We have repeated the above calculations for 15 more cases, which correspond to some of the reactions quoted in Refs. 1a, 1b, 5. The quality of the results are as good as the ones shown in Figs. 2-10, and for the sake of completeness we present in Fig. 11 the predicted values of the maximum fusion cross section. It is found that our result come out quite reasonable, and follow closely the trend of the empirically determined σ^{max}_F of Ref. 2). Also shown in the same figure are the results of the statistical yrast line model of Ref. 7).

IV. ANGULAR DISTRIBUTIONS AND ANGULAR CROSS-CORRELATION FUNCTIONS

In the previous sections, we have presented the results of the calculations of the heavy ion fusion cross sections for several light and medium HI systems. The results show that the model does supply a consistent description of σ_F both in "region I" and II. In this section we present another feature of our model connected with the localization, in angular momentum space, of the partial, compound, and doorway cross sections, namely the angular distribution for a given transition. We consider the case $^{16}O + ^{12}C \rightarrow ^{28}Si \rightarrow \alpha + ^{24}Mg$.

Let us consider, first, the evolution, with increasing bombarding energy, of the partial cross sections σ_J. In Fig. 10, we show the energy dependence of σ_J for the transition $^{16}O + ^{12}C \rightarrow ^{28}Si \rightarrow \alpha + ^{24}Mg$ (g.s.), involving the compound σ^{CN}_J and doorway σ^D_J configurations, respectively. The σ_J distribution have been coined "statistical windows" in Ref. 8, and the same nomenclature is used here. We have found that the compound statistical window maintains its width roughly constant. Its center of gravity, on the other hand, increases with increasing energy as expected. The width of the doorway statistical window is found to decrease with energy with a corresponding shift to higher values of its centroids. At this point, it is worthwhile to discuss the characteristics of the compound statistical window, in the absence of the coupling to the doorway considered here, where the results of a standard Hauser-Feshbach calculation indicate that the widths of the σ_J's <u>increase</u> with increasing CM energy with a corresponding increase in their centers of gravity.

It has been pointed out in Ref. 7 that as a consequence of the well-localized nature of the CN statistical window, the angular distribution of transitions to low-spin states in the residual nucleus, reflect clearly the two most important characteristics of this localization: the center of gravity of the window, L_o, and its width, ΔL. The period of the regular oscillations in $\frac{d\sigma}{d\Omega}$ is approximately given by $\frac{\pi}{L_o}$, whereas their gradual damping as the angle increases is related to ΔL. The $\frac{1}{\sin\theta}$ dependence is also present. Simple closed expressions were obtained in Ref. 8. In Ref. 9 it was also found that, in general, the inclusion of a doorway configuration results in a smaller period of the angle oscillations, as a result of the larger value of the center of gravity of doorway statistical window. In the same reference, changes in the angular cross-correlation function were also discussed.

We now use our calculated windows to discuss the angular distribution $\frac{d\sigma}{d\Omega}$ and the angular cross correlation further $C(\theta,\theta')$ (see below). Instead of using the closed expression of Ref. 8, we calculate $\frac{d\sigma}{d\Omega}$ and $C(\theta,\theta')$ exactly.

The differential cross section $\frac{d\sigma}{d\Omega}$ and the angular cross-correlation function $C(\theta,\theta')$ for the transitions to zero spin find state can be easily obtained from the fluctuation

amplitude (for zero spin transitions)

$$f^{fl}(\theta) = \frac{1}{2ik} \sum_J (2J+1)\, S^{fl}(J)\, P_J(\cos\theta) \tag{45}$$

where $S^{fl}(J)$ is rapidly fluctuating function of energy. Then

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left\langle \left| f^{fl}(\theta) \right|^2 \right\rangle \tag{46}$$

$$C(\theta,\theta') \equiv \frac{\left\langle \frac{d\sigma}{d\Omega}(\theta)\, \frac{d\sigma}{d\Omega}(\theta') \right\rangle}{\left\langle \frac{d\sigma}{d\Omega}(\theta) \right\rangle \left\langle \frac{d\sigma}{d\Omega}(\theta') \right\rangle} - 1 \tag{47}$$

where $< >$ indicates average over energy. From the statistical condition

$$\langle f^{fl.}(\theta) \rangle = 0 \tag{48}$$

and assuming a normal (Gaussian) distribution, we can rewrite $C(\theta,\theta')$ in the following form

$$C(\theta,\theta') = \frac{\left| \langle f^{fl.}(\theta)\, f^{fl.}(\theta') \rangle \right|^2}{\left\langle \frac{d\sigma}{d\Omega}(\theta) \right\rangle \left\langle \frac{d\sigma}{d\Omega}(\theta') \right\rangle} - 1 \tag{49}$$

Since eq. (48) implies $\langle S^{fl}(\ell) \rangle = 0$ and using again a normal distribution we obtain

$$\langle S^{f\ell}(J)\, S^{f\ell}(J') \rangle = \delta_{JJ'} \langle |S^{f\ell}(J)|^2 \rangle \tag{50}$$

The quantity $\langle |S^{f\ell}(J)|^2 \rangle$ is identified with the statistical window considered earlier. The expressions for $\langle \frac{d\sigma}{d\Omega} \rangle$ and $C(\theta,\theta')$ read

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \frac{1}{4k^2} \sum_J (2J+1)^2 \langle |S^{f\ell}(J)|^2 \rangle \left(P_J(\cos\theta)\right)^2 \tag{51}$$

$$C(\theta,\theta') = \left[\left\langle \frac{d\sigma}{d\Omega}(\theta) \right\rangle \cdot \left\langle \frac{d\sigma}{d\Omega}(\theta') \right\rangle \right]^{-1} \cdot \left| \frac{1}{4k^2} \sum_J (2J+1)^2 \langle |S^{f\ell}(J)|^2 \rangle P_J(\cos\theta)\, P_J(\cos\theta') \right|^2 \tag{52}$$

For the two-class case under discussion here we merely substitute $|S^{f\ell}(J)|^2$ by $|S^{f\ell}_{CN}(J)|^2 + |S^{f\ell}_{D}(J)|^2$. From here on we shall denote $|S^{f\ell}_{CN}(J)|^2 \equiv \sigma^{C.N.}$ and $|S^{f\ell}_{D}(J)|^2 \equiv \sigma^{D}$.

At this point it is instructive to follow the evolution of $C(\theta,\theta')$ as a function of the bombarding energy for both $\sigma^{C.N.}$ and σ^{D}, discussed in the previous section.

As figure 12 shows, the compound nuclear $C(\theta,\theta')$ changes little with energy implying that the corresponding statistical window maintains its widths as has been discussed earlier. This is, in fact, a clear demonstration of the influence of the doorway configurations. A pure compound

nuclear statistical window becomes significantly wider as the energy increases.

The above discussion can be summarized by saying that the coherence angle defined through

$$C((\theta-\theta')_{coh.}) = \frac{1}{2} \tag{53}$$

maintains its value, e.g. for $^{16}O + {}^{12}C$, $\theta_{coh} \sim 16^{\circ}$ in our two-class model, whereas in a pure compound nuclear calculation, this is a clear change of θ_{coh}. In fact empirical observations[10] have shown that θ_{coh} depends on the width of the window as

$$\theta_{coh.} \simeq \frac{K}{\Delta L} \tag{54}$$

where K is $\cong 1.4$.

In contrast to the compound piece of our cross section, the doorway contribution exhibits a cross correlation function $C(\theta,\theta')$ which changes drastically with energy. Whereas at low energies one finds the doorway contribution behaving like the pure compound nucleus one, at high energies more rapid oscillations appear in $C(\theta,\theta')$. This fact indicates that a higher degree of coherence is being attained. Fig. 13 shows the behavior of $C_{DOOR}(\theta,\theta')$ for $^{16}O + {}^{12}C \to \alpha + Mg$. It is thus possible to suggest that at $E = 40$ MeV the doorway in $^{16}O + {}^{12}C$ reaches a stage where one might consider it as a "bridge" to more direct process; e.g. deep inelastic. The coherence angle at this energy is actually ill-defined.

