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INSTITUTO DE FÍSICA
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by

A.C.C. Villari , F.I.A. Almeida and
A. Lépine-Szily

Instituto de Física, Universidade de São Paulo

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A.C.C. Villari, F.I.A. Almeida and A. Lépine-Szily

Instituto de Física, Universidade de São Paulo
C. Postal 20.516, São Paulo, Brasil

A model independent method, derived from the optical theorem, valid even in the presence of nuclear forward glory, is suggested for the obtention of total reaction cross section σ_R from elastic scattering data. A new, graphical way of interpreting the optical theorem is also presented.

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The precise determination of the total reaction cross section σ_R is of critical importance since it contains all information about the colliding system, including all possible channels with the exception of elastic scattering. The direct, angle-integrated measurement of all channels is very time consuming and for this reason alternative methods have been proposed, where σ_R is obtained from the elastic scattering. Some of these methods are model-dependent as the "quarter-point recipe"[1] which uses the semi-classical model, or the optical-model calculations of σ_R.

A model-independent method[2,3,4] derived from the optical theorem, relates σ_R to the "sum-of-differences" integral where the difference $\left[\sigma_{Ruth}(\theta) - \sigma_{e\ell}(\theta)\right]$ is integrated from θ_o to π.

Recently it has been shown[5,6] that this method may not be valid in the case of light heavy ion systems due to contribution from the forward nuclear glory in the amplitude $f_N(0)$. But the inclusion of $f_N(0)$ in the calculations turns again the method model-dependent, since in the absence of the precise experimental knowledge of $f_N(0)$, which by itself would furnish precious information about the nuclear interaction at short distances, it can only be calculated in the framework of a model.

One is still tempted, however, to ask whether it is still possible to obtain σ_R in a model-independent way even in cases where $f_N(0)$ is important.

The aim of this letter is to answer this question and suggest that even for cases where the forward glory amplitude is important, the reaction cross-section σ_R can be obtained in a practical way from a modified version of the "sum-of-differences" method for certain values of θ_o , in a model-independent manner.

If we take into account the possible contribution of nuclear forward glory in the case of charged particle scattering, with screened Coulomb potential, the optical theorem can be rewritten in the following way[7]:

$$\Delta\sigma_T = \sigma_R - 2\pi\int_0^{\pi}\left[\sigma_{sc}(\theta) - \sigma(\theta)\right]\sin\theta\, d\theta \qquad (1)$$

where the quantity $\Delta\sigma_T$ is defined[7] as

$$\Delta\sigma_T = \frac{4\pi}{k}\,\mathrm{Im}\, f'(0) \equiv \frac{4\pi}{k}\,\mathrm{Im}\left[f(0) - f_{sc}(0)\right] \qquad (2)$$

where $f(0)$ and $f_{sc}(0)$ are total and screened Coulomb amplitudes and where $\sigma(\theta)$ and $\sigma_{sc}(\theta)$ are respectively the elastic and screened Coulomb differential cross-sections.

The integral in equation (1) will be decomposed in two parts:

$$2\pi\int_0^{\pi}\left[\sigma_{sc}(\theta') - \sigma(\theta')\right]\sin\theta'\, d\theta' = 2\pi\int_0^{\theta}\left[\sigma_{sc}(\theta') - \sigma(\theta')\right]\sin\theta'\, d\theta' + 2\pi\int_{\theta}^{\pi}\left[\sigma_{sc}(\theta') - \sigma(\theta')\right]\sin\theta'\, d\theta' \qquad (3)$$

The first term of the RHS will be called

$$I(\theta) = 2\pi\int_0^{\theta}\left[\sigma_{sc}(\theta') - \sigma(\theta')\right]\sin\theta'\, d\theta' \qquad (4)$$

For heavy ions, where semiclassical concepts have meaning, the screening radius R can be related to a screening

angle θ_{sc} by $2\,\mathrm{cotg}\,\frac{\theta_{sc}}{2} = \frac{k}{n}R$. For the system $^{16}O + {}^{12}C$ at $E_{CM} \approx 18.0$ MeV, $\theta_{sc} \sim 0.13^o$. If $\theta > \theta_{sc}$, the screened Coulomb cross section in the second term of the RHS of equation (3) can be substituted by the usual Rutherford cross section $\sigma_{Ruth}(\theta)$. Then equation (1) can be written as:

$$I(\theta) = \sigma_R - \Delta\sigma_T - 2\pi\int_{\theta}^{\pi}\left[\sigma_{Ruth}(\theta') - \sigma(\theta')\right]\sin\theta'\, d\theta' \qquad (5)$$

The screening effects of the integral $I(\theta)$ are contained only in $\Delta\sigma_T$.

