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THE COLOR DIELECTRIC PARAMETER IN 2+1 DIMENSIONAL QCD

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ABSTRACT

We describe a study of the color dielectric parameter ε in 2+1 dimensional QCD, based on the Schwinger-Dyson equations. With the help of the Ward identities in the axial gauge, we conclude that $\varepsilon(q^2)$ displays a $(q^2)^{-\frac{1}{2}}$ singularity in the infrared limit. This behaviour yields confinement of color charges via constant chromoelectric fields at large distances.

August - 1986

I. INTRODUCTION

In the past years there have been many investigations of color dielectric models of confinement[1,2,3]. This approach gives an intuitive picture of the QCD vacuum as a dielectric medium, the properties of which give rise to confinement. In a recent publication[4], we have studied an effective dielectric model of QCD in two space dimensions and investigated the structure of its confinement domain.

In this work we wish to show that under certain conditions, the effective dielectric parameter which characterizes this model arises as a consequence of the quantum fluctuations of the Yang-Mills fields. To this end we will make use of the methods developed in an important series of papers[5] by Baker, Ball and Zachariassen, where the behaviour of the gluon propagator in the axial gauge is investigated in the infrared limit. These authors have made an essential use of the Schwinger-Dyson equations for the gluon propagator and presented arguments that its wave function in the axial gauge is directly related to the color dielectric parameter in the infrared limit. Furthermore, it has been shown perturbatively[6] that in physical gauges the gluon wave function describes the dominant infrared behaviour of S-matrix elements in Yang-Mills theory. For these reasons, we will assume that the long-distance properties of the gluon propagator in the axial gauge is relevant to the confinement problem, via the behaviour of the color dielectric parameter in the infrared region.

The plan of the paper is as follows. In section II we discuss the general properties of the gluon propagator for the 2+1 dimensional Yang-Mills theory in the axial gauge. We derive exact sum rules describing its behaviour and which yield important boundary condition for the gluon propagator at high energy. To study its infrared behaviour, we describe in section III the Schwinger-Dyson equations with help of the Ward identities in the axial gauge. To this end we use in this work dimensional regularization[7] which yields a gauge invariant calculation and also allows us to understand the behaviour of the gluon propagator in any number N of space-time dimensions. We show that the Schwinger-Dyson equations yield a wave function whose leading behaviour in the infrared domain behaves like $(q^2)^{\frac{2-N}{2}}$ for small q^2. We also analyse for the three dimensional theory the next-to-leading corrections which behave like $(q^2)^{0.3422}$. In the last section we show that this behaviour yields a chromoelectric field which is constant at large distances for any value N of the space-time dimensions. We also present an analytically soluble model reflecting the properties of the true Schwinger-Dyson equations, which allows a simple understanding of the transition between the perturbative expansion at high energy and the non-perturbative description in the infrared limit.

II. THE GLUON PROPAGATOR

In two space dimensions, the chromoelectric field E_i^a, where a stands for color indices and i denotes the

spatial dimensions x and y, is given by the expression:

$$E_i^a = \partial_0 A_i^a - \partial_i A_0^a + g f^{abc} A_0^a A_i^c \quad (1)$$

Here g represents the effective coupling constant, defined for instance in the infinite momentum limit[8], which has dimensions of $(\text{mass})^{\frac{1}{2}}$. In this case the magnetic field B^a is expressed as follows:

$$B^a = \partial_x A_y^a - \partial_y A_x^a + g f^{abc} A_x^b A_y^c \quad (2)$$

Let us now consider the axial gauge characterized by the condition:

$$n_\mu A_\mu^a = 0 \quad (3)$$

We observe that, if n is pure space-like, this gauge condition implies that the magnetic field reduces to the same expression as in the free-field case. Therefore all the effects connected with the behaviour of the QCD vacuum must come in this case from the properties connected with the chromoelectric fields which can be effectively characterized by a color dielectric parameter ε. To study these effects, consider the gluon propagator $D_{\mu\nu}$ which in view of the gauge condition (3), will have the following general form

$$D_{\mu\nu}(q,n) = \frac{Z}{q^2}\left[\delta_{\mu\nu} - \frac{n_\mu q_\nu + n_\nu q_\mu}{n.q} + \frac{n^2 q_\mu q_\nu}{(n.q)^2}\right] +$$

$$+ \frac{Z'}{q^2} \left[\delta_{\mu\nu} - \frac{n_\mu n_\nu}{n^2} \right] \tag{4}$$

