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DOUBLE NN SCATTERING EFFECTS IN THE ION-ION INTERACTION AT INTERMEDIATE ENERGIES

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ABSTRACT

The double scattering correction to the ion-ion double folding interaction is calculated for the $^{12}C + ^{12}C$ system. The multiple scattering theory was employed for the purpose. The effect of this correction on the total reaction cross section at intermediate energies (~100 MeV/nucleon) was found to be less than 10%.

Microscopic approaches to the Ion-Ion potential invariably employ the single or double folding prescription[1]. At intermediate energies, the multiple scattering theory constitutes an excellent framework within which the optical potential can be discussed[2),3)]. The expansion involved is in ascending order of correlation. The first-order term in this expansion is, in the context of heavy-ions, the usual "$t_{\rho_1\rho_2}$" expression, which constitutes the above mentioned double folding potential. Of course, in contrast to the usually employed DF potential, the "$t_{\rho_1\rho_2}$" interaction carries a well defined energy dependence. Further its reactive content is well established in single nucleon knock out.

During the last few years, several authors have addressed the question of whether the imaginary part of "$t_{\rho_1\rho_2}$" appropriately corrected for the Pauli principle does account reasonably well for the total reaction cross section of heavy ions at intermediate energies[4]. The analysis of the most recent data on $^{12}C + ^{12}C$ at E = 100(MeV/nucleon)/300(MeV/nucleon)[5], does indicate that the "$t_{\rho_1\rho_2}$" interaction is reasonably adequate. On the other hand, one knows that high order terms in the multiple scattering expansion of the optical potential should play a role at not too high energies. In fact, Ray[7], has shown that in proton-nucleus scattering the second-order term (double scattering contribution) contributes appreciably to the scattering. In fact, one of the conclusions reached by Ray is that an over-simplified application of the "$t\rho$" interaction for proton-nucleus may result in a fortuitously good agreement with data. Several effects are to be considered. It is our aim in this paper to assess the importance of the second-order, double NN scattering contribution to the ion-ion interaction in so far as the total reaction cross section is concerned. Our application will be restricted to intermediate energies where the multiple scattering theory is expected to work best. For the purpose of completeness, and the presentation of a general theoretical framework, within which corrections to the first order approximation may be constructed and discussed, we present below the essential ingredients of this approach.

The Hamiltonian for the projectile nucleon-target nucleus system is written as,

$$H = -\frac{\hbar^2\nabla^2}{2m} + H_N + V \qquad (1)$$

where H_N is the target nucleus Hamiltonian and V is the interaction between the incident nucleon and the target nucleus, which can be written as a sum of individual nucleon-nucleus interactions

$$V = \sum_{i=1}^{A} V_{pi} \qquad (2)$$

The solution of the scattering problem is represented by the full nucleon-nucleus T-matrix

$$T = V + V\frac{1}{E-(H-V)+i\varepsilon}T\,, \qquad (3)$$

where E is the C.M. energy . The solution of (3) is facilitated by the decomposition

$$T = \sum_i \tau_{pi}(\epsilon)\,\eta_i(\epsilon) \qquad (4)$$

with

$$\tau_{pi}(\epsilon) = V_{pi} + V_{pi}\frac{1}{E-(H-V)+i\varepsilon}\tau_{pi}(\epsilon) \qquad (5)$$

substituting (4) into (3) gives

$$\eta_i(\epsilon) = 1 + \frac{1}{E-(H-V)+i\varepsilon}\sum_{j\neq i}^{A}\tau_{pj}(\epsilon)\,\eta_j(\epsilon) \qquad (6)$$

The set of equations, 4-6 constitutes the basis of the multiple scattering series, which results in the following

$$T = \sum_i \tau_{pi}(\epsilon) + \sum_{i,j\neq i}\tau_{pi}(\epsilon)\frac{1}{E-(H-V)+i\varepsilon}\tau_{pj}(\epsilon) \qquad (7)$$

At this point it is important to emphasize that τ_{pi} are not two-body projectile-nucleon transition matrices; the propagator $1/(E-(H-V)+i\varepsilon) \equiv G(E)$ contains the full nuclear Hamiltonian H_N (see eq. (1)) and consequently τ_i is an (A+1)-body operator.

