UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
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T. Frederico
Centro Técnico Aeroespacial, Instituto de Estudos Avançados, 12200 São José dos Campos, SP, Brazil, and
Departamento de Física, Universidade Federal de Pernambuco, 50000 Recife, PE, Brazil

Sadhan K. Adhikari
Departamento de Física, Universidade Federal de Pernambuco, 50000 Recife, PE, Brazil

M.S. Hussein
Instituto de Física, Universidade de São Paulo

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T. FREDERICO
Centro Técnico Aeroespacial, Instituto de Estudos Avançados*.
12.200 São José dos Campos, São Paulo, Brazil
and
Departamento de Física, Universidade Federal de Pernambuco
50.000 Recife, Pernambuco, Brazil

Sadhan K. ADHIKARI
Departamento de Física, Universidade Federal de Pernambuco
50.000 Recife, Pernambuco, Brazil

M.S. HUSSEIN
Instituto de Física, Universidade de São Paulo,
01.498 São Paulo, São Paulo, Brazil.

The role of deuteron asymptotic D to S normalization ratio η^D on the calculation of triton observables is imphasized. Studying the new correlations among triton asymptotic normalizations and η^D we make a new estimate of triton asymptotic normalizations C_S and C_D: $C_S = 1.85 \pm 0.04$, $C_D/C_S = 0.0463 \pm 0.0013$, for $\eta^D = 0.0271$ (4), and $C_D/C_S\eta^D = 1.71 \pm 0.04$.

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* Permanent and present address.

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A strong correlation among the three-nucleon low-energy observables - especially among the spin doublet neutron deuteron (n-d) scattering length $^2a_{nd}$ and the triton binding energy E_T - was first pointed out explicitly by Phillips [1], though the pioneering calculation by Aaron, Amado, and Yam [2] indicated such a trend. Such correlations have great practical importance in that they have proved to be very useful in predicting experimental values of yet unmeasured observables or in confirming or refining known experimental quantities. For example, Phillips [1] suggested the correct experimental value of $^2a_{nd}$ studying such correlations which was later confirmed experimentally [3]. Recently, the Los Alamos group [4] suggested the value of spin doublet proton deuteron (p-d) scattering length $^2a_{pd}$ which contradicts the existing experimental values [5]. The value of S wave asymptotic normalization C_S of triton was suggested by Girard and Fuda [6] from a study of its variation with E_T using the partial wave dispersion relations. More recently, Ishikawa and Sasakawa [7] predicted the value of triton asymptotic D to S normalization ratio η^T from a study of its variation with E_T.

The correct approach for predicting such experimental values of triton is the following. First, one has to find out all relevant independent few nucleon observables on which a particular "unknown" triton observable is sensitive in a realistic calculation. When a dynamical model calculation yields the correct experimental values of these few nucleon observables it is expected to yield the correct "experimental" value of the unknown triton observable. For example, the S wave dispersion model of Girard and Fuda [6] suggests that the S wave triton asymptotic normalization C_S be a function of E_T and $^2a_{nd}$. Then by forcing their moodel to yield the correct E_T and $^2a_{nd}$ they predicted the "experimental" value of C_S: $C_S=1.82\pm0.03$. The triton asymptotic D to S normalization ratio η^T, for reasons discussed later, seems to be insensitive to the exact value of $^2a_{nd}$ (within the width of $^2a_{nd}-E$ Phillips plot) and only sensitive to E_T. Using this simple correlation among η^T and E_T recently Ishikawa and Sasakawa [7] predicted the "experimental" value of η^T: $\eta^T=0.0432\pm0.0015$, which is close to the experimental estimates [8].

It has been suggested by Kim and Tubis [9] that triton asymptotic normalization parameters C_S and C_D should be given the same importance as other triton observables, such as, binding energy E_T, quadrupole moment, magnetic moment etc, in any trinucleon calculation. This is why the correct "experimental" value of C_S and C_D or alternatively of C_S and η^T appears to be very important from a theoretical standpoint. Here we reexamine the correlations among C_S and C_D and other few nucleon observables, from which we make a new estimate of C_S and C_D. In particular we show that a correct representation of deuteron asymptotic normalizations is important in addition to other few nucleon observables for a precise evaluation of C_S and C_D. The works of Girard and Fuda [6] and of Ishikawa and Sasakawa [7] do not have the correct information about deuteron asymptotic normalization built into their model. We suggest new correlation among deuteron and triton asymptotic D to S normalization ratios η^D and η^T from which we make new estimates about C_S and C_D for triton which are expected to be more precise than previous estimates.