In order to study the behaviour of Z and Z' let us recall[9] that the gluon propagator can be represented in the spectral form:

$$D_{\mu\nu}(q,n) = \int_0^\infty \frac{\rho_{\mu\nu}}{q^2 + \sigma^2 - i\varepsilon} \, d\sigma^2 \tag{5.a}$$

where in view of the form (4) which is of zero degree in n, the spectral function $\rho_{\mu\nu}$ has the structure:

$$\rho_{\mu\nu}(\sigma^2, n, q) = \rho_1\left(\sigma^2, \frac{n.q}{|n|}\right)\left[\delta_{\mu\nu} - \frac{n_\mu q_\nu + n_\nu q_\mu}{n.q} + \frac{n^2 q_\mu q_\nu}{(n.q)^2}\right] +$$

$$+ \rho_2\left(\sigma^2, \frac{n.q}{|n|}\right)\left[\delta_{\mu\nu} - \frac{n_\mu n_\nu}{n^2}\right] \tag{5.b}$$

We will now derive two exact sum rules satisfied by the form factors ρ_1 and ρ_2. To this end we use the fact[9] that ρ_{ij} is connected to the vacuum expectation value of an equal-time commutator via the relation:

$$\delta^{ab} \int_0^\infty d\sigma^2 \, \rho_{ij}(\sigma^2, n, q) = \int d^2x \, e^{i\vec{q}.\vec{x}} \langle 0 | [A_j^b(0), \partial_0 A_i^a(\vec{x}, 0)] | 0 \rangle \tag{6.a}$$

In a non-abelian theory, $\partial_0 A_i^a$ is not canonical to A_i^a. In fact, from (1) we see that, in terms of the canonical momenta E_i^a we have:

$$\partial_0 A_i^a = E_i^a + \partial_i A_0^a - g f^{abc} A_0^b A_i^c \tag{6.b}$$

Since A_0 is not a dynamical field it must be eliminated by solving the equations of motion. In the axial gauge, it is possible to accomplish this exactly to all orders in the coupling constant. We then obtain for the equal-time commutator in (6.a) the result

$$\delta_{ab}\left[\delta_{ij} - \frac{n_i q_j + n_j q_i}{n.q} + \frac{n^2 q_i q_j}{(n.q)^2}\right] + C_{YM}\, g^2 n^2 \sum_Q \frac{\langle 0|A_i^a(0)|Q\rangle \langle Q|A_j^b(0)|0\rangle}{[n.(q+Q)]^2}$$

$$= \delta^{ab} \int_0^\infty d\sigma^2 \, \rho_{ij}(\sigma^2, n, q) \tag{7}$$

where $|Q\rangle$ denotes a complete set of intermediate states and C_{YM} stands for the Casimir invariant of the Yang-Mills fields. The second expression on the left-hand side of (7) depends on q only through the combination $n.q$ and in view of the gauge condition (3) will be proportional to $\delta_{ij} - n_i n_j / n^2$. This combination is independent from the first term in (7) and therefore, comparing with equation (5.b) we obtain two sum rules satisfied respectively by the form factor ρ_1 and ρ_2:

$$\int_0^\infty d\sigma^2 \, \rho_1\left(\sigma^2, \frac{n.q}{|n|}\right) = 1 \tag{8.a}$$

$$\int_0^\infty d\sigma^2 \, \rho_2\left(\sigma^2, \frac{n.q}{|n|}\right) = g^2 \, C_{YM} \, n^2 \sum_Q \frac{|\langle 0|A_i^a(0)|Q\rangle|^2}{[n.(q+Q)]^2} \tag{8.b}$$

We are now in a position to study the structure of the gluon propagator $D_{\mu\nu}$. Comparing equations (4) and (5), and making use of relations (8), we see that in the high-energy domain Z and Z' have the following behaviour:

$$Z(q\to\infty) = 1 \qquad (9.a)$$

$$Z'(q\to\infty) = g^2 C_{YM} n^2 \sum_Q \frac{|\langle 0|A_i^a(0)|Q\rangle|^2}{[n.(q+Q)]^2} \qquad (9.b)$$

It can be verified that, although Z' vanishes to order g^2, it is in general a non-zero and gauge dependent quantity in this limit.

