UNIVERSIDADE DE SÃO PAULO

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PUBLICAÇÕES

IFUSP/P-574

A NONLINEAR ELECTRODYNAMICS VIA GRAVITATIONAL NONMINIMAL COUPLING

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Março/1986

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PACS. 04.50. - Unified field theories and other theories of gravitation.

Summary.- A nonlinear electrodynamics is generated via gravitational nonminimal coupling. As a result, a neutral test particle will not follow geodesics on the whole. A general static spherically symmetric solution is also obtained. An interesting feature of the solution is the presence of a parameter similar to the "Scharzschild radius", which separates the solutions with a negative "effective Einstein constant" from those with a positive one.

(*) This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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Einstein's ultimate goal of a unified theory of gravitation and electromagnetism has been pursued by a legion of physicists since the last few decades. On the other hand, the study of electromagnetic phenomena, in the framework of general relativity, is mainly restricted to the consideration of Einstein-Maxwell equations. Such equations are derived with the help of the so-called minimal coupling principle. According to it, the equations of motion of electrodynamics in curved space must not have terms containing the curvature of space-time. Usually these terms are disregarded by a priori reasons. As a consequence, the resulting electrodynamics is linear. Here we are interested in breaking such linearity, allowing nonminimal couplings with gravity. In doing so, we introduce a more realistic interaction between the gravitational and electromagnetic fields. Furthermore, the fact that the electromagnetic energy should also be responsible for the curvature of the manifold is reinforced by consideration of such a kind of coupling. To some extent, our model is more than a mere amalgamation of gravitation and electromagnetism at the macroscopic level.

To accomplish this, we add to the usual Einstein - Maxwell Lagrangian

$$L = \sqrt{-g}\,(R/\varkappa - 1/2\, F_{\alpha\beta}F^{\alpha\beta}) + \sqrt{-g}\,L_m, \qquad (1)$$

where

$$(2) \qquad F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} ,$$

$F_{\mu\nu}$, A_{μ}, L_m being the electromagnetic field tensor, the potential vector and the Lagrangian for matter, respectively, an interaction term of the form

$$(3) \qquad \sqrt{-g}\ \lambda I .$$

I is anyone of the following electromagnetic invariants regarding the curved space-time

$$(4) \quad RA^{\mu}A_{\mu},\ RF^{\mu\nu}F_{\mu\nu},\ R_{\mu\nu}A^{\mu}A^{\nu},\ R_{\alpha\beta\gamma\delta}A^{\alpha}A^{\gamma}F^{\beta\delta},\ R_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} .$$

λ , as usual, is a coupling constant.

It is not difficult to show that among the previous invariants, $RF^{\mu\nu}F_{\mu\nu}$ and $R_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$ are the only two which give off an electrodynamics consonant with charge conservation. Appealing to simplicity as a guide, we choose the former one to analyze. Although the criterion we have adopted is not an orthodox one, it is of common practice in Physics. It is worth mentioning that the invariants $RA^{\mu}A_{\mu}$, $R_{\mu\nu}A^{\mu}A^{\nu}$, $R_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$, have already been investigated by Novello and Salim ([1]), Novello and Heintzmann ([2]) and Prasana ([3,4]).

([1]) M.NOVELLO and J.M.SALIM: Phys.Rev. **20D**, 377 (1979).
([2]) M.NOVELLO and H.HEINTZMANN: Gen.Rel.Grav. **16**, 527 (1984).
([3]) A.R.PRASANA: Phys.Lett. **37A**, 331 (1971).
([4]) A.R.PRASANA: Lett. Nuovo Cimento **6**, 420 (1973).

The corresponding field equations can be presented in the form ([5])

$$(5) \qquad G_{\mu\nu} = -\kappa(E_{\mu\nu} + T_{\mu\nu}),$$

$$(6) \qquad \nabla_{\nu}\ [(2\lambda R - 1)F^{\mu\nu}] = 0 ,$$

where

$$(7) \qquad G_{\mu\nu} \equiv R_{\mu\nu} - 1/2\ R\ g_{\mu\nu} ,$$

$$(8) \quad E_{\mu\nu} \equiv \frac{1}{4} F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu} - F_{\mu\alpha}F_{\nu}{}^{\alpha} + \lambda(F_{\alpha\beta}F^{\alpha\beta}G_{\mu\nu} + 2RF_{\mu\alpha}F_{\nu}{}^{\alpha}).$$

Tracing Eq. (5), we get ([6]):

$$(9) \qquad T + R(\lambda F^{\alpha\beta}F_{\alpha\beta} - 1/\kappa) = 0.$$

Eqs. (6) and (9) exhibit, in an obvious way, the nonlinearity of the model.