According to Holdeman and Thaler[7], the residual scattering amplitude $f'(\theta)$ can be expanded as

$$f'(\theta) = e^{-2i\Lambda}\bar{f}'(\theta) = (2ik)^{-1} e^{-2i\Lambda}\sum_{\ell}(2\ell+1)e^{2i\sigma_\ell} \times \left[e^{2i\hat{\delta}_\ell} - 1\right] P_\ell(\cos\theta) \equiv e^{-2i\Lambda} f_N(\theta) \qquad (6)$$

Λ being the phase-shift due to screening $\Lambda = n\,\ell n\, 2kR$, σ_ℓ and $\hat{\delta}_\ell$ the Coulomb and nuclear phase shifts respectively, without screening.

Then $f'(\theta)$ is the usual nuclear scattering amplitude, multiplied by $e^{-2i\Lambda}$ due to the screening effects. Then equation (2) can be written as:

$$\Delta\sigma_T = \frac{4\pi}{k}\,\mathrm{Im}\left[e^{-2i\Lambda} f_N(0)\right] \qquad (7)$$

The phase Λ is very large for a screening radius R of the order of atomic radius (e.g. $\Lambda = 61.0$ for $^{16}O + {}^{12}C$ at 18 MeV) and makes no physical difference in any measurable quantity. Λ may take on any value and for convenience, as do Holdeman and Thaler[7], we will take it to be zero in the following calculations. Therefore equation (5) can be written as:

$$I(\theta) = \sigma_R - \frac{4\pi}{k} \operatorname{Im} f_N(0) - 2\pi \int_{\theta}^{\pi} [\sigma_{Ruth}(\theta') - \sigma(\theta')] \sin\theta' d\theta' \tag{7}$$

It is this equation which is used to calculate $I(\theta)$. Both σ_R and $f_N(\theta)$ were calculated by an optical model code[10] up to 1°. $f_N(\theta)$ has a slow variation with angle[7] and was extrapolated to zero degree. The third term in the RHS of eq. (7) was also calculated using optical model elastic cross-section instead of experimental data.

In figure 1 we present $I(\theta)$ calculated for the system $^{16}O + {}^{12}C$ at $E_{CM} = 18.0$ MeV, using the optical potential of ref. [11], which gives $\sigma_R = 289\ fm^2$ and $\Delta\sigma_T = 114\ fm^2$. We can observe on figure 1 that $I(\theta)$ oscillates around a constant value in the small angle region ($\theta_{sc} < \theta < \theta_{1/4}$). It is clear on figure 1 that this constant value is $-\Delta\sigma_T$, obtained by the substraction of σ_R from $I(\pi)$. This is expected from the equation (7) in the case of $\theta = \pi$

$$I(\pi) - \sigma_R = -\Delta\sigma_T \tag{8}$$

Figures 2 and 3 respectively present similar calculations for the systems $^{18}O + {}^{58}Ni$ at $E_{CM} = 48.4$ MeV (optical potential of ref. [12]) and $^{16}O + {}^{28}Si$ at $E_{CM} = 44.0$ MeV (optical potential E-18[13]). These figures 2 and 3 show the same behaviour, namely the small angle oscillations in $I(\theta)$ are around $I(\pi) - \sigma_R = -\Delta\sigma_T$, which may take positive or negative values. In the case of $^{18}O + {}^{58}Ni$, this value is rather small, $\Delta\sigma_T = -5\ fm^2$, and the oscillations also have a small amplitude, while in the case of $^{16}O + {}^{12}C$ the oscillations have a much larger amplitude.

So the behaviour of $I(\theta)$ is the following: at small angles ($\theta_{sc} < \theta < \theta_{1/4}$) it oscillates around $-\Delta\sigma_T$ due to oscillations in the integral $2\pi \int_{\theta}^{\pi} [\sigma_{Ruth}(\theta') - \sigma(\theta')] \sin\theta' d\theta'$, for increasing angles this integral decreases and $I(\theta)$ increases towards $I(\pi) = \sigma_R - \Delta\sigma_T$.

A practical way of obtaining σ_R from elastic scattering angular distributions becomes evident in the light of the above.

