UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
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01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-594

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Julho/1986

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ABSTRACT

We develop a scheme for computing the free energy of topological defects at finite temperature. We compute, as an application, the free energy of topological defects (monopoles and domain walls) for the minimal SU(5) model, by using the semiclassical approach at high temperatures. This procedure allows us to get a better picture of the phase diagram of gauge theories at finite temperatures and obtain the critical temperatures associated to the various phase transitions.

*Work partially supported by CNPq, CAPES, and FAPESP.

I. INTRODUCTION

One of the most important problems one faces in field theory is the determination of the phase diagram of gauge theories. The difficulty in this context relies upon the question on how to distinguish the different phases of gauge theories when coupled to matter fields. Bricmont and Fröhlich[1] have proposed that the distinction (between different phases in gauge theories) is achieved by analyzing the free energies of "topological defects".

The defect free energy approach relies upon the study of the change in the free energy that takes place when one forces a topological defect to appear in the systems through the use of convenient boundary conditions. Comparing the free energies of the systems, when one imposes different boundary conditions, one can learn about its phase diagram: when the free energy becomes insensitive to certain boundary conditions the system has reached a new "phase".

One can have a better picture of this method by applying it to the Ising model: the analogue of the free energy defect is the surface tension. The surface tension is nothing but the free energy of the topological defect of the ising model (domain walls). The temperature at which the surface tension vanishes is the critical temperature of the model, since one can prove that the spontaneous magnetization vanishes at the same temperature[2].

Now suppose that the system, originally at a given

phase at zero temperature, is in contact with a heat bath at temperature $T = \beta^{-1}$. At low temperatures, defects with positive free energy are rare or nonexistent (those which require an infinite amount of energy). However, for sufficiently high temperatures, quantum (entropy) effects come into play in such a way that the free energy of a given topological defect vanishes[3-4]. Hence, there is no energy cost to introduce an extra topological defect into the system, which implies that one has reached another phase of the theory - that is, the one in which the condensation of defects takes place.

Topologically nontrivial structures (defects) emerge in field theories whose symmetry is spontaneously broken. At the classical level, these defects correspond to topologically nontrivial solutions of the Euler-Lagrange equations.

In this paper, we shall be concerned with the computation of defect free energy for nonabelian gauge theories at finite temperature. Explicit results are derived in the one loop approximation. We have illustrated how the scheme works by considering the minimal SU(5) model, which exhibits two types of topological defects: domain walls and magnetic monopoles. The extension of this method to other models and different defects is straightforward.

This paper is organized as follows. Section II deals with the general framework and specially with a formal expansion which allows us to implement the semiclassical approximation and an explicit separation of the zero and finite temperature terms of the free energy. In section III we apply the scheme to

obtain the free energy of the topological defects of the minimal SU(5) model. In the high temperature limit, we obtain closed expressions for the free energy of domain walls and magnetic monopoles. We end this paper with conclusions in section IV. This paper is supplemented by two appendices.

II. FORMAL EXPRESSION FOR DEFECT FREE ENERGY

The partition function for a given gauge theory, whose Euclidean Lagrangean density is L, may be expressed as a functional integral[5]

$$Z(\beta) = N^{-1}(\beta) \oint [D\varphi] \exp\left\{ - \int_0^\beta d\tau \int d^3\vec{x} \, [L - J(x)\varphi(x)] \right\} \times$$

$$\times \text{ gauge fixing terms} \quad , \tag{2.1}$$

where τ is the euclidean time, φ stands for all fields in the theory and the integral over the fields is subject to the following boundary condition in φ:

$$\varphi(\vec{x},0) = \varphi(\vec{x},\beta) \quad \text{for bosonic fields}$$

and $$\varphi(\vec{x},0) = -\varphi(\vec{x},\beta) \quad \text{for fermionic fields} \quad .$$

N is a normalization constant which may be chosen such that $Z(\infty) = 1$.