In accordance with reference 5 we will now make the ansatz, to be verified later, that Z' is irrelevant in the infrared limit for the determination of physical quantities like the color dielectric parameter. In this case, only Z will give the dominant behaviour of the propagator in the infrared region. Then, the vacuum polarization tensor $\Pi_{\mu\nu}$ will be essentially proportional to the inverse of the free propagator, having the structure:

$$\Pi_{\mu\nu}(q\to 0) = Z^{-1}(\delta_{\mu\nu} q^2 - q_\mu q_\nu) \qquad (10)$$

This procedure is consistent provided Z is a physical, gauge invariant quantity in this region. This has been verified perturbatively in [6] and also checked numerically in reference 5 where it has been argued that Z is directly connected to the static dielectric parameter ε via the relation:

$$\varepsilon(\vec{q}) = Z^{-1}(\vec{q}) \qquad (11)$$

In what follows we shall take advantage of the gauge independence of Z in the dominant infrared region and make $n.q = 0$, a choice that will enable us to study analitically the behaviour of ε in this region. With this choice, we can determine Z by looking at the scalar equation obtained by multiplying (10) by $n_\mu n_\nu/n^2$. We obtain:

$$Z^{-1} = \left.\frac{n_\mu \Pi_{\mu\nu} n_\nu}{n^2 q^2}\right|_{n.q=0} \qquad (12)$$

In our study it will be important to determine the boundary conditions satisfied by the wave function Z. In the high-energy limit these conditions are given by expression (9.a), while the lowest order corrections are obtained from the second order vacuum polarization graph $\Pi_{\mu\nu}$ shown in Fig. 1.

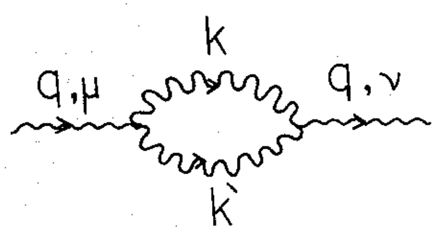

Fig. 1 - Diagram representing the second order contributions to the gluon self-energy.

With the help of equation (12) we obtain:

$$Z(q\to\infty) = 1 + \frac{23}{32} C_{YM} \frac{g^2}{(q^2)^{\frac{1}{2}}} \qquad (13)$$

III. THE SCHWINGER-DYSON EQUATION

We now turn to the determination of the infrared behaviour of Z via the S-D equation for the polarization tensor $\Pi_{\mu\nu}$. The relation expressed in equation (12) allows us to reduce it to a single scalar equation involving the 3-gluon vertex as illustrated in Fig. 2. In this equation there are no four gluon terms attaching to the external vertices since these contain delta functions like $\delta_{\mu\beta}$ where β is some internal index. Then n_μ becomes transferred to a gluon propagator and therefore $n_\beta D_{\beta\beta'}$ vanishes in consequence of gauge condition (3).

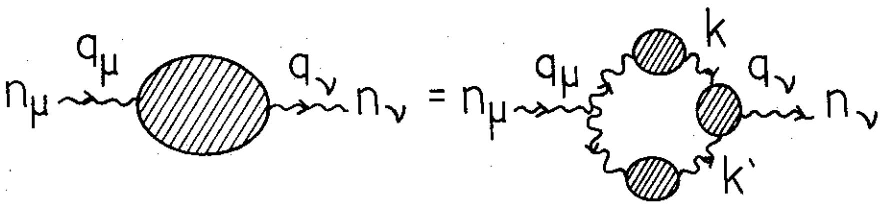

Fig. 2 - Diagrammatic representation of the Schwinger-Dyson equation for the vacuum polarization tensor in the axial gauge.

