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TAMING THE TWO-PION EXCHANGE THREE-NUCLEON POTENTIAL

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ABSTRACT

In this work we argue that the two-pion exchange three-nucleon potential obtained by means of chiral symmetry has to be modified in order to allow reliable results to be obtained from realistic calculations. We do this by discussing the main ideas used in the construction of the potential for the case of point-like nucleons. The relevance of chiral symmetry is shown to be independent on whether it is implemented by means of current algebra or effective Lagrangians. We argue that the inclusion of form factors into the potential distorts its form in a rather large region around the origin, violating our expectation that it should just produce modifications at small internucleon distances. The cause of this peculiar behaviour is traced back to a term in the potential that becomes a δ-function in the absence of form factors and which can be interpreted as a "contact" interaction between extended nucleons. The taming of the two-pion exchange three-nucleon potential is achieved by redefining it in such a way as to exclude contact interactions.

1. INTRODUCTION

The two-pion exchange three-nucleon potential (ππE-3NP) has influence over both static and dynamic properties of trinuclei. For instance, several recent calculations have shown that this potential can account for the difference of about 1.5 MeV, between the measured binding energies of ^3H and ^3He and the corresponding values calculated by means of realistic nucleon-nucleon potentials[1,2]. The dominant attractive nature of the ππE-3NP can also be felt into the electric and magnetic form factors of trinuclei, in the region around the first minimum[1,2].

An interesting feature of the calculations that include the ππE-3NP is their extreme sensitivity to the πN form factors embodied into the potential. For instance, Chen, Friar, Gibson and Payne[3] have studied the triton by means of the Reid[4] two-nucleon interaction and the Tucson-Melbourne[5,6] (TM) three-nucleon potential. In their work we learn that the calculated binding energy becomes 7.46 MeV, 8.86 MeV or 11.16 MeV when the cutoff parameter of the πN form factor is respectively 575 MeV, 800 MeV or 1000 MeV. Also, in a work by Ishikawa and Sasakawa[7], a change of the cutoff parameter from 700 MeV to 800 MeV is reported to shift the triton binding energy calculated by means of the Paris[8] and TM potentials from 8.32 MeV to 9.18 MeV. This sort of behaviour is clearly problematic, since it does not allow stable conclusions to be drawn concerning the role of the ππE-3NP in a given physical process. We have recently shown that this sensitivity to form factors can be traced back to terms in the potential which can be interpreted as contact interactions between nucleons[9,10].

We have also argued that this problem is not present in two alternative versions of the $\pi\pi$E-3NP, which exclude part[9,11] or all[10] of these contact interactions.

However, our ideas about the problem have had a piecemeal development, through a series of works where aspects regarding both the form of the potential and its application were tackled at the same time. During this process we have produced two alternative versions of the $\pi\pi$E-3NP, one in 1983[11] and the other in 1986[10], the latter being supposed to exclude the former. Clearly, this situation may promote confusion among readers. In order to remedy it, we present here a summary of our argumentation concerning the $\pi\pi$E-3NP, setting earlier developments into an unified conceptual framework.

The main motivation for this work is to argue that the taming of the $\pi\pi$E-3NP, needed in order to allow reliable results to be produced in realistic calculations, passes through the redefinition procedure proposed in ref. (10). An evidence of the possible numerical consequences of this redefinition is produced by the fact that results obtained by means of the TM potential are rather different from those corresponding to our already superseeded 1983 version. In order to see this we note that the latter can be obtained from the former when its parameters a and c are replaced by -a and zero respectively (see section 4.1 for details). When this transformation is applied to the work of Bömelburg[12], we obtain a net 3NP contribution to the triton binding energy of -0.793 MeV instead of $+0.203$ MeV, whereas it changes the corresponding result of Ishikawa, Sasakawa, Sawada and Ueda[13] from -0.89 MeV to -1.24 MeV. The sensitivity of numerical results to the form

of the potential has been investigated in detail by Wiringa, Friar, Gibson, Payne and Chen[14]. In their work we find, among many other results that, when the πN cutoff parameter is $\Lambda = 5.8\,\mu$, the 3NP contribution to the triton binding energy is -0.41 MeV for the TM potential and -1.10 MeV for our 1983 alternative form. In more recent works[3,15] Chen, Friar, Gibson and Payne have claimed that an increase in the number of Faddeev channels is able to offset partially the differences between these two potentials. Nevertheless, there remains a noticeable difference between the value of 8.86 MeV for the TM potential and 9.09 MeV for our earlier version.

In this work we are mostly interested in the conceptual aspects of the problem, and therefore we divide our presentation into small blocks, each of them centred around an important idea. The algebraic details are not considered, since they can be found in the original references. Our presentation is organized as follows. In section 2 we give the generic form of the $\pi\pi$E-3NP, that depends on an intermediate πN amplitude. This amplitude is discussed in section 3 within the context of chiral symmetry, whereas in section 4 we combine these results in order to produce the $\pi\pi$E-3NP for point-like nucleons. Form factors are introduced in section 5 and the potential for extended nucleons is shown in section 6. Our expectations regarding the introduction of form factors are displayed in section 7, whereas in section 8 we discuss the way they fail to be fulfilled. The causes of this problem are identified at a dynamical level in section 9, motivating our redefinition of the potential. The redefined version of the $\pi\pi$E-3NP is discussed in section 10. Finally, in section 11, we briefly summarize our argumentation, exhibit the redefined potential and present our conclusions.

2. ππÊ-3NP: GENERIC FORM

The ππE-3NP is the longest range three-nucleon potential and corresponds to the process whereby the pion emitted by one of the nucleons is scattered by a second one before being absorbed by the third nucleon. A 3NP describes only proper three-nucleon interactions, corresponding to diagrams that cannot be split into two parts by cutting only forward propagating nucleon lines, in order to avoid double counting an iterated two-body force. Moreover, the 3NP is designed to be applied to loosely bound systems, such as ^3H and ^3He, which means that off-shell nucleon effects can be neglected in its derivation. Therefore, in constructing the ππE-3NP, we consider T_{3N}, the amplitude describing the elastic scattering of three unbound nucleons due to the exchange of two pions, and then subtract the part associated with nucleons propagating forward in time. This process is represented in figure 1, where g is πNN vertex and $T_{\pi N}$ is the pion-nucleon scattering amplitude for off-shell pions.

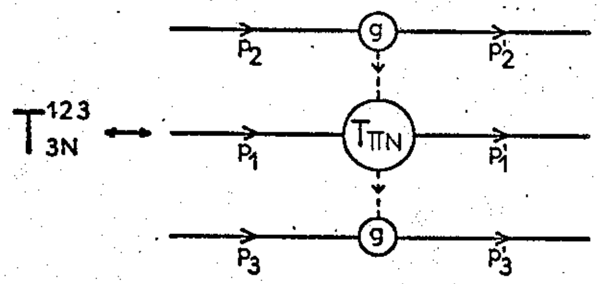

Fig.1. Diagram corresponding to the ππE-3NP.

Adopting the pseudo-vector coupling for the πNN

vertex, we obtain[11]

$$T_{3N}^{123} = \left[\bar{u}(\underset{\sim}{p}_2')\not{k}\,\gamma_5\,\tau_a\,u(\underset{\sim}{p}_2)\right]\frac{g/2m}{k^2-\mu^2}\left\{T_{\pi N}^{ab}\right\}\frac{g/2m}{k'^2-\mu^2}\left[\bar{u}(\underset{\sim}{p}_3')\not{k}'\,\gamma_5\,\tau_b\,u(\underset{\sim}{p}_3)\right] \quad (1)$$

where g is the πN coupling constant, m and μ are the nucleon and pion masses, $\underset{\sim}{\tau}$ are matrices that produce isovectors when sandwhiched between nucleon states, and $T_{\pi N}$ has the general form

$$T_{\pi N}^{ab} = \bar{u}(\underset{\sim}{p}_1')\left[\left(A^+ + \frac{\not{k}'+\not{k}}{2}B^+\right)\delta_{ab} + \left(A^- + \frac{\not{k}'+\not{k}}{2}B^-\right)i\,\epsilon_{bac}\,\tau_c\right]u(\underset{\sim}{p}_1)\ . \quad (2)$$

The amplitudes $A^{\pm}$ and $B^{\pm}$ are relativistic invariants and depend on the details of the πN dynamics. The nucleons are assumed to be on shell and hence each of these amplitudes is a function of four variables, namely k^2, k'^2, ν and t, where

$$\nu \equiv (p_1' + p_1)\cdot(k' + k)/4m$$
$$t \equiv (k' - k)^2\ . \quad (3)$$

The variable ν is related to the usual Mandelstam variables $s \equiv (p_1+k)^2$ and $u \equiv (p_1-k')^2$ by $\nu \equiv (s-u)/4m$.

An alternative expression for $T_{\pi N}^{ab}$ is[5]

$$T_{\pi N}^{ab} = \bar{u}(\underset{\sim}{p}_1')\left[\left(F^+ - \frac{[\not{k}',\not{k}]}{4m}B^+\right)\delta_{ab} + \left(F^- - \frac{[\not{k}',\not{k}]}{4m}B^-\right)i\,\epsilon_{bac}\,\tau_c\right]u(\underset{\sim}{p}_1) \quad (4)$$

where

$$F^{\pm} \equiv A^{\pm} + \nu B^{\pm} \quad . \tag{5}$$

The potential is a meaningful concept for low-energy processes and, in constructing the 3NP, we assume that the momenta of the nucleons bound in nuclei are of the order of the pion mass. Thus we need only to consider the non-relativistic limit of eq. (1), that is given by

$$t_{3N}^{123} = (2m)^3 \frac{g/2m}{\underset{\sim}{k}^2 + \mu^2} \frac{g/2m}{\underset{\sim}{k}'^2+\mu^2} (\underset{\sim}{\sigma}^{(2)} \cdot \underset{\sim}{k})(\underset{\sim}{\sigma}^{(3)} \cdot \underset{\sim}{k}') \tau_a^{(2)} \tau_b^{(3)} \times$$

$$\times \left\{ \left[f^+ + i\, \underset{\sim}{\sigma}^{(1)} \cdot (\underset{\sim}{k}' \times \underset{\sim}{k})\, b^+/2m \right] \delta_{ab} + \left[f^- + i\, \underset{\sim}{\sigma}^{(1)} \cdot (\underset{\sim}{k}' \times \underset{\sim}{k})\, b^-/2m \right] i\epsilon_{bac} \tau_c^{(1)} \right\} \tag{6}$$

where $\underset{\sim}{\sigma}^{(i)}$ and $\underset{\sim}{\tau}^{(i)}$ indicate expectation values,

$$f^{\pm} = a^{\pm} + \nu b^{\pm} \tag{7}$$

and $a^{\pm}$ and $b^{\pm}$ are the non-relativistic versions of the amplitudes $A^{\pm}$ and $B^{\pm}$.