The usual procedure is to replace τ_{pi} by the corresponding, nucleon-nucleon T-matrix in free space

$$t_{pi}(\epsilon) = V_{pi} + V_{pi}\frac{1}{E+\frac{\hbar^2\nabla^2}{2m^2}+i\varepsilon}t_{pi}(\epsilon) \qquad (8)$$

The corrections to the replacement $\tau \to t$ reside in corrections

to the free Green's function (the replacement of an (A+1) operator by a two-body operator, and the use of the C.M. energy of the p+A system in the p+N system (this is reasonable if $A^{-1} \ll 1$)).

The stage is now set for the obtention of the optical potential operator which is formally defined by the equation

$$T = V + V \frac{P_0}{E + \frac{\nabla^2}{2m} - K_A} T \tag{9}$$

where $P_0 \equiv |\Psi_0\rangle\langle\Psi_0|$, is the projection operator, onto the target nucleus ground state, and K_A represents the kinetic energy of the C.M. of the nucleus. Thus

$$V = T(E) - T(E)\, g_0(E)\, T(E) + \cdots$$
$$= \sum_i t_i(E) + \sum_{i \neq j} t_i(E) \frac{1 - P_0}{E + \frac{\nabla^2}{2m} - K_A} t_j(E) + \cdots \tag{10}$$

The ground state matrix element of V gives us the optical potential for elastic scattering, viz

$$V(E) = \langle \vec{k}'; \vec{m}'; 0 | V | \vec{k}; 0; 0 \rangle$$
$$= (2\pi)^3 \delta(\vec{k}' + \vec{m}' - \vec{k})\, V(\vec{k}'; \vec{k}; E) \tag{11}$$

the first order potential obtained from eq.10 reads

$$v^{(1)}(\vec{k}'; \vec{k}; E) = A \int \frac{d\vec{p}_i}{(2\pi)^3}\, \phi_0(\vec{p}_i + \vec{q}; \vec{p}_i)\, t(\vec{k}_i'; \vec{k}_i; \varepsilon) \tag{12}$$

where ϕ_0 is the target nucleus density matrix, which is related to the density by

$$\rho(q) = \int \frac{d\vec{p}_i}{(2\pi)^3}\, \phi_0(\vec{p}_i + \vec{q}; p_i)$$
$$q = \vec{k}' - \vec{k}$$
$$\vec{k}_i = \vec{k} - \frac{\nu}{m_N}(\vec{k} + \vec{p}_i) \tag{13}$$
$$E' = E - \frac{(\vec{k} + \vec{p}_i)^2}{2(m_N + m_p)}$$

The next step is to set $\vec{p}_i = 0$ in t, wich results in the "to" expression

$$v^{(1)}(\vec{k}'; \vec{k}; E) = A\rho(q)\, t(\vec{k}_i', \vec{k}_i; \varepsilon')$$
$$\approx A\rho(q)\, t(\theta = 0^\circ; E) \tag{14}$$

The last form ignores off-shell effects. It has the advantage of supplying a model independent procedure for discussing nucleon-nucleus elastic scattering. The reactive content of $v^{(1)}$, as is known, is quasi-free knock-out.

It is to be expected that the impulse approximation form of $v^{(1)}$, eq. (14) would be valid at intermediate proton energies (E ≳ 200 Mev), at these energies, the nucleon-nucleon scattering is practically purely elastic (except for very small bremsstrahlung emission). At higher energies, pion production becomes important. Clearly medium effects modify this picture to some extent (e. g. shifting the pion production threshold to lower energies). Further, these same nuclear medium effects like Pauli blocking. Fermi motion of the target nucleus, bring about changes in the form of $v^{(1)}$ (validity of impulse approximation) as well as make higher order corrections, related to nucleon-nucleon correlations, more important.

Among the numerous corrections required for a better treatment of the scattering process, the second order, double-scattering, effect seems to be the easiest to estimate.