We now use the Ward identity satisfied by the 3-gluon vertex to determine its low momentum behaviour in terms of Z^{-1}. Due to the absence of ghosts this identity is very simple in the axial gauge[10]:

$$i\,\Gamma_{\sigma\sigma'\nu}(k,k',-q)(-q_\nu) = \Pi_{\sigma\sigma'}(k) - \Pi_{\sigma\sigma'}(k') \qquad (14)$$

We assume, following reference 5, that the transverse part of

this vertex does not give the dominant contributions to the infrared singularity of the propagator, and therefore replace the vertex by its longitudinal part. Then the solution for the longitudinal part of the 3-gluon vertex following from equation (14) has the form:

$$\Gamma^L_{\sigma\sigma'\nu}(k,k',-q) = \delta_{\sigma\sigma'}\left[Z^{-1}(k)\,k_\nu - Z^{-1}(k')\,k'_\nu\right]$$
$$- \frac{Z^{-1}(k) - Z^{-1}(k')}{k^2 - k'^2}\left[k\,.\,k'\,\delta_{\sigma\sigma'} - k'_\sigma k_{\sigma'}\right](k-k')_\nu +$$
$$+ \text{ cyclic permutations} \qquad (15)$$

In this way the S-D equation becomes a non-linear integral equation for Z whose infrared behaviour will be the same as that of the true propagator. Using dimensional regularization in a space-time of dimension N, one finds:

$$\left[Z^{-1}(q) - 1\right]q^2 = g^2\, C_{YM} \int \frac{d^N k}{(2\pi)^N}\,\frac{(n.k)^2}{n^2}\, D^0_{\lambda\sigma}(k)\, D^0_{\lambda\sigma'}(k')\ .$$
$$\left\{\left[-\frac{Z(k)Z(k')}{Z(q)} + Z(k)\right]\frac{(q+k')_\sigma\, q_{\sigma'}}{k'^2 - q^2} + \frac{Z(k) - Z(k')}{k^2 - k'^2}\,(k.k'\,\delta_{\sigma\sigma'} - k'_\sigma k_{\sigma'}) + \right.$$
$$\left. + Z(k')\,\delta_{\sigma\sigma'} + (k \leftrightarrow k', \sigma \leftrightarrow \sigma')\right\} \qquad (16)$$

where $D^0_{\mu\nu}$ denotes the free gluon propagator. Using the fact that $D^0_{\mu\nu}$ has a vanishing angular average: $\int d\Omega_N\, D^0_{\mu\nu} = 0$, it is straightforward to verify that in the infrared limit, the right-hand side of equation (16) becomes:

$$R(q\to 0) \simeq \frac{8\,g^2\,C_{YM}}{(4\pi)^{N/2}}\,\frac{N-1}{N\Gamma(\frac{N}{2})}\int dk\,k^{N-3}\left[k^2\,\frac{\partial Z(k)}{\partial k^2}+\frac{N-2}{2}\,Z(k)\right] \qquad (17)$$

This expression vanishes provided the wavefunction Z has the following leading behaviour:

$$Z(k) \simeq A\left(\frac{\mu^2}{k^2}\right)^{\frac{N-2}{2}} \qquad (18)$$

where μ denotes a unit of mass which sets the scale of the theory. Since in a N-dimensional space-time the Yang-Mills coupling constant has dimensions of $(\text{mass})^{\frac{4-N}{2}}$, we can take without loss of generality $\mu \equiv g^{\frac{2}{4-N}}$.

For consistency we also need that the -1 on the left hand side of eq. (16) to be cancelled. This requires to consider the next to leading term in the infrared limit. Our goal here is to determine analitically this correction which we denote by z. To this end, let us write:

$$Z(q) = A\left(\frac{\mu^2}{q^2}\right)^{\frac{N-2}{2}} + z(q) \qquad (19)$$

and linearize (16) in z, since as $q\to 0$ the first term in (19) will dominate.

With the help of the following relations:

$$\int d^Nk\, f(k,k')\,\frac{k.k'\,n^2}{(n.k)^2} = (N-2)\int d^Nk\, f(k,k') \qquad (20.a)$$

and

$$\int d^Nk\, f(k,k')\,\frac{(n.k)^2}{n^2} = \frac{1}{N-1}\int d^Nk\, f(k,k')\left[k^2-\frac{(q.k)^2}{q^2}\right] \qquad (20.b)$$

which are valid when $n.q=0$, we find that equation (16) becomes:

$$Z^{-1}(q) = 1 + g^2\,C_{YM}\int\frac{d^Nk}{(2\pi)^N}\,\frac{1}{k^2}\,\frac{1}{k'^2}\,.$$