V. TEMPORAL EVOLUTION AND CORRELATION WIDTHS

So far, we have discussed only the angular characteristics of the heavy ion system through the analysis of the two coupled statistical windows (see Section IV). Another important feature of a multi-step compound process is the overall temporal evolution of the system. Once formed, the time evolution of the compound system, represented by $A(t)$ (see below), can be easily found by Fourier transforming the S-matrix correlation function[11]. In the eigenclass representation[10,12] which diagonalizes π (Eq. (3)) we have, following McVoy and Tang[13],

$$A(t) = a_+ e^{-\lambda_+ t} + a_- e^{-\lambda_- t} \tag{55}$$

where $\lambda_\pm$ are the eigenvalues of the matrix $\rho^{-1} \Pi \rho^{-1}$, namely

$$\lambda_\pm = \frac{\Gamma_c + \Gamma_d}{2} + \left(\sqrt{\frac{\rho_d}{\rho_c}} + \sqrt{\frac{\rho_c}{\rho_d}}\right)\overline{V_0^2} \pm \sqrt{\left(\frac{\Gamma_c - \Gamma_d}{2}\right)^2 + \left(\overline{V_0^2}\right)^2} \tag{56}$$

The λ_+ and λ_- have very simple physical meaning; they correspond, respectively, to the correlation length (inverse life time) of the doorway (dinuclear) and compound nucleus configurations[12]. The coefficients a_+ and a_-, are "eigenclass" cross sections, both having the one-class Hauser-Feshbach form[13] on the calculation of these cross

sections, but rather concentrate on the more interesting correlation widths, λ_+ and λ_-.

We recall at this point, that these widths have recently been extracted, through a generalized Ericson analysis of the type proposed in 12), for the system $^{15}N + {}^{12}C \to \alpha + {}^{23}Na$ [14] and using the spectral density method, for the system $^{16}O + {}^{12}C \to \alpha + {}^{24}Mg$ [3].

From our results of Section IV, we have extracted the correlation widths of the doorway configuration, λ_+ and for the equilibrated system, λ_-. We have chosen the process $^{16}O + {}^{12}C \to \alpha_o + {}^{24}Mg$, for definiteness. We have found that if we maintain the value of $\overline{V_o^2}$, in the coupling, equal to 21.5 (MeV) we obtain a reasonable value for λ_- ($\sim$ 70 keV), however, λ_+ comes out extremely large. This shows that the lifetime of the dinuclear system is very short. According to the findings of Ref. 3) λ_+ for $^{16}O + {}^{12}O \to \alpha + {}^{24}Mg$ is about 250 keV and varies slowly with increasing excitation energy. To get the expected values of λ_+ and λ_- (70 and 250 keV, respectively) we had to reduce $\overline{V_o^2}$ by a factor 10^4 ! The resulting fusion cross sections, however come out in sharp disagreement with the data.

The above findings clearly indicate that our model, though fully adequate for the description of heavy ion fusion as well as the angular distribution of emitted particles, cannot simultaneously describe the time evolution of the system. Presumably the details of the equilibration process, which are not fully accounted for in our model, is the necessary missing ingredient!

VI. DISCUSSION AND CONCLUSION

In this paper, a multi-step compound model of heavy-ion fusion reaction is developed. Two distinct configurations of the compound system were considered; a dinuclear system which is allowed to decay through break-up into two fragments as well as by particle emission, and an assumed equilibrated system having the chance to deexcite through particle emission. The summed particle emission cross section from the equilibrated stage is used to define the fusion cross section.

The result of our calculation of σ_F, using the Agassi et al formulation of multi-step compound reaction, generalized to heavy ions, was found to be in good agreement with the data of numerous heavy ion fusion systems.

The explicit consideration of the competition between fusion and doorway break-up and particle emission channels in our model is an important feature of the model, which helps account naturally and consistently for the downward drop of σ_F, seen in light-heavy systems, at higher energies, avoiding thus the introduction of a "region III"[15], in complete agreement with Ohta et al[16].

In fact symmetrical break-up (or fast fission) seems to be a common occurance in HI reactions at energies higher than those corresponding to σ_F^{max}[17]. In our calculation, the existence of these decay channels consistently reduce the values of σ_F^{max}, making them closer to the data (Fig. 11).

The angular distributions of emitted particles from the two configurations as well as their respective correlation widths were also considered and analysed. It was found that whereas the calculated angular characterisitcs are reasonable, the correlation width of the dinuclear system comes out to be much too large. This may indicate that other intermediate configurations of the composite system besides the one we considered here, should be taken into account in order to get a more consistent picture of the time evolution of the system.

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FIGURE CAPTIONS

Figure 1. A schematic representation of the two-step compound fusion process.

Figure 2. σ_F for the system $^{12}C + {}^{16}O$. Full curve corresponds to our calculated σ_F of Eq. (40). Dashed curve represents $\sigma_F + \sigma_{p,D}$ (Eqs. (40) and (39a)). The dashed dotted curve is the total reaction cross section, calculated from the entrance channel transmission coefficient. The data points were collected from Ref. 1.

Figure 3. Same as Fig. 2 for the system $^{12}C + {}^{18}O$. Data were taken from Ref. 1.

Figure 4. Same as Fig. 2 for the system $^{12}C + {}^{19}F$. Data were taken from Ref. 1.

Figure 5. Same as Fig. 2 for the system $^{12}C + {}^{27}Al$. Data were taken from Ref. 1b.

Figure 6. Same as Fig. 2 for the system $^{16}O + {}^{24}Mg$. Data were taken from Ref. 1.

Figure 7. Same as Fig. 2 for the system $^{16}O + {}^{27}Al$. Data were taken from Ref. 1.

Figure 8. Same as Fig. 2 for the system $^{16}O + {}^{40}Ca$. Data were taken from Ref. 1.

Figure 9. Same as Fig. 2 for the system $^{24}Mg + {}^{32}S$. Data were taken from Ref. 1.

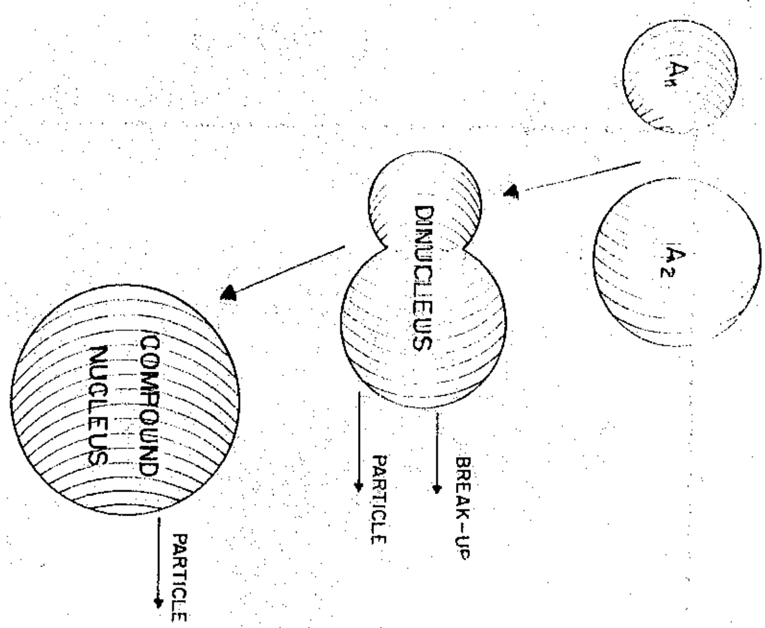
Figure 10. Same as Fig. 2 for the system $^{40}Ca + {}^{40}Ca$. Data were taken from Refs. 5 and 6.

Figure 11. Maximum fusion cross section σ_F^{max} measured for various systems (closed circles). Data were taken from original papers cited in Refs. 1 and 5.The open circles are our calculated σ_F^{max}. The full curve is the empirically found σ_F^{max} from the modified statistical yrast line model[2]. The dashed curve is the statistical yrast line model prediction of Ref. 7.