The relationship between the amplitude t_{3N} and the three-nucleon potential $W_{\pi\pi}$ is the following[16]

$$\langle \underset{\sim}{p}_1' \underset{\sim}{p}_2' \underset{\sim}{p}_3' | W_{\pi\pi} | \underset{\sim}{p}_1 \underset{\sim}{p}_2 \underset{\sim}{p}_3 \rangle \equiv -(2\pi)^3\, \delta^3(\underset{\sim}{p}_f - \underset{\sim}{p}_i)(1/2m)^3 t_{3N} \quad . \tag{8}$$

The factor $(1/2m)^3$ has been introduced because the non-relativistic momentum space states are normalized as $\langle \underset{\sim}{p}' | \underset{\sim}{p} \rangle = (2\pi)^3 \delta^3(\underset{\sim}{p}' - \underset{\sim}{p})$.

The potential in coordinate space is obtained by Fourier transforming that in momentum space

$$\langle \underset{\sim}{r}_1' \underset{\sim}{r}_2' \underset{\sim}{r}_3' | W_{\pi\pi} | \underset{\sim}{r}_1 \underset{\sim}{r}_2 \underset{\sim}{r}_3 \rangle =$$

$$= -\frac{(2\pi)^3}{(2m)^3} \int \frac{d\underset{\sim}{p}_1'}{(2\pi)^3} \cdots \frac{d\underset{\sim}{p}_3}{(2\pi)^3}\, \delta^3(\underset{\sim}{p}_f - \underset{\sim}{p}_i) \exp\left[i(-\underset{\sim}{p}_1' \cdot \underset{\sim}{r}_1' \ldots + \underset{\sim}{p}_3 \cdot \underset{\sim}{r}_3) \right] t_{3N} \quad . \tag{9}$$

When velocity dependent terms are neglected in t_{3N}, this expression becomes

$$\langle \underset{\sim}{r}_1' \underset{\sim}{r}_2' \underset{\sim}{r}_3' | W_{\pi\pi} | \underset{\sim}{r}_1 \underset{\sim}{r}_2 \underset{\sim}{r}_3 \rangle \equiv \delta^3(\underset{\sim}{r}_1' - \underset{\sim}{r}_1) \delta^3(\underset{\sim}{r}_2' - \underset{\sim}{r}_2) \delta^3(\underset{\sim}{r}_3' - \underset{\sim}{r}_3)\, W \quad , \tag{10}$$

where

$$W = W(1) + W(2) + W(3) \quad , \tag{11}$$

$$W(i) = -\int \frac{d\underset{\sim}{k}'}{(2\pi)^3} \frac{d\underset{\sim}{k}}{(2\pi)^3}\, e^{i\underset{\sim}{k}' \cdot \underset{\sim}{r}_{ki}}\, e^{i\underset{\sim}{k} \cdot \underset{\sim}{r}_{ij}} (1/2m)^3\, t_{3N}^{ijk} \quad , \tag{12}$$

$$\underset{\sim}{r}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j \tag{13}$$

and i, j, k correspond to cyclic permutations of the integers 1, 2, 3.

The generic form of the potential is obtained by combining eqs. (12) and (6)

$$W(i) = -\int \frac{d\underset{\sim}{k}'}{(2\pi)^3} \frac{d\underset{\sim}{k}}{(2\pi)^3}\, e^{i\underset{\sim}{k}' \cdot \underset{\sim}{r}_{ki}}\, e^{i\underset{\sim}{k} \cdot \underset{\sim}{r}_{ij}} \frac{g/2m}{\underset{\sim}{k}^2 + \mu^2} \frac{g/2m}{\underset{\sim}{k}'^2+\mu^2} \times$$

$$\times (\underset{\sim}{\sigma}^{(j)} \cdot \underset{\sim}{k})(\underset{\sim}{\sigma}^{(k)} \cdot \underset{\sim}{k}') \tau_a^{(j)} \tau_b^{(k)} \left\{ \left[f^+ + i\sigma^{(i)} \cdot (\underset{\sim}{k}' \times \underset{\sim}{k})\, b^+/2m \right] \delta_{ab} + \right.$$

$$\left. + \left[f^- + i\sigma^{(i)} \cdot (\underset{\sim}{k}' \times \underset{\sim}{k})\, b^-/2m \right] i\epsilon_{bac} \tau_c^{(i)} \right\} \quad . \tag{14}$$

This is the basic equation in the calculation of the $\pi\pi$E-3NP. It is worth noting that it has been derived under three assumptions, namely:

i) off-shell nucleon effects can be neglected;

ii) bound nucleons are non-relativistic, their momenta being of the order of the pion mass;

iii) velocity dependent terms in t_{3N} can be neglected.

3. THE INTERMEDIATE πN AMPLITUDE

The expression for W(i) given by eq. (14) determines both the spin and isospin structures of the $\pi\pi$E-3NP. So, the explicit form of the potential depends on the knowledge of the subamplitudes $f^{\pm}$ and $b^{\pm}$, describing the intermediate πN scattering for off-shell pions. The kinematical variables associated with the various particles have the following orders of magnitude: $|\underset{\sim}{p}| \sim |\underset{\sim}{p}'| \sim |\underset{\sim}{k}| \sim |\underset{\sim}{k}'| \sim \mu$, $p_0 \sim p_0' \sim m$, and the pion energies k_0 and k_0' are comparable to μ^2/m. Hence the variables that determine $f^{\pm}$ and $b^{\pm}$ assume the following approximate values: $k^2 \cong -\underset{\sim}{k}^2$, $k'^2 \cong -\underset{\sim}{k}'^2$, $\nu \cong [2m(k_0'+k_0) - (\underset{\sim}{p}_1'+\underset{\sim}{p}_1)\cdot(\underset{\sim}{k}'+\underset{\sim}{k})]/4m \sim \mu^2/m$, $t \cong -(\underset{\sim}{k}'-\underset{\sim}{k})^2$. If the pions were on shell the values of the kinematical variables would be very different, since the pion energies would be $\omega \equiv (\mu^2+\underset{\sim}{k}^2)^{1/2}$ and $\omega' \equiv (\mu^2+\underset{\sim}{k}'^2)^{1/2}$, of order μ, and we would have: $k^2 = \mu^2$, $k'^2 = \mu^2$, $\nu = [(E'+E)(\omega'+\omega) - (\underset{\sim}{p}_1'+\underset{\sim}{p}_1)\cdot(\underset{\sim}{k}'+\underset{\sim}{k})]/4m \sim \mu$, $t = (\omega'-\omega)^2 - (\underset{\sim}{k}'-\underset{\sim}{k})^2$. The fact that the intermediate pions are off-shell means, therefore, that the subamplitudes $f^{\pm}$ and $b^{\pm}$ cannot be directly extracted from experiment. Thus, the construction of the potential requires the use of a theoretical amplitude

that is constrained by on-shell πN data and suited for off-shell extrapolation. Chiral symmetry provides a consistent theoretical framework where such a πN amplitude can be constructed. This is the reason why this symmetry is so relevant in the determination of the $\pi\pi$E-3NP.

The elastic πN scattering has been extensively studied by means of chiral symmetry, and agreement with experiment is good both below threshold and for pion energies up to 350 MeV[17-19]. This symmetry has also been tested in many other situations[20] and is the most successful theory for describing the interactions of low-energy pions with other hadrons. Its basic assumptions are the approximate invariance of these interactions under transformations of the group $SU(2) \times SU(2)$ and that the symmetric limit is attained when the four-momenta of the pions vanish. The implementation of chiral symmetry in low-energy pionic processes can be performed by means of either current algebra or effective Lagrangians. In the TM[5,6] derivation of the $\pi\pi$E-3NP, the former method has been adopted, whereas our approach to the problem[11] is based on the latter. Both ways of implementing the symmetry are equivalent as far as physics is concerned, but correspond to rather different calculational techniques.

3.1. CURRENT ALGEBRA

The Tucson-Melbourne potential[5] was derived within the context of current algebra, for a particular frame of reference. In the more general version of Coon and Glöckle[6] it corresponds, in the case of point-like nucleons, to the following expressions for the non-relativistic πN subamplitudes

$$f^{+} = -\frac{\sigma}{f_\pi^2}\left[1 - \frac{2\mathbf{k}'\cdot\mathbf{k}}{\mu^2} + \frac{\mathbf{k}'^2+\mathbf{k}^2}{\mu^2}\right] - \frac{2}{\mu^2}\,\bar{F}^{+}(0,\mu^2;\mu^2,\mu^2)\,\mathbf{k}'\cdot\mathbf{k}$$

$$+ \frac{g^2}{4m^3}\,(\mathbf{k}'^2+\mathbf{k}^2) + \frac{g^2}{2m^3}\,\mathbf{p}_1'\cdot\mathbf{p}_1 \qquad (15)$$

$$b^{+} = 0 \qquad (16)$$

$$f^{-} = \frac{g^2}{8m^3}\left[-\mathbf{p}_2'^2 + \mathbf{p}_2^2 + \mathbf{p}_3'^2 - \mathbf{p}_3^2 + (\mathbf{p}_1'+\mathbf{p}_1)\cdot(\mathbf{k}'+\mathbf{k})\right]$$

$$+ \left[\frac{1}{2f_\pi^2} - \frac{g^2}{2m^2} - g_\Delta^2(\ldots)\right]\nu \qquad (17)$$

$$b^{-} = \frac{1+\mu_p-\mu_n}{2f_\pi^2} + g_\Delta^2(\ldots) \qquad (18)$$

In these results, σ is the πN sigma term, f_π is the pion decay constant, μ_p and μ_n are the anomalous magnetic moments of the proton and the neutron, whereas g_Δ is the $\pi N\Delta$ coupling constant. The terms proportional to g^2 are due either to the backward propagating nucleon or to relativistic effects.

The structure of f^+ is determined by low energy theorems. It is convenient to express it as a sum of two terms, one representing the πN sigma term and the other associated with a background ascribed to different dynamical processes. Only the former contains leading off-shell effects, through the explicit dependence on k^2 and k'^2. Even in the current algebra approach the expression describing the sigma contribution is not the outcome of a complete theory and hence should be understood as a good mathematical model. The numerical values of the parameter σ and of the background can be derived in a model independent way from the subthreshold coefficients of Höhler, Jacob and Strauss[21,22]. These coefficients are

obtained through the extrapolation, by means of dispersion relations, of the "experimental" amplitudes $A^{\pm}(\nu,t)$ and $B^{\pm}(\nu,t)$ to regions below threshold, where they are expanded in a power series of ν and t.