The free energy of the system is defined through

the following equations[6]:

$$F(\beta,J) = -\beta^{-1}\,\ell n\, Z \quad , \tag{2.2}$$

$$M(x,J) \equiv M_J(x) = -\frac{\delta(\beta F)}{\delta J(x)} \quad , \tag{2.3}$$

$$\Gamma(\beta,M_J) = F(\beta,J) + \beta^{-1}\int_0^\beta d\tau \int d^3\vec{x}\; M_J(x)J(x) \tag{2.4}$$

$\Gamma(\beta,M_J)$ is the generating functional of one-particle irreducible Green's functions and is the free energy of the field configuration M_J. The effective potential method analyzes Γ for constant field configurations M_J in order to obtain the phase diagram of the model[6].

One can define the free energies of the different types of topological defects[1,4,7] by

$$F_M = -\beta^{-1}\,\ell n\left(\frac{Z_M}{Z_V}\right) \quad , \tag{2.5}$$

$$F_S = -\frac{\beta^{-1}}{L}\,\ell n\left(\frac{Z_S}{Z_V}\right) \quad , \tag{2.6}$$

and

$$F_W = -\frac{\beta^{-1}}{L^2}\,\ell n\left(\frac{Z_W}{Z_V}\right) \quad , \tag{2.7}$$

where F_W, F_S and F_M are, respectively, the free energy for domain walls, strings and magnetic monopoles. Usually a given model does not exhibit all the three different topological

defects, so one must consider only the relevant ones. Z_M, Z_S and Z_W stands for the partition function of the system evaluated when one imposes boundary conditions that force the existence of a magnetic monopole, string, and domain wall defect in the system, while Z_{VAC} is the partition function obtained using topologically trivial boundary conditions (vacuum sector). L is the size of the system.

The various thermodynamical functions can be written, in the one loop approximation, as shown in the appendix A, as differences of the effective action of the theory evaluated at certain field configurations. Let $\Gamma(\varphi)$ be the effective action of the theory and φ_V be the constant field configuration associated to the vacuum of the theory. In terms of the effective action one can write the effective potential

$$Veff \equiv \frac{1}{L^3\beta}\left\{\Gamma(\bar{\varphi}) - \Gamma(\varphi_V)\right\} \tag{2.8}$$

where the bar stands for constant field configurations.

Whereas for the defects that we are concerned in GUTS (monopole, string and wall) one has

$$F_M = \left[\Gamma(\varphi_M) - \Gamma(\varphi_V)\right] \quad , \tag{2.9}$$

$$F_S = \frac{1}{L}\left[\Gamma(\varphi_S) - \Gamma(\varphi_V)\right] \quad , \tag{2.10}$$

and

$$F_W = \frac{1}{L^2}\left[\Gamma(\varphi_W) - \Gamma(\varphi_V)\right] \quad , \tag{2.11}$$

that is, all thermodynamical parameters can be written as differences between the effective action computed at some special field theoretical configurations and those associated to the vacuum of the theory. These special field theoretical configurations, within the semiclassical scheme, are the defects associated to the classic solutions to the Euler-Lagrange equations of the model.

The general structure of $\Gamma[\beta,\varphi_D(x)]$ is

$$\Gamma[\beta,\varphi_D(x)] = \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \left(\int_0^\beta d\tau_j \int d^3x_j \, \varphi_D(x_j) \right) \Gamma^{(n)}(\tau_1\vec{x}_1,\ldots,\tau_n\vec{x}_n) \tag{2.12}$$

where $\Gamma^{(n)}(\tau_1\vec{x}_1,\ldots,\tau_n\vec{x}_n)$ are the one-particle irreducible Green's functions, φ_D stands for the fields associated to the defect. If one uses the Fourier transform of $\Gamma^{(n)}$, given by