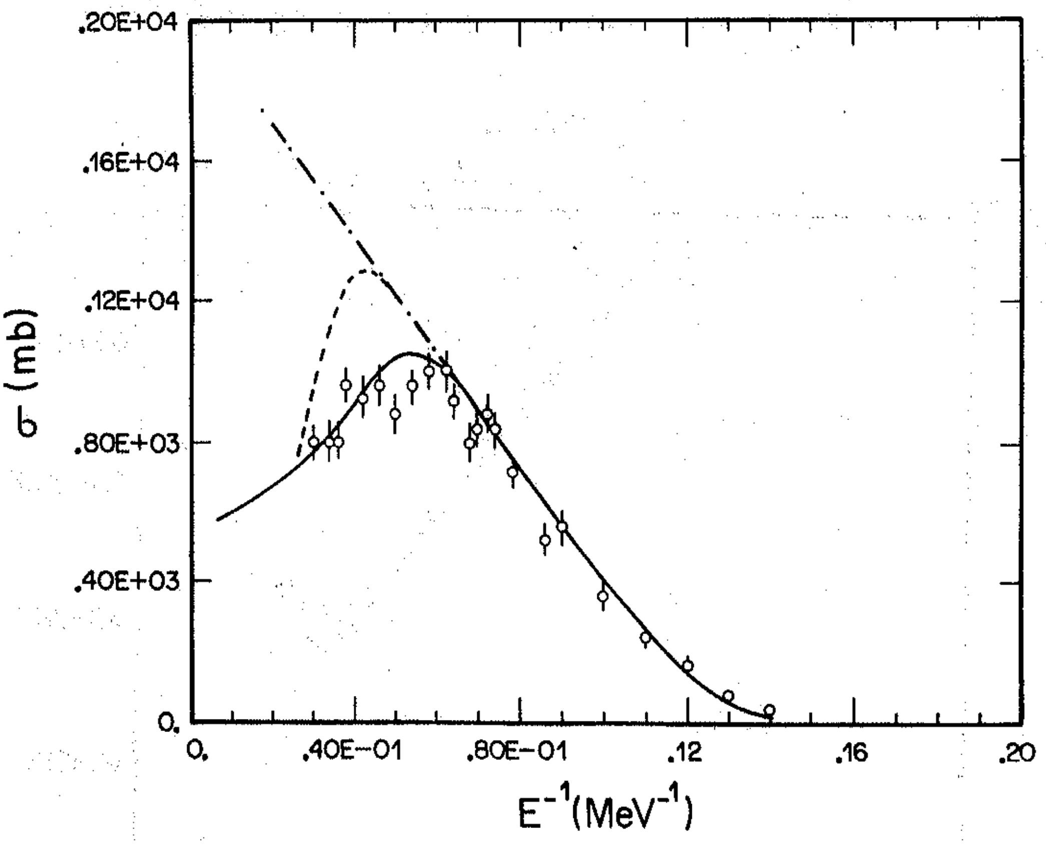
Figure 12. The angular cross-correlation function for the compound α_0-decay channel at a) E = 10 MeV, b) E = 20 MeV, c) E = 33.3 MeV and d) E = 40 MeV, see text for details.

Figure 13. Same as Fig. 11 for the dinucleus ("doorway") α_0-decay channels.

Fig 1



A1
A2
DINUCLEUS
BREAK-UP
PARTICLE
COMPOUND NUCLEUS
PARTICLE



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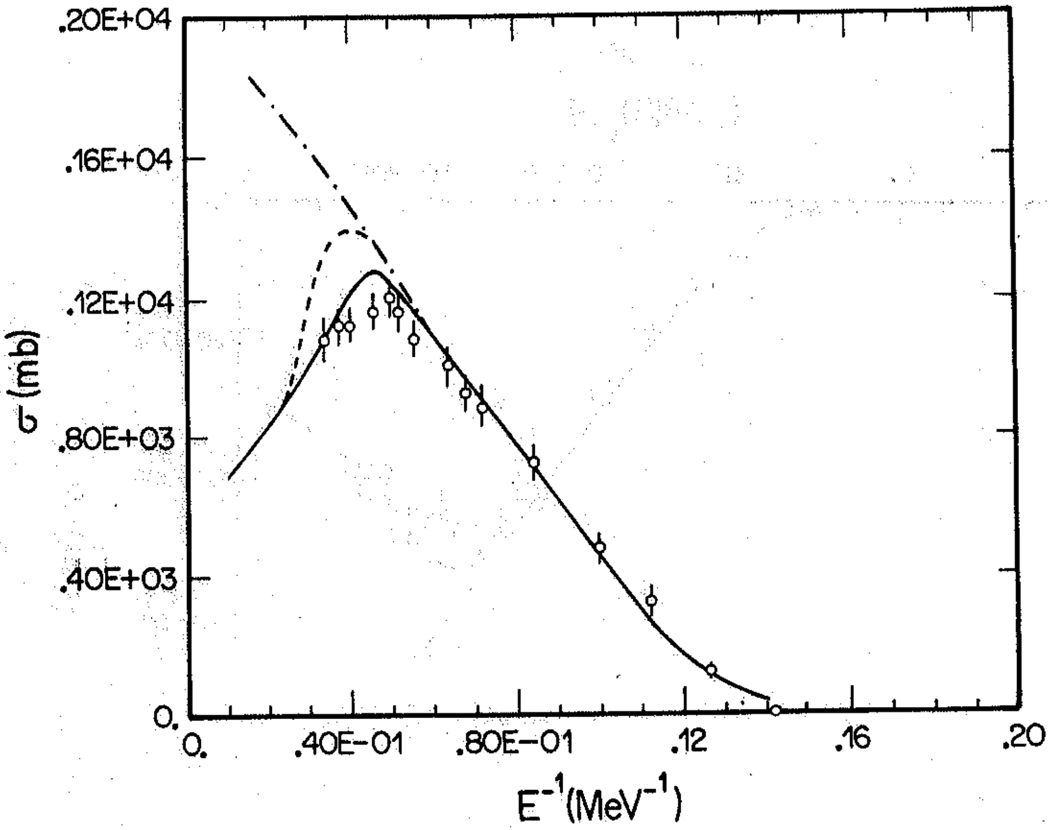
E^{-1}(MeV^{-1})

Fig. 2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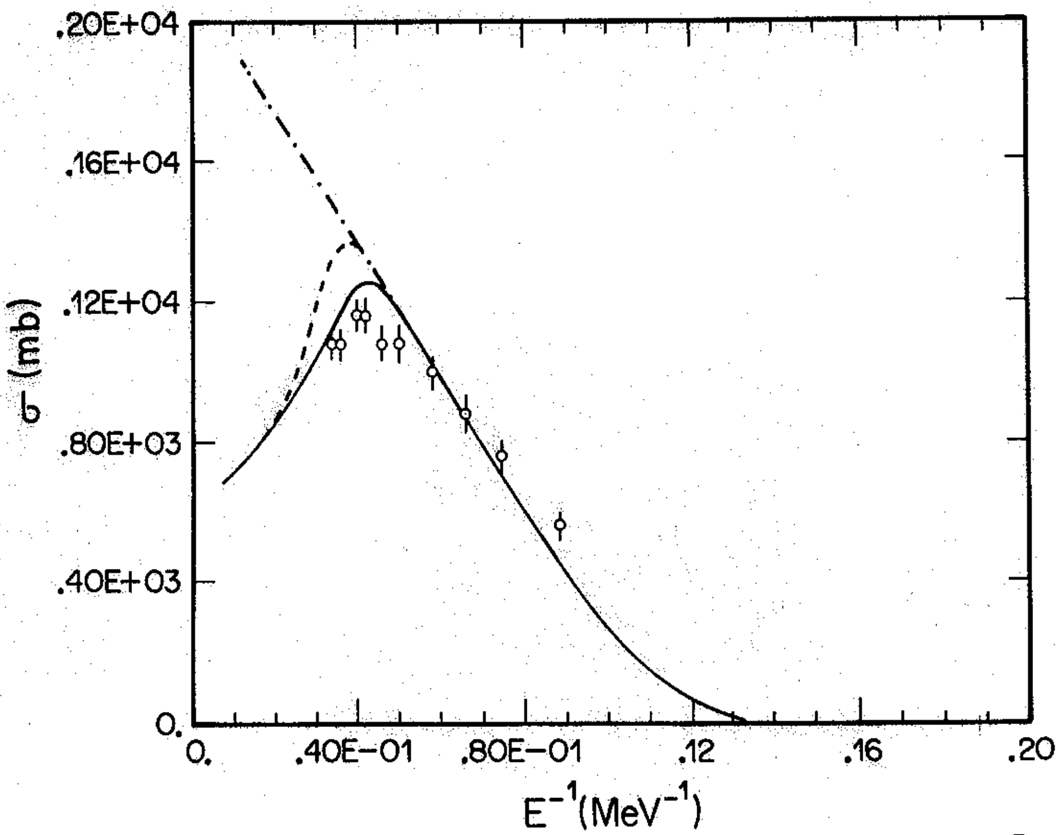
E^-1(MeV^-1)

Fig. 3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E^-1(MeV^-1)

