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Abstract:

The Hartree-Bose-Fermi and the adiabatic approximations are used to derive analytic formulas for the moment of inertia and the decoupling parameter of the interacting boson fermion approximation for deformed systems. These formulas are applied to the SU(3) dynamical symmetry. In the large N limit we obtain perfect agreement with the exact results.

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One of the characteristic features of the interacting boson approximation (IBA)[1)] is the occurrence of dynamical symmetries. In the IBA-1, where no distinction between proton-bosons and neutron-bosons is made,three limiting symmetries arise from the U(6) group structure of the model. When the boson hamiltonian corresponds to one of the three limits, U(5), SU(3) or O(6), it can be analytically diagonalized by means of group-theoretical techniques[2)]. Mean field techniques (MFT) like Hartree-Bose(HB)[3)] provide a geometrical interpretation of the three limiting cases[4)] as spherical vibrator, axially symmetric rotor and unstable rotor respectively. The new phenomena that appear in the IBA-II due to the interplay between proton and neutron degrees of freedom are also understood using MFT[5,6)].

The role of the MFT is not only restricted to a geometrical interpretation, but the MFT also give approximate solutions when the model space increases, due to the inclusion of higher spin bosons or non collective degrees of freedom, and exact diagonalizations are not feasible. More accurate results can be obtained by using the random phase approximation[3)] for the band heads and self-consistent cranking[7)] or angular momentum projection[8)] for the rotational bands.

The interacting boson-fermion approximation (IBFA) is the extension of the IBA to odd-even nuclei. This model is based on the coupling of an ideal fermion to the boson core. As the odd fermion commutes with the bosons, the boson-fermion interaction should not only reflect the nuclear interaction, but also the Pauli principle related to the fermion nature of the bosons. Both aspects were considered in a semi-microscopic derivation of the boson-fermion interaction through a direct and an exchange force[9)].

The increase of the model space due to the inclusion of the single particle degrees of freedom makes it useful, and in some cases necessary, to derive approximate solutions.

Several dynamical boson-fermion symmetries[10] related with the three limits of the IBA are known. In particular, it was shown that there is a correspondence between the SU(3) boson-fermion symmetry and the Nilsson model[11]. However, a direct connection with the Nilsson model and the particle plus rotor model would allow better insight into the physical properties of the IBFA in the deformed region.

In a recent work[12] the Hartree-Bose-Fermi approximation (HBF) and the Tamm-Dankoff approximation (TDA) were derived for systems of bosons and fermions. The validity of these approaches was well established in the deformed limit, where good agreement with the exact results for the ground state and the band-heads was obtained[12].

The purpose of this note is to extend further these approximations so as to determine the inertial parameters of the rotational bands from the mean field solutions.

We begin by summarizing the HBF[12] approximation for the case of axial symmetry.

The most general boson-fermion hamiltonian is:

$$H = \sum_{LM} \varepsilon^{B}_{LM} \gamma^{\dagger}_{LM} \gamma_{LM} + \sum_{\substack{L_1L_2L_3L_4 \\ M_1M_2M_3M_4}} V^{BB}_{\substack{L_1L_2L_3L_4 \\ M_1M_2M_3M_4}} \gamma^{\dagger}_{L_1M_1} \gamma^{\dagger}_{L_2M_2} \gamma_{L_3M_3} \gamma_{L_4M_4} \tag{1}$$

$$+ \sum_{jm} \varepsilon^{F}_{j} c^{\dagger}_{jm} c_{jm} + \sum_{\substack{L_1L_2j_1j_2 \\ M_1M_2m_1m_2}} V^{BF}_{\substack{L_1L_2j_1j_2 \\ M_1M_2m_1m_2}} \gamma^{\dagger}_{L_1M_1} \gamma_{L_2M_2} c^{\dagger}_{j_1m_1} c_{j_2m_2}$$

Where $\gamma^{\dagger}_{LM}\left(c^{\dagger}_{jm}\right)$ creates a boson (fermion) with angular momentun $L\,(j)$ and projection $M\,(m)$.