$$\Gamma^{(n)}(\tau_1\vec{x}_1,\ldots,\tau_n\vec{x}_n) = \beta^{-n} \prod_{j=1}^{n} \sum_{n_j=-\infty}^{\infty} \int \frac{d^3\vec{k}_j}{(2\pi)^3} \, \tilde{\Gamma}^{(n)}(\omega_1\vec{k}_1,\ldots,\omega_n\vec{k}_n) \times$$

$$\times \exp\left[- i \sum_{\ell=1}^{n} (\omega_\ell\tau_\ell + \vec{k}_\ell\vec{x}_\ell) \right] \tag{2.13}$$

where $\omega_\ell = 2\pi\ell\beta^{-1}$, and remembering that translational symmetry allows us to set

$$\tilde{\Gamma}^{(n)}(\{\omega_i\vec{k}_i\}) = \beta(2\pi)^3\delta(\Sigma\omega_i)\delta^3(\Sigma\vec{k}_i)\bar{\Gamma}^{(n)}(\{\omega_i\vec{k}_i\}) \tag{2.14}$$

then, for static field configurations (those with which we will be concerned in this paper), the general structure of $\Gamma(\beta,\varphi_D)$ is

$$\Gamma(\beta,\varphi_D) = \beta \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int d^3\vec{k}_j \, \tilde{\varphi}(-\vec{k}_j) \, \bar{\Gamma}^{(n)}(\{\vec{k}_j,\omega_j=0\})\delta^3(\Sigma\vec{k}_j) \, . \tag{2.15}$$

The graphs that contribute to $\bar{\Gamma}^{(n)}$ will involve sums over the discrete ω_j which, once performed, yield a term independent of temperature plus one which has the full T dependence. This separation can always be implemented if one uses identities of the form

$$\beta^{-1} \sum_{n=-\infty}^{\infty} \frac{1}{\left(\frac{2n\pi}{\beta}\right)^2 + z^2} = \frac{1}{2z} + \frac{1}{z(e^{\beta z}-1)} \, . \tag{2.16}$$

One can then split $\bar{\Gamma}^{(n)}$ into two parts

$$\bar{\Gamma}^{(n)}(\{\vec{k}_i,\omega_i=0\}) = \bar{\Gamma}_0^{(n)}(\{k_i\}) + \bar{\Gamma}_T^{(n)}(\{\vec{k}_i\},\omega_i=0) \tag{2.17}$$

where the second term contains all the T-dependence. The general structure of this dependence can be inferred by making a change in all internal momenta integration variables. This change is just a replacement $\vec{p} \to \vec{p}' = \vec{p}\beta$. After this scaling in the internal momenta one can predict, from pure dimensional analysis,

that $\bar{\Gamma}_T^{(n)}(\{\vec{k}_i,\omega_i=0\})$ have the following structures[6]

$$\bar{\Gamma}_T^{(n)}(\{\vec{k}_i\},\omega_i=0) = \sum_{\gamma_n} T^{d(\gamma_n)} G_{\gamma_n}\left[\frac{\vec{k}_i}{T},\frac{m}{T}\right] \qquad (2.18)$$

where $d(\gamma_n)$ is the superficial degree of divergence of a graph γ_n contributing to $\bar{\Gamma}$ and G_{γ_n} is dimensionless. Putting (2.15), (2.17) and (2.18) together, we have

$$\Gamma(\beta,\varphi_D) = \Gamma_0(\varphi_D) + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int d^3\vec{k}_j\, \tilde{\varphi}_D(-\vec{k}_j) \sum_{\gamma_n} T^{d(\gamma_n)} \times$$

$$\times\, G_{\gamma_n}\left[\frac{\vec{k}_j}{T},\frac{m}{T}\right] \delta^3(\Sigma\vec{k}_j) \qquad (2.19)$$

where $\Gamma_0(\varphi_D)$ is the effective action computed at the background field φ_D at zero temperature.