Fig. 4

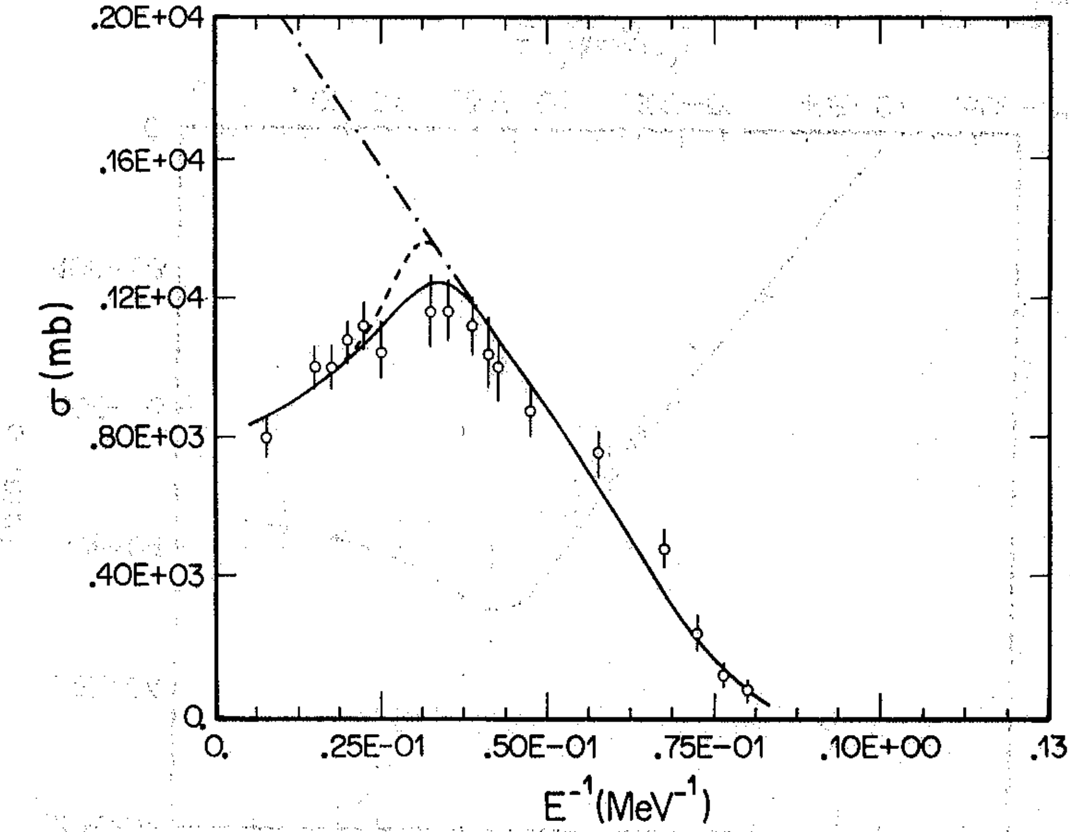
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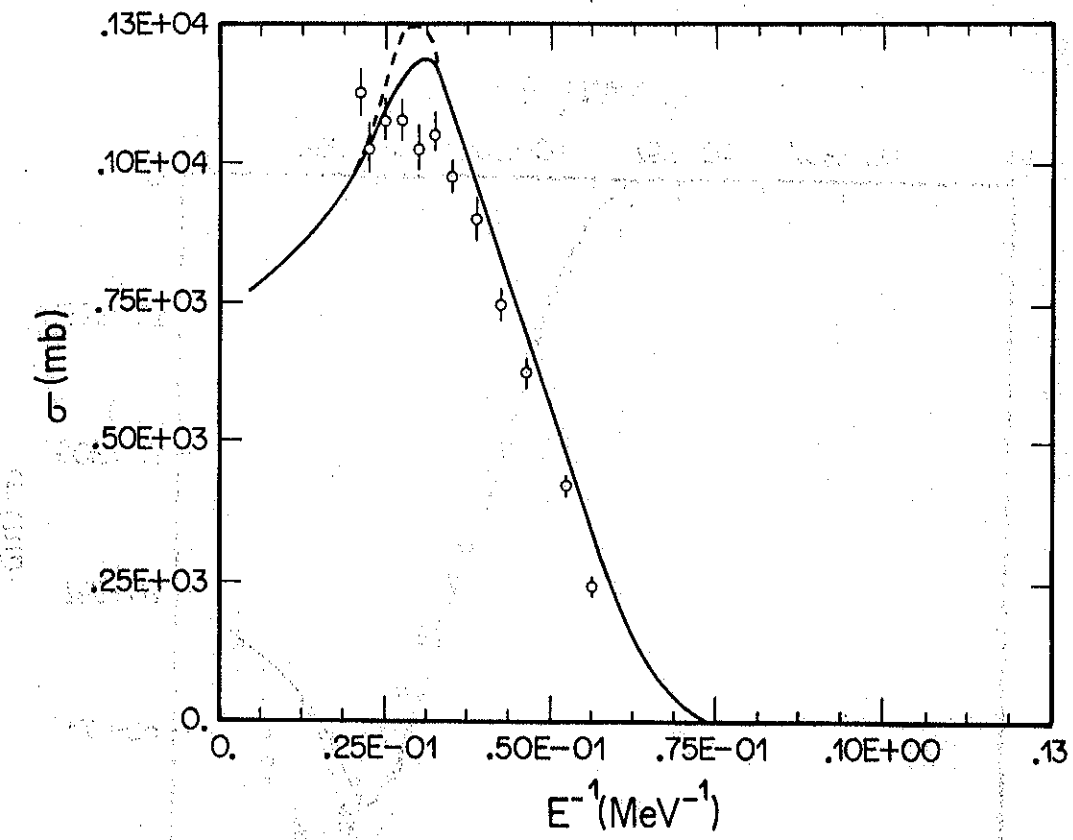
Fig. 5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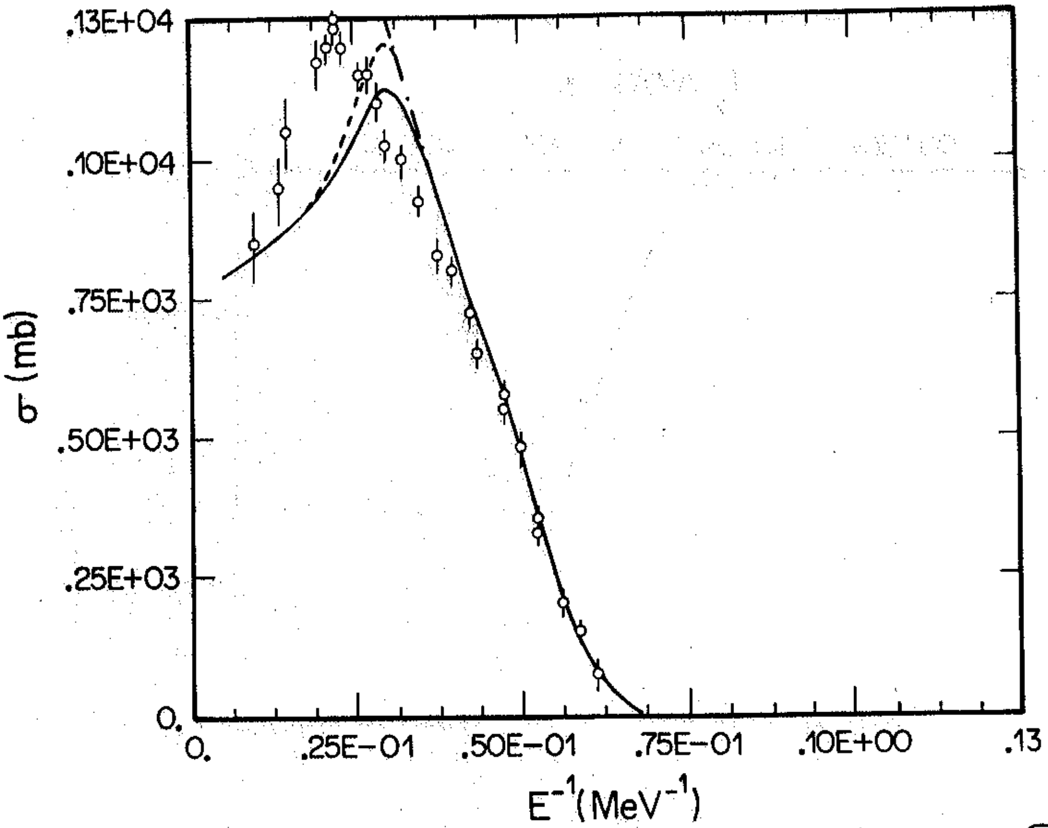
E⁻¹(MeV⁻¹)

