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NUCLEAR MATTER KINETIC COEFFICIENTS AND DAMPING OF FINITE NUCLEAR COLLECTIVE MODES

by

A.F.R. de Toledo Piza
Instituto de Física, Universidade de São Paulo

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A.F.R. de Toledo Piza
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brazil

1. INTRODUCTION

The description of nuclear structure properties in terms of nucleon degrees of freedom interacting through realistic two-body potentials develops around an accurate lowest order description given in the form of an effective mean-field theory[1]. The relevant mean-field itself is, however, necessarily a rather complicated theoretical construct, due to specific properties of the realistic nucleon-nucleon potential. Essential short range correlations are dealt with in terms of Brueckner's G-matrix. The use of the G-matrix as an effective interaction for the definition of a self-consistent mean-field requires moreover a careful consideration of its density and energy dependence, implying need for further renormalization for the construction of the effective mean-field. This can be rather accurately expressed, eventually, as the Hartree-Fock (HF) mean-field of an effective, density

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dependent, two-body interaction, which is taken as independent of energy[2].

The complicated nature of the effective mean-field also reflects itself on the description of the simplest excitations of finite nuclei. In fact, for the description of single-particle (hole) states in odd-A nuclei and of collective particle-hole vibrations of even-A nuclei, careful consideration of single-particle mass operators and their byproducts (such as the so called induced interaction) is required. This carries the description beyond the pure single-particle or particle-hole levels in a context complicated by the circumstance that the underlying effective mean-field already embodies certain correlation effects, albeit in approximate, energy-averaged form[3].

In what follows these matters will be approached from the point of view that became fashionable after kinetic phenomena on the nuclear scale were brought into evidence by the study of deep-inelastic collisions between heavy-ions[4]. By carrying the general description of one-body observables beyond the mean-field approximation one identifies those correlation terms responsible for kinetic phenomena and, at the same time, those involved in the renormalization of the G-matrix mean-field in finite nuclei. The procedure described in refs. [5-7] is reviewed, leading to a kinetic equation for the one-body density. This will in general contain an effective mean-field including a dispersive component, in addition to a collision term which gives rise to equilibration and transport phenomena. By sorting out collision effects and identifying the effective interaction which is responsible for them, estimates for transport coefficients and for the damping of zero sound are obtained in terms of available Fermi liquid results. These estimates point once more to the inadequacy of hydrodynamical (local equilibrium) descriptions of collective nuclear modes and indicate that collisional damping in large nuclei may account for one or a few tenths of the observed widths.

2. KINETIC EQUATION FOR THE ONE-BODY DENSITY

A complete derivation is beyond the limits of this presentation, and has been given elsewhere[5,6]. The exact effective dynamical law for the one-body density is best written in Liouville space, where the time displacement operator is a one-body liouvillian (super-)operator $\hat{\hat{\ell}}(t)$. The time-evolution equation is then obtained in the form

$$i\dot{\hat{\rho}}(t) = \hat{\hat{\ell}}(t)\hat{\rho}(t) + \hat{r}(t). \tag{1}$$

The liouvillian $\hat{\hat{\ell}}(t)$ is in fact a nonlinear functional of the one-body density $\hat{\rho}$. It splits as $\hat{\hat{\ell}}_0(t) + \hat{\hat{\ell}}_1(t)$ with

$$\hat{\hat{\ell}}_0(t)\hat{\rho}(t) = [\hat{h}[\hat{\rho}], \hat{\rho}(t)] \tag{2}$$

where $\hat{h}$ is the HF hamiltonian associated with $\hat{\rho}(t)$. A many body hamiltonian consisting of a kinetic energy term plus a two-body potential is being assumed. The two-body potential will be discussed shortly. The remaining part of $\hat{\hat{\ell}}(t)$, $\hat{\hat{\ell}}_1(t)$, is a complicated object containing the effects of time evolution of correlations in the system. It plays the role of a generalized, non-markovian collision term, so that eq. (1) looks in fact as a kinetic equation. The simplest approximation to this term will suffice for the present discussion. It appears as

$$\left[\hat{\hat{\ell}}_1(t)\hat{\rho}(t)\right]_{\lambda\mu} = \frac{i}{2}\sum_{\beta\gamma\delta}\int_0^t dt' \Big[\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_t \langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_{t'} (p_\lambda p_\beta q_\gamma q_\delta - p\leftrightarrow q)_{t'}$$

$$+ \langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_t \langle\gamma\delta|\tilde{v}|\mu\beta\rangle_{t'} (p_\mu p_\beta q_\gamma q_\delta - p\leftrightarrow q)_{t'}\Big]. \tag{3}$$