Using (2.9)-(2.11) and (2.19), the free energies of the various topological defects can then be written as

$$F^D(\beta) = \left[\Gamma_0(\varphi_D) - \Gamma_0(\varphi_V)\right]\cdot\frac{1}{L^\alpha} + \frac{1}{L^\alpha}\left\{\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{j=1}^{\infty}\int d^3\vec{k}_j\,\tilde{\varphi}_D(-\vec{k}_j)\times\right.$$

$$\times\,\delta^3(\Sigma\vec{k}_j)\sum_{\gamma_n} T^{d(\gamma_n)} G_{\gamma_n}\left(\frac{\vec{k}_j}{T},\frac{m}{T}\right) - \sum_{n}\frac{1}{n!}\varphi_V^n \sum_{\gamma_n} T^{d(\gamma^n)} \times$$

$$\left.\times\, G_{\gamma_n}\left(0,\frac{m}{T}\right)L^3\right\} \qquad (2.20)$$

where α is an index that, in accordance to (2.9)-(2.11), runs from 0 to 2.

To get a formal series for the free energy from any solution associated to a particular defect, we just introduce it in (2.20). Just for the sake of completeness, we write the expression for the effective potential. From (2.8) and (2.19) it follows that

$$\mathrm{Veff}(\bar{\varphi}) = \frac{1}{V}\left[\Gamma_0(\bar{\varphi}) - \Gamma_0(\varphi_V)\right] + \sum_{n=1}^{\infty}\frac{1}{n!}(\bar{\varphi}^n - \varphi_V^n)\sum_{\gamma_n} T^{d(\gamma_n)} G_{\gamma_n}\left(0,\frac{m}{T}\right) \qquad (2.21)$$

Once the general formalism is set we shall apply it to our specific model: the minimal SU(5) GUT.

III. PHASE DIAGRAM FOR THE MINIMAL SU(5) GUT[8]

We shall consider the minimal SU(5) GUT at finite temperature. Its Euclidean Lagrangean density is*

$$L = -\frac{1}{4}\,\mathrm{Tr}\left[G_{\mu\nu}G_{\mu\nu}\right] + \frac{1}{2}\,\mathrm{Tr}\left[(D_\mu\Phi)^2\right] + V(\Phi) \qquad (3.1)$$

where Φ is the Higgs multiplet belonging to the adjoint representation,

$$V(\Phi) = -\frac{\mu^2}{2}\,\mathrm{Tr}(\Phi^2) + \frac{a}{4}\left[\mathrm{Tr}(\Phi^2)\right]^2 + \frac{b}{2}\,\mathrm{Tr}[\Phi^4]\ , \qquad (3.2)$$

*In our calculations, we are assuming that the coupling constants are such that the phase transitions are expected to be of second order.

$$G_{\mu\nu} = \sum_{i=1}^{24} G^i_{\mu\nu} \frac{\lambda^i}{\sqrt{2}} \quad , \qquad W_\mu = \sum_{i=1}^{24} W^i_\mu \frac{\lambda^i}{\sqrt{2}} \quad ,$$

$$\Phi = \sum_{i=1}^{24} \varphi^i \frac{\lambda^i}{\sqrt{2}} \quad , \qquad D_\mu \Phi = \partial_\mu \Phi - \frac{ig}{\sqrt{2}} \mathrm{Tr}[W_\mu, \Phi] \quad \text{and}$$

λ^i $(i = 1,\ldots,24)$ are the generators of SU(5) in the fundamental representation (normalized so that $\mathrm{Tr}[\lambda^i \lambda^j] = 2\delta^{ij}$). We also impose that $b > 0$ and $a > -\frac{7}{15} b$. The notation used is the one in ref. (18).