Fig. 6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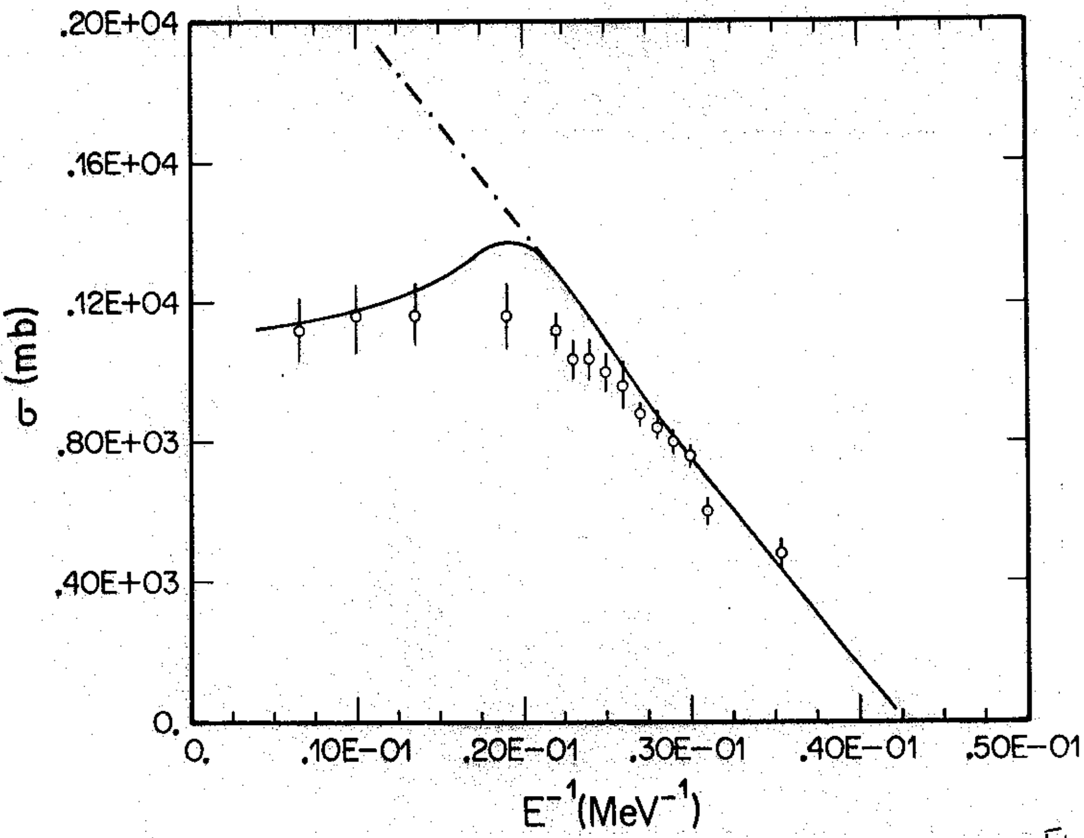
$E^{-1}(MeV^{-1})$

Fig. 7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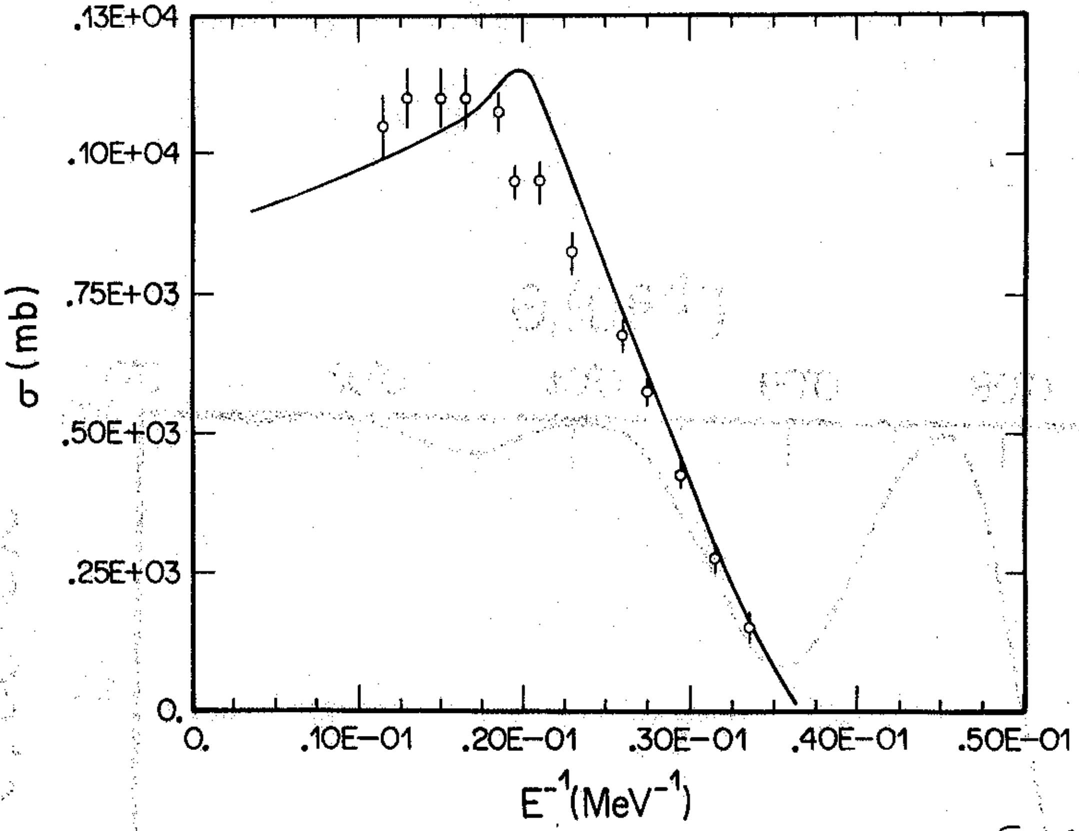
$E^{-1}(MeV^{-1})$

Fig. 8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E^{-1}(MeV^{-1})

Fig. 9

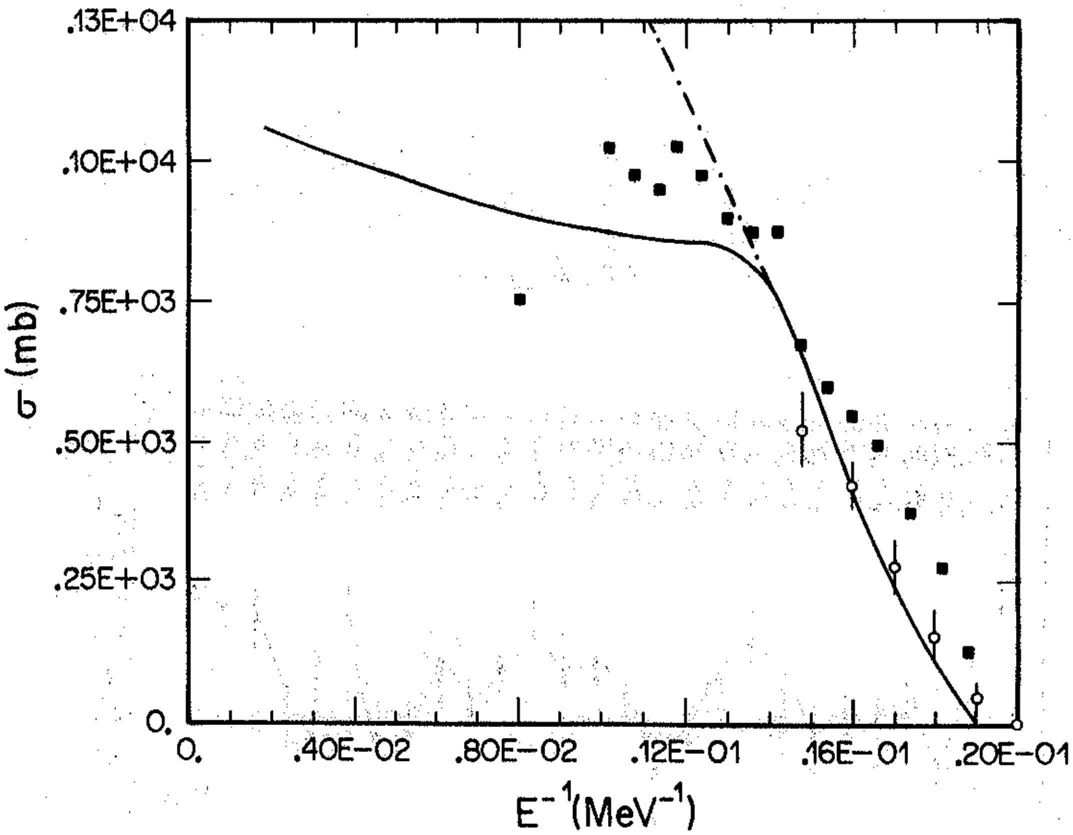
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Fig. 10