Nucleon-nucleon tensor force and the deuteron D state has been essential in producing the electric quadrupole moment and refining the value of magnetic dipole moment, which are considered to be basic deuteron properties. Any realistic nucleon nucleon interaction to be used in trinucleon calculation must have the correct deuteron properties built in, in addition to the on-shell properties of the singlet state. Will the trinucleon observables - especially C_S and C_D - be sensitive to any spcific property of deuteron D state besides those informations about deuteron D state which are contained in deuteron observables? The study of Gibson and Lehman [10] suggests that this specific property is deuteron D wave percentage P_D which should be considered important in a triton calculation. They studied the correlation among C_S (and C_D) and P_D which should be considered important for predicting a precise value of C_S and C_D of triton.

The deuteron D state probability P_D has been considered to be an important property of deuteron eversince the D state was introduced and until very recently when Amado and collaborators [11] have commented that P_D should not be considered an observable of the deuteron. Because of theoretical difficulties [11] no experiment, however accurate, will ever be able to fix the value of P_D. A theooretical calculation of P_D will be model dependent as it will be sensitive to the deuteron wave function in the interior region. Thus a classification of triton properties as a function of P_D will be model dependent and a study of correlation among C_S (and C_D) and P_D as has been done in ref. [10] will yield model dependent result for C_S and C_D even if the "correct" value of P_D is known. To make matters worse there is no hope for determining P_D correctly. Amado and collaborators have suggested that P_D should not be given the status it has enjoyed and have suggested that it is the deuteron asymptotic D to S normalization ratio η^D which should be given the "experimental" status of a single quantity to "measure" the D state. Analysis of experimental data leads to a precise value of $\eta^D : \eta^D$=0.0271 (4) [12]. So if certain triton observable is sensitive to the value of η^D apart from other few nucleon observables one has to have the correct value of η^D built in a model in order to make a model independent estimate of the particular triton observable. We shall see that triton asymptotic normalization parameters are sensitive to η^D and this fact should be remembered while making model independent estimates for triton asymptotic normalizations.

Though we use results of realistic calculations to extract the values of C_S and C_D, in order to understand the dependence of C_S and C_D on two nucleon (particularly η^D) and three nucleon proprties we preformed Faddeev calculation with separable $^1S_0$ and $^3S_1-^3D_1$ interaction with Yamaguchi [13] form factors. In our calculation we always keep triplet deuteron parameters – deuteron binding, triplet scattering length a_1, and electric quadrupole moment – fixed. We keep the provision of varying the singlet effective range r_0 and scattering length a_C. We use a_0=-17.0 fm and -23.7 fm to simulate neutron-neutron and the proton-neutron systems, respectively. The variation of r_0 is motivated in order to understand the origin of certain low energy correlations better. We shall comment on this variation in appropriate place. Apart from this flexibility our model is very similar to that of ref. [10]. The model uses, deuteron binding energy E_D=2.225 MeV, a_1=5.424 fm, deuteron quadrupole moment 0.2859 fm .

It is naturally suggested that triton asymptotic D to S ratio η^T be given the status of a single experimental quantity to "measure" the triton D state. The triton D state appears in a model only when we include a tensor $^3S_1-^3D_1$ nucleon-nucleon interaction which produces a D wave component for the deuteron. The single quantity which measures the deuteron D state is the asymptotic quantity η^D. In any model where η^D=0, automatically η^T=0. So it is intutively expected that η^T will be sensitive to the value of η^D. In order to study this sensitivity in our model, we varied r_0 in a controlled way keeping the deuteron parameters and a_0 fixed, such that our model always produces the experimental triton binding energy of 8.48 MeV. We plotted the resulting η^T versus η^D in fig.1 which gives a linear plot passing through the origin $\eta^D=\eta^T=0$. As the relevant values ofo η^D are small and as $\eta^D=\eta^T=0$ is a point on the correlation, it is intutively expected that in the lowest order of approximation the $\eta^T-\eta^D$ correlation is linear. (The calculated points had values of η^D ranging from 0.025 to 0.030 in steps of 0.001). We also calculated the neutron-deuteron scattering length $^2a_{nd}$ for these points. The variation of r_0 in the range 1.8 fm to 3.6 fm generated small variation in the value of $^2a_{nd}$, which contributes to the width of Phillips plot at a fixed energy. So we conclude from fig.1 that keeping deuteron proprties, a_0 and E_T fixed if we vary η^D, η^T varies linearly with η^D, and is insensitive to small variations of $^2a_{nd}$. This conclusion will be confirmed in the plot of fig.2, which is in part motivated by this conclusion. So any attempt of theoretical evaluation of η^T must take into consideration the correct value of η^D. η^T being an asymptotic observable is expected to be insensitive to the interior part of triton wave function and hence to $^2a_{nd}$ except the information about $^2a_{nd}$ which is indirectly contained in E_T.