This term looks like, in momentum space (using the free nucleon-nucleon t-matrix as basic input)

$$\langle \vec{k}'; 0 | v^{(2)} | \vec{k}; 0 \rangle \equiv \sum_{i=1}^{A} \sum_{j \neq i} \sum_{\alpha \neq 0} \int \langle \vec{k}'; 0 | t_{ij} | \vec{k}''; \alpha \rangle \cdot$$
$$\cdot \frac{1}{E - \frac{\nabla^2 k''^2}{2m} - E_\alpha + i\eta} \langle \vec{k}'', \alpha | t_{ij} | \vec{k}, 0 \rangle \frac{d^3 k''}{(2\pi)^3} \tag{15}$$

Several approximations are usually employed to simplify the above. Use an average nuclear excitation energy in the free Green function, $E_\alpha + \bar{E}_\alpha \equiv \bar{E}$, employ closure to get rid of the α-sum

$$\sum_{\alpha \neq 0} \equiv \sum_{\alpha} - |0\rangle\langle 0| \tag{16}$$

and employ the eikonal (high energy) approximation in

evaluating the Green's function. Introducing the two particle correlation function

$$P^{(2)}(\vec{r}',\vec{r}) \equiv \frac{1}{A(A-1)} \int \Psi_0^+(\vec{r}_1,\ldots,\vec{r}_A) \sum_{A=1}^{A} \sum_{j\neq A} \delta(\vec{r}'-\vec{r}) \delta(\vec{r}-\vec{r}_j) \Psi_0(\vec{r}_1,\ldots,\vec{r}_A)\, d\vec{r}_1 \ldots d\vec{r}_A \tag{17}$$

one can thus write an approximate form for the double scattering contribution, which in coordinate space looks like

$$v^{(2)}(r) = \frac{2\pi_A}{k}\left(-\frac{2m}{4\pi\hbar^2}\right)(tA\rho)\left(\frac{A-1}{A}\right) R_{CORR}\, v^{(1)}$$

$$\simeq -\frac{i k}{2\epsilon}\left[v^{(1)}(r)\right]^2 R_{CORR} \tag{18}$$

where $V^{(1)}$ is the first-order "t_ρ" potential and R_{CORR} is the two-particle correlation length, given by

$$R_{CORR} \equiv \int_0^\infty \left[\frac{P^{(2)}(\vec{r}',\vec{r})}{\rho(\vec{r}')\rho(\vec{r})} - 1\right] d(\vec{r}-\vec{r}') \tag{19}$$

where a further assumption on the quantity "$P^{(2)}/\rho\rho$" has been made namely that it depends only on the relative separation between the two nucleons and not on their individual positions. In the absence of two-body correlations, $R_{CORR} = 0$. In general, it is expected that $P^{(2)}(\vec{r}',\vec{r})$ would approach the no-correlation form at separations larger than the hard core radius ($\sim$0.4 fm) at smaller separations $P^{(2)} = 0$. Thus $R_{CORR} = -0.4$ fm[6].

We see clearly from our estimate for $V^{(2)}$ that the multiple scattering series is an expansion in order of correlation. The third and higher order terms, would depend on three and many body correlations. No simple expressions are found for these terms.

Our recipe for the second-order nucleus-nucleus potential (double scattering, contribution) is to perform a symmetrized single folding with the projectile and target densities. This in turn suggests

$$U^{(2)}_{A_1A_2}(r) = -\frac{i k}{4\epsilon} R_{CORR} \left\{ \int \left[v_{NA_1}(\vec{r}-\vec{r}')\right]^2 \rho_{A_2}(\vec{r}')\, d\vec{r}' + \int \left[v_{NA_2}(\vec{r}-\vec{r}')\right]^2 \rho_{A_1}(\vec{r}')\, d\vec{r}' \right\} \tag{20}$$

where $V^{(1)}_{NA_1}(r)$ is the nucleon-nucleus (A_1) "t" type optical potential discussed earlier.