The subamplitude f^- is dominated by the term proportional to $1/f_\pi^2$, which is associated with the vector current originating from the commutator of two axial currents. It is velocity dependent, since it is linear in ν, and hence it does not contribute to the local potential. Finally, b^- receives a contribution from the vector current, proportional to $1/f_\pi^2$, and another one from a background, indicated by $g_\Delta(\ldots)$. The latter can be either extracted from the HSJ coefficients[5] or from a model associated with the delta resonance[5,6].

3.2. CHIRAL DYNAMICS

The implementation of chiral symmetry by means of current algebra is technically envolved and has the disadvantage of hiding the dynamical content of the soft pion limit. These problems are not present in the approach based on effective or phenomenological Lagrangians, which are constructed in such a way as to reproduce the results of current algebra when used in lowest order perturbation theory[23]. As pointed out by Weinberg[24], these phenomenological Lagrangians may even render unnecessary at all the use of current algebra. It is important to stress that these Lagrangians, which constitute the basis of the so-called chiral dynamics, are different in spirit, for instance, from those appearing in quantum electrodynamics. The use of effective Lagrangians should not, therefore, be understood as an attempt to employ ordinary perturbation theory in calculations of strong processes. Rather than being fundamental objects, effective Lagrangians are quick and efficient tools that allow chiral symmetry to be implemented with the help of all the nice features of a covariant field theory. Another advantage of the use of Lagrangians is that it makes possible a clear understanding of the dynamical origins of the various contributions to a given process and hence is well suited for guiding our intuition.

An effective Lagrangian was used by Yang[16] in the evaluation of the $\pi\pi$E-3NP. However, his model did not take into account breaking effects of chiral symmetry as well as did not reproduce on-shell πN data, since he disregarded both the σ-term and the nucleon magnetic moments. The Lagrangian approach was also adopted by ourselves in a recent derivation of the $\pi\pi$E-3NP[11], where the constraints produced by πN scattering data were properly taken into consideration. We used a non-linear Lagrangian, associated with a pseudo-vector πN coupling, and the amplitude for the process $\pi N \to \pi N$ was assumed to be given by the diagrams of fig. 2, corresponding to nucleon and delta poles, rho and sigma exchanges.

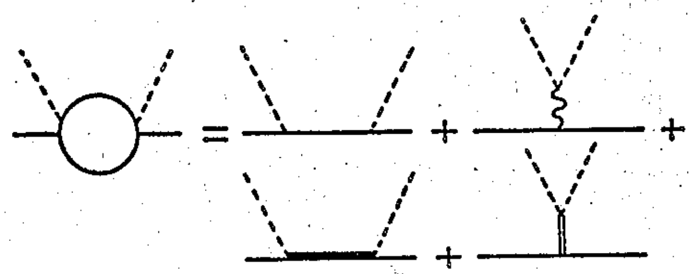

Fig.2. Diagrams corresponding to πN scattering; continuous and thick lines represent nucleons and deltas, whereas broken, wavy and double ones represent pions, rhos and sigma.

The diagram associated with the sigma does not describe the exchange of a real particle or resonance, since no serious candidate for the sigma field seems to exist. So, it must be understood as representing the πN sigma-term that, in the context of current algebra, comes from the equal-time commutator of an axial current and its divergence. The usual procedure consists in considering this contribution by means of a parametrized form[17].

The sigma term contributes only to the function A^+ of the relativistic πN scattering amplitude (eq. (2)). In our derivation of the $\pi\pi$E-3NP (ref. 11), we have used the following form for the σ contribution

$$A^+_\sigma = \alpha_\sigma + \beta_\sigma \, k' . k \tag{19}$$

where α_σ and β_σ are constants extracted from experiment. This form has been taken from ref. 17 and is adequate for the case of free pions. When the pions are not free, off-shell effects must be included and this parametrization of A^+_σ has to be modified, as it has been correctly pointed out in ref. 2. The parametrization adopted in the TM potential does not suffer from this difficulty and is consistent with the theoretical single and double soft pion limits of the intermediate πN amplitude. The parametrization used in the TM potential is equivalent to the following form for A^+_σ:

$$A^+_\sigma = \frac{\sigma}{f^2_\pi} \left\{ (1-\beta) \left[(k'^2+k^2)/\mu^2 - 1 \right] + \beta \left[t/\mu^2 - 1 \right] \right\}$$

$$= \frac{\sigma}{f^2_\pi} \left[1 - 2\beta \, k' . k/\mu^2 + (k'^2-\mu^2)/\mu^2 + (k^2-\mu^2)/\mu^2 \right]$$

$$\equiv \alpha_\sigma + \beta_\sigma \, k' . k + \gamma_\sigma \left[(k'^2-\mu^2) + (k^2-\mu^2) \right] \tag{20}$$

where we have made the identifications: $\alpha_\sigma \equiv (\sigma/f_\pi^2)$, $\beta_\sigma \equiv -(2\beta/\mu^2)\alpha_\sigma$, $\gamma_\sigma \equiv \alpha_\sigma/\mu^2$. Comparing eqs. (19) and (20) it is possible to see that they differ by the terms proportional to γ_σ, which describe off-shell effects.

The effective nonlinear Lagrangian, combined with the appropriate parametrization for the σ-term, produces the following individual contributions for the subamplitudes $f^\pm$ and $b^\pm$ entering the intermediate πN amplitude

<u>nucleon propagating backward in time</u>:

$$f_N^\pm = b_N^\pm = 0 \tag{21}$$

<u>rho</u>:

$$\begin{aligned} f_\rho^+ &= b_\rho^+ = 0 \\ f_\rho^- &= \nu/2f_\pi^2 \\ b_\rho^- &= (1+\mu_p-\mu_n)/2f_\pi^2 \end{aligned} \tag{22}$$

<u>delta</u>:

$$\begin{aligned} f_\Delta^+ &= 8\,g_\Delta^2\,\mathbf{k}'\cdot\mathbf{k}/9(M_\Delta-m) \\ b_\Delta^+ &= f_\Delta^- = 0 \\ b_\Delta^- &= 4\,g_\Delta^2\,m/9(M_\Delta-m) \end{aligned} \tag{23}$$

<u>sigma</u>:

$$\begin{aligned} f_\sigma^+ &= \alpha_\sigma - \beta_\sigma\,\mathbf{k}'\cdot\mathbf{k} - \gamma_\sigma\left[(\mathbf{k}'^2+\mu^2) + (\mathbf{k}^2+\mu^2)\right] \\ b_\sigma^+ &= f_\sigma^- = b_\sigma^- = 0 \end{aligned} \tag{24}$$

The results of Yang[16] correspond to setting $\mu_p = \mu_n = \alpha_\sigma$, $\beta_\sigma = \gamma_\sigma = 0$ in these expressions.

3.3. COMPARISON BETWEEN THE TWO APPROACHES

It is instructive to remark that the only important differences between these results and those entering the TM potential are the nucleon contributions present in the latter. In order to see this, we note that in the Lagrangian approach f^+ is given by

$$\begin{aligned} f^+ &= \frac{\sigma}{f_\pi^2}\left\{1 + 2\beta\,\mathbf{k}'\cdot\mathbf{k}/\mu^2 - \left[(\mathbf{k}'^2+\mu^2) + (\mathbf{k}^2+\mu^2)\right]/\mu^2\right\} + 8g_\Delta^2\,\mathbf{k}'\cdot\mathbf{k}/9(M_\Delta-m) \\ &= -\frac{\sigma}{f_\pi^2}\left\{1 - 2\,\mathbf{k}'\cdot\mathbf{k}/\mu^2 + (\mathbf{k}'^2+\mathbf{k}^2)/\mu^2\right\} - \left[\frac{2(1-\beta)}{\mu^2}\,\frac{\sigma}{f_\pi^2} - \frac{8g_\Delta^2}{9(M_\Delta-m)}\right]\mathbf{k}'\cdot\mathbf{k}\ . \end{aligned} \tag{25}$$

The term within curly brackets is identical, by construction, to that of the TM potential, given by eq. (15). The square bracket, on the other hand, corresponds to an explicit evaluation of $\left[\frac{2}{\mu^2}\,\bar{F}^+(0,\mu^2;\mu^2,\mu^2)\right]$ in the context of the Lagrangian model[17]. On the other hand, the nucleon contributions in eq. (15) represent just small corrections to the leading terms, meaning that f^+ is essentially the same in both calculations. This is also valid for b^+, whose value is zero.

As far as the isospin odd amplitudes f^- and b^- are concerned, we note that the contributions proportional to $1/f_\pi^2$ are the same in both the current algebra and effective Lagrangian calculations. In the former case these terms are attributed to the isovector current arising from the commutator of two axial currents, whereas in latter it is due to the rho meson that is supposed to dominate this isovector current. In the TM calculation, f^- also contains a contribution from the nucleon and a negligible term associated with the delta. These differences do not influence the final form of the local potential because the terms present in f^- are velocity dependent. Finally, there are no important differences between the results for b^-, since the delta contributions in both calculations are essentially the same.

This comparison of the intermediate πN amplitude corresponding to the TM potential and that arising from the Lagrangian model shows that the only important structural difference between them concerns the contribution of backward propagating nucleons. As we have seen, this difference is not determinant, since it affects very little numerical results. Nevertheless, the discussion of this point may prove to be conceptually interesting. In both calculations a forward propagating nucleon term has been subtracted from the intermediate πN amplitude, in order to avoid double counting an iterated one-pion exchange potential (OPEP). The difference between both results has its origin in the fact that the TM calculation is based on a pseudoscalar (PS) πN coupling, whereas in the Lagrangian approach a pseudovector (PV) coupling is used. It is well known that in the PV case the pion coupling to a nucleon-antinucleon pair is much smaller than in the PS case. This is the reason why there is no contribution, in leading order, from backward propagating nucleons in the Lagrangian calculation.

It is not a trivial matter to decide which subtraction procedure is the best. The fact that the evaluation of the OPEP is usually based on the PS coupling does not mean much, since in it the nucleons are considered to be on shell, a

situation where the PS and PV couplings become equivalent. On the other hand, the suppression of antinucleons produced in the PV case may make this coupling more coherent with the physical picture provided by non-relativistic nuclear physics, where only positive frequency nucleons are present. It is useful, however, to bear in mind that this is not a strong argument, which does not suffice for deciding this matter.

As a concluding remark to this section, we would like to stress that the treatments of the intermediate πN amplitude by means of either current algebra or effective Lagrangians produce essentially the same results because these are just different methods for implementing chiral symmetry in accordance with on shell πN data.