Motivated by the above conclusion we plot in fig.2 η^T/η^D versus E_T. We not only plot results of our calculations for a_c=-17.0 fm and a_c=-23.7 fm by varying both r_0 and η^D but also of various realistic calculations [7,14,15] of other authors using sophisticated nucleon

nucleon potentials with and without three nucleon force. Our results with a_0=-17.0 and -21.7 fm fall on two straight lines and the calculation of Gibson and Lehman [10] for a_0=-16.85 fm coincides with our calculation with a_0=-17.0 fm. We see that all the calculations fall on a very narrow band. In our calculation we have varied r_0 and η^D. Variation of r_0 arround r_0=2.8 fm generates the width of $E_T - {}^2a_{nd}$ Phillips plot. η^T, as expected, seems to be completely insensitive to variations in r_0 and associated variations of ${}^2a_{nd}$. From the plot of fig.2 we can easily read off the correct value of η^T/η^D consistent with E_T=8.48 MeV: $\eta^T/\eta^D = 1.71 \pm 0.04$. Using the experimental value of η^D (=0.0271 (4)) [12], one easily obtains our theoretical estimate for η^T: $\eta^T = 0.0463 \pm 0.0013$.

Ishikawa and Sasakawa plotted η^T versus E_T. In so doing they generated a coorrelation of larger width as the effect of variation of η^D of various model nucleon nucleon interactions used in their calculation was not compensated. As the two nucleon forces they used had a η^D smaller than the experimental value a neglect of correction due to η^D resulted in a smaller η^T: $\eta^T = 0.0432 \pm 0.0015$ [7]. Also it seems that their prediction of η^T is lower than that given by the average straight line one can plot near E_T=8.48 MeV. The present value is in better agreement with the experimental values: $\eta^T = 0.48 \pm 0.007$ and 0.051 ± 0.005 [15], than that of Ishikawa and Sasakawa. This is clearly exhibited in fig.2. The point marked IS is Ishikawa and Sasakawa's estimate of η^T/η^D using their estimate of η^T and the experimental value of η^D [12]. This point lies clearly outside the band of fig.2, through the calculated points of Ishikawa and Sasakawa fall perfectly in the band. At this point we note that they [7] also modified the three nucleon force artificially to produce the coorrect E_T. This arbitrary modification of the three nucleon potential (by adding a term called W_c) possibly destroys the low energy correlations to some extent and generates points which fall outside the narrow band of fig.2 and which lead to a somewhat lower value of η^T. We do not show these points in fig.2.

Once we have determined η^T in a "model independent" manner we would like to determine the values of C_S and C_D separately. For this purpose we plot C_D/η^D versus E_T in fig.3. Results of realistic calculations are taken from refs. [7,14,15]. In this case $C_D - \eta^D$ plot at a fixed E_T does not lead to a straight line as in fig.1. However, in the lowest order of approximation this plot can again be considered linear passing through the origin $C_D = \eta^D = 0$; as $\eta^D \to 0$, $C_D \to 0$. Hence the C_D/η^D versus E_T plot as shown in fig.3 is expected to be less sensitive (but not completely insensitive) to η^D. This means, in other words, that C_S will have some sensitivity to the correct value of η^D, which is not implicitly contained in the correct value of E_T and ${}^2a_{nd}$, and which was not considered in ref.[6]. In order to eliminate the effect of variation of η^D on C_D/η^D (at a fixed E_T), in fig.3 we only vary r_0 keeping η^D constant in the present separable potential model. If we vary η^D too much we generate points which fall outside the band of fig.3. (But in fig.2 we varied both η^D and r_0 and generated points which fall inside the band. The reason is that η^T/η^D is not sensitive to variation of η^D when E_T is held fixed). We, however, expect that the $(C_D/\eta^D - E_T)$ correlation be less universal than the $(\eta^T/\eta^D - E_T)$ correlation and be more sensitive to other trinucleon observables, such as, for example, ${}^2a_{nd}$. This is obvious from fig.3 which leads to a plot of larger width than fig.2. To predict the exact value of C_D/η^D one has to use calculations which produce the correct value of ${}^2a_{nd}$.