Here an explicit single-particle representation has been introduced. The basic one-body state vectors have been chosen for convenience as the orthonormal time-dependent natural orbitals (eigenvectors) of $\hat{\rho}(t)$, i.e.

$$\hat{\rho}(t) = \sum_\lambda |\lambda\rangle_t \, p_\lambda(t) \, \langle\lambda|_t \, , \tag{4}$$

$p_\lambda(t)$ being the associated (time-dependent) occupation probabilities, and $q_\lambda(t) = 1 - p_\lambda(t)$. The nonlinearity in $\hat{\rho}$ is thus explicit in eq. (3).

The remaining term of (1), $\hat{r}(t)$, vanishes if one starts with a determinantal state at $t=0$[6]. In general it is also a nonlinear functional of $\hat{\rho}$ which contains effects of initial (i.e., at $t=0$) correlations. When studying e.g. the response of the system to an external perturbation, this term is required to ensure the stationarity of $\hat{\rho}$ in the absence of the perturbation but in presence of the correlation term $\hat{\hat{\ell}}_1 \hat{\rho}$, which would give rise to nontrivial evolution of an uncorrelated initial state[8]. In this context, and to the same approximation used in eq. (3), it appears as

$$\hat{r}_{\lambda\mu}(t) = \frac{1}{2}\sum_{\beta\gamma\delta}\left[\frac{\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_t\langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_0}{\epsilon_\gamma+\epsilon_\delta-\epsilon_\lambda-\epsilon_\beta}\left(p_\gamma p_\delta q_\lambda q_\beta - p\leftrightarrow q\right)_0\right.$$

$$\left.+\frac{\langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_t\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_0}{\epsilon_\mu+\epsilon_\beta-\epsilon_\gamma-\epsilon_\delta}\left(p_\gamma p_\delta q_\mu q_\beta - p\leftrightarrow q\right)_0\right]. \quad (5)$$

Here the ϵ's are HF energies such that the time evolution under $\hat{h}[\hat{\rho}]$ is $|\lambda\rangle_t = e^{-i\epsilon_\lambda t}|\lambda\rangle_0$.

One of the conveniences of the natural orbital representation appears in full light when one analyses eqs. (3) and (5) as contributions to the time-derivative of eq. (4): off diagonal ($\lambda\neq\mu$) terms correct the time-evolution of the natural orbitals themselves, while diagonal ($\lambda=\mu$) terms relate to the time rate of change of the eigenvalues $p_\lambda(t)$, a non-unitary effect. Since $\sum_\lambda \dot{p}_\lambda = 0$, moreover, this particular nonunitarity is probability conserving[5].

3. THE STATIC EFFECTIVE MEAN-FIELD

Consider now stationary solutions of eq. (1). Using single-particle energy phase factors for the time dependence of the $|\lambda\rangle_t$, and setting $t'=0$ in $p_\lambda(t')$ to perform the time integral in (3) one obtains ($\lambda\neq\mu$ only)

$$0 = [\hat{h}[\hat{\rho}],\hat{\rho}]_{\lambda\mu} + \frac{1}{2}\sum_{\beta\gamma\delta}\left[\frac{\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_c\langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_c}{\epsilon_\gamma+\epsilon_\delta-\epsilon_\lambda-\epsilon_\beta}\left(p_\lambda p_\beta q_\gamma q_\delta - p\leftrightarrow q\right)_c\right.$$

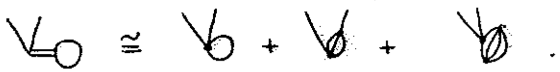
$$\left.+\frac{\langle\lambda\beta|\tilde{v}|\gamma\delta\rangle_c\langle\gamma\delta|\tilde{v}|\mu\beta\rangle_c}{\epsilon_\mu+\epsilon_\beta-\epsilon_\gamma-\epsilon_\delta}\left(p_\mu p_\beta q_\gamma q_\delta - p\leftrightarrow q\right)_c\right] \quad (7)$$

which corresponds precisely to what is obtained by varying an energy functional involving the G-matrix[2] with obvious changes related to the particular structure of the G-matrix in that case. Pictorially, the right-hand side of eq. (7) appears as

An additional term  also appears, together with the first graph above, when renormalized occupation probabilities from eq. (5) are used in $\hat{h}[\hat{\rho}]$.

