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PUBLICAÇÕES

ON THE EQUIVALENCE OF DILUTE ANTIFERROMAGNETS
AND FERROMAGNETS IN RANDOM EXTERNAL FIELDS:
CURIE-WEISS MODELS

by

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IN RANDOM EXTERNAL FIELDS: CURIE-WEISS MODELS

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ABSTRACT

Using a method proposed by van Hemmen we compute the free energy of the Curie-Weiss version of the site-dilute antiferromagnetic Ising model in the presence of an uniform magnetic field. The solution displays an exact correspondence between this model and the Curie-Weiss version of the Ising model in the presence of a random magnetic field.   The phase diagrams are discussed and a tricritical point is shown to exist.

Considerable attention has been devoted in the literature to the study of the Ising model in the presence of random magnetic fields (RMF) [1,6,7]. The critical behavior and the phase diagram of this model are believed to be related to those of a site dilute antiferromagnetic Ising model in the presence of an applied uniform magnetic field (DAF) [2,3]. This is specially relevant since site-dilute magnetic materials can be prepared and the magnetic-ordering transition as a function both of the external field and of the dilution of the magnetic species has been experimentally investigated [4].

In this letter we use an approach introduced by van Hemmen [5] to compute the free energy and the phase diagram of a Curie-Weiss mean field version of a DAF model. We then compare our results to those obtained by Salinas and Wrezinski [6] for the Curie-Weiss version of a RMF model and establish an exact correspondence between the parameters and phase diagrams of the two-models, as to show their complete equivalence.

Our mean field DAF model is described in a finite-volume $\Lambda \subset \mathbb{Z}^\nu$ by the hamiltonian

$$H_{DAF} = -\frac{J}{N} \sum_{i,j \in \Lambda_e} \epsilon_i \epsilon_j \sigma_i \sigma_j - \frac{J}{N} \sum_{i,j \in \Lambda_o} \epsilon_i \epsilon_j \sigma_i \sigma_j$$

$$+ \frac{J}{N} \sum_{\substack{i \in \Lambda_e \\ j \in \Lambda_o}} \epsilon_i \epsilon_j \sigma_i \sigma_j + H \sum_{i \in \Lambda} \epsilon_i \sigma_i \qquad (1)$$

where $\Lambda_e = \Lambda \cap \mathbb{Z}^\nu_e$, $\Lambda_o = \Lambda \cap \mathbb{Z}^\nu_o$ with $\mathbb{Z}^\nu_e (\mathbb{Z}^\nu_o)$ being the sublattice of $\mathbb{Z}^\nu$ for which the sum of coordinates of each site are even (odd) integers. The interaction is anti-ferromagnetic (J > 0) between sites in different sublattices and ferromagnetic between sites in the same sublattice. The random

variables $\epsilon_i \in \{0, 1\}$ describe the site dilution and they are taken to independent and identically distributed, with

$$\epsilon_i = \begin{cases} 1, & \text{probability} \quad p \\ 0, & \text{probability} \quad 1-p \end{cases}$$

The spin variables σ_i are, for simplicity, taken to be of Ising type: $\sigma_i = \pm 1$. The external magnetic field H is uniform and deterministic. N denotes the number of points in Λ.

Introducing the sub-lattice magnetizations:

$$m_e = \frac{1}{N} \sum_{i \in \Lambda_e} \epsilon_i \sigma_i \quad , \quad m_o = \frac{1}{N} \sum_{i \in \Lambda_o} \epsilon_i \sigma_i$$

the hamiltonian (1) can be written in the form

$$H_{DAF} = -N \frac{J}{2} (m_e - m_o)^2 + NH (m_e + m_o) \tag{2}$$

As proved by van Hemmen [5] the quenched free energy density $f(\beta)$ of the system at inverse temperature β is given by:

$$-\beta f(\beta) = \max_{\vec{m} \in \mathbb{R}^2} \{ Q(\vec{m}) + c^*(\vec{m}) \} \tag{3}$$

where $\vec{m}$ is the two-component vector $\vec{m} = (m_e, m_o)$,

$$Q(\vec{m}) = -\frac{\beta}{N} H_{DAF} \tag{4}$$

$$c^*(\vec{m}) = -\sup_{\vec{t} \in \mathbb{R}^2} (\vec{m} \cdot \vec{t} - c(\vec{t})) \tag{5}$$

and

$$c(\vec{t}) = \lim_{N \to \infty} \frac{1}{N} \ln \left[\frac{1}{2^N} \sum_{\{\sigma_i = \pm 1\}} \exp N \vec{t} \cdot \vec{m} \right] \tag{6}$$