The present separable model calculation produces the correct $^2a_{nd}$. This fact has been used to reduce the error of C_D/η^D obtained from fig.3. From the plots we extract the "experimental" value of C_D/η^D= 3.16±0.02, or C_D=0.0856±0.002. This result together with our estimate for η^T yields C_S=1.85±0.04. This is slightly higher than that of Girard and Fuda [6]. Girard and Fuda did not have the correct η^D built into their model, consequently they obtained C_S=1.82±0.03. The fact that they had the correct $^2a_{nd}$ and E_T already included some effect of η^D in their result. The difference between our result for C_S and that of Girard and Fuda should, in part, be attributed to the correlation between C_D/η^D and η^D at a fixed value of E_T.

In conclusion, we have studied new correlations between triton and deuteron asymptotic normalization parameters. These correlations are important for making theoretical estimates about triton asymptotic normalization parameters. For the first time in this letter we take into consideration the effect of these correlations in extraction of triton asymptotic normalizations. Our findingS —

$$C_S = 1.85\pm0.04, \qquad C_D = 0.0856\pm0.002,$$

and

$$\eta^T = 0.0463\pm0.0013,$$

— take into account the correct value of E_T and η^D, and are slightly different from earlier theoretical estimates [6,7]. The value of η^T is in good agreement with experimental results of refs.[8].

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FIGURE CAPTIONS:

1. $\eta^T-\eta^D$ correlation at a fixed E_T=8.48 MeV using the present separable potential model for a_0=-17 fm and =-23.7fm. We vary r_0 to generate the points which lie on the straight lines passing through the origin as shown in figure.

2. η^T/η^D-E_T correlation. The straight lines are generated by varying both r_0 and η^D in the present separable potential model for a_0=-17 fm and =-23.7 fm. The open circles are from ref.[7], full circles from ref. [15], triangles from ref.[14], the point marked IS is the estimate of ref.[7] and the points marked E are exprimental results of ref.[8].

3. C_D/η^D-E_T correlation. The full straight line is generated by varying r_0 with η^D=0.027 in the present separable model for both a_0=-17 fm and =-23.7 fm. The open circles are from ref.[7], full circles from ref. [15] and triangles from ref. [14].