This model exhibits two different topological defects: domain walls and magnetic monopoles. The background field describing a domain wall is

$$\bar{\Phi}_W = \frac{\mu}{\sqrt{\lambda}} \tanh\left(\frac{\mu}{\sqrt{2}} x\right) \frac{\lambda_{24}}{\sqrt{2}} \quad ,$$
$$\bar{W}^{c\ell}_\mu = 0 \qquad (3.3)$$

with $\lambda = a + \frac{7}{15} b$. Note that this solution depends only on one spatial coordinate, which we choose to be the x one. The classical field configuration associated to a magnetic monopole satisfies the ansatz[9]

$$\bar{W}^a_4 = 0 \quad \text{for} \quad a = 1,\ldots,24 \; ; \; \bar{W}^a_\mu = 0 = \bar{\varphi}^a \quad \text{for} \quad a = 1,2,\ldots,20,24 \quad ,$$
$$\bar{W}^{2a}_k = \frac{1}{g} \varepsilon^{akj} \frac{x^j}{r^2} F(r) \quad \text{for} \quad a,k = 1,2,3 \qquad (3.4)$$
$$\bar{\varphi}^{2a} = \frac{x^a}{r} \eta(r) \varphi_V \quad \text{for} \quad a = 1,2,3 \quad ,$$

and the boundary conditions

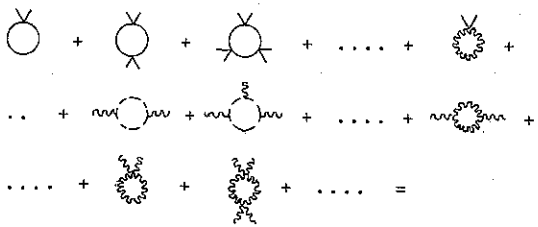
$$F(r) \xrightarrow[r\to\infty]{} 1 \quad ; \quad \eta(r) \xrightarrow[r\to\infty]{} 1 \; .$$

Let us exhibit the structure of the free energies of the system under these background fields in the one-loop approximation. In the zero-loop approximation one has, from (2.20),

$$F^D_{(0)}(\beta) = \frac{T[S(\varphi_D) - S(\varphi_V)]}{L^\alpha} \equiv \Delta\varepsilon_{(0)} \; . \qquad (3.5)$$

That is, in the zero-loop approximation, the free energy of the topological defect is just the difference between the classical action associated to the defect and the energy of vacuum. For the monopole, $\Delta\varepsilon_{(0)}$, defined in (3.5), is its mass whereas for the domain wall $\Delta\varepsilon_{(0)}$ is the mass per unit area.

Within the one-loop approximation $\Gamma(\bar{\varphi}, \bar{W}_\mu)$ will have the structure predicted from (2.12) which, for the example that we are considering, has the structure

$\Gamma(\bar{\varphi}, \bar{W}_\mu) = S c\ell(\bar{\varphi}, \bar{W}_\mu)$ +  =

$$= S c\ell(\bar{\varphi}, \bar{W}_\mu) - \frac{1}{2!} \sum^{ab}(T) \int_0^\beta d\tau \int d^3\vec{x} \; \bar{\varphi}^a \bar{\varphi}^b -$$

$$- \frac{1}{2!} \Pi^{ab}_{\mu\nu}(T) \int_0^\beta d\tau \int d^3\vec{x}\ \overline{W}^a_\mu\ \overline{W}^b_\nu + \ldots \qquad (3.6)$$

Scℓ is the classical action associated to the background field, $\sum^{ab}(T)$ can be represented graphycally as

$$\sum^{ab}(T) = \quad a \quad b \quad + \quad a \quad b \qquad (3.7)$$

whereas $\Pi^{\mu\nu}_{ab}(T)$ can be represented as

$$\Pi^{ab}_{\mu\nu}(T) = \quad \mu\ a \quad \nu\ b \quad + \quad \mu\ a \quad \nu\ b \quad + \quad \mu\ a \quad \nu\ b \quad +$$