A more refined treatment of R_{CORR} presented by Ray[7] shows that it is actually composed of four distinct contributions.

$$R_{CORR} = R_{PAULI} + R_{SRD} + R_{PSR} + R_{C.M.} \tag{21}$$

where, following Boridy and Fesbach[8], R_{PAULI} is related to the Pauli exclusion principle correlations, R_{SRD} is related to the short range dynamical correlations and R_{PSR} is connected to a combination of Pauli and short-range dynamic terms. Finally $R_{C.M.}$ arises form center of mass correlations. We give below the approximate expressions for these four contributions to R_{CORR} derived by Ray[7].

$$-R_{PAULI} = \frac{1}{2}\left[1 - \frac{5}{A} + \frac{4}{A^2}\right] \frac{3\tilde{\pi}}{10 k_F(r)} \frac{1}{1 + \frac{8}{5}\bar{B} k_F^2(r)}$$

$$-R_{SRD} = \frac{1}{2}\left[1 - \frac{2}{A} + \frac{1}{A^2}\right] \sqrt{\pi}\, \frac{b^3}{b^2 + 8\bar{B}}$$

$$R_{PSR} = \frac{1}{2}\left[1 - \frac{5}{A} + \frac{4}{A^2}\right] \frac{3\tilde{\pi}}{10} \frac{1}{\sqrt{k_F^2(r) + 5/b^2}} \frac{1}{\left[1 + 8\bar{B}\left(\frac{k_F^2(r)}{5} + \frac{1}{b^2}\right)\right]} \tag{22}$$

$$-R_{C.M.} = \left[1 - \frac{2}{A} + \frac{1}{A^2}\right] \ell_c$$

where the parameters A, $k_F(r)$, $\bar{B}$, b, ℓ_c are the target mass numbers, local Fermi momentum, finite range parameter of nucleon-nucleon elastic t-matrix, short-range dynamical correlation parameter and the effective "correlation length", respectively. We should mention that $\bar{B}$ exhibits a non-negligible energy dependence: 0.66 at E_{LAB} = 100 MeV and dropping to about 0.1 at E_{LAB} = 2200 MeV.

We have evaluated the second order (double scattering) correction to the "$t_{\rho_1\rho_2}$", potential, according to Eq. (20), with R_{CORR} given by Eqs. (22) for the system $^{12}C + ^{12}C$ at the following lab energies per nucleon 100, 200, 300 and 500 MeV. In figure 1, we show the radial distributions of the second order correction to the optical potential for the $^{12}C + ^{12}C$ system at the above energies. For comparison, we also show the contribution of the dominant, "$t_{\rho_1\rho_2}$", DF, potential at 100 MeV/nucleon. The range of the second-order potential is appreciably shorter than that of the first one owing to the higher-order density dependence; "$(t\rho_1)^2\rho_2$" vs. "$(t\rho_2)\rho_1$". It

is interesting to note that the imaginary part of the optical potential changes sign at 100 MeV/nucleon namely, $W^{(2)}$ is regenerative whereas at the other cited energies it is absorptive. We should stress, though, that the summed contributions of $W^{(1)}$ and $W^{(2)}$ are guaranteed to be absorptive.

In our calculation we have used Pauli-blocking corrected nucleon-nucleon total cross-sections[9]. The Pauli blocking effect was, however, better treated here than in ref. (9), as we, here, used an energy-dependent σ_{NN}, when performing the average. Further, we have also taken into account the Pauli-blocking of the projectile nucleons[10].

Owing to the fact that the radial average of $V^{(2)}$ is 0.3 of that of $V^{(1)}$, it is expected that the effect of $V^{(2)}$ in the total reaction cross section is small. We have verified this by evaluating, within the WKB approximation, the total reaction cross section of $^{12}C+^{12}C$ using $V^{(1)}+V^{(2)}$ for an optical potential and have found a less than 10% influence with respect to the calculation with only $V^{(1)}$ included. In our calculation we have included the Pauli blocking effect mentioned above, and performed an adequate average over the Fermi motion of the nucleons.

In conclusion, we have calculated in this paper the double scattering contribution to the ion-ion double folding potential. We have found that this correction is of shorter range than the dominant DF interaction. The energy dependence of the calculated correction is quite important in the sense that the potential changes character from regenerative at E = 100 MeV/nucleon to absorptive at higher energies: The effect of this double scattering contribution on the total reaction cross section was found to be less than 10% at E_{LAB} = 100 MeV/nucleon.