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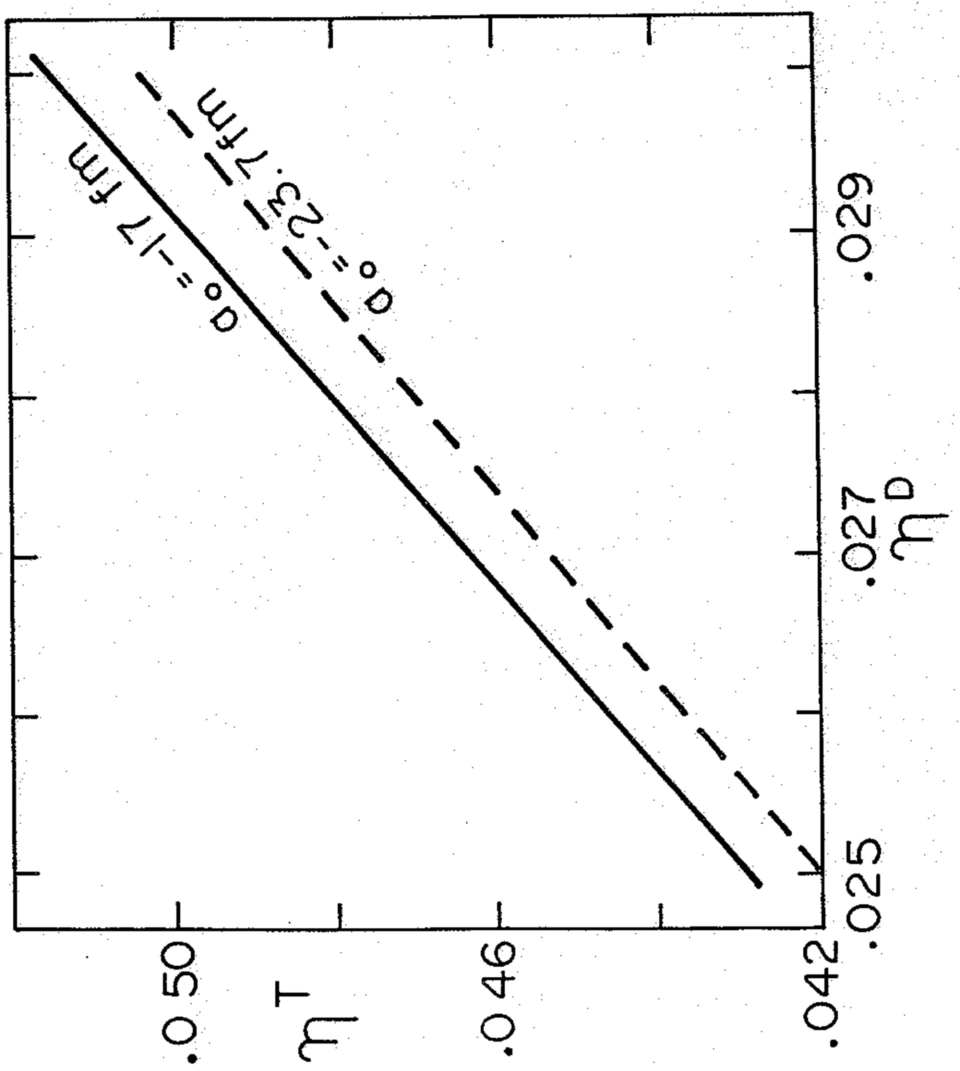
Fig 1



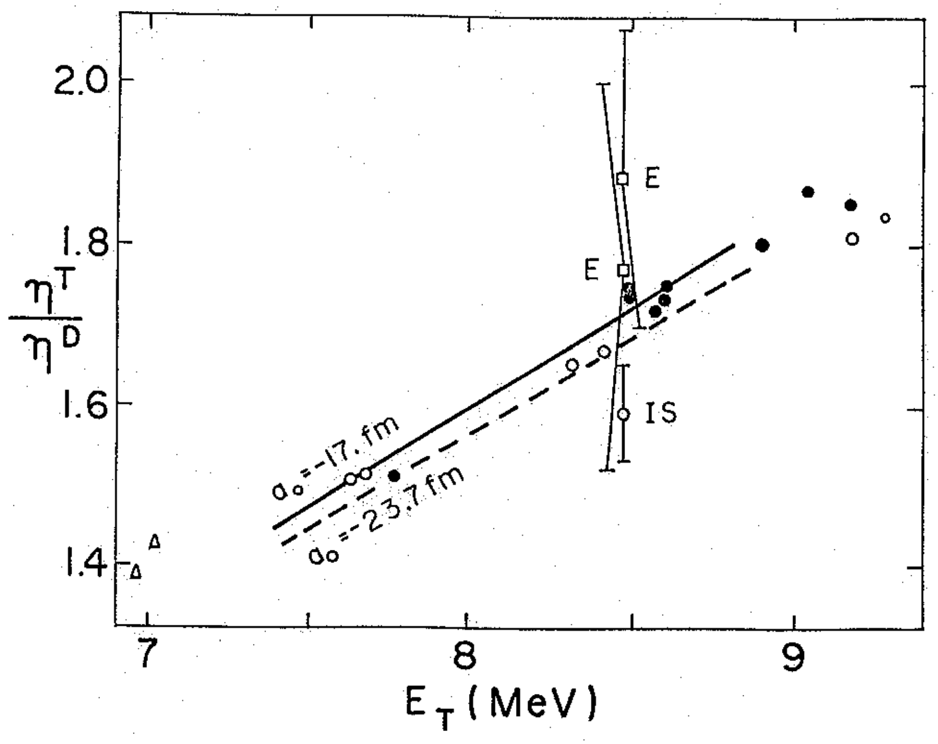

2.0
1.8
1.6
1.4
η^T / η^D
E
E
IS
$a_0 = -17.$ fm
$a_0 = -23.7$ fm
7
8
9
E_T (MeV)