$$\Big\{z(k)\left[N\left(\frac{q^2}{k'^2}\right)^{\frac{N-2}{2}} + \frac{N-2}{N-1}\left(\frac{k^2}{q^2}-\frac{(k.q)^2}{q^4}\right)+\right.$$

$$+\frac{2}{N-1}\left(\left(\frac{q^2}{k'^2}\right)^{\frac{N-2}{2}}-1\right)\frac{(N-2)(k'.q)^2+q^2k'^2}{q^2(q^2-k'^2)}+$$

$$\left.+\frac{2}{N-1}\,\frac{(N-1)(k.k')^2+(k^2-(q.k)^2/q^2)(k^2+k'^2+(4-N)k.k')}{q^2(k^2-k'^2)}\right]+$$

$$+\frac{2}{N-1}\left(\frac{q^2}{k'^2}\right)^{\frac{N-2}{2}}\left(z(k)-\left(\frac{q^2}{k^2}\right)^{\frac{N-2}{2}}z(q)\right)\frac{q^2k^2+(q.k)^2\,(N-2)}{q^2(q^2-k^2)}+$$

$$+\,k\leftrightarrow k' - N\,z(q)\left(\frac{q^4}{k^2k'^2}\right)^{\frac{N-2}{2}}\Big\} \qquad (21)$$

We expect that the form of the correction term will be a power: $z(q) = B\left(\frac{q^2}{\mu^2}\right)^{\alpha}$, as $q\to 0$. With the help of the method used in reference 5, we will determine α by the following consistency condition. On one hand, Z^{-1} as determined by equation (19) will have the form:

$$Z^{-1}(q) \simeq \frac{1}{A}\left(\frac{q^2}{\mu^2}\right)^{\frac{N-2}{2}} - \frac{B}{A^2}\left(\frac{q^2}{\mu^2}\right)^{N-2+\alpha} \qquad (22)$$

On the other hand, it is easy to see by dimensional reasoning that by substituting the form for the correction $z(q)$ into equation (21), its right hand side will yield a contribution proportional to $(q^2)^{\frac{N-4}{2}+\alpha}$. Consistency with the form (22) requires that the coefficient of this power must be zero, condition which will be used in order to determine α. To this end we will perform the integration in eq. (21) in an N-dimensional Euclidean space and rescale its modulus by writing $|k| = |q^2|^{\frac{1}{2}}\, x$. Furthermore we choose the N^{th} axis in the direction of q and denote by y the cosine of the angle between k and q. In terms of these variables it is straightforward to show that the above consistency condition yields the equation

$$F(N,\alpha) \equiv \int_0^\infty dx \int_{-1}^{1} dy (1-y^2)^{\frac{N-3}{2}} \frac{1}{f(x,y)} \cdot$$

$$\left\{\frac{N}{2x}\left(f^\alpha - f^{\frac{2-N}{2}}\right) + \frac{N}{2} x^{2\alpha+N-3} f^{\frac{2-N}{2}} + \right.$$

$$+ \frac{x^{2\alpha+N-3}}{N-1}\left[(N-2)x^2(1-y^2) + \frac{2x^2}{1-x^2} f^{\frac{2-N}{2}} (1-x^{2-N-2\alpha})(1+(N-2)y^2) + \right.$$

$$+ \frac{2}{x^2+2xy}\left(1 - f^{\frac{2-N}{2}}\right)\left(x^2 - x^2y^2 + (N-1)(1+xy)^2\right) +$$

$$\left.\left. + \frac{2x^2}{1+2xy}\left((1-y^2)(1+(N-2)(x^2+xy)) + (N-1)(x+y)^2\right)\right]\right\} = 0 \qquad (23)$$

where $f = f(x,y) \equiv x^2 + 2xy + 1$.