To solve the variational problem is equivalent to find the transformation coefficients η^{B} and η^{F} that define new boson and fermion states:

$$\Gamma^{\dagger}_{PM} = \sum_{L} \eta^{BM}_{PL} \gamma^{\dagger}_{LM} \quad ; \quad a^{\dagger}_{\alpha m} = \sum_{j} \eta^{Fm}_{\alpha j} c^{\dagger}_{jm} \tag{2}$$

The trial wave function is that of an N boson condensate times a one fermion state:

$$|\phi_{N\alpha m}) = \frac{1}{\sqrt{N!}}\, a^{\dagger}_{\alpha m} \left(\Gamma^{\dagger}_{0 0}\right)^{N} |0> \tag{3}$$

The boson of the condensate is labelled by its lowest state p=o with angular momentum projection m=0, because axial symmetry has been imposed.

Minimization of the energy leads to two self-consistent coupled equations for the η^{B} and η^{F}:

$$\sum_{L_2} h^{B}_{L_1L_2} \eta^{B}_{0L_2} = E^{B}_{0} \eta^{B}_{0L_1}$$

with (4)

$$h^{B}_{L_1L_2} = \varepsilon^{B}_{L_1} \delta_{L_1L_2} + 2(N-1) \sum_{L_3L_4} V^{BB}_{\substack{L_1L_3L_4L_2 \\ 0\,0\,0\,0}} \eta^{B}_{0L_3} \eta^{B}_{0L_4} + \sum_{j_1j_2} V^{BF}_{\substack{L_1L_2j_1j_2 \\ 0\,0\,m\,m}} \eta^{Fm}_{\alpha j_1} \eta^{Fm}_{\alpha j_2}$$

and

$$\sum_{j_2} h^{Fm}_{j_1j_2} \eta^{Fm}_{\alpha j_2} = E^{F}_{\alpha m} \eta^{Fm}_{\alpha j_1} \tag{5}$$

with

$$h^{Fm}_{j_1j_2} = \varepsilon^{F}_{j_1} \delta_{j_1j_2} + N \sum_{L_1L_2} V^{BF}_{\substack{L_1L_2j_1j_2 \\ 0\,0\,m\,m}} \eta^{B}_{0L_1} \eta^{B}_{0L_2}$$

Equation (4) for the bosons differs from the usual HB theory in the appearance of the third term which couples the system to the odd-fermion. In the limit of $N \to \infty$ this term can be neglected and the two equations are decoupled.

Equation (5) for the fermion represents a one body potential generated by the mean field of the bosons. It resembles very much Hartree-Fock theory.

For a well deformed system, it is possible to make use of the adiabatic assumption, in the sense that the slow collective rotation of the boson core can be separated from the much faster particle motion in the distorted mean field. This assumption is consistent with the decoupling of the HBF equations in the large N limit.

If this is the case, the boson-fermion hamiltonian can be approximated by a collective part which describes the rotation of an inert boson core plus an intrinsic hamiltonian as in the particle plus rotor model.

$$H = H_{coll} + H_{intr} \tag{6}$$

where:

$$H_{coll} = \sum_{i=1}^{3} \frac{L_i^2}{2\mathcal{J}_i} \tag{7}$$

L_i are the three components of the boson angular momentun and $\mathcal{J}_i$ the corresponding moments of inertia.