Fig 2

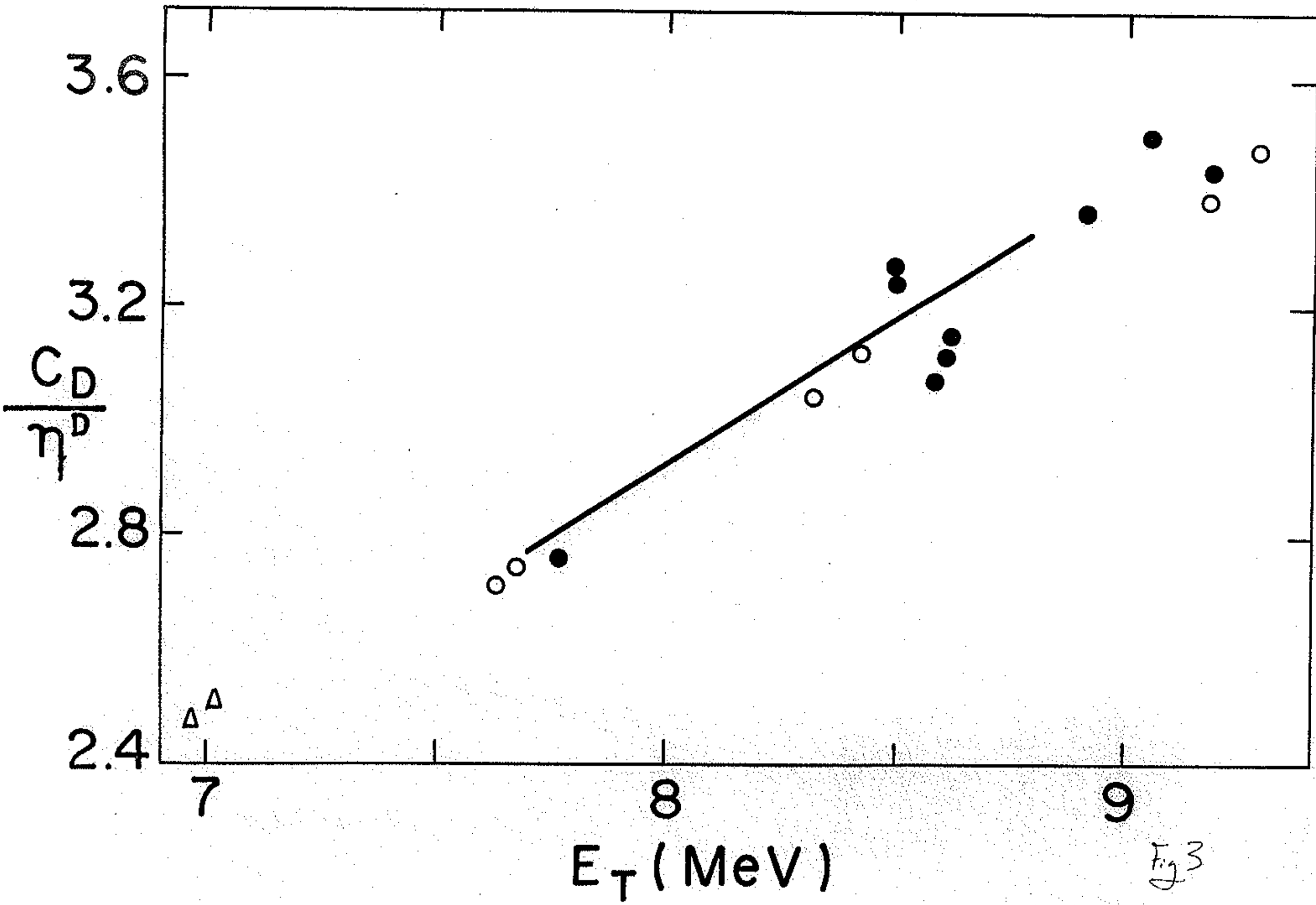

3.6
3.2
2.8
2.4
C_D / η^D
7
8
9
E_T (MeV)

Fig 3