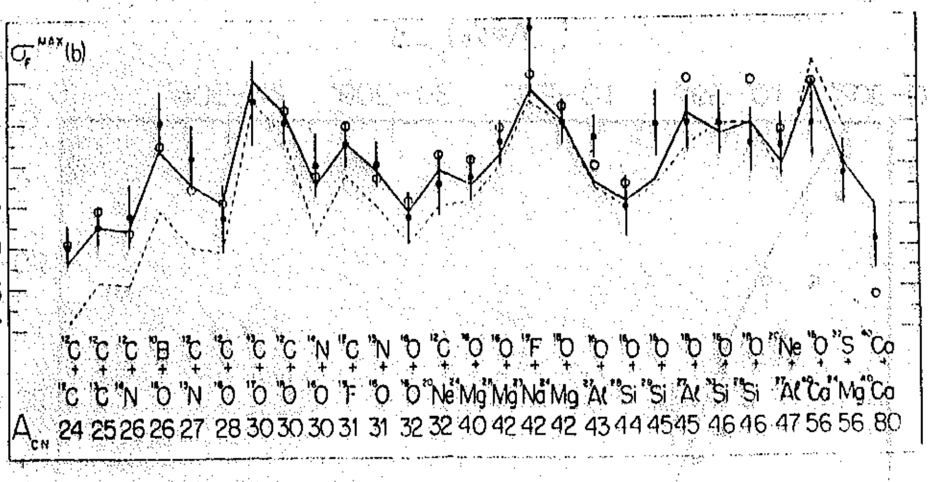


Fig. 11

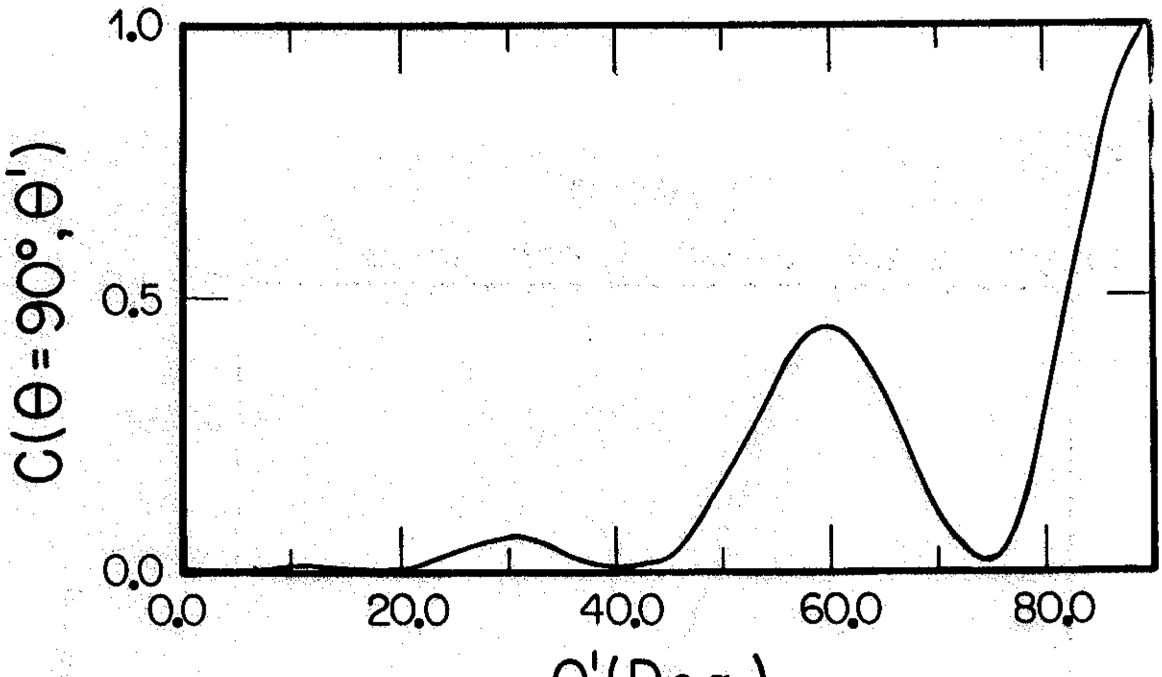


Fig. 12a

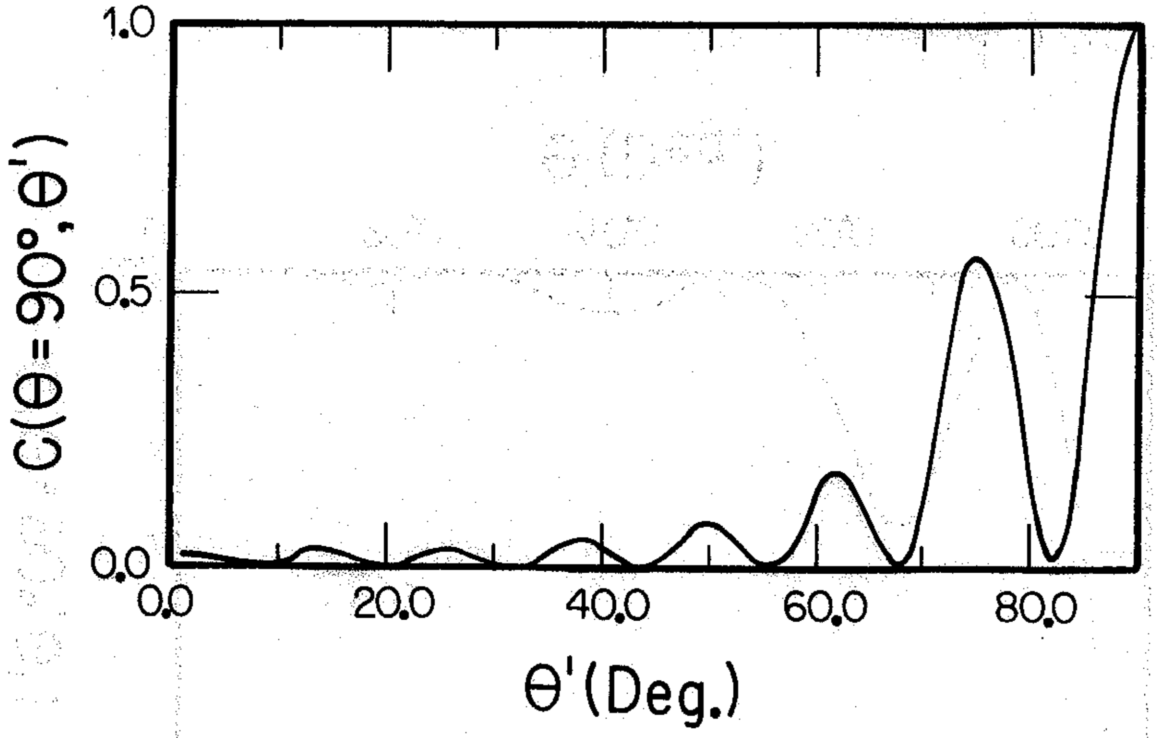


Fig. 12 b