For angles where $I(\theta_i) = -\Delta\sigma_T$

$$\sigma_{R_i} = 2\pi \int_{\theta_i}^{\pi} [\sigma_{Ruth}(\theta') - \sigma(\theta')] \sin\theta' d\theta' \tag{9}$$

If the elastic scattering angular distribution is measured in forward angles, where the oscillations in $I(\theta)$ are well defined, the angles θ_i are those at which the function $I(\theta)$ crosses the mean value $-\Delta\sigma_T$. In other words, inflection points of $I(\theta)$ are good candidates for θ_i, to initiate the integration of equation (9). Giordano[9] suggested the same criterium for θ_i, comparing σ_{SOD} with σ_R obtained from optical model calculations.

In a real application of the method to experimental data, the function $I(\theta)$ cannot be determined directly, since only the integral

$$2\pi \int_{\theta}^{\pi} \left[\sigma_{Ruth}(\theta') - \sigma(\theta') \right] \sin\theta' d\theta' = I(\pi) - I(\theta) \qquad (10)$$

is obtained with $I(\pi)$, an unknown constant. On the other hand, the oscillations in the integral (10) are the same as those in $I(\theta)$ of equation (7), in particular both have the same inflection points θ_i. Accordingly σ_R can still be extracted in real data situation through the knowledge of θ_i and using equation (9). This, of course, leaves $\Delta\sigma_T$ undetermined.

Optical model calculations were performed for the systems $^{16}O + {}^{12}C$ in the energy range $E_{CM} = 10 - 24$ MeV, $^{14}N + {}^{12}C$ at $E_{CM} = 9 - 25$ MeV and $^{16}O + {}^{28}Si$ at $E_{CM} = 20 - 52$ MeV, initiating the integral of equation (9) at the inflection point before the last one (the last inflection point in some cases is not a good choice).

The results are presented in table 1, where the third column is σ_R calculated by optical model, the fourth column σ_{R_i} is the integral of equation (9) calculated from θ_i to π, the fifth column $\Delta\sigma_T$ is calculated by optical model from equation (7) (with the restriction on Λ mentioned in the text) and the sixth column is the angle θ_i. Comparing the third and fourth columns one sees that this method gives reaction cross-sections σ_{R_i} in good accord with the optical model σ_R, even if $\Delta\sigma_T$ is important.

The main result of this work was a better understanding of the optical theorem through the study of the function

$I(\theta)$ of equation (7). The graphical decomposition of $I(\theta)$ into σ_R and $\Delta\sigma_T$ shows clearly that

$$I(\theta) = 2\pi \int_{0}^{\theta} \left[\sigma_{SC}(\theta') - \sigma(\theta') \right] \sin\theta' d\theta'$$

does not have a zero mean value for small angles, as it was generally suggested, and for this reason $I(\pi)$ is not σ_R, but $\sigma_R - \Delta\sigma_T$.

When this work was completed, we received a preprint from J. Barrette and N. Alamanos where they give a different interpretation to $\Delta\sigma_T$.

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FIGURE CAPTIONS

Fig. 1 - Optical model calculation of function $I(\theta)$ using eq. (12), for the system $^{16}O + {}^{12}C$ at $E_{CM} = 18$ MeV, using optical potential of ref. [11]. σ_{SOD}, indicated in figure, is defined as $\sigma_{SOD} = 2\pi \int_0^\pi \left[\sigma_{sc}(\theta) - \sigma(\theta)\right] \sin\theta \, d\theta$.

Fig. 2 - Optical model calculation of function $I(\theta)$ using eq. (12), for the system $^{18}O + {}^{58}Ni$ at $E_{CM} = 48.4$ MeV, using optical potential of ref. [12]. σ_{SOD} is defined as in fig. 1.

Fig. 3 - Optical model calculation of function $I(\theta)$ using eq. (12), for the system $^{16}O + {}^{28}Si$ at $E_{CM} = 44.0$ MeV, using optical potential of ref. [13]. σ_{SOD} is defined as in fig. 1.