In order to treat the singularities which arise in this expression at $x = 1$, we will use the principal value prescription. We shall now consider more specifically the case when $N \to 3$. In this case, the x and y integrations can be evaluated analitically. After a long calculation, (23) leads in this limit to the following equation for $F(3,\alpha) \equiv F(\alpha)$:

$$F(\alpha) \equiv (4\alpha - \tfrac{14}{3})\,\psi(\alpha+\tfrac{1}{2}) + (4\alpha - \tfrac{16}{3})\left[\psi(\alpha+1) + \pi \operatorname{ctg} \alpha\pi\right] -$$

$$- \frac{17}{6}\frac{1}{\alpha} + \frac{5}{3}\frac{1}{2\alpha+1} - \frac{1}{2}\frac{1}{\alpha+1} - \frac{1}{2\alpha+3} - 8(1-\gamma)\alpha - \frac{5}{3}\gamma + \frac{125}{12} +$$

$$+ \left[2^{-2\alpha}\left(\frac{3}{2\alpha+2} - \frac{7}{2\alpha} + \frac{3}{\alpha-1}\right) + \frac{1}{2}\frac{1}{\alpha+1} + \frac{2}{\alpha} - \frac{3}{2\alpha+2}\right]\frac{\pi}{4}\operatorname{tg}\alpha\pi +$$

$$+ \left(\frac{1}{2\alpha} - \frac{1}{\alpha+1} + \frac{1}{\alpha+2}\right)\frac{2^{2\alpha}4\pi}{\operatorname{sen} 2\alpha\pi} - \frac{1}{2}\left[\frac{1}{4}\frac{1}{(\alpha+2)^2} - \frac{1}{(\alpha+1)^2} + \frac{2}{\alpha^2}\right] -$$

$$- \frac{1}{2}\left[I(2\alpha+3) - 4\,I(2\alpha+1) + 8\,I(2\alpha-1)\right] = 0 \qquad (24.a)$$

where ψ denotes the logarithmic derivative of the gamma function, γ is the Euler constant and $I(\alpha)$ is given by the series:

$$I(\alpha) = \frac{1}{\alpha+1}\sum_{k=0}^{\infty}\frac{1}{2^k(k+\alpha+1)} \qquad (24.b)$$

The behaviour of the expression $F(\alpha)$ is represented graphically in Fig. 3. This function has poles at $\alpha = 0, \frac{1}{2}, 1$ and a zero which to four figure accuracy turns out to be at $\alpha = 0.3422$.

Using this value in equation (19) for N=3, we see that the leading and next to leading benaviour of Z in the infrared region is given by the expression:

$$Z(q\to 0) \simeq A\left(\frac{\mu^2}{q^2}\right)^{\frac{1}{2}} + B\left(\frac{q^2}{\mu^2}\right)^{0.3422} \qquad (25)$$

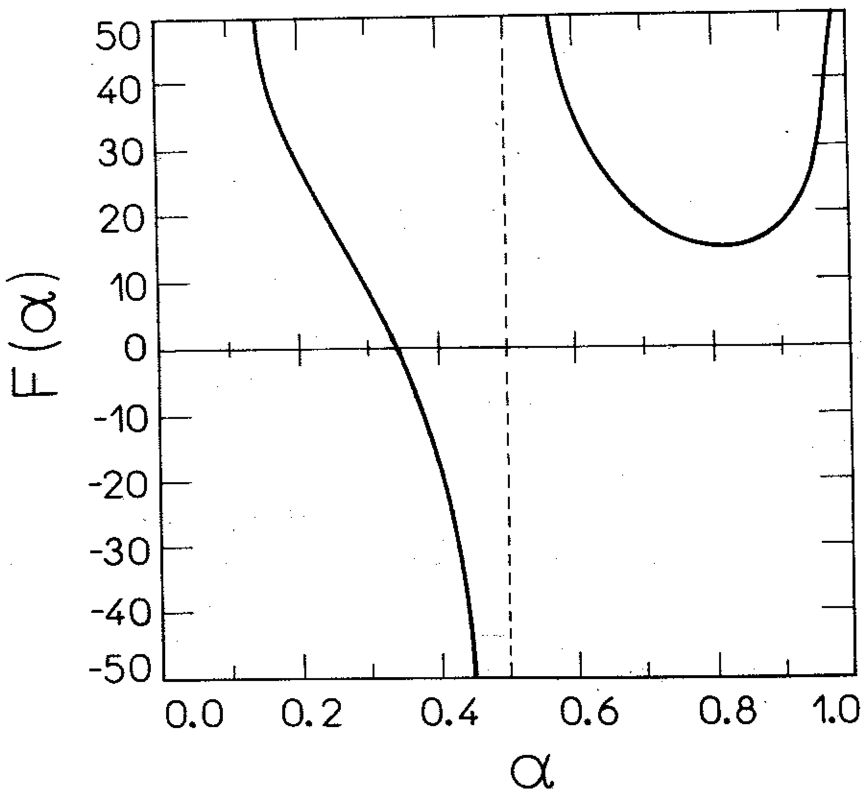

Fig. 3 - The function F(α) plotted versus α in the range $0 < \alpha < 1$.