4. $\pi\pi$E-3NP: POINT-LIKE NUCLEONS

The $\pi\pi$E-3NP for point-like nucleons is derived by using the intermediate πN subamplitudes $f^\pm$ and $b^\pm$ into the generic expression for the local potential, given by eq. (14). Adopting the results of eqs. (21-24), we obtain

$$W(i) = W_s(i) + W_p(i) + W_p'(i) \tag{26}$$

where the subscripts s and p refer to partial waves in the intermediate πN amplitude and

$$W_s(i) = (C_s/\mu^2)(\underset{\sim}{\tau}^{(j)}\cdot\underset{\sim}{\tau}^{(k)})(\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\nabla}_{ij})(\underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\nabla}_{ki})\left[1 + (\underset{\sim}{\nabla}^2_{ij} - \mu^2)/\mu^2 + (\underset{\sim}{\nabla}^2_{ki} - \mu^2)/\mu^2\right] U(r_{ij})U(r_{ki}) \tag{27}$$

$$W_p(i) = (C_p/\mu^4)\,(\underset{\sim}{\tau}^{(j)}\cdot\underset{\sim}{\tau}^{(k)})\,(\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\nabla}_{ij})\,(\underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\nabla}_{ki})\,(\underset{\sim}{\nabla}_{ij}\cdot\underset{\sim}{\nabla}_{ki})\,U(r_{ij})\,U(r_{ki}) \tag{28}$$

$$W_{p'}(i) = (C_p'/\mu^4)\,(i\,\underset{\sim}{\tau}^{(i)}\cdot\underset{\sim}{\tau}^{(j)}\times\underset{\sim}{\tau}^{(k)})\,(\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\nabla}_{ij})\,(\underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\nabla}_{ki})\,(i\,\underset{\sim}{\sigma}^{(i)}\cdot\underset{\sim}{\nabla}_{ij}\times\underset{\sim}{\nabla}_{ki})\times$$
$$\times\, U(r_{ij})\,U(r_{ki}) \tag{29}$$

The strength parameters of the potential are given by[11]

$$C_s = \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \mu^2\, \frac{\sigma}{f_\pi^2}$$

$$C_p = -\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \mu^4 \left[\frac{8 g_\Delta^2}{9(M_\Delta - m)} - \beta_\sigma\right] \tag{30}$$

$$C_p' = -\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \mu^4 \left[\frac{1}{2 f_\pi^2}\,\frac{1+\mu_p-\mu_n}{2m} + \frac{2 g_\Delta^2}{9(M_\Delta - m)}\right]$$

and $U(r)$ is the usual Yukawa function

$$U(r) = \frac{4\pi}{\mu}\int \frac{d\underset{\sim}{k}}{(2\pi)^3}\,\frac{e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{\underset{\sim}{k}^2+\mu^2}$$
$$= \frac{e^{-\mu r}}{\mu r} \quad . \tag{31}$$

The terms proportional to C_p and C_p' in eqs. (28) and (29) are the same as those of eq. (61) of our derivation of the $\pi\pi$E-3NP[11]. The term with coefficient C_s, on the other hand, has been modified by the inclusion of the factors $(\vec\nabla^2 - \mu^2)$, as the result of the discussion presented in the previous section.

4.1. COMPARISON WITH THE TM POTENTIAL

Before discussing the role of form factors in this problem, it is convenient to compare the preceding expressions with those corresponding to the TM potential. In order to do this, we note that the function $U(r)$ is such that

$$(\underset{\sim}{\nabla}^2 - \mu^2)\, U(r) = -\frac{4\pi}{\mu}\,\delta^3(\underset{\sim}{r}) \quad . \tag{32}$$

This result allows us to rewrite $W_s(i)$ as

$$W_s(i) = (C_s/\mu^2)\,(\underset{\sim}{\tau}^{(j)}\cdot\underset{\sim}{\tau}^{(k)})\,(\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\nabla}_{ij})\,(\underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\nabla}_{ki})\times$$
$$\times\left[U(r_{ij})U(r_{jk}) - \frac{4\pi}{\mu^3}\,\delta^3(\underset{\sim}{r}_{ij})U(r_{ki}) - \frac{4\pi}{\mu^3}\,U(r_{ij})\,\delta^3(\underset{\sim}{r}_{ki})\right] . \tag{33}$$

Eqs. (33), (28) and (29) can be directly compared with the often used eq. (3.1) of Coon and Glöckle[6], if we note that their function $Z_1(r)$ is our $U(r)$ whereas their $Z_0(r)$ becomes $\frac{4\pi}{\mu}\,\delta^3(\vec r)$ in the case of point-like nucleons. Thus we obtain the following structural relations among the various coefficients:

$$\frac{C_s}{\mu^2} = -\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 (a - 2\mu^2 c)$$
$$= \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 c\,\mu^2 \quad , \tag{34}$$

$$\frac{C_p}{\mu^4} = \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 b \quad , \tag{35}$$

$$\frac{C_p'}{\mu^4} = \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 (d_3 + d_4) \quad . \tag{36}$$

The simultaneous enforcement of both equalities of eq. (34) means that the relationship between C_s and a is

$$\frac{C_s}{\mu^2} = + \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 a \quad . \qquad (37)$$

Therefore the structure of our 1983 alternative version of the ππE-3NP can be obtained from the TM potential by replacing a by −a and c by zero.

In table 1 we display the values of the various strength parameters mentioned above, as found in refs. (11) and (5,6). The inspection of this table shows that these parameters do not fulfill exactly eqs. (34-37). This happens partly because the values of Coon and Glöckle incorporate some relativistic corrections and contributions from the antinucleon and partly because the two sets are based on different phenomenological inputs. Nevertheless, the discrepancies are not large, being of the order of 10%, and hence comparable to the uncertainties in the data employed into the construction of the potential.

TABLE 1

Strength parameters of the ππE-3NP.

Coelho, Das and Robilotta[11]		Coon and Glöckle[6]	
parameter	value in MeV	parameter	value in MeV multiplied by $\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2$
C_s	+ 0.92	$a\ \mu^2$	0.99
		$c\ \mu^4$	0.88
C_p	− 1.99	$b\ \mu^4$	− 2.26
C'_p	− 0.67	$(d_3+d_4)\mu^4$	− 0.66

As a concluding remark to this section, it is worth stressing that the expression for the ππE-3NP given by eqs. (26-30) is a direct consequence of chiral symmetry applied to the intermediate πN amplitude. But this is as far as chiral symmetry can safely guide us in the present problem.

5. FORM FACTORS

The ππE-3NP for point like nucleons, exhibited in the previous section, diverges for very small relative distances. The reason for this behaviour is that the small distance region is outside the domain of validity of the potential. In order to see this, it is convenient to remind ourselves that, in the derivation of the ππE-3NP, the nucleons were consistently assumed to be non-relativistic, their momenta being of the order of the pion mass. This allows us to expect the expression for the potential to be reliable only for internucleon distances larger than μ^{-1}. Hence, as these distances decrease, the potential becomes progressively less realistic.

The short distance problem is usually tackled with the help of πN form factors, which are introduced into the potential with a double purpose. The first of them is pragmatic, in the sense that the regularization of the potential at the origin is essential for realistic applications. For instance, as discussed in refs. (11) and (25), the singular behaviour of a ππE-3NP for point-like nucleons is responsible for unphysical nodes in the trinucleon wave function. The second reason for introducing form factors is that nucleons do have structure. At the hadron level, this structure corresponds to the meson clouds that dress the point-like nucleon whereas, at a more fundamental level, it is associated with the fact that the nucleon is made up of quarks.

Form factors can be introduced into the $\pi\pi$E-3NP by allowing all the coupling constants entering figs. 1 and 2 to become dependent of the pion four-momentum. In the case of the πN coupling constant, for instance, we would have $g \to g(k^2) \equiv g\,\overline{G}(k^2)$, where the function $\overline{G}$ is such that $\overline{G}(\mu^2) \equiv 1$. Similar relations should hold for all the other vertices and, in order to implement this program, we would need to have either theoretical or phenomenological information concerning the structure of these form factors. However, this information is rather incomplete, and hence it may become a serious source of uncertainty into the problem.

The present state of the art concerning applications of the $\pi\pi$E-3NP consists in trying to understand the gross features of its contributions to various physical processes[1]. Therefore we choose to introduce form factors into the potential in a simpler, although less rigorous way. We do this by allowing each pion line, consisting of a vertex-propagator-vertex, to be multiplied by

$$\overline{G}(k^2) = \left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 - k^2}\right)^2 , \qquad (38)$$

where Λ is a phenomenological parameter, supposed to represent effectively the dynamics associated with the size of the nucleon. This procedure has been adopted in a large number of works[3,5,6,7,9,10,11,12,13,14,15,26,27] and can be interpreted either as the use of an "universal" form factor for all the interactions or, alternatively, as the correction of only the

πN vertex by a dipole form factor. When needed, the elimination of the form factor can be achieved through the limit $\Lambda \to \infty$. In the sequence we adopt the form of $\overline{G}$ given by eq. (38), but our conclusions are general enough to be easily generalized to other parametrizations.

As far as the value of Λ concerned, various studies have produced a wide spectrum of indications, some of which are displayed in table 2. Inspecting it, we note that there is a group of values around $\Lambda \sim 900$ MeV whereas another corresponds to $\Lambda \gtrsim 1250$ MeV. The compatibility between these two sets of values may be restored when we recall that the latter is associated with pion-exchange processes between nucleons. As pointed out by Holinde[28], values extracted from these processes may also include other meson exchanges and hence correspond to effective quantities. If this is the case, the second group of values may be more suited to the $\pi\pi$E-3NP. In this work, however, we prefer to consider Λ as a free parameter rather than to commit ourselves to a particular value.

6. $\pi\pi$E-3NP: EXTENDED NUCLEONS

The introduction into the $\pi\pi$E-3NP of the form factors given by eq. (38) corresponds to employing the following modified Yukawa functions:

$$U(r) = \frac{4\pi}{\mu} \int \frac{d\underset{\sim}{k}}{(2\pi)^3} \frac{e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{\underset{\sim}{k}^2 + \mu^2} \left(\frac{\Lambda^2 - \mu^2}{\Lambda^2 + \underset{\sim}{k}^2}\right)^2 =$$

$$= \frac{e^{-\mu r}}{\mu r} - \frac{\Lambda}{\mu} \frac{e^{-\Lambda r}}{\Lambda r} - \frac{1}{2} \frac{\mu}{\Lambda} \left(\frac{\Lambda^2}{\mu^2} - 1\right) e^{-\Lambda r} . \qquad (39)$$

TABLE 2

Parameters of the πN form factor, eq. (38).