At this point it is necessary to discuss the nature of the two-body force $\tilde{v}$ more explicitly. It has been thus far assumed to be smooth, in the sense of producing a reasonable enough HF field. In order to accomodate strong short range repulsion, the preceding argument can still be carried through after replacing $\tilde{v}$ by an effective object, satisfying in general a Bethe-Salpeter equation, which reduces to ladder sums in the static limit. It can be obtained[9] by expansion and suitable ressumation of eq. (3.17a) of ref. [5].

Through eq. (7) the generalized collision term of eq. (1) becomes thus related to density dependence of nuclear mean-fields. Density fluctuations give rise to mean-field fluctuations through an interaction  between quasi-particles (in the sense of Landau) which can be explicitly obtained by taking functional derivatives of the effective mean-field[2]



The complete stationarity condition for $\hat{\rho}_0$ can now be written as

$$i\dot{\hat{\rho}}_0 = 0 = \left[\tilde{h}[\hat{\rho}_0], \hat{\rho}_0\right] + \left(\hat{\hat{\ell}}_1 \hat{\rho}_0 + \hat{r}\right)_{\{\lambda=\mu\}} \qquad (8)$$

where the effective mean-field hamiltonian $\tilde{h}$ includes the correlation corrections discussed so far. The last, nonunitary term still contains the bare interaction $\tilde{v}$. Since the comutator term has vanishing diagonal elements, this diagonal

term must also be made to vanish. This is what is accomplished, in particular, by the initial correlations term $\hat{r}$ as written in eq. (5). In terms of the ground state energy, the effect of this term in the case of a two-body potential $\tilde{v}$ is to add the second order correlation energy to the HF energy.

4. NON-STATIONARY STATES: DISPERSION AND COLLISIONS

As a result of the non-Markov nature of the correlation term, the chief modification affecting the above picture when one deals with non-stationary states is that the correlation contributions acquire a dispersive character. In particular, the linearized dynamics of small disturbances from equilibrium, $\hat{\rho} = \hat{\rho}_0 + \delta\hat{\rho}$, where $\delta\hat{\rho}$ is assumed to have a harmonic time dependence, is described by the equation

$$i\delta\dot{\hat{\rho}} = \left[\hat{h}_0, \delta\hat{\rho}\right] + \left[\frac{\delta\hat{h}}{\delta\hat{\rho}}\Big|_{\hat{\rho}_0}\delta\hat{\rho}, \hat{\rho}_0\right] + \frac{\delta\left[\hat{\hat{\ell}}_1\hat{\rho} + \hat{r}\right]}{\delta\hat{\rho}}\Big|_{\hat{\rho}_0}\delta\hat{\rho}. \qquad (9)$$

When evaluating the memory integral appearing in the last term (cf. eq. (3)), the time dependence of $\delta\hat{\rho}$ will modify the energy phase factors. In the case of a harmonic time-dependence of frequency ω, one essentially collects an additional factor $\exp -i\omega(t-t')$ thus causing a shift of $\hbar\omega$ e.g. in energy denominators (cf. eq. (7)) and energy conservation δ-functions.

In order to illustrate this, and in preparation of the application in sect. 5 below, consider the case of a weakly inhomogeneous perturbation of an extended, homogeneous system. In this case it is useful to represent eq. (9) in momentum space, where $\hat{\rho}_0$ is diagonal and $\delta\hat{\rho}$ is strongly peaked around its diagonal. Following usual practice, it is convenient do express $\delta\hat{\rho}$ in terms of a quantity $\hat{\psi}$ such that

$$\delta\hat{\rho} = -\hat{\rho}_0(1-\hat{\rho}_0)\hat{\Psi} \; . \tag{10}$$

The linearized last term of eq. (9) contains several contributions of similar structure. The $\vec{k}, \vec{k}'$ matrix element of a typical-contribution at time t appears as

$$\propto \int_0^t dt' \sum_{\vec{k}_i} \langle \vec{k}_3 \vec{k}_4 | \tilde{v} | \vec{k}' \vec{k}_2 \rangle \langle \vec{k} \vec{k}_2 | \tilde{v} | \vec{k}_3 \vec{k}_4 \rangle \hat{\rho}_0(\vec{k}) \hat{\rho}_0(\vec{k}_2) \times$$
$$\times [1-\hat{\rho}_0(\vec{k}_3)][1-\hat{\rho}_0(\vec{k}_4)] \, e^{i\frac{(k^2+k_2^2-k_3^2-k_4^2)}{2m}(t-t')} \hat{\Psi}(\vec{k}_4 \vec{k}_4' ; t') \; . \tag{11}$$