IV. DISCUSSION

With the help of equations (11) and (18) we see that in an N-dimensional space-time the static color dielectric parameter has the leading infrared behaviour given by:

$$\varepsilon(\vec{q}\to 0) = \frac{1}{A}\left(\frac{\vec{q}^2}{\mu^2}\right)^{\frac{N-2}{2}} \qquad (26)$$

The constant A depends in general on the dimensions of space-time, and should be unity for N=2, since in this case the Yang-Mills theory effectively reduces, in the axial gauge, to a free-field theory where $\varepsilon=1$.

We are now in a position to estimate the behaviour of the chromoelectric field associated with a quark of color charge Q:

$$E(r,N) = \frac{Q}{r^{N-2}\varepsilon(r)} \qquad (27)$$

Going over from equation (26) to coordinate space we find:

$$E(r\to\infty,N) \simeq \frac{\sqrt{\pi}\,(\mu/2)^{N-2}\,AQ}{\Gamma(\frac{N-1}{2})} \qquad (28)$$

where Γ denotes the gamma function. This expression shows that the chromoelectric field is constant at large distances, behaviour which leads to confinement. Although this result was to be expected in two space-time dimensions, it represents as we have seen a non-trivial behaviour which is consistent with the Schwinger-Dyson equations also in higher dimensions.

In the particular case when $N=3$, using the fact that $\mu = g^2$, equation (28) yields in the infrared region the result:

$$E(r) \simeq \frac{\sqrt{\pi}}{2} g^2 AQ \qquad \text{for} \qquad rg^2 >> 1 \qquad (29.a)$$

On the other hand, with the help of equations (11), (13) and (27), we obtain that the behaviour of the chromoelectric field at small distances is given in this case by:

$$E(r) = \frac{Q}{r} + \frac{23\sqrt{\pi}}{64} g^2 C_{YM} Q \qquad \text{for} \qquad rg^2 << 1 \qquad (29.b)$$

Of course, its exact form in the whole range of r is unknown until we solve completely the QCD theory. Even the value of the parameter A is in fact unknown, since it is affected by the transverse part of the 3-gluon vertex which has been neglected in this approach. For this reason, by making the simplest possible ansatz consistent with the boundary conditions and the continuity of the field we can write:

$$E(r) = \frac{Q}{r\varepsilon(r)} = \frac{Q}{r} + a\, g^2 C_{YM} Q \qquad (30)$$

where $a \equiv \sqrt{\pi} A/2C_{YM} \sim 23\sqrt{\pi}/64$. We can solve this system by expressing the color dielectric parameter ε as a function of the chromoelectric field E, obtaining:

$$\varepsilon(E) = 1 - \frac{a\, C_{YM}\, g^2 Q}{E} \equiv 1 - \frac{E_0}{E} \qquad (31)$$

which is precisely of the same form as the one assumed in

reference 4.

We will finally discuss the behaviour in the infrared region implied by the inclusion of the next to leading corrections. To this end we write equation (25) in the form:

$$Z(q,N=3) \simeq A \frac{g^2}{(q^2)^{\frac{1}{2}}} \left[1 + \frac{B}{A} \left(\frac{q^2}{g^4}\right)^{0.8422} \right] \qquad (32)$$

This represents a special case of the general expression which follows from equation (19) with $g^2 = \mu^{4-N}$, namely:

$$Z(q,N) = A\left(\frac{\mu^2}{q^2}\right)^{\frac{N-2}{2}} \left[1 + \frac{B}{A} \left(\frac{q^2}{\mu^2}\right)^{1+\nu} + \dots \right] \qquad (33)$$

The form (32) is obtained in the particular case with $N=3$ and $\nu = -0.1578$. Furthermore, the result obtained in Ref. 5 by Baker, Ball and Zachariassen corresponds to the case $N=4$ with $\nu = 0.1737$. It is interesting to inquire for the reason why the bracket in eq. (33) seems to have, apart from small corrections ν, a polynomial expansion in powers of q^2/μ^2.