Reference	Process	value of Λ		
		MeV	μ	fm^{-1}
29	dispersion relations	700	5.0	3.5
30	dispersion relations	1200	8.6	6.0
31	$np \to pn$ and $\bar{p}p \to \bar{n}n$	890	6.4	4.5
32	$\gamma p \to \pi^+ n$	1000	7.2	5.1
33, 2	dispersion relations (half of the G.T. discrepancy)	800	5.7	4.0
34	$pp \to n\Delta^+$, $pp \to p\Delta^+$, $\pi^+ p \to \rho^0 \Delta^{++}$	800 1000	5.7 7.2	4.0 5.1
35	dispersion relations (half of the G.T. discrepancy)	800	5.7	4.0
28	NN phase shifts deuteron quadrupole moment	> 1530 > 1200	> 11.0 > 8.6	> 7.7 > 6.1
36	deuteron asymptotic D/S ratio deuteron quadrupole moment	> 1000 > 1400	> 7.2 > 10.0	> 5.1 > 7.1
37	$\gamma d \to pn$	1250	9.0	6.3
	full G.T. discrepancy (6%)	570	4.1	2.9

In the evaluation of the various derivatives of the function $U(r)$ we use the following results and definitions

$$\frac{\partial U(r)}{\partial x_i} = \frac{x_i}{r}\,\frac{\partial U(r)}{\partial r}$$

$$\equiv \frac{x_i}{r}\,\mu\, U_1(r) \quad , \tag{40}$$

$$\frac{\partial^2 U(r)}{\partial x_i\,\partial x_j} = \delta_{ij}\,\frac{1}{r}\,\frac{\partial U(r)}{\partial r} + \frac{x_i x_j}{r^2}\left[\frac{\partial^2 U(r)}{\partial r^2} - \frac{1}{r}\,\frac{\partial U(r)}{\partial r}\right]$$

$$\equiv \delta_{ij}\,\frac{\mu}{r}\,U_1(r) + \frac{x_i x_j}{r^2}\,\mu^2\,U_2(r) \quad , \tag{41}$$

where the x_i are the Cartesian components of $\underset{\sim}{r}$ and

$$U_1(r) \equiv -\left(1+\frac{1}{\mu r}\right)\frac{e^{-\mu r}}{\mu r} + \frac{\Lambda^2}{\mu^2}\left(1+\frac{1}{\Lambda r}\right)\frac{e^{-\Lambda r}}{\Lambda r} + \frac{1}{2}\left(\frac{\Lambda^2}{\mu^2}-1\right)e^{-\Lambda r} \quad , \tag{42}$$

$$U_2(r) \equiv \left(1+\frac{3}{\mu r}+\frac{3}{\mu^2 r^2}\right)\frac{e^{-\mu r}}{\mu r} - \frac{\Lambda^3}{\mu^3}\left(1+\frac{3}{\Lambda r}+\frac{3}{\Lambda^2 r^2}\right)\frac{e^{-\Lambda r}}{\Lambda r}$$

$$-\frac{1}{2}\,\frac{\Lambda}{\mu}\left(\frac{\Lambda^2}{\mu^2}-1\right)\left(1+\frac{1}{\Lambda r}\right)e^{-\Lambda r} \quad . \tag{43}$$

The action of the Laplacian can be obtained either from eq. (41)

$$\underset{\sim}{\nabla}^2 U(r) = 3\,\frac{\mu}{r}\,U_1(r) + \mu^2\,U_2(r) \quad , \tag{44}$$

or from eq. (39)

$$\underset{\sim}{\nabla}^2 U(r) = \frac{4\pi}{\mu}\int \frac{d\underset{\sim}{k}}{(2\pi)^3}\,\frac{\mu^2-(\underset{\sim}{k}^2+\mu^2)}{(\underset{\sim}{k}^2+\mu^2)}\,e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\underset{\sim}{k}^2}\right)^2$$

$$\equiv \mu^2\,[U(r) - G(r)] \quad . \tag{45}$$

In this expression, $G(r)$ is a function proportional to the Fourier transform of the form factor

$$G(r) = \frac{4\pi}{\mu^3}\int \frac{d\underset{\sim}{k}}{(2\pi)^3}\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}} \left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\underset{\sim}{k}^2}\right)^2$$

$$= \frac{1}{2}\,\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2}-1\right)^2 e^{-\Lambda r} \qquad (46)$$

In order to establish contact with our earlier work, it is worth pointing out that in ref.(11) the combination $[U(r) - G(r)]$ was denoted by $U_0(r)$. Comparing eqs. (44) and (45), we obtain

$$\frac{3}{\mu r}\, U_1(r) = U(r) - G(r) - U_2(r) \qquad (47)$$

Finally, it is useful to introduce the notation

$$G_1(r) \equiv \frac{1}{\mu}\,\frac{\partial G(r)}{\partial r} \qquad (48)$$

This function is given by

$$G_1(r) = -\frac{1}{2}\left(\frac{\Lambda^2}{\mu^2}-1\right)^2 e^{-\Lambda r} \qquad (49)$$

6.1. THE POTENTIAL

Using the results given above, it is possible to rewrite the various parts of the ππE-3NP, given by eqs. (27-29) as follows

$$W_s(i) = \frac{C_s}{3}\,\underset{\sim}{\tau}^{(j)}\cdot\underset{\sim}{\tau}^{(k)} \Big\{ U_1(r_{ij})U_1(r_{ki}) - G_1(r_{ij})\,U_1(r_{ki}) - U_1(r_{ij})\,G_1(r_{ki}) \Big\} \times$$
$$\times \Big[\cos\theta_i\,\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\sigma}^{(k)} + S_{jk}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ki})\Big] \qquad (50)$$

$$W_p(i) = \frac{C_p}{9}\,\underset{\sim}{\tau}^{(j)}\cdot\underset{\sim}{\tau}^{(k)} \Big\{ \Big[(U(r_{ij}) - G(r_{ij}))\,(U(r_{ki}) - G(r_{ki}))$$
$$+ 2P_2(\cos\theta_i)\,U_2(r_{ij})\,U_2(r_{ki}) \Big]\,\underset{\sim}{\sigma}^{(j)}\cdot\underset{\sim}{\sigma}^{(k)}$$
$$+ \Big[U(r_{ij}) - G(r_{ij}) - U_2(r_{ij})\Big]\,U_2(r_{ki})\,S_{jk}(\hat{\underset{\sim}{r}}_{ki},\hat{\underset{\sim}{r}}_{ki})$$
$$+ U_2(r_{ij})\Big[U(r_{ki}) - G(r_{ki}) - U_2(r_{ki})\Big]\,S_{jk}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ij})$$
$$+ 3\cos\theta_i\,U_2(r_{ij})\,U_2(r_{ki})\,S_{jk}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ki}) \Big\} \qquad (51)$$

$$W_p'(i) = \frac{C_p'}{9}\,(i\,\underset{\sim}{\tau}^{(i)}\cdot\underset{\sim}{\tau}^{(j)}\times\underset{\sim}{\tau}^{(k)}) \Big\{ \Big[(U(r_{ij}) - G(r_{ij}))\,(U(r_{ki}) - G(r_{ki})) \times$$
$$\times (i\,\underset{\sim}{\sigma}^{(i)}\cdot\underset{\sim}{\sigma}^{(j)}\times\underset{\sim}{\sigma}^{(k)}) \Big]$$
$$+ \Big[U(r_{ij}) - G(r_{ij})\Big]\,U_2(r_{ki})\,\frac{1}{2}\Big[S_{jk}(\hat{\underset{\sim}{r}}_{ki},\hat{\underset{\sim}{r}}_{ki})\,\underset{\sim}{\sigma}^{(i)}\cdot\underset{\sim}{\sigma}^{(j)}$$
$$- \underset{\sim}{\sigma}^{(i)}\cdot\underset{\sim}{\sigma}^{(j)}\,S_{jk}(\hat{\underset{\sim}{r}}_{ki},\hat{\underset{\sim}{r}}_{ki})\Big]$$
$$+ U_2(r_{ij})\Big[U(r_{ki}) - G(r_{ki})\Big]\,\frac{1}{2}\Big[-S_{jk}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ij})\,\underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\sigma}^{(i)}$$
$$+ \underset{\sim}{\sigma}^{(k)}\cdot\underset{\sim}{\sigma}^{(i)}\,S_{jk}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ij})\Big] +$$

$$+ U_2(r_{ij})\, U_2(r_{ki})\, \frac{1}{2}\Big[S_{ij}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ij})\, S_{ki}(\hat{\underset{\sim}{r}}_{ki},\hat{\underset{\sim}{r}}_{ki}) - S_{ki}(\hat{\underset{\sim}{r}}_{ki},\hat{\underset{\sim}{r}}_{ki})\, S_{ij}(\hat{\underset{\sim}{r}}_{ij},\hat{\underset{\sim}{r}}_{ij}) \Big]\Big]\Big\} \tag{52}$$

where the tensor S_{ij} for two unit vectors $\hat{\underset{\sim}{u}}$ and $\hat{\underset{\sim}{v}}$ is defined as

$$S_{ij}(\hat{\underset{\sim}{u}},\hat{\underset{\sim}{v}}) \equiv 3(\underset{\sim}{\sigma}^{(i)}\cdot\hat{\underset{\sim}{u}})(\underset{\sim}{\sigma}^{(j)}\cdot\hat{\underset{\sim}{v}}) - \hat{\underset{\sim}{u}}\cdot\hat{\underset{\sim}{v}}\,\underset{\sim}{\sigma}^{(i)}\cdot\underset{\sim}{\sigma}^{(j)} \tag{53}$$

and

$$\cos\theta_i \equiv \frac{\underset{\sim}{r}_{ij}\cdot\underset{\sim}{r}_{ki}}{r_{ij}\, r_{ki}} = \hat{\underset{\sim}{r}}_{ij}\cdot\hat{\underset{\sim}{r}}_{ki} \;. \tag{54}$$

The expressions for W_p and W_p' are the same as those of eq. (67) of ref. (11), except for an unfortunate misprint in the sign of the term proportional to C_p' in that equation. On the other hand, the expression for W_s includes now $G_1(r)$, the derivative of the function $G(r)$, due to the factor $(\nabla^2 - \mu^2)$.

7. EXPECTATIONS FROM FORM FACTORS

The introduction of form factors in to the ππE-3NP derived by means of chiral symmetry is accompanied by expectations at various levels.