$$+ \quad \mu\ a \quad \nu\ b \quad + \quad \mu\ a \quad \nu\ b \qquad (3.8)$$

The wavy, solid and dotted lines stand, respectively, for the gauge bosons, Higgs and ghost fields (for the fluctuations we are working in the Landau gauge). $\Pi^{ab}_{\mu\nu}$ can be identified as the polarization tensor for zero external momenta[19]. Following our earlier prescription (2.17), we can also write

$$\sum^{ab}(T) = \sum_0^{ab} + \overline{\sum}_T^{ab(n)} \quad (\{\vec{k}_i\}, \omega_i = 0) \qquad (3.9)$$

$$\Pi^{ab}_{\mu\nu}(T) = \Pi^{ab}_{\mu\nu 0} + \overline{\Pi}^{ab}_{\mu\nu}(T) \qquad (3.10)$$

First of all one notes, looking at (3.6), the

appearance of ultraviolet divergences. These, however, can be treated, as usual, by adding appropriate renormalization counterterms which are just the usual ones at zero temperature[6]. This means that the zero temperature renormalization scheme suffices for getting finite expression to free energies of topological defects. Substituting (3.6) into (2.15), one can obtain the topological defect free energies of the SU(5) model.

$$F_{wall}(T) = \Delta\epsilon_W - \frac{1}{2!} \frac{\overline{\sum}^{24,24}(T)}{L^2} \int_0^\beta d\tau \int d^3\vec{x} \left[\overline{\varphi}^W_{24}(x)\overline{\varphi}^W_{24}(x) - \right.$$

$$\left. - \overline{\varphi}^2_V \right] + \ldots \qquad (3.11)$$

where $\Delta\epsilon_W$ is the energy density of the wall taking into account quantum corrections at zero temperature up to one loop; $\overline{\varphi}^W_{24}(x)$ is given in (3.3), $\overline{\varphi}_V = \frac{\mu}{\sqrt{2\lambda}} \lambda_{24}$, $\overline{\sum}^{24,24}(T)$ is given by (3.9), and the dots represent one loop contributions not included explicitly in (3.11).

On the other hand, for the magnetic monopole one obtains

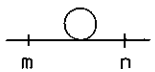
$$F_M(T) = M - \frac{1}{2!} \overline{\sum}^{ab}(T) \int_0^\beta d\tau \int d^3\vec{x} \left[\overline{\varphi}^a(\vec{x})\overline{\varphi}^b(\vec{x}) - \varphi^2_V \delta_{a_{24}} \delta_{b_{24}} \right] -$$

$$- \frac{1}{2!} \overline{\Pi}^{ab}_{\mu\nu}(T) \int_0^\beta d\tau \int d^3\vec{x}\ \overline{W}^a_\mu(\vec{x})\overline{W}^b_\nu(\vec{x}) + \ldots \qquad (3.12)$$

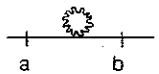
where now M stands for the renormalized mass of the monopole at the one loop level, $\bar{\sum}^{ab}(T)$ and $\Pi^{ab}_{\mu\nu}(T)$ are given in (3.9) and (3.10), the fields $\bar{\varphi}^a$ and $\bar{W}^a_\mu$ are defined in (3.4) and the dots represents contributions that are not shown in (3.12).

One could go further and write down similar expression for all the one-loop graphs for the topological structures of the SU(5) model. However, instead of doing this explicitly, we will just analyze the high temperature limit of the free energy. In this limit, the form (2.20) is particularly useful, since the leading power in T of series (2.20) is easily obtained. Property (2.18) permits us to identify these contributions, which are the ones with higher superficial degree of divergence. These contributions are precisely the ones we have written explicitly.