To this end, motivated by the work of these authors, we will now consider a model which illustrates the important features of Schwinger-Dyson equation and which is exactly soluble. Considering the non-linear term in Z in equation (16) and using dimensional arguments, we take our model to be:

$$Z^{-1}(q,N) = 1 - C g^2 Z^{-1}(q) \int_q^\infty \frac{dk}{k^{5-N}} Z^2(k) + f(q,N) \qquad (34.a)$$

where $f(q,N)$ represents the effect of all other terms in that equation. These terms guarantee that when $N=2$, the solution of the S-D equation must be $Z=1$, since in this case the theory is effectively free. We can ensure this boundary condition by considering the function $f(q,N)$ given by:

$$f(q,N) = \frac{C\,D^{N-2}}{N}\,\frac{\mu^2}{q^2} \qquad (34.b)$$

In this case, taking the derivative with respect to q, the integral equation (34) can be converted into the following non-linear differential equation:

$$\left[1 + \frac{C\,D^{N-2}}{N}\,\frac{\mu^2}{q^2}\right]\frac{dZ}{dq} - 2\,\frac{C\,D^{N-2}}{N}\,\frac{\mu^2}{q^3}\,Z + \frac{C\,\mu^{4-N}}{q^{5-N}}\,Z^2 = 0 \qquad (35)$$

This is a Bernoulli type of non-linear equation which can be solved using well known methods[11]. Imposing, in accordance with (9.a), the boundary condition $Z(q\to\infty) = 1$, the solution of the equation (35) turns out to be:

$$Z^{-1}(q,N) = \left(\frac{q^2}{D^2\mu^2}\right)^{\frac{N-2}{2}} \cdot$$

$$\left\{\left[1 + \frac{N}{C\,D^{N-2}}\,\frac{q^2}{\mu^2}\right]{}_2F_1\left(2\,,\frac{N}{2}\,;\frac{N}{2}+1\,;\frac{-N}{C\,D^{N-2}}\,\frac{q^2}{\mu^2}\right)\right\} \qquad (36.a)$$

where ${}_2F_1$ denotes the hypergeometric function[12] and D is determined by:

$$D = \left\{\left[\Gamma\left(\frac{N}{2}+1\right)\,\Gamma\left(2-\frac{N}{2}\right)\right]^{\frac{2}{N-2}}\,\frac{C}{N}\right\}^{\frac{1}{4-N}} \qquad (36.b)$$

We see that the solution (36) has, as $q\to 0$, a power-like behaviour given by:

$$Z^{-1}(q\to 0,N) = \left(\frac{q^2}{D^2\mu^2}\right)^{\frac{N-2}{2}}\left[1 + \frac{(2-N)N}{2+N}\,\frac{1}{C\,D^{N-2}}\,\frac{q^2}{\mu^2} + \ldots\right] \qquad (37)$$

This form which is very similar to expression (33) with $\nu = 0$, explains why the actual value of this parameter in the solution of the true S-D equation is small. It is important to note that for $N<4$, expression (37) represents in the infrared limit a non-perturbative expansion in the coupling constant, since $\mu^2 \equiv g^{\frac{4}{4-N}}$ occurs in the denominators.

On the other hand, making use of the transformation properties of the hypergeometric functions[12], which relate ${}_2F_1(x)$ to ${}_2F_1(1/x)$, we obtain from eq. (36) the following perturbative behaviour valid at high energy for $N<4$:

$$Z^{-1}(q\to\infty,N) = 1 - \frac{C}{4-N}\left(\frac{\mu^2}{q^2}\right)^{\frac{4-N}{2}} + \frac{C\,D^{N-2}}{N}\,\frac{\mu^2}{q^2} + \ldots \qquad (38)$$

We remark that the pole at $N=4$ is associated with the ultraviolet singularities which in this case arise in the theory. Furthermore, as expected from previous considerations at $N=2$ we obtain the free field solution $Z=1$. On the other hand, when $N=3$ the solution corresponding to equations (36-38) with $\mu = g^2$ is non-trivial.

In conclusion we hope that, despite its limitations, the approach based on the Schwinger-Dyson equations might correctly describe the transition mechanism between the low and high energy domain of 2+1 dimensional QCD.

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