Expectation 1: At the first place, we expect that form factors should render the potential regular at the origin. For the form factor of eq. (38) this is indeed the case, as we can see

when inspecting the explicit forms of the functions U, U_1, U_2, G and G_1, responsible for the spacial content of the potential and given in the previous section. In this case, the regularization is caused by the powers of $\underset{\sim}{k}^2$ present in its denominator, that act effectively as a cut off for high values of the momenta. It is interesting to note that the regularization of the potential would not be achieved if only a single monopole form factor were used, since this would correspond to correcting the contribution of a given pion as follows

$$\frac{1}{\underset{\sim}{k}^2+\mu^2} \longrightarrow \frac{1}{\underset{\sim}{k}^2+\mu^2}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\underset{\sim}{k}^2}\right) = \frac{1}{\underset{\sim}{k}^2+\mu^2} - \frac{1}{\underset{\sim}{k}^2+\Lambda^2} \;. \tag{55}$$

Thus, a single monopole form factor is equivalent to the introduction of a particle of mass Λ, with the same quantum numbers and coupling constant as the pion. The various derivatives of the Fourier transform of an amplitude "corrected" in this way are not regular at the origin.

Expectation 2: In the parametrization of the form factor adopted here, two values of Λ are of special significance, namely $\Lambda\to\infty$ and $\Lambda=\mu$. The former represents the absence of the form factor, whereas the latter causes the vanishing of the whole ππE-3NP. Realistic values of Λ correspond to a compromise between these two extreme situations, allowing us to understand the introduction of form factors as the result of a continuous variation of Λ, from infinity up to the desired phenomenological value. As discussed in section 5, there is no consensus as to which value of Λ should be preferred in the πN form factor, since several values are suggested by different studies. Therefore we adopt the standpoint that the equations representing a useful potential must be valid for generic values of Λ. This means that, in the construction of this potential, we are concerned with expressions where Λ is to be considered as a free parameter, whose value could either be switched off or fixed, according to the particular case where they are being applied. So, our second expectation is that this variation should be followed by a somehow monotonic variation of the potential. In other words, we expect that nothing very dramatic should happen when form factors are turned on.

Expectation 3: The third thing we expect is the modifications produced by the introduction of form factors to be confined to small internucleon distances. This is due to the fact that the potential for point-like nucleons is already suited for distances of the order or greater than μ^{-1} and cannot change much depending on whether form factors are present or not.

Expectation 4: Finally, we expect the contributions of ad hoc form factors to be restricted to playing a secondary role in the potential, namely that of a mere correction to non-relativistic results. This means that the hyerarchy between what is being corrected and the correction must be preserved: the latter cannot be more important than the former.

8. THE PROBLEM

A striking feature of the $\pi\pi$E-3NP for extended nucleons, as given by eqs. (50-52), is that it fails to fulfil the expectations 2-4 presented in the previous section. In the sequence we reproduce an instance[9] where the introduction of form factors into the

potential does not just regularize it at the origin. In this case, instead, it determines the behaviour of the 3NP in a much larger region, strongly distorting its original form. In order to show this, we display the qualitative features of the expectation value of the operator W_s on the principal S wave of a trinucleon system, which is characterized by totally anti-symmetric spin and isospin states[9]. This expectation value is expressed as

$$\langle S|W_s|S\rangle = -S^2(\underset{\sim}{x},\underset{\sim}{y})\, C_s\Big\{\cos\theta_i \Big[U_1(r_{ij})\, U_1(r_{ki}) - G_1(r_{ij})\, U_1(r_{ki}) - U_1(r_{ij})\, G_1(r_{ki})\Big] + \text{cyclic permutations}\Big\} \qquad (56)$$

where $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are Jacobi coordinates, and $S(\underset{\sim}{x},\underset{\sim}{y})$ is a fully symmetric function under nucleon permutations.

The main features of $\langle S|W_s|S\rangle$ can be understood with the help of equipotential plots[27,9,10], which are very useful due to their simplicity. These equipotential plots are constructed by fixing the positions of two of the nucleons and using the third one as a probe. The coordinate system employed in the description of the trinucleon is shown in fig. 3. All diagrams are symmetric under rotations around the X direction and under reflection about the Y direction. Hence the specification of a single quadrant is enough to determine uniquely the complete spacial diagram. We adopt the value $r_{12} = 0.88$ fm for the fixed internucleon distance, corresponding to the minimum of the Reid two-body potential[4].

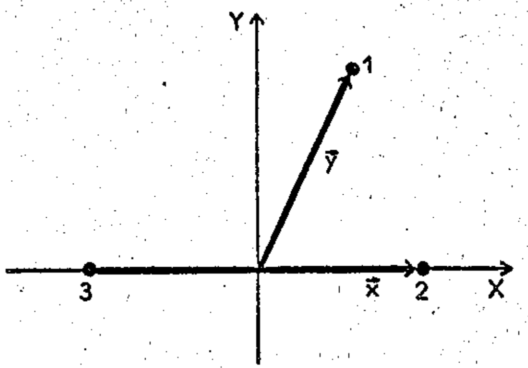

Fig.3. Coordinates of the trinucleon system.

In order to follow the dependence of eq. (56) on Λ, in fig. 4 we show the equipotential plots representing $\langle S|W_s|S\rangle$ for six values of this parameter, ranging between infinity and the still realistic value of $4\,\mathrm{fm}^{-1}$. As pointed out before, the first value corresponds to the absence of form factors. The first and last plots of this figure are very different, meaning that the diagrams for realistic values of Λ are not just corrected versions of that without form factors. The modifications produced by the variation of Λ on the ππE-3NP are not confined to small internucleon distances, showing that the form factor, that should just correct the potential, as a matter of fact, determines its most important features. A closer inspection of this figure shows that the potential for point-like nucleons is so different from that corresponding to $4\,\mathrm{fm}^{-1}$, that the latter can be thought as a new potential. In both cases, the lines along which the matrix element $\langle S|W_s|S\rangle$ is zero are roughly parallel, but they delimit regions that have changed sign and magnitude when form factors were introduced.

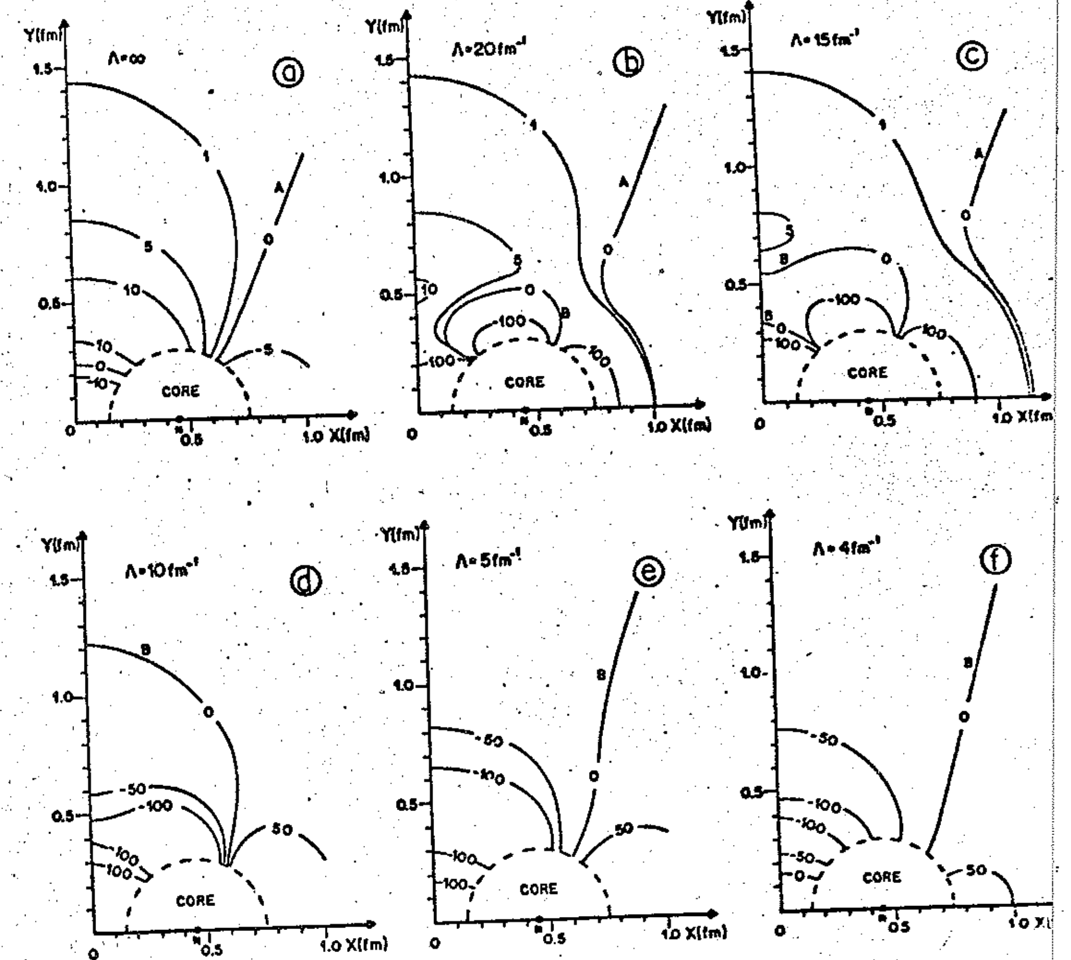

Fig.4. The influence of form factors on $\langle S|W_s|S\rangle$. The equipotential curves are symmetric about the X and Y axes. The point N indicates the position of one of the fixed nucleons; the other one is located symmetrically about the origin. All energies are given in MeV. The various values of Λ are shown in the figures; Λ=∞ corresponds to the absence of form factors. The understanding of these figures becomes easier when the evolution of the lines labeled A and B is followed.

Considering the whole sequence of figures, it is possible to notice an interesting feature, namely that the "new" potential springs from within the core region as the form factor is turned on. It is barely visible in fig.4b, gradually expels that corresponding to point-like nucleons, until it becomes completely dominant in the last two figures. This analysis can provide a hint as to what is the cause of this odd behaviour.

The fact that deformations emerge from the core region means that, at short distances, the potential is too sensitive to the value of Λ. On the other hand, the matrix element given by eq. (56) depends on Λ through two functions, namely $U_1(r)$ and $G_1(r)$. When r tends to zero, these functions behave as follows:

$$\lim_{r\to 0} U_1(r) = 0 \qquad (57)$$

$$\lim_{r\to 0} G_1(r) = -\frac{1}{2}\left(\frac{\Lambda^2}{\mu^2} - 1\right)^2 .$$

These results suggest strongly that the excessive sensitivity of the potential to the value of Λ is associated with the functions $G(r)$ and its derivative, $G_1(r)$.

9. THE DYNAMICAL MEANING OF G(r)

The physical meaning of the terms proportional to $G(r)$ entering the $\pi\pi$E-3NP is given by the dynamical content of the πN form factor. When no form factors are present, the function $G(r)$, given by eq. (46), becomes

$$G(r) = \frac{4\pi}{\mu^3}\,\delta^3(\underset{\sim}{r}) , \qquad (58)$$

whereas eq. (45) indicates that $U(r)$ turns into the Green's function associated with the propagator of point-like particles. In this case, the function $G(r)$ describes an interaction which is effective only when two nucleons concur at the same point in space and have contact.