We then compute:

$$c(\vec{t}) = \frac{1}{2} \lim_{N \to \infty} \left\{ \frac{2}{N} \sum_{i \in \Lambda_e} \ln \cosh(t_e \epsilon_i) + \frac{2}{N} \sum_{i \in \Lambda_o} \ln \cosh(t_o \epsilon_i) \right\}$$

$$= \frac{1}{2} \left\{ \overline{\ln \cosh(t_e \epsilon)} + \overline{\ln \cosh(t_o \epsilon)} \right\} \tag{7}$$

where $\bar{A}$ denotes the average of the quantity $A(\epsilon)$ with respect to the probability distribution of the random variable ϵ. The last equality in (7) is a consequence of the strong law of large numbers, which is a key ingredient in the technique of van Hemmen. Therefore

$$c(\vec{t}) = \frac{p}{2} \ln \cosh t_e + \frac{p}{2} \ln \cosh t_o \tag{8}$$

If we now introduce the order parameter $m = m_e - m_o$, the free energy is given by

$$f_{DAF}(\beta, J, p, H) = \frac{1}{2} J m^2 - \frac{p}{2\beta} \ln[\cosh \beta (Jm - H)]$$

$$- \frac{p}{2\beta} \ln[\cosh \beta (Jm + H)] \tag{9}$$

with m determined by the equation

$$\frac{2m}{p} = tgh[\beta(Jm \cdot H)] + tgh[\beta(Jm - H)] \tag{10}$$

or equivalently

$$f_{DAM}(\beta, J, p, H) = \inf_m \, g_{DAM}(\beta, J, p, H, m) \tag{11}$$

where g_{DAM}, the free energy density in the magnetization ensemble, is defined by the r.h.s. of (9).

Let us now consider the Curie-Weiss RMF hamiltonian

$$H_{RMF} = -\frac{J}{N} \sum_{i,j \in \Lambda} \sigma_i \sigma_j + \sum_{i \in \Lambda} h_i \sigma_i \tag{12}$$

where the random variables h_i, $i \in \Lambda$ are independent and identically distributed, being equal to $\pm h$ $(h > 0)$ with probability $\frac{1}{2}$. Applying the method of van Hemmen to H_{RMF}, the free energy density can be computed [6]:

$$f_{RMF}(\beta, J, h) = \frac{1}{2} J m^2 - \frac{1}{2\beta} \ln[\cosh\beta(Jm-h)]$$
$$- \frac{1}{2\beta} \ln[\cosh\beta(Jm+h)] \tag{13}$$

with the order parameter m determined by

$$2m = \operatorname{tgh}[\beta(Jm-h)] + \operatorname{tgh}[\beta(Jm+h)] \tag{14}$$

or equivalently

$$f_{RMF}(\beta, J, h) = \inf_m \, g_{RMF}(\beta, J, h, m) \tag{15}$$

with g_{RMF} defined as the r.h.s. of (13).

Comparing expressions (9) and (13) we see that:

$$f_{DAF}(\beta, J, p, H) = p \, f_{RMF}(\beta, pJ, H) \tag{16}$$

and

$$g_{DAF}(\beta, J, p, H, m) = p \, g_{RMF}\left(\beta, pJ, H, \frac{m}{p}\right) \tag{17}$$

which establishes the correspondence between the two models.

As in [6] the phase diagram of the DAF model can be analysed by expanding g_{DAM} in powers of m around $m=0$:

$$g_{DAF}(m) = A + B m^2 + C m^4 + D m^6 + \cdots \tag{18}$$

where

$$A = -\frac{p}{\beta} \ln \cosh \beta H$$

$$B = \frac{J}{2} - \frac{p}{2\beta}(\beta J)^2 \operatorname{sech}^2 \beta H$$

$$C = \frac{p}{2\beta}(\beta J)^4 (\operatorname{sech}^2 \beta H)(1 - 3\operatorname{tgh}^2 \beta H) \tag{19}$$

$$D = -\frac{p}{6\beta}(\beta J)^6 (\operatorname{sech}^2 \beta H)\left(\operatorname{tgh}^4 \beta H - \operatorname{tgh}^2 \beta H + \frac{2}{15}\right)$$

There is a critical line determined by the condition $B=0$ i.e.

$$p\beta J = \cosh^2 \beta H \tag{20}$$

On this critical line, there is a tri-critical point determined by the conditions $B=C=0$, $D>0$:

$$p\beta J = \frac{3}{2} \quad , \quad \operatorname{tgh}^2 \beta H = \frac{1}{3} \tag{21}$$

The critical line represents a first order phase transition

for $p\beta J > \frac{3}{2}$ (since $c < 0$) and a second order phase transition

for $p\beta J < \frac{3}{2}$ (since $c > 0$).

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