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by

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MAGNETIC ISLANDS CREATED BY RESONANT HELICAL WINDINGS*

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ABSTRACT

The triggering of disruptive instabilities by resonant helical windings in large aspect-ratio tokamaks is associated to destruction of magnetic surfaces. The Chirikov condition is applied to estimate analytically the helical winding current thresholds for ergodization of the magnetic field lines.

1. INTRODUCTION

Disruptive instabilities limit the tokamak operation and their causes are still unclear. There are experimental evidences that in several cases these instabilities are preceded by some plasma perturbation modes[1,2]. To investigate it, disruptive instabilities can be triggered by resonant magnetic fields created by helical windings[2]. In this paper, we suppose that magnetic surface break-up, caused by these resonances, triggers the tokamak disruptions. We estimate analytically helical winding current thresholds for these disruptions in a large aspect-ratio tokamak with circular cross-section. As an example, we calculate these thresholds for the brazilian TBR tokamak[3].

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2. MAGNETIC SURFACES

Plasma equilibrium in tokamaks is described by MHD equations. The differential equations for the magnetic field lines are

$$\frac{dr}{dz} = \frac{B_r}{B_z} \quad , \quad \frac{d\vartheta}{dz} = \frac{B_\vartheta}{r B_z} \quad , \tag{1}$$

where r, ϑ and z are cylindrical coordinates. For helical symmetry equilibria these lines lie on magnetic surfaces given by the equation

$$\vec{B} \cdot \nabla\psi = 0 \quad , \tag{2}$$

where $\psi = \psi(r,u)$, $u = \vartheta - \alpha z$ and $\alpha = \frac{n}{mR}$ is a constant. In this work n and m are rational numbers.

We consider a large aspect-ratio tokamak with circular cross-section, represented by a periodical cylinder with length $2\pi R$, and assume the tokamak scaling

$$\frac{B_z}{B_\vartheta} \approx \frac{R}{a} >> 1 \quad , \tag{3}$$

where R and a are respectively the major and minor plasma radii. The unperturbed equilibrium is determined by B_z and the plasma current density $\vec{j}$:

$$\nabla B_z = 0 \quad , \quad \vec{j} = j_0 (1 - r^2/a^2)^\gamma \, \hat{e}_z \quad , \tag{4}$$

where j_0 and γ are constants. In this case magnetic line helicities depend on r and are characterized by the safety factor q given by

$$q = \frac{r B_z}{R B_\vartheta} \quad . \tag{5}$$

At $r=a$, $q(a) \sim B_z/I_p$, where I_p is the plasma current. At rational surfaces with $q(r_{m,n}) = m/n$, the magnetic field lines close on themselves after m trips along the cylinder and n trips in the poloidal direction.

The unperturbed function ψ_0 is[4]

$$\psi_o = \frac{n r^2 B_z}{2 m R} - \frac{\mu_o I_p}{2\pi} \int_0^r \frac{dr'}{r'} \left[1 - (1 - r'^2/a^2)^{\gamma+1}\right] \qquad (6)$$

The magnetic field created by electrical currents I flowing in m pairs of helical windings, equally spaced, with radius b wounded on a circular cylinder (corresponding to a large aspect-ratio tokamak) exhibits helical symmetry. This field depends on the coordinates r and u. The constant α characterizes the helicity of the windings. The stream function ψ_1 for I flowing in opposite directions in adjacent windings, is[4,5]

$$\psi_1(r,u) = \frac{\mu_o I}{\pi} \left(\frac{r}{b}\right)^m \cos m u \quad . \qquad (7)$$

We consider $|\psi_1/\psi_o| << 1$ and the linear superposition of an unperturbed equilibrium described by $\psi_o(r)$ with a resonant helical perturbation described by $\psi_1(r,u)$. Within this approximation, the stream function

$$\psi(r,u) = \psi_o(r) + \psi_1(r,u) \qquad (8)$$

satisfy the Eq.(2). This perturbation in resonance with the period of the equilibrium magnetic lines creates m islands around the rational magnetic surface with $q(r_{m,n}) = m/n$. This approximation is not valid for marginally stable states when the plasma response should not be neglected.

Fig. 1 illustrates the structures of the magnetic islands created by $m=2$ pairs of helical windings ($n=1$). The diagram shows the intersections of the magnetic surfaces with a plane $z = const$. All magnitudes were adjusted to fit TBR data[3]. Thus $a = 0{,}08\,m$, $b = 0{,}11\,m$ e $R = 0{,}3\,m$.