These contact interactions yield a 3NP corresponding to permutations of the configuration space diagrams of fig. 5a. In it, the black dot represents the term associated with G, whereas the broken line is a pion-propagation given by the regularized Yukawa function $U(r)$.

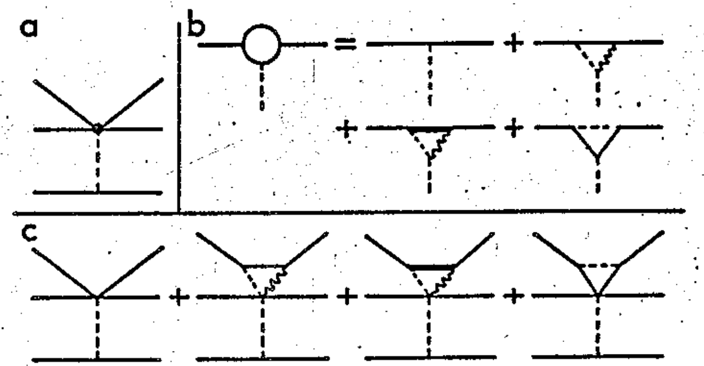

Fig.5. Configuration space diagrams: (a) contribution to the 3NP due to a contact interaction between two point-like nucleons; (b) contributions to the πN form factor; (c) contributions to the 3NP due to "contact" interactions between extended nucleons. In these figures, continuous and thick lines represent nucleons and deltas, whereas broken and wavy lines represent pions and rhos.

When we consider form factors, the function $\bar{G}(k^2)$ is not equal to one and $G(r)$, its Fourier transform, describes the distribution of hadronic matter within the nucleon. Therefore the terms containing this function in the $\pi\pi$E-3NP correspond to "contact" interactions between extended objects. In order to make this statement more precise, we consider the dynamical content of the πN form factor at the hadron level. In the context of the chiral $SU(2) \times SU(2)$ group, it corresponds to diagrams such as those of fig. 5b. So, the "contact" interactions between extended objects contribute to the 3NP represented in fig. 5c. This last figure means that the terms in the potential containing $G(r)$ cannot be associated with the propagation of pions between different points in space and do not correspond to a proper $\pi\pi$E-3NP. Rather, they could be called π?E-3NP, since the parameter of the form factor does not let us know the type of particles being exchanged.

In other words, the inclusion of the function $G(r)$ into the potential means that we are considering forces whose dynamical content remains hidden behind a parametrization. This makes it difficult to understand which are the Feynman diagrams one is including in the potential. Of course, the diagrams shown in fig. 5c should be evaluated at some stage of the research program on three-body forces, but, their inclusion should be the result of explicit calculations, using an appropriate dynamics such as chiral symmetry. Moreover, in this research program, the study of many other processes such as pion-rho[38,39], rho-rho[38], pion-omega and three-pion exchanges should precede those of fig. 5c, since they correspond to forces of longer range.

10. THE REDEFINITION OF THE $\pi\pi$E-3NP

The discussion of the preceding section shows that the three-nucleon potential given by eqs. (50-52) is composed of two types of terms. One of them, containing the Fourier transform of the form factor, describes a contact interaction between two extended nucleons and is associated with the exchange of several different particles. The other one contains only the functions $U(r)$, that correspond to the spacial propagation of pions. The considerations produced above allow us to conclude that only the latter deserves the name of $\pi\pi$E-3NP, as opposed to π?E-3NP.

All the problems that we have mentioned regarding $W_{\pi\pi}$ can be avoided when we redefine the $\pi\pi$E-3NP as the potential associated only with the propagation of pions described by the regularized Yukawa functions $U(r)$ and its derivatives. This corresponds to the full elimination of $G(r)$ and $G_1(r)$ from eqs. (50-52), producing the potential $\hat{W}_{\pi\pi}$, where the caret indicates this modification. The exclusion of $G(r)$ and $G_1(r)$ from the original $\pi\pi$E-3NP for extended nucleons amounts to stating that we should regularize the results of chiral symmetry by eliminating all possible δ-function before the inclusion of the form factors. In doing the opposite we would be using form factors to regularize a δ-function.

In order to produce a feeling about the numerical meaning of the redefinition of the potential, we show in fig. 6 the behaviour of the functions $U, -U_1, U_2, G$ and $-G_1$ for two values of Λ, namely ∞ and $5\ fm^{-1}$. In this figure we learn that the introduction of the form factor into the functions U, U_1 and U_2 suppresses considerably their short distance behaviour, whereas a δ-function and its derivative become G and G_1 respectively. Moreover, when $\Lambda = 5\ fm^{-1}$, we note that G and G_1 are strongly dominant in the short and intermediate distance regions, whereas U, U_1 and U_2 determine the behaviour of the potential for large distances. The main consequence of the introduction of form factors is, therefore, the partial substitution of short distance behaviour of U, U_1 and U_2 by that of G and G_1. Thus the proposed redefinition scheme produces a potential that has the long distance properties associated with the pion chiral dynamics and is weak at small distances. In other words, the redefinition amounts to keeping only the tail of the ππE-3NP and deliberately disregarding the uncertainties associated with the short distance region.

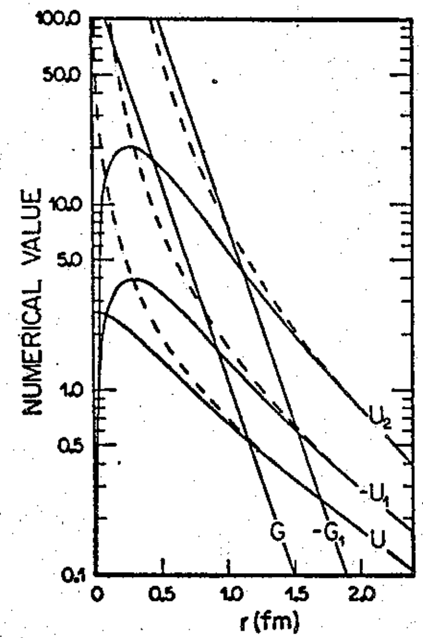

Fig. 6 (revised version)

10.1. THE QUALITATIVE BEHAVIOUR OF $\tilde{W}\pi\pi$

The problems associated with the inclusion of form factors into the ππE-3NP, which were discussed in section 8, are no longer present in the redefined version of the potential. In order to see this, we consider the modifications induced in $\langle S|\tilde{W}_s|S\rangle$ by the inclusion of form factors, that can be followed by inspecting fig. 7, the analog of fig. 4 for the redefined version of the potential. We note that now the differences between the various plots are much less pronounced, and thus the influence of the form factor tends to be confined, as it must, to small internucleon distances.

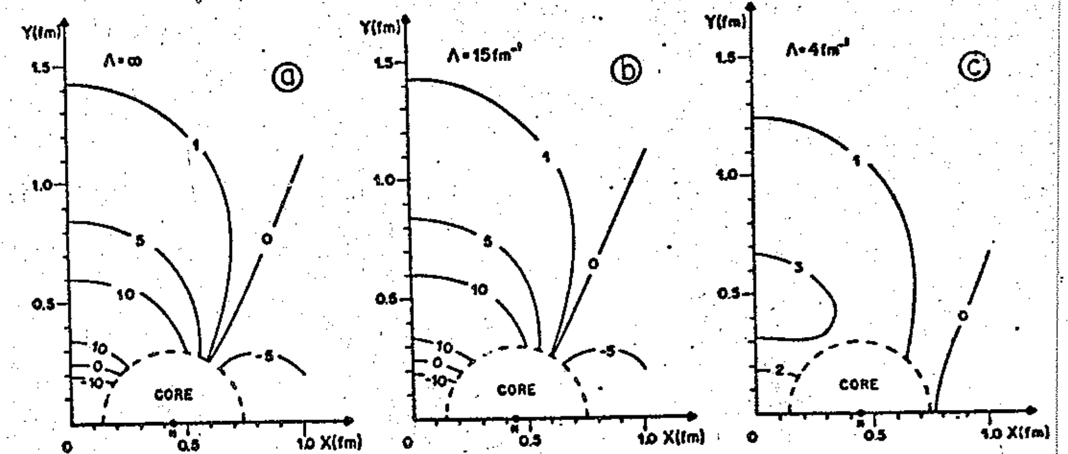

Fig 7. The influence of form factors on $\langle S|W_s|S\rangle$. Conventions as in fig.4.

The redefinition of the potential also affects other results. For instance, the couplings between the principal S and D states of trinucleon systems[10] due to the original components W_s, W_p and W'_p of the $\pi\pi$E-3NP are given in figs. 8a-c, whereas those corresponding to the redefined version are given in figs. 8d-f. These diagrams show once more that considerable changes are produced by the redefinition of the potential. The matrix elements $\langle D|W_s|S\rangle$ and $\langle D|\hat{W}_s|S\rangle$, in particular, have opposite signs and quite different magnitudes, similarly to the case of $\langle S|W_s|S\rangle$ and $\langle S|\hat{W}_s|S\rangle$, discussed above. These differences are entirely caused by the form factor, showing that it completely determines the contribution of the original potential.

11. SUMMARY AND CONCLUSIONS

In this work we have analyzed the construction of the $\pi\pi$E-3NP in the cases of both point-like and extended nucleons. In the former case, we have emphasized that the crucial ingredient is the treatment of the intermediate πN amplitude by means of chiral symmetry, in consistency with on shell data. We have compared the current algebra and effective Lagrangian approaches to the implementation of the symmetry and shown that they produce essentially the same results.