TABLE 1

The results of optical model calculations for the systems: $^{16}O + ^{12}C$ using optical potential parameters[11] $V = 100.0$ MeV, $r_V = 1.91$ fm, $a_V = 0.48$ fm, $W = 10.0$ MeV, $r_W = 1.26$ fm, $a_W = 0.26$ fm, $^{14}N + ^{12}C$ using optical potential parameters[14] $V = 30.0$ MeV, $r_V = 1.02$ fm, $a_V = 0.57$ fm, $W = 7.1$ MeV, $r_W = 1.20$ fm, $a_W = 0.79$ fm, $^{16}O + ^{28}Si$ using optical potential parameters[13] $V = 10.0$ MeV, $r_V = 1.35$ fm, $a_V = 0.618$ fm, $W = 23.4$ MeV, $r_W = 1.23$ fm, $a_W = 0.552$ fm. σ_R is optical model reaction cross section, σ_{R_i} is calculated from eq. (14), $\Delta\sigma_T$ is calculated from eq. (11) and θ_i is the inflection point used to calculate σ_{R_i}.

TABLE 1

System	E_{CM} (MeV)	σ_R (fm^2)	σ_{R_i} (fm^2)	$\Delta\sigma_T$ (fm^2)	θ_i (degrees)
$^{16}O + ^{12}C$	10.0	194.5	208.4	− 19.09	24.4
	12.0	231.9	222.0	− 54.77	15.9
	14.0	257.4	248.7	− 13.56	12.9
	16.0	275.6	273.7	76.82	11.0
	18.0	288.9	296.7	114.30	9.6
	20.0	299.1	318.3	46.04	8.5
	22.0	307.3	336.6	− 64.93	7.7
	24.0	313.4	357.4	−150.26	7.1
$^{14}N + ^{12}C$	9.0	41.6	42.4	0.0	37.7
	11.0	67.0	68.6	0.85	27.7
	13.0	84.4	87.2	− 1.18	21.7
	15.0	96.5	100.5	− 8.07	17.6
	17.0	105.3	109.6	− 16.79	14.4
	19.0	111.8	109.5	− 22.74	11.1
	21.0	116.8	110.0	− 22.82	9.1
	23.0	120.7	111.4	− 15.30	7.8
	25.0	123.8	112.2	− 2.54	6.9
$^{16}O + ^{28}Si$	20.0	41.1	43.3	0.0	61.2
	24.0	75.3	79.0	0.0	43.3
	28.0	99.0	103.1	− 3.41	33.1
	32.0	116.1	120.6	− 2.39	26.7
	36.0	128.9	135.0	10.50	22.6
	40.0	138.9	147.7	2.67	19.6
	44.0	146.9	155.3	− 18.67	17.0
	48.0	153.2	153.9	− 7.31	14.4
	52.0	158.5	159.5	23.11	12.9