H_{intr} is given in the HBF approximation by:

$$H_{intr} = E_0^B \Gamma_0^+ \Gamma_0 + \sum_{\alpha m} E_{\alpha m}^F a_{\alpha m}^+ a_{\alpha m} \tag{8}$$

Where E_c^B and $E_{\alpha m}^F$ are the eigenvalues of the HBF equations (4) and (5). In the case of axial symmetry the hamiltonian (6) can be worked out in the usual way to give:

$$H = H_{rot} + H'_{intr} + H_{cor} \tag{9}$$

$$H_{rot} = \frac{I^2}{2\mathcal{J}} \tag{10}$$

$$H'_{intr} = H_{intr} + \frac{j^2}{2\mathcal{J}} \tag{11}$$

$$H_{cor} = -\frac{1}{2\mathcal{J}} (j_+ I_- + j_- I_+) \tag{12}$$

Here I is the total angular momentum and j the angular momentum of the fermion.

The collective hamiltonian (7) has been factored into a pure rotational hamiltonian (10), a recoil term included in the intrinsic hamiltonian (11) and the Coriolis interaction (12).

In the strong coupling limit the Coriolis interaction is considered as a perturbation, and the eigenfunctions of the system are the product of the eigenfunctions of H_{rot} and H_{intr}.

$$\Psi_{N\alpha\lambda\mu m} = \left(\frac{2\lambda+1}{16\pi^2}\right)^{1/2} \left[D^{\lambda}_{\mu m}\, \phi_{N\alpha m} + (-)^{\lambda+m} D^{\lambda}_{\mu -m}\, \phi_{N\alpha -m} \right] \tag{13}$$

where is the intrinsic eigenfunction given by (3).

The moment of inertia of the boson core $\mathcal{J}$ that appears in the hamiltonian (10) is calculated using the Inglis prescription[3].

$$\mathcal{J} = \sum_{p} \frac{N\, |\langle 0|\Gamma_p L_x \Gamma_0^+|0\rangle|^2}{E_p^B - E_0^B} \tag{14}$$

Taking into account the Coriolis interaction to first order in perturbation theory, a correction for the m=1/2 bands is obtained

$$E_{N\alpha\lambda\mu m} = E^F_{\alpha m} + \frac{1}{2\mathcal{J}} \left\{ \lambda(\lambda+1) + a_\alpha (-)^{\lambda+1/2} (\lambda + 1/2)\, \delta_{m,1/2} \right\} \tag{15}$$

where the decoupling parameter a is given by:

$$a_\alpha = -\sum_{j} \left| \eta^{F\,1/2}_{\alpha j} \right|^2 (-)^{j+1/2} (j+1/2) \tag{16}$$

The accuracy of the HBF approximation has already been tested for the band-heads in the SU(3) limit[12]. We would like to see here how the model works for the moment of inertia and the decoupling parameter. The model is applied to the SU(3) dynamical symmetry of the IBFA with the fermion occupying the orbits j=1/2, 3/2[10].

The boson-fermion hamiltonian is:

$$H = -K\, Q_{BF} \cdot Q_{BF} \tag{17}$$

where

$$Q_{BF} = Q_B + Q_F$$

$$Q_{B\mu} = (s^\dagger d + d^\dagger s)^2_\mu - \frac{\sqrt{7}}{2}(d^\dagger d)^2_\mu \tag{18}$$

$$Q_{F\mu} = \frac{1}{\sqrt{2}}\left\{ (c^\dagger_{3/2} c_{3/2})^2_\mu + (c^\dagger_{3/2} c_{1/2})^2_\mu - (c^\dagger_{1/2} c_{3/2})^2_\mu \right\}$$

The total angular momentum is:

$$I_\mu = L_\mu + j_\mu$$

$$L_\mu = \sqrt{3}\,(d^\dagger d)^1_\mu \tag{19}$$

$$j_\mu = -\frac{1}{\sqrt{2}}(c^\dagger_{1/2} c_{1/2})^1_\mu - \sqrt{5}\,(c^\dagger_{3/2} c_{3/2})^1_\mu$$

The exact solution of the hamiltonian (17) is:

$$E(N, M, \lambda_B, \mu_B, \lambda, \mu, \mathcal{J}, I) = -\frac{K}{2}\left[\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)\right] + \frac{3}{8} K\, \mathcal{J}(\mathcal{J}+1) \tag{20}$$

Where $\mathcal{J}(\mathcal{J}+1)$ is the eigenvalue of the pseudo-orbital angular momentum of the system[10].