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Figure Captions

Figure 1: The real part of the double NN scattering contribution to the ion-ion potential at four laboratory energies per nucleon: 100 MeV (dashed curve), 200 MeV (dotted curve), 300 MeV (dashed-dotted curve) and 500 MeV (dashed-double dotted curve). For reference the usual double-folding potential is shown by the full curve.

Figure 2: Same as in Figure 1 for the imaginary part. Note that the potential at 100 MeV is regenerative. The W(Im V) equivalent to the double folding model, namely Im "$t_{\rho_1\rho_2}$" shown in the full curve was calculated for comparison.

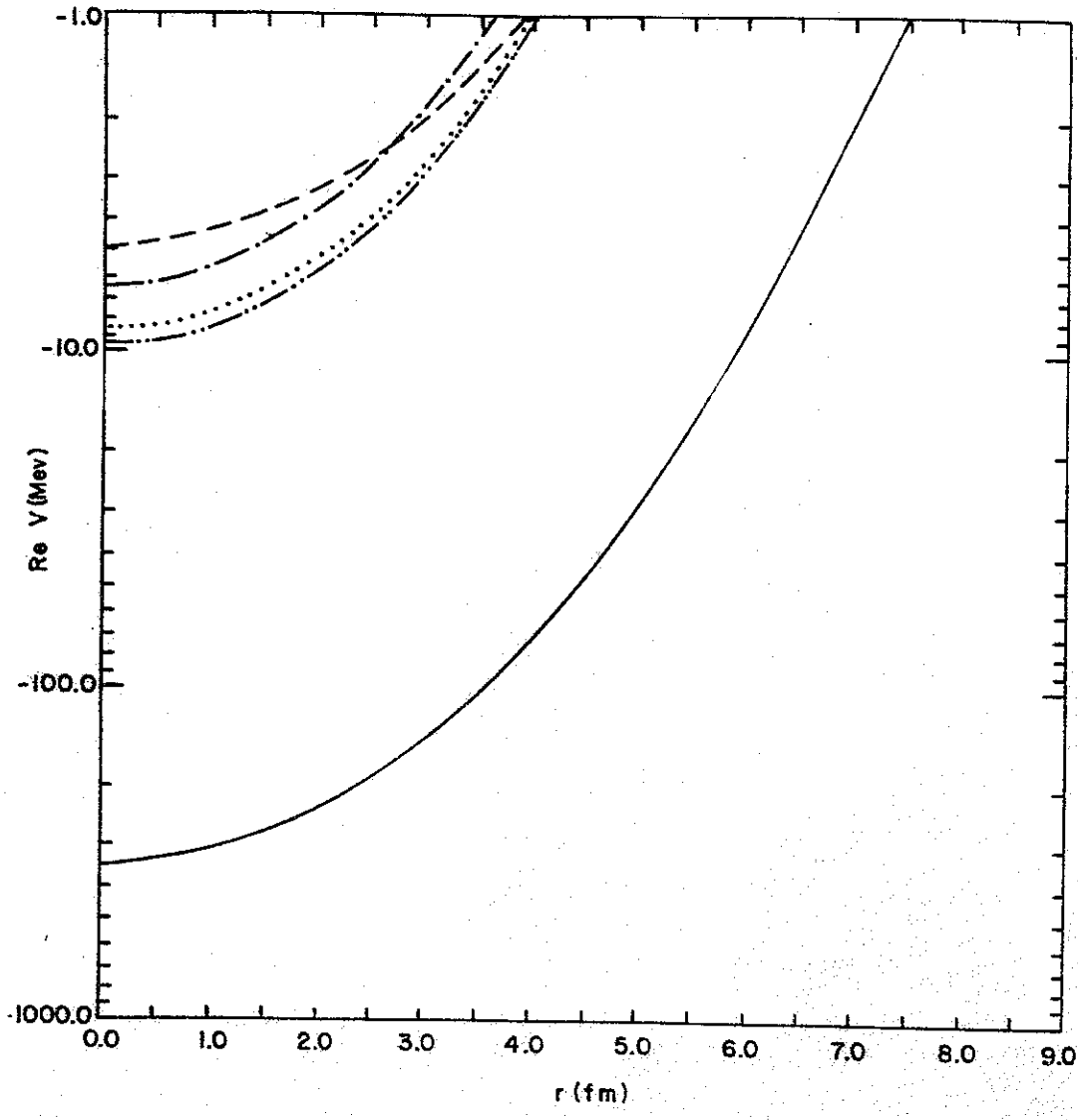

Fig. 1

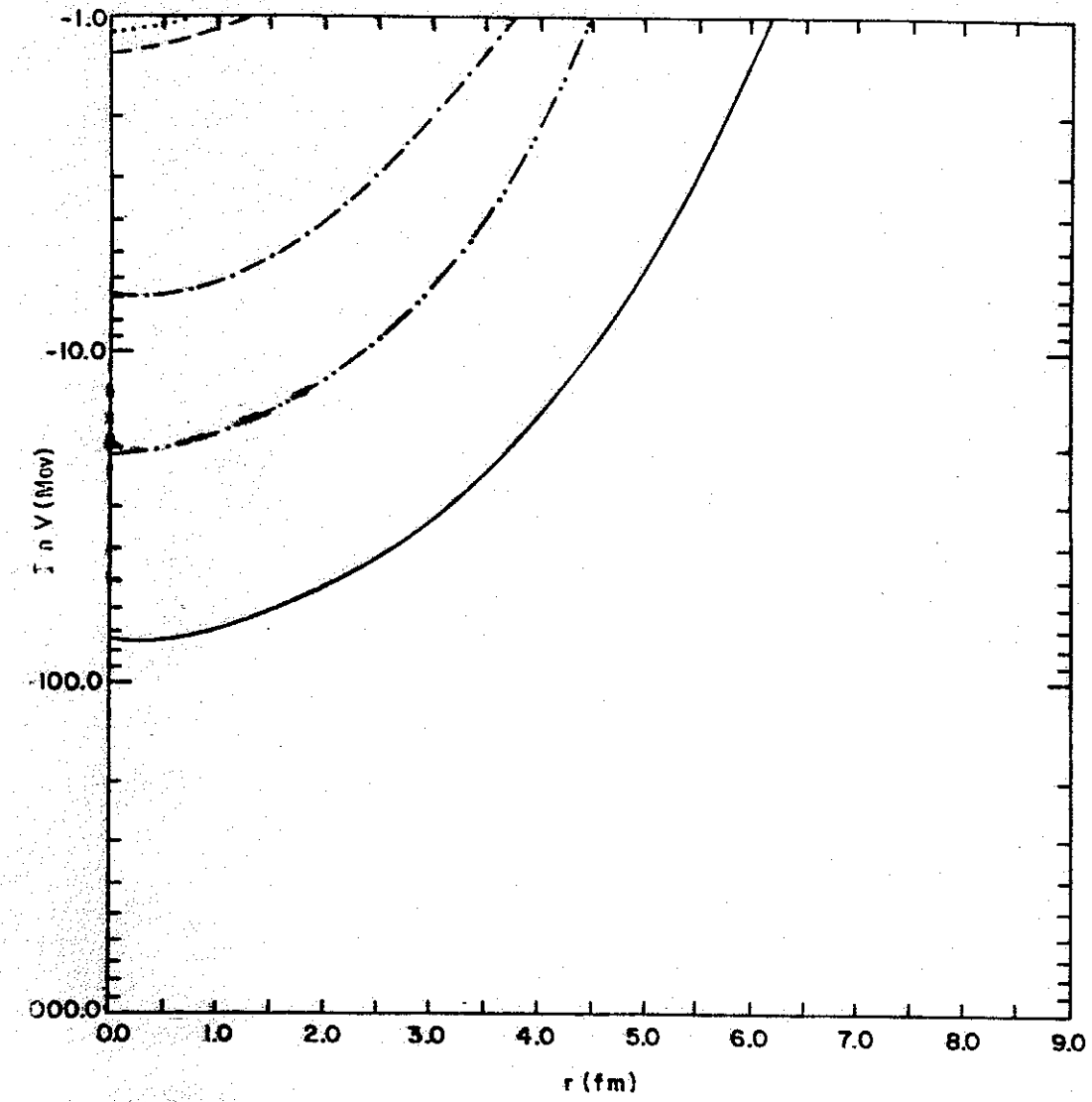

Fig. 2