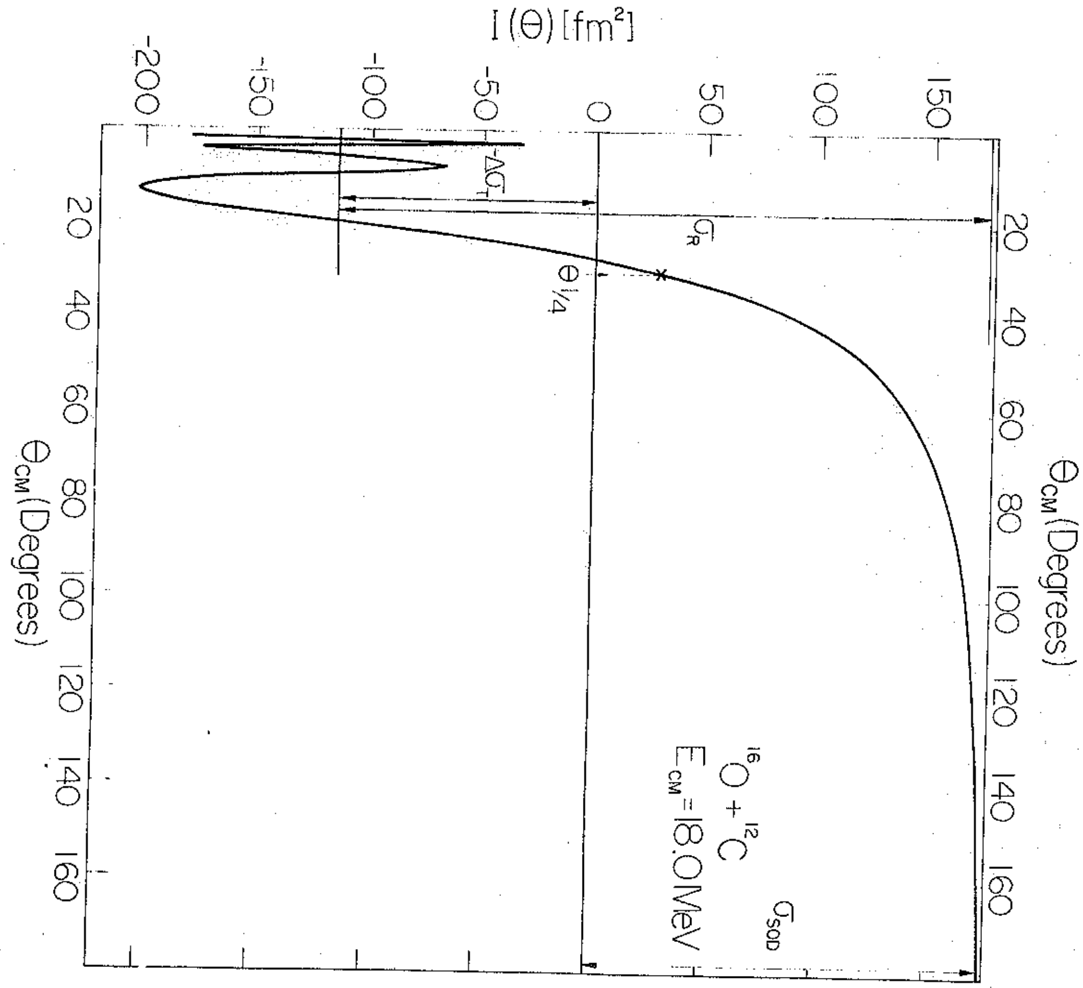
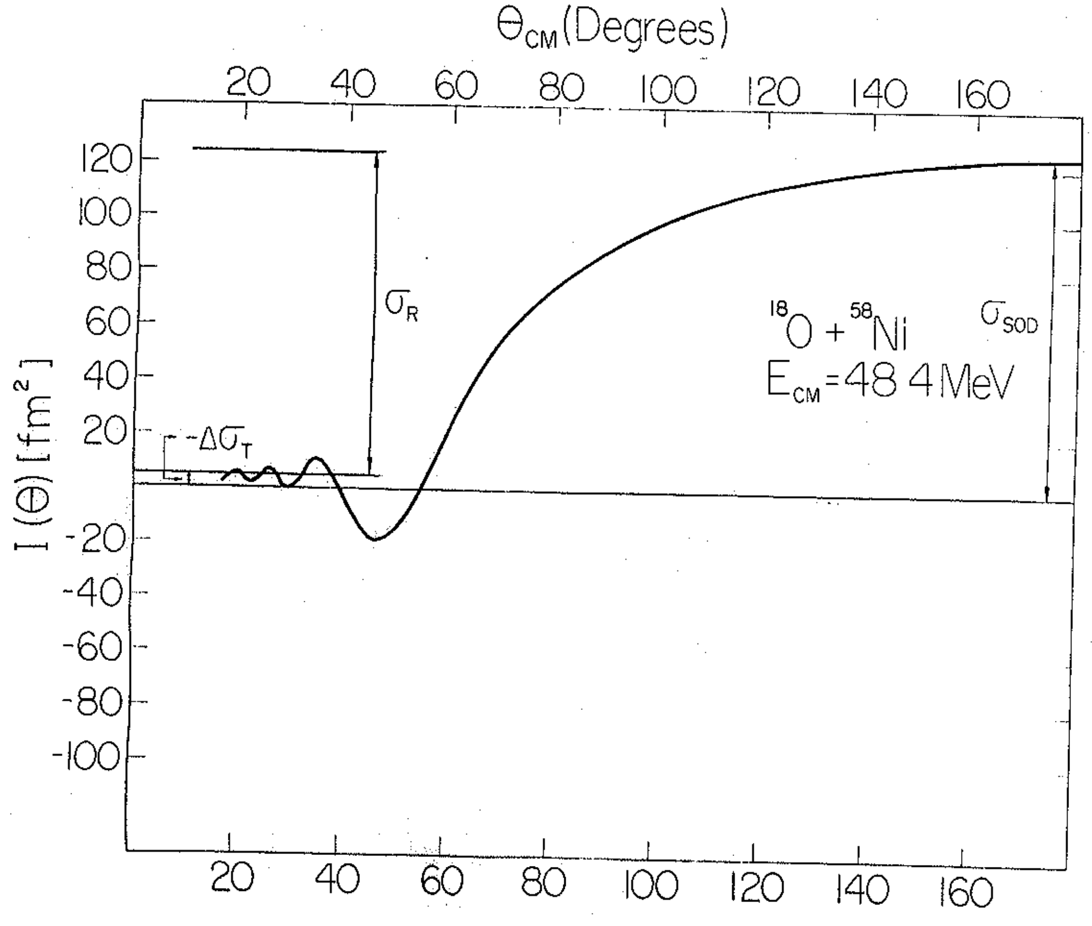