For (λ_B, μ_B) = (2N,0) the coupling of the fermion with the boson core gives rise to three bands, the ground band with m=1/2 and two degenerate excited bands with m=1/2, 3/2. The three bands are characterized by the same moment of inertia $\mathcal{J} = 4/3K$, and different decoupling parameters. The decoupling parameter of the ground band is a=-1 while it is zero for the excited bands.

In the limit $N \to \infty$ the first HBF equation is decoupled from the fermion, and for the hamiltonian (17) it reduces to the usual HB approximation of a pure SU(3) bosonic limit.

The solution of this equation is known to give[4]:

$$\Gamma_0^\dagger = \frac{1}{\sqrt{3}}\left(s^\dagger + \sqrt{2}\, d_0^\dagger\right) \tag{21}$$

for the boson of the condensate.

The moment of inertia calculated using the Inglis prescription (14) in the SU(3)[3,13] limit is

$$\mathcal{J} = \frac{4}{3K} \tag{22}$$

and coincides with the exact result. To leading order in N, the moment of inertia depends only on the structure of the boson intrinsic state.

The new aspects of the HBF approximation appear in the second equation which for a quadrupole-quadrupole interaction can be written as

$$h^{Fm}_{j_1 j_2} = \varepsilon^F_{j_1} \delta_{j_1 j_2} - 2K \langle Q^B \rangle\, q^{Fm}_{j_1 j_2} \tag{23}$$

This is the Nilsson hamiltonian where the deformation parameter β is expressed in terms of the mean value of the quadrupole operator in the boson intrinsic state. Although the correspondence between bands in the SU(3) boson-fermion symmetry and the Nilsson orbits has been already established[11], eq.(23) states the direct relation between the two approximations. For the hamiltonian (17), single fermion energies are degenerate and the mean value of the quadrupole operator in a boson intrinsic state with structure (21) gives:

$$\langle Q^B \rangle = N\sqrt{2} \tag{24}$$

Replacing the matrix elements of the quadrupole fermion operator (18) in eq.(23) and diagonalizing we obtain:

$$\mathcal{E}_1(1/2) = -2KN \quad ; \quad \mathcal{E}_2(1/2) = \mathcal{E}_1(3/2) = KN \tag{25.a}$$

$$a_1^{\dagger}(3/2) = c^{\dagger}_{3/2\,3/2}$$

$$a_1^{\dagger}(1/2) = \frac{1}{\sqrt{3}}\left[c^{\dagger}_{1/2\,1/2} - \sqrt{2}\,c^{\dagger}_{3/2\,1/2}\right] \tag{25.b}$$

$$a_2^{\dagger}(1/2) = \frac{1}{\sqrt{3}}\left[\sqrt{2}\,c^{\dagger}_{1/2\,1/2} + c^{\dagger}_{3/2\,1/2}\right]$$

The eigenvalues give a m =1/2 ground state and two degenerate excited bands m=1/2 and m=3/2 at an energy of 3 KN, which agrees with the exact result to leading order in N.

A first test of the fermion eigenfunction is provided by the calculation of the decoupling parameter a. By using the eigenfunctions (25) and eq.(16), we obtain for the decoupling parameters:

$$a_1 = -1 \quad ; \quad a_2 = 0 \tag{26}$$

which again coincides with the exact result.

In conclusion, we have derived the formulas for the moment of inertia and the decoupling parameter of the IBFA model. The HBF approximation for the intrinsic state and the adiabatic assumption have been used. Calculations in the large N limit of an SU(3) boson-fermion symmetry give perfect agreement with the exact results. Also, a direct connection with the Nilsson hamiltonian and the particle plus rotor model has been established. This relation could be fruitful for obtaining a better understanding of the boson-fermion interaction in the deformed region.

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