Assuming harmonic time dependence for $\hat{\psi}(t')$ and extracting an overall $\exp{-i\omega t}$ phase one gets an additional term $\omega(t-t')$ on the exponent of the energy phase factor. The time integral can now be performed with the result

$$\frac{1}{i\Delta_{\vec{k}_i\omega}}(1-\cos\Delta_{\vec{k}_i\omega}t) + \frac{1}{\Delta_{\vec{k}_i\omega}}\sin\Delta_{\vec{k}_i\omega}t$$

which, for large t reduces to a principal value operator plus a δ-function part. While the former is the dispersive counterpart of the correlation corrections to the mean-field, the latter emerges with the characteristic structure of the (linearized) dissipative Uehling-Uhlembeck collision term, with the energy conservation δ-function as modified originally by Landau[10].

5. FERMI LIQUID ESTIMATES OF COLLISION EFFECTS

The discussion of the preceding section indicates that one may try and estimate collision effects in nuclei extending the treatment of ref. [11] by adding the collision term to the linearized dynamical equation. A crucial point for this lies in the realization that different Landau parameters are involved in the description of mean-field fluctuations and of transition probabilities in the collision term. In fact, in the case of the former one must account for the shielding effects of medium correlations, while for the latter a G-matrix residual interaction should be used.

Contact with the standard treatment found in the literature[12] is made as usual by taking the Wigner transform of eq. (9) in the long wavelength limit. Assuming an equilibrium Fermi distribution $\hat{\rho}_0$ with temperature T, and using the Landau parameters F_0, F_0', G_0 and G_0' of the G-matrix residual interactions of. ref. [13] one obtains for the shear viscosity coefficient η the results shown in Table 1. Note that $\omega \to 0$ in the quasi-static situation involved in this case.

Table 1

Interaction (Ref. [13])	F_0	F'_0	G_0	G'_0 (302 MeV fm^3)	ηT^2 (MeV^{3}s fm^{-3})
HM3A	- 1.12	0.28	0.17	0.73	1.1×10^{-20}
HEA(NR)	- 1.17	0.32	0.20	0.63	1.2×10^{-20}
HEA(R)	- 0.72	0.42	0.12	0.63	2.0×10^{-20}

The damping of zero sound, on the other hand, can be described in terms of a width Γ defined as the imaginary part of the frequency variable. This has been obtained using $f_0 \sim 0.2$ as the only Landau parameter associated with mean-field fluctuations, and by converting the temperature dependence to a frequency dependence simply by multiplication by the correction factor[10]

$$1 + \left(\frac{\hbar\omega}{2\pi T}\right)^2 \simeq \left(\frac{\hbar\omega}{2\pi T}\right)^2 .$$

Results for the indicated residual interactions, obtained in the approximation of a single relaxation time[12], are shown in Table 2.

Table 2

Interaction (Ref. [13])	$\Gamma/(\hbar\omega)^2$ (MeV^{-1})
HM3A	4.6×10^{-3}
HEA(NR)	4.5×10^{-3}
HEA(R)	2.6×10^{-3}

6. DISCUSSION

Differences in the results obtained for each of the residual interactions illustrates a sensitivity to the Landau parameters. The values shown for the shear viscosity are about three orders of magnitude ($T \sim 1$ MeV) larger than the fitted value $\eta = 1 \times 10^{-23}$ MeV s fm^{-3} from phenomenological hydrodynamic treatments of nuclear collectivity[14]. This is consistent with the weakly collisional character of the nuclear fluid, which invalidates local equilibrium assumptions for the treatment of nuclear collective motion.

The values obtained for the widths Γ, on the other hand, though smaller than typical observed widths of zero sound states, indicates that collisional effects are <u>not</u> negligibly small. Note that the opposite conclusion would have been reached had one used anything like $F_0 \sim 0.2$ in the collisional transition probability. This would in fact have reduced the value of Γ by roughly two orders of magnitude. As for possible additional sources of damping, one would have clearly to consider the dispersive nature of the dynamical medium effects. On the other hand, the semiclassical approximation inherent to the Landau limit eliminates non-collisional dispersive effects which appear in the quantum treatment of zero sound[15].

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