As far as the case of extended nucleons is concerned, we have argued that the expressions derived by the Tucson-Melbourne group[5,6] and by ourselves in 1983[11] need to be modified in order to be used in realistic calculations. The problem with these potentials is that they become strongly distorted when form factors are present. These distortions are

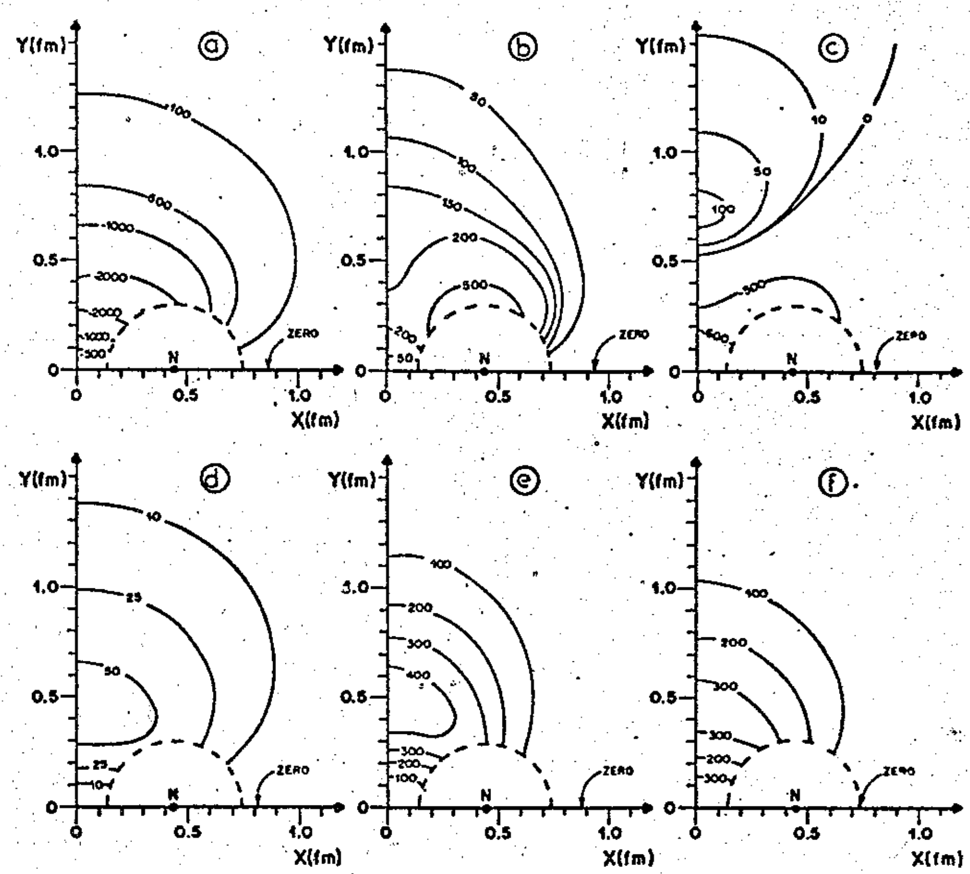

Fig. 8. Equipotentials for (a) $\langle D|W_s|S\rangle$, (b) $\langle D|W_p|S\rangle$, (c) $\langle D|W'_p|S\rangle$, (d) $\langle D|\hat{W}_s|S\rangle$, (e) $\langle D|\hat{W}_p|S\rangle$, (f) $\langle D|\hat{W}'_p|S\rangle$. Conventions as in fig.4.

incompatible with our expectation that form factors should just modify the potential for point-like nucleons at small distances. As discussed in sects. 8 and 9, this odd behaviour is associeated with terms in the potential that can be interpreted as "contact" interactions between extended nucleons. We have argued that the elimination of these "contact" terms from the potential makes it much better behaved. Therefore we have proposed the redefinition of the $\pi\pi$E-3NP as the potential associated only with the propagation of pions described by Yukawa functions regularized at the origin. This new version of the potential is formally obtained by eliminating $G(r)$ and $G_1(r)$ from eqs. (50-52).

Before displaying the redefined potential, however, it is useful to compare it with other forms. We note that eqs. (50-52) describe the generic TM potential, whereas our version of 1983 can be obtained from these equations by keeping the function $G(r)$ and disregarding $G_1(r)$. Thus our 1983 potential can be understood as a hybrid situation between the TM and the fully redefined version presented in this paper. As we have discussed in the main text, the motivations for the redefinition of the potential are such that the present version excludes that of 1983 and should replace it as an alternative to the TM force.

Except for a qualitative study[10], at present there are no calculations considering the effects of the full redefinition of the potential on trinucleon observables. However, as we have discussed in the introduction, great sensitivity of numerical results to a change from the TM to our 1983 version of the $\pi\pi$E-3NP entitles us to believe that the same will hold for the redefined potential proposed here. It is given by

$$\hat{W} = \sum_{i=1}^{3} \left(\hat{W}_s(i) + \hat{W}_p(i) + \hat{W}'_p(i)\right) \tag{59}$$

where

$$\hat{W}_s(i) = \frac{C_s}{3}\, \boldsymbol{\tau}^{(j)} \cdot \boldsymbol{\tau}^{(k)}\, U_1(r_{ij})\, U_1(r_{ki}) \left[\cos\theta_i\, \boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} + S_{jk}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ki})\right] \tag{60}$$

$$\begin{aligned} \hat{W}_p(i) = \frac{C_p}{9}\, \boldsymbol{\tau}^{(j)} \cdot \boldsymbol{\tau}^{(k)} \Big\{ & \left[U(r_{ij})\, U(r_{ki}) + 2P_2(\cos\theta_i)\, U_2(r_{ij})\, U_2(r_{ki})\right] \boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} \\ & + \left[U(r_{ij}) - U_2(r_{ij})\right] U_2(r_{ki})\, S_{jk}(\hat{\boldsymbol{r}}_{ki}, \hat{\boldsymbol{r}}_{ki}) \\ & + U_2(r_{ij}) \left[U(r_{ki}) - U_2(r_{ki})\right] S_{jk}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ij}) \\ & + 3\cos\theta_i\, U_2(r_{ij})\, U_2(r_{ki})\, S_{jk}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ki}) \Big\} \end{aligned} \tag{61}$$

$$\begin{aligned} \hat{W}'_p(i) = \frac{C'_p}{9} \left(i\, \boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)} \times \boldsymbol{\tau}^{(k)}\right) \Big\{ & \left[U(r_{ij})\, U(r_{ki})\; i\, \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)} \times \boldsymbol{\sigma}^{(k)}\right] \\ & + U(r_{ij})\, U_2(r_{ki})\, \frac{1}{2} \left[S_{jk}(\hat{\boldsymbol{r}}_{ki}, \hat{\boldsymbol{r}}_{ki})\, \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)} - \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)}\, S_{jk}(\hat{\boldsymbol{r}}_{ki}, \hat{\boldsymbol{r}}_{ki})\right] \\ & + U_2(r_{ij})\, U(r_{ki})\, \frac{1}{2} \left[- S_{jk}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ij})\, \boldsymbol{\sigma}^{(k)} \cdot \boldsymbol{\sigma}^{(i)} + \boldsymbol{\sigma}^{(k)} \cdot \boldsymbol{\sigma}^{(i)}\, S_{jk}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ij})\right] \\ & + U_2(r_{ij})\, U_2(r_{ki})\, \frac{1}{2} \left[S_{ij}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ij})\, S_{ki}(\hat{\boldsymbol{r}}_{ki}, \hat{\boldsymbol{r}}_{ki}) - S_{ki}(\hat{\boldsymbol{r}}_{ki}, \hat{\boldsymbol{r}}_{ki})\, S_{ij}(\hat{\boldsymbol{r}}_{ij}, \hat{\boldsymbol{r}}_{ij})\right] \end{aligned} \tag{62}$$

The meaning of the various functions entering this expression can be found in sections 4 and 6, whereas the values of the strength parameters are given in table 1.

As a final comment, we would like to stress that $\hat{W}$, the redefined potential, is not the outcome of indisputable formal derivations. Instead, it is closely associated with a particular way of interpreting a physical picture allowed by the mathematical formalism. However, we believe that this is as far as we can go for the time being. The test in realistic calculations of the conclusions presented in this work would be extremely useful in improving our understanding of the problem.

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TABLE 1

Strength parameters of the ππE-3NP.

Coelho, Das and Robilotta[11]		Coon and Glöckle[6]	
parameter	value in MeV	parameter	value in MeV multiplied by $\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2$
c_s	+ 0.92	a μ^2	0.99
		c μ^4	0.88
c_p	− 1.99	b μ^4	− 2.26
c'_p	− 0.67	$(d_3+d_4)\mu^4$	− 0.66

TABLE 2

Parameters of the πN form factor, eq. (38).

Reference	Process	value of Λ		
		MeV	μ	fm^{-1}
29	dispersion relations	700	5.0	3.5
30	dispersion relations	1200	8.6	6.0
31	$np \rightarrow pn$ and $\bar{p}p \rightarrow \bar{n}n$	890	6.4	4.5
32	$\gamma p \rightarrow \pi^+ n$	1000	7.2	5.1
33, 2	dispersion relations (half of the G.T. discrepancy)	800	5.7	4.0
34	$pp \rightarrow n\Delta^+$, $pp \rightarrow p\Delta^+$, $\pi^+ p \rightarrow \rho^0 \Delta^{++}$	800 1000	5.7 7.2	4.0 5.1
35	dispersion relations (half of the G.T. discrepancy)	800	5.7	4.0
28	NN phase shifts deuteron quadrupole moment	> 1530 > 1200	> 11.0 > 8.6	> 7.7 > 6.1
36	deuteron asymptotic D/S ratio deuteron quadrupole moment	> 1000 > 1400	> 7.2 > 10.0	> 5.1 > 7.1
37	$\gamma d \rightarrow pn$	1250	9.0	6.3
	full G.T. discrepancy (6%)	570	4.1	2.9

FIGURE CAPTIONS

Fig.1. Diagram corresponding to the ππE-3NP.

Fig.2. Diagrams corresponding to πN scattering; continuous and thick lines represent nucleons and deltas, whereas broken, wavy and double ones represent pions, rhos and sigma.

Fig.3. Coordinates of the trinucleon system.

Fig.4. The influence of form factors on $\langle S|W_s|S\rangle$. The equipotential curves are symmetric about the X and Y axes. The point N indicates the position of one of the fixed nucleons; the other one is located symmetrically about the origin. All energies are given in MeV. The various values of Λ are shown in the figures; $\Lambda=\infty$ corresponds to the absence of form factors. The understanding of these figures becomes easier when the evolution of the lines labeled A and B is followed.

Fig.5. Configuration space diagrams: (a) contribution to the 3NP due to a contact interaction between two point-like nucleons; (b) contributions to the πN form factor; (c) contributions to the 3NP due to "contact" interactions between extended nucleons. In these figures, continuous and thick lines represent nucleons and deltas, whereas broken and wavy lines represent pions and rhos.

Fig.6. Behaviour of the functions U, $-U_1$, U_2, G and $-G_1$ for $\Lambda\to\infty$ (broken lines) and $\Lambda = 5\ fm^{-1}$ (continuous lines).

Fig.7. The influence of form factors on $\langle S|W_s|S\rangle$. Conventions as in fig.4.

Fig.8. Equipotentials for (a) $\langle D|W_s|S\rangle$, (b) $\langle D|W_p|S\rangle$, (c) $\langle D|W_p'|S\rangle$, (d) $\langle D|\hat{W}_s|S\rangle$, (e) $\langle D|\hat{W}_p|S\rangle$, (f) $\langle D|\hat{W}_p'|S\rangle$. Conventions as in fig.4.