The wave equation for the potential vector A^{μ} is as follows:

$$(10) \quad [\Box A^{\mu} + R^{\mu}{}_{\alpha}A^{\alpha} - \nabla^{\mu}\nabla_{\nu}A^{\nu}]\ (1-2\lambda R) = 2\lambda[A^{\mu,\nu} - A^{\nu,\mu}]R_{,\nu} ,$$

([5]) $\delta\int\sqrt{-g}\ L_m\ d^4x \equiv \int\sqrt{-g}\ T_{\mu\nu}\delta g^{\mu\nu}d^4x$.
([6]) $T \equiv T^{\alpha}{}_{\alpha}$.

where $\Box A^{\mu} + R^{\mu}{}_{\alpha}A^{\alpha}$ is the Rahm's wave operator in curved space.

The Einstein tensor being divergenceless, the right-hand side of Eq. (5) is also divergenceless. But in general,

$$\nabla_{\nu}E^{\mu\nu} \neq 0 \quad , \tag{11}$$

therefore also

$$\nabla_{\nu}T^{\mu\nu} \neq 0 \quad , \tag{12}$$

and a **neutral** test particle will not follow geodesics on the whole. Thus, we can consider that there is a supplementary force which deviates the test body from the geodesics. This situation is completely different from that of the usual Einstein-Maxwell theory. Of course, under conditions such as $\lambda = 0$ this extra force vanishes and the two theories lead to the same results.

In the static spherically symmetric case, with a line element given by

$$ds^2 = e^{2\nu}dt^2 - e^{2\mu}dr^2 - r^2d\Omega^2 \quad , \tag{13}$$

where ν and μ depend on r only, the field equations can be written as

$$[-\nu'' - \nu'^2 + \nu'\mu' - 2\nu'/r](1 - a^4/r^4) = - \varkappa e^{2\mu}\epsilon^2/2r^4 \quad , \tag{14}$$

$$[\nu'' + \nu'^2 - \nu'\mu' - 2\mu'/r](1 - a^4/r^4) = \varkappa e^{2\mu}\epsilon^2/2r^4 \ , \tag{15}$$

$$\{e^{-2\mu}[(\nu' - \mu')r + 1] - 1\}\,(1 - a^4/r^4) = - \varkappa\epsilon^2/2r^2 \ , \tag{16}$$

$$r^2\Phi'\, e^{-(\nu+\mu)} = \epsilon \ , \tag{17}$$

wherein

$$A_{\mu} = (\Phi(r),0,0,0), \ a^4 \equiv 2\varkappa\lambda\epsilon^2,$$

ϵ is an integration constant and the primes denote derivatives with respect to r.

The asymptotically flat solution of these equations is given by

$$e^{2\nu} = e^{-2\mu} = 1 - \frac{2m}{r} + \frac{\varkappa\epsilon^2}{4ar}\left[\tfrac{1}{2}\ \ln\left|\frac{r+a}{r-a}\right| + \mathrm{ctn}^{-1}\frac{r}{a}\right] \quad , \tag{18}$$

$$\Phi = -\epsilon/r \ . \tag{19}$$

In case $r > a$, Eq. (18) takes the form

$$e^{2\nu} = e^{-2\mu} = 1 - \frac{2m}{r} + \frac{\varkappa\epsilon^2}{2r^2} + \frac{(\varkappa\epsilon^2)^2}{5}\,\frac{\lambda}{r^6} + \frac{2(\varkappa\epsilon^2)^3}{9}\,\frac{\lambda^2}{r^{10}} + \ldots \tag{20}$$

The prior solution includes the Schwarzschild solution ($\epsilon \to 0$) as well as the Reisner-Nordström one ($\lambda \to 0$) [7].

[7] S.W.HAWKING and G.F.R.ELLIS: The Large Scale Structure of Space-Time, Cambridge University Press (1974).

Here the parameter a plays a role similar to that of the "Schwarzschild radius". On the other hand, an interesting physical interpretation can be attached to it, if we rewrite Eq.7 in the form

$$(21)\qquad G_{\mu\nu} = \varkappa_{ef}\left[\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu} - F_{\mu\alpha}F_{\nu}^{\ \alpha} + 2\lambda RF_{\mu\alpha}F_{\nu}^{\ \alpha}\right] ,$$

where the "effective Einstein constant" is given by

$$(22)\qquad \varkappa_{ef} = \frac{\varkappa}{1 - a^4/r^4} .$$

It is clear from Eq. 22 that $\varkappa_{ef}$ will be negative or positive provided that $r < a$ or $r > a$, respectively. So, a "phase--transition" associated with a change of sign of the "effective Einstein constant" will occur at the "critical point" $r = a$.

The nonminimal coupling term we have investigated in this note represents an exciting deviation from the equivalence principle in the sense that it respects the geometrical nature of gravity as well as the gauge symmetry of electromagnetism. Taking into account that all physical principles like the principle of equivalence ought to be tested and that the standard experiments used to verify it are performed in space-time regions of small curvature, what means that they do not fit the present model, we can conclude that the consideration of such kind of coupling is justifiable.

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One of the authors (A.J.Accioly) is very grateful to C.M.do Amaral for stimulating discussion.

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