Expanding ψ near $r = r_{m,n}$, the radius of resonant magnetic surfaces, we obtain a formula for the small island half-width $\Delta_{m,n}$

$$\Delta_{m,n} \simeq \left[\frac{4\mu_o I}{\pi \psi_o''(r_{m,n})} \left(\frac{r_{m,n}}{b}\right)^m\right]^{\frac{1}{2}} \quad . \qquad (9)$$

To obtain the island width, taking into account the radial dependence of the

perturbation ψ_1 and the change in the safety factor q over the island width, we use the equation for the island separatrix

$$\psi_o(r) + \psi_1(r,u) = \psi_o(r_{m,n}) + \psi_1(r_{m,n}, u_o) \quad . \qquad (10)$$

The accuracy of the formula (9) is illustrated by the comparisons, in Figs. (2), (3) and (6), of helical current thresholds calculated using Eqs. (9) and (10). The agreement is found to be good, specially for smaller islands.

3. THE BREAK-UP OF THE MAGNETIC SURFACES

There are experimental evidences that hard disruptive instabilities observed in tokamaks can be provocated by the interaction of the $m=2$ magnetic islands with the plasma edge (the limiter is at $r=a$)[2]. We calculated the current I in $m=2$ pairs of helical windings that causes the contact between the magnetic islands and the limiter. The dependence of I with $q(a)$ is shown in the Fig. 2. The perturbation due to this current would trigger hard disruptions in the TBR tokamak.

We consider also the hypothesis that soft disruptive instabilities observed in tokamaks are caused by the ergodic wandering of magnetic field lines. Magnetic surfaces break-up occurs due to the destruction of the system symmetry. As a symmetry breaking perturbation due to magnetic islands created by different resonant helical fields grows, the magnetic surfaces are destroyed and a disruption may occur. The degree of ergodic behaviour depend upon the strength of this perturbation. The helical winding current threshold for ergodization of the magnetic field lines was estimated applying the Chirikov condition[6]. Two of magnetic islands with mode numbers (m,n) and (m',n') are ergodized when

$$\Delta_{m,n} + \Delta_{m',n'} > |r_{m,n} - r_{m',n'}| \quad . \qquad (11)$$

This happens for I greater than the values plotted in the Fig. 2 as a function of $q(a)$. In this example $m=2$ and $m'=3$ ($n=n'=1$). The helical current threshold increases with $q(a)$ although the distance between the rational magnetic surfaces with $q=2$ and $q=3$ remain almost the same, as one can see in the Fig. 4. This happens because the width islands decrease when the islands move inward (see Fig. 5), where the shear of the magnetic field lines, represented by $\psi_0''(r_{m,n})$ in Eq. (9), is smaller.

When a single helical perturbation is superimposed upon an equilibrium with toroidal symmetry both symmetries are broken, and therefore the magnetic surfaces should be expected to disappear. However for a small helical perturbation, i.e., $I/I_p \ll 1$, the major effect, on a large-aspect ratio tokamak, is the appearance of secondary magnetic islands on the rational magnetic surfaces with $q=m\pm1$.

We calculated the helical current I on a single winding set required for ergodization of the magnetic field lines. The toroidal effect was considered multiplying the unperturbed B_z in Eq. (1) by the factor $(1+\frac{r}{R}\cos\vartheta)^{-1}$. Expanding the resulting expressions, neglecting terms of the order $(a/R)^2$ or higher and selecting the resonant terms at the magnetic surfaces with $q = m' = m\pm1$ we obtained

$$\vec{B} \cdot \nabla X = 0 \tag{12}$$

where

$$X = \int^{r} dr' \left(\frac{r}{a}\right)^{m'-m} \left(nB_z - \frac{m' R B_\vartheta}{r}\right) + \frac{m\,\mu_0\, I\, r^{m'}}{2\pi\, a^{m'-m}\, b^m} \cos u' \quad , \tag{13}$$

$u' = \vartheta - \frac{nz}{m'R}$. This function was used, in the same manner as the stream function ψ, to calculate the width of m' magnetic islands at the $q=m'$ surfaces. The ratio between the widths of the primary and secondary islands is proportional to $(a/R)^{\frac{1}{2}}$. An example of winding currents required for ergodization is plotted in the Fig. 6 as a function of $q(a)$.

4. CONCLUSIONS

It was assumed that magnetic surface destruction caused by resonant helical windings trigger the disruptive instabilities in tokamaks. Within this hypothesis the helical current that causes the contact between the $m=2$ magnetic island and the limiter was calculated. The helical current threshold for magnetic surface break-up caused by overlapping of magnetic islands with two helicities was estimated analytically. Secondary magnetic islands on neighboring rational surfaces, due to the action of toroidicity, were also considered. The calculations can be applied to tokamak experiments with resonant helical windings.

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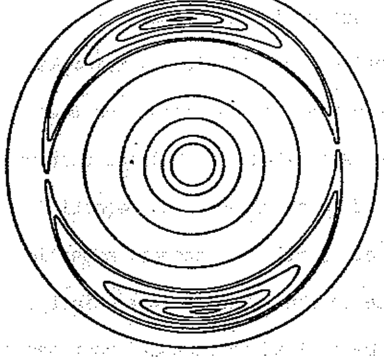

Fig. 1-Island structures produced by two pairs of helical windings ($m=2$, $n=1$, $q(a)=3$, $\gamma=2$, $I_p=10kA$ and $I=100A$).