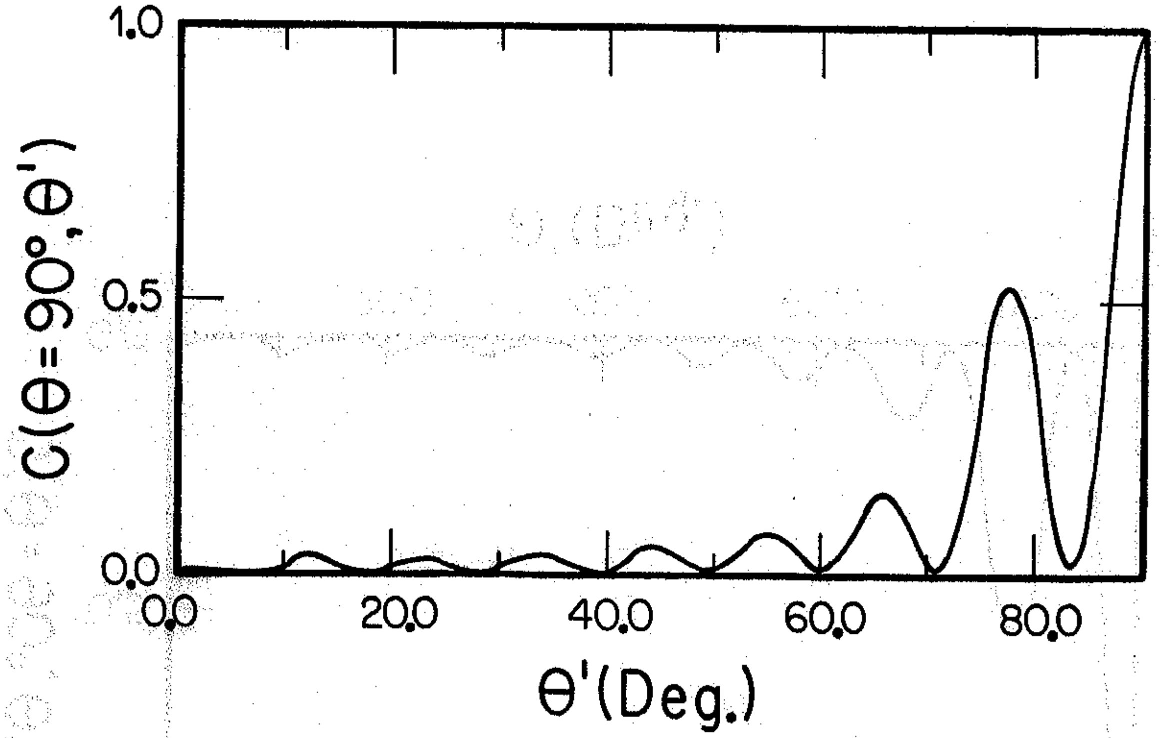


Fig. 12 c

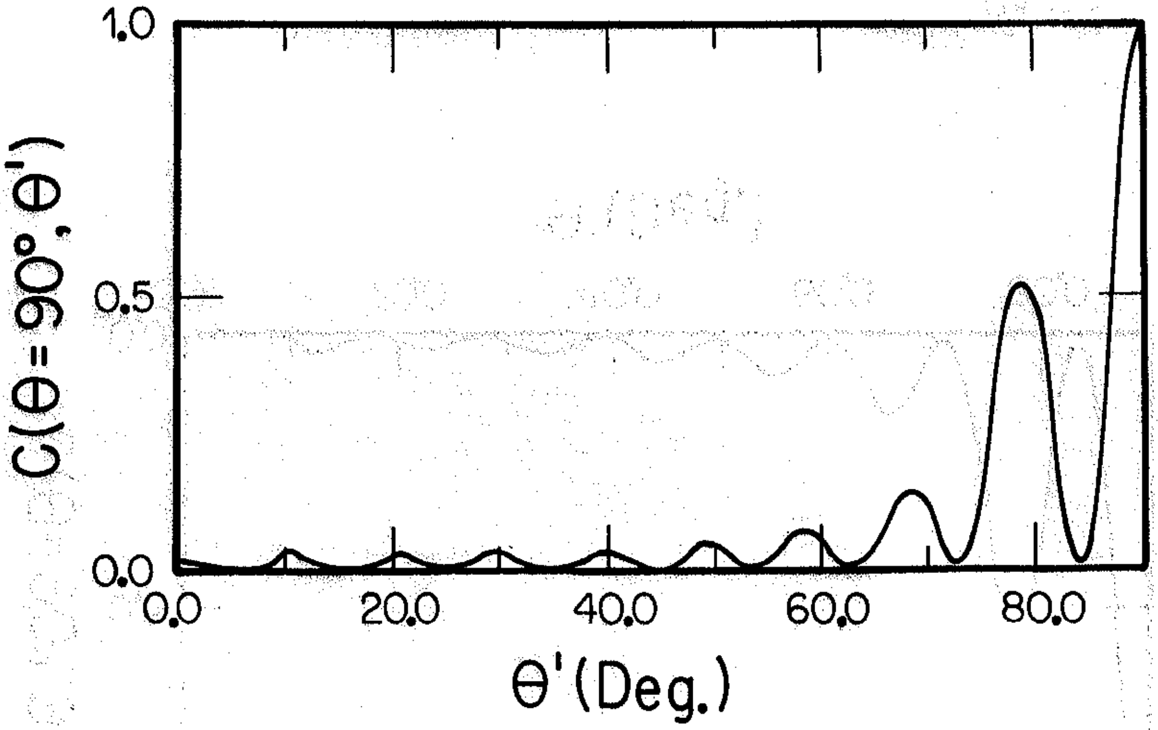


Fig. 12d

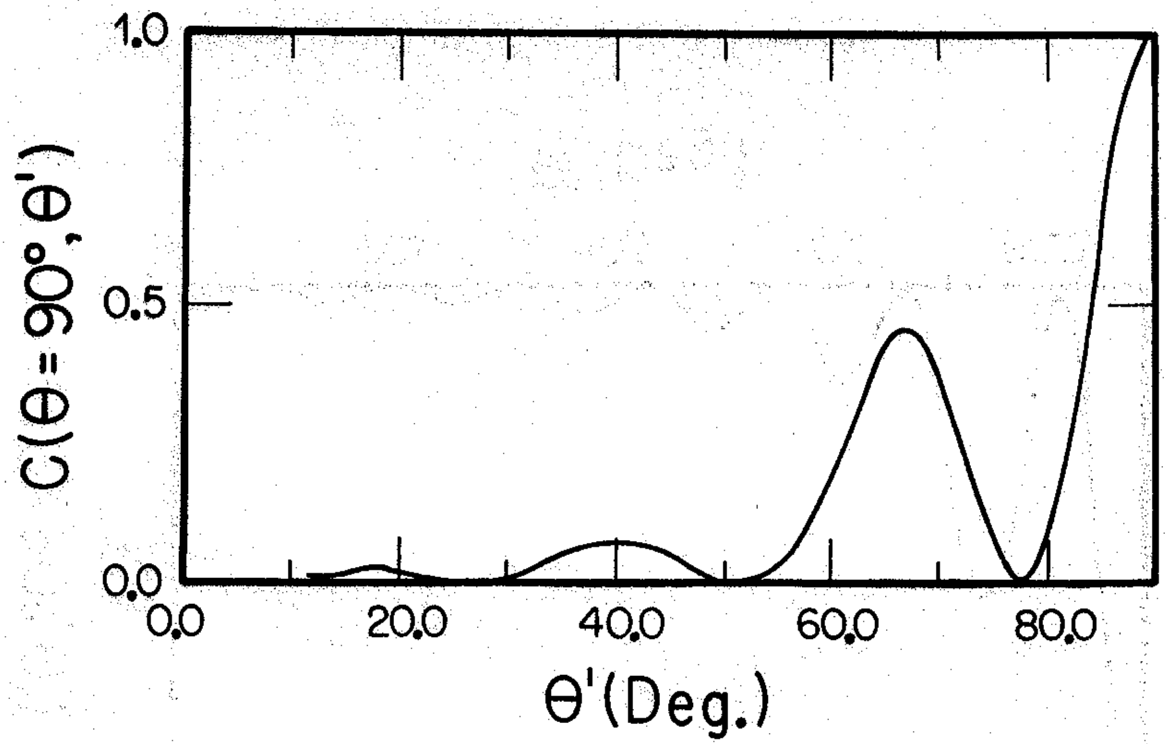
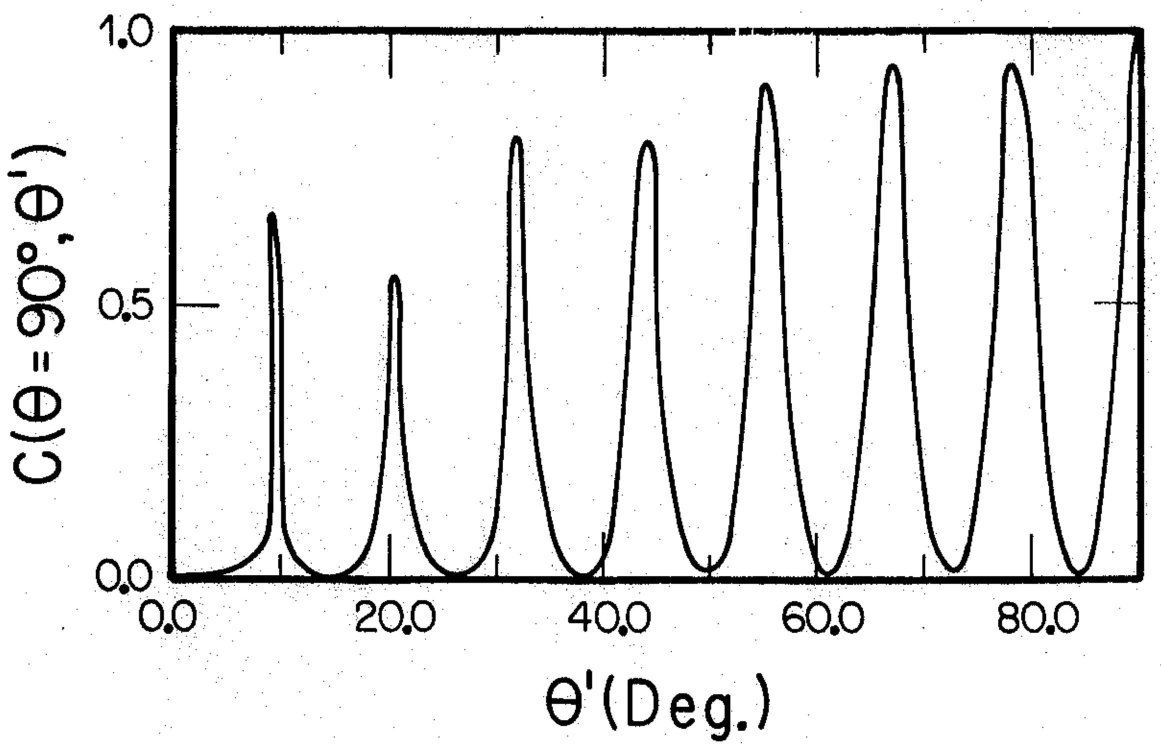