Figure 1



ΘCM (Degrees)

Figure 2

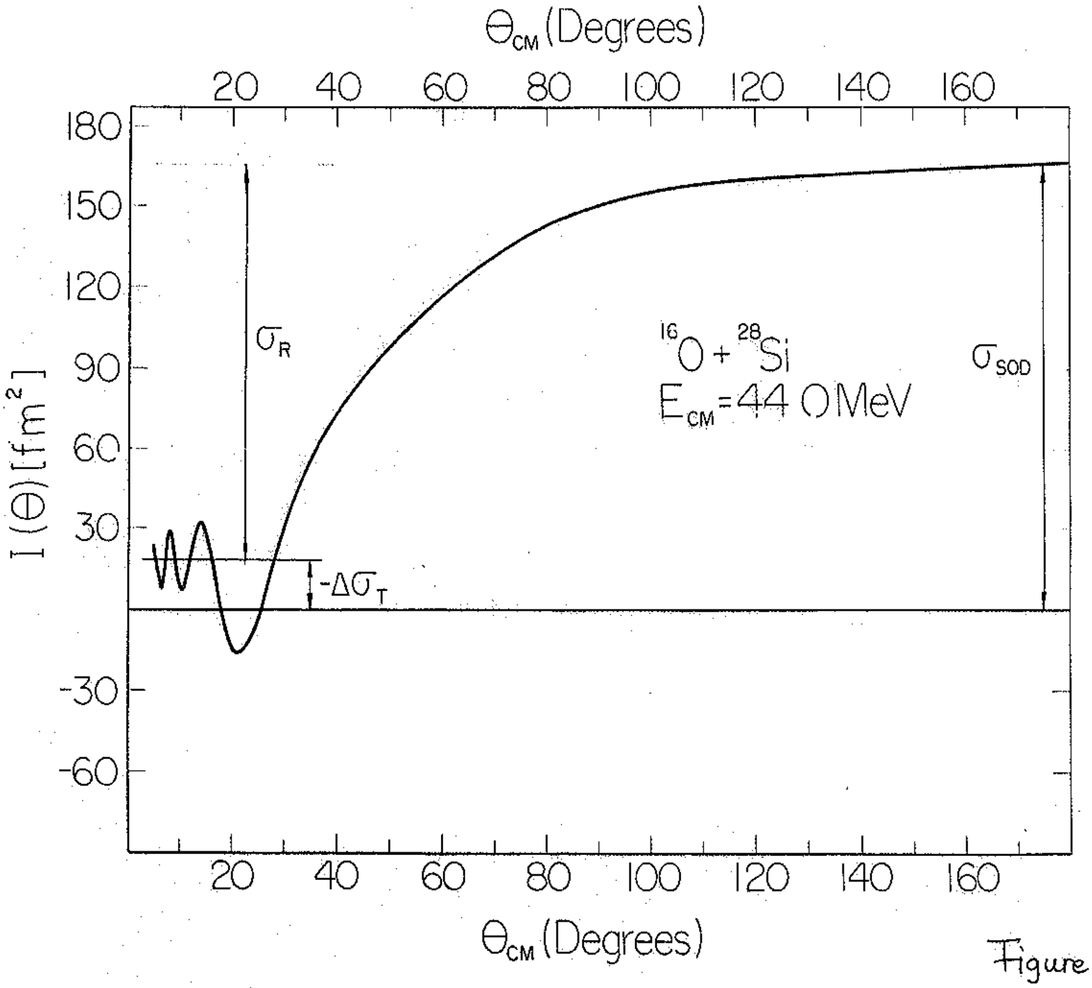

Θ_{CM} (Degrees)
20 40 60 80 100 120 140 160
180
150
120
90
60
30
-30
-60
$I(\Theta)\,[fm^2]$
σ_R
$^{16}O + ^{28}Si$
$E_{CM} = 44.0$ MeV
σ_{SOD}
$-\Delta\sigma_T$
20 40 60 80 100 120 140 160
Θ_{CM} (Degrees)

Figure 3