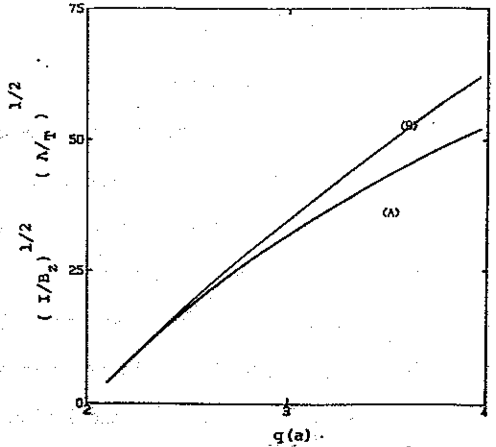

Fig. 2-Helical winding current for the $m=2$ islands touch the limiter. The curves A and B were obtained using, respectively, the Eqs. (9) and (10) ($q(0)=1$, $I_p=10kA$).

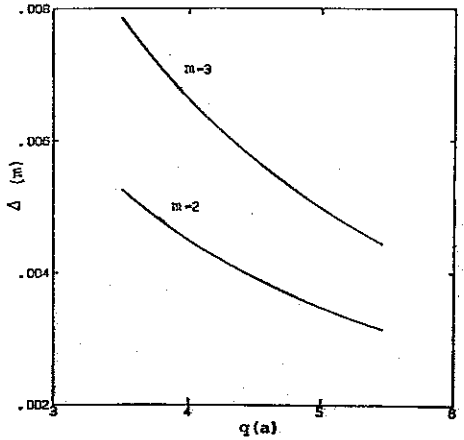

Fig. 5-Island widths of the $m_1=2$ and $m_2=3$ resonances ($n=1$) as a function of $q(a)$ for the same numerical values of Fig. 4 .

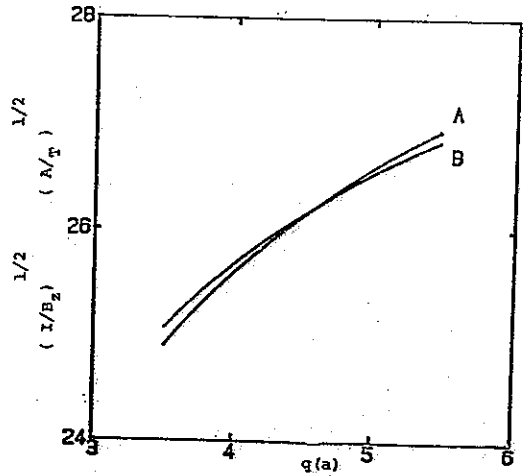

Fig. 6-Helical current for overlap of $m=2$ (primary) and $m'=3$ (secondary) resonances. The curves A and B were obtained using, respectively, the Eqs. (9) and (10). $q(0)=1$, $I_p=10kA$).

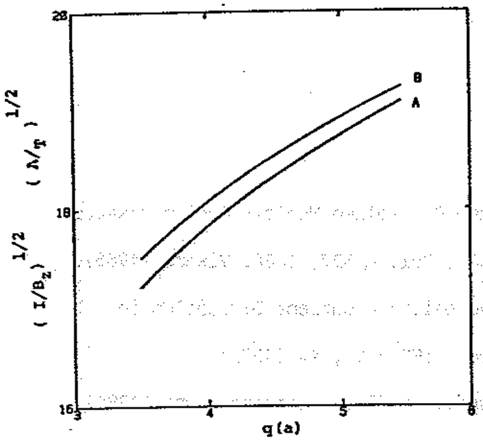

Fig. 3-Helical current in two sets of helical windings for overlap of $m_1=2$ and $m_2=3$ resonances. The curves A and B were obtained using, respectively, the Eqs.(9) and (10). ($q(0)=1$, $I_p=10kA$) .

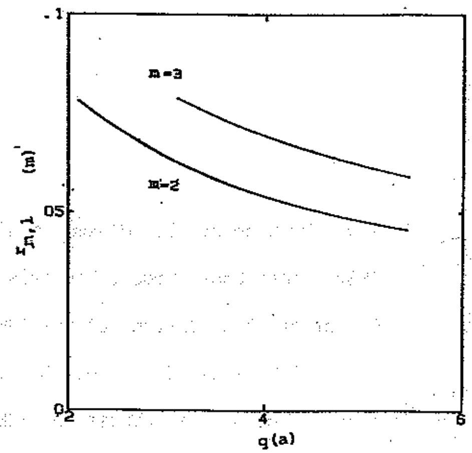

Fig. 4-Rays of the $q=2$ and $q=3$ magnetic surfaces as a function of $q(a)$ for $I=97A$ $I_p=10kA$ and $q(0)=1$.