Fig. 13a



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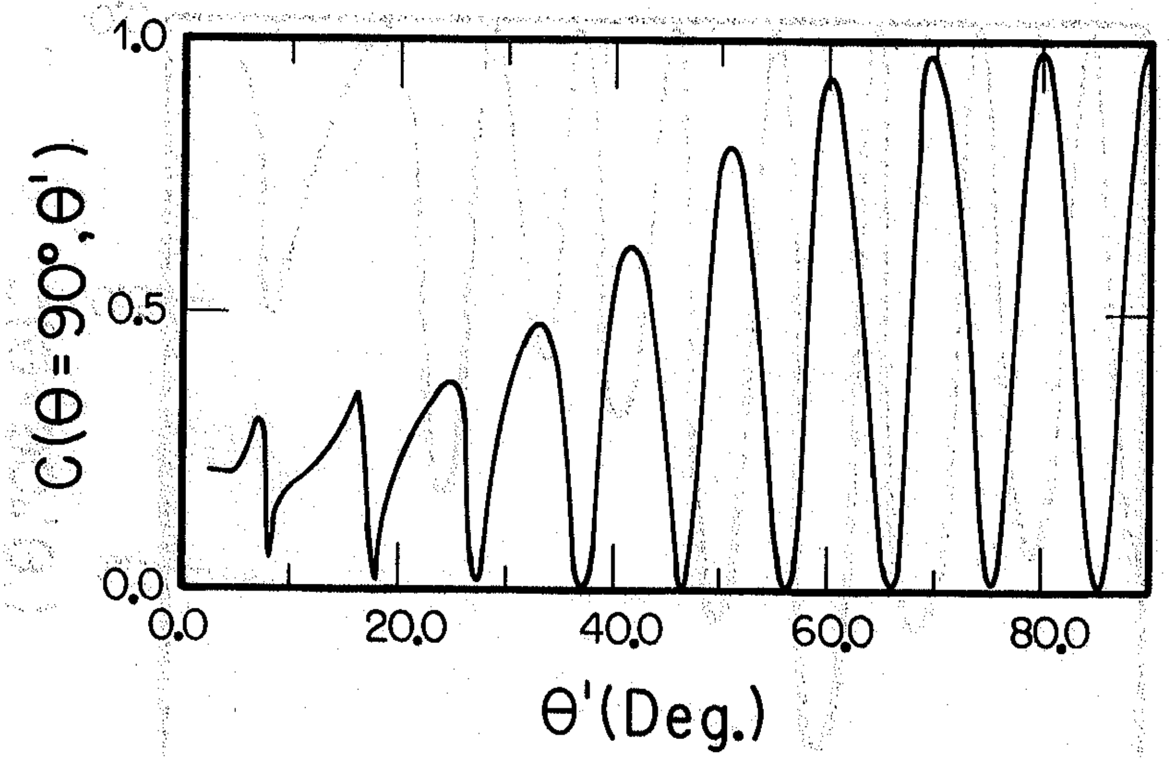

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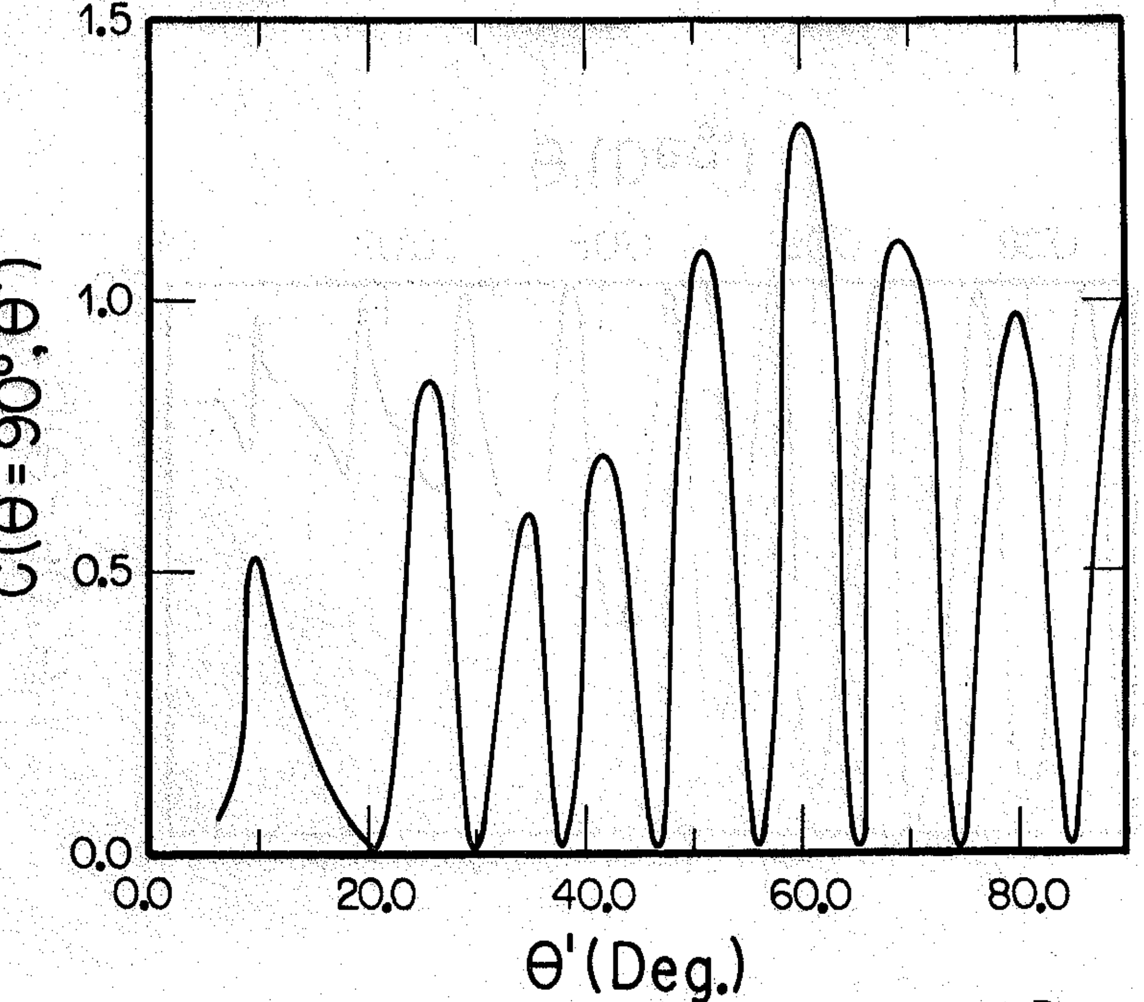

Fig. 13 d