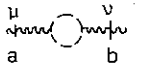
In the high temperature limit, the graphs appearing in (3.7) and (3.8) yield



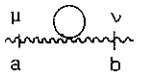
$$= -\left(26a + \frac{282}{15}b\right)\frac{T^2}{12}\delta^{mn} \tag{3.13}$$



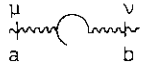
$$= -\frac{5}{4}g^2T^2\delta^{ab} \tag{3.14}$$



$$= \begin{cases} -\frac{5}{24}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu,\nu = 1,2,3 \\ \frac{5}{24}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu \text{ and/or } \nu=4 \end{cases} \tag{3.15}$$



$$= -\frac{5}{12}g^2T^2\delta^{ab}\delta_{\mu\nu} \tag{3.16}$$



$$= \begin{cases} \frac{5}{12}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu,\nu = 1,2,3 \\ -\frac{5}{12}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu \text{ and/or } \nu=4 \end{cases} \tag{3.17}$$

$$= \begin{cases} \frac{5}{4}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu,\nu = 1,2,3 \\ -\frac{5}{4}g^2T^2\delta^{ab}\delta_{\mu\nu} & \text{for} \quad \mu \text{ and/or } \nu=4 \end{cases} \tag{3.18}$$

$$= -\frac{25}{24}g^2T^2\delta^{ab}\delta_{\mu\nu} \quad . \tag{3.19}$$

From (3.13)-(3.19), (3.7) and (3.8) we have the asymptotic expressions for $\bar{\sum}^{cd}(T)$ and $\Pi^{cd}_{\mu\nu}(T)$

$$\bar{\sum}^{cd}(T) \sim -\frac{T^2}{4}\left[5g^2 + \frac{1}{3}\left(26a + \frac{282}{15}b\right)\right]\delta^{cd} \tag{3.20}$$

$$\bar{\Pi}^{cd}_{\mu\nu}(T) \sim -\frac{35}{12}g^2T^2\delta^{cd}\delta_{\mu4}\delta_{\nu4} \quad . \tag{3.21}$$

One obtains from (3.11)-(3.21) the high temperature behaviour

$$F_{wall}(T) \sim \Delta\epsilon_w + \frac{T^2}{8}\left[5g^2 + \frac{1}{3}\left(26a + \frac{282}{15}b\right)\right]\int dx\left[\varphi^2_W(x) - \varphi^2_V\right] \tag{3.22}$$

$$F_M(T) \sim M + \frac{T^2}{8}\left[5g^2 + \frac{1}{3}\left(26a + \frac{282}{15}b\right)\right] \times$$

$$\times \int d^{3\div}x\left\{\sum_{a=1}^{24}\bar{\varphi}^a\bar{\varphi}^a - \varphi^2_V\right\} + \frac{35}{24}g^2T^2\int d^{3\div}x\sum_{a=1}^{24}\bar{W}^a_4\bar{W}^a_4 \quad . \tag{3.23}$$

At first sight, the appearance of the term $\int d^3\vec{x}(\overline{W}_4^a)^2$ in the last expression, could seem to be a problem: (3.23) is not explicitly gauge invariant with respect to gauge transformations of the background fields (we have just fixed the gauge for the fluctuations[7]). At this point we are forced to adopt a "physical" gauge with respect to the magnetic monopole degrees of freedom or generalize our calculation to include a Jacobian for the ghost like degrees of freedom associated to the magnetic monopole[10] When considering the background (3.4) for the magnetic monopole we have decided for the former strategy, since this background satisfies $\overline{W}_4^a = 0$ Therefore, in this gauge, we have for the monopole

$$F_M(T) \sim M + \frac{T^2}{8}\left[5g^2 + \frac{1}{3}(26a + \frac{282}{15}b)\right]\int d^3\vec{x}\left\{\sum_{a=1}^{24}\overline{\varphi}^a\overline{\varphi}^a - \varphi_V^2\right\} \tag{3.24}$$

The substitution of (3.3) into (3.22) leads to

$$F_{wall}(T) \sim \Delta\varepsilon_W - \frac{T^2}{12}\frac{\mu\sqrt{2}}{\lambda}(26a + \frac{282}{15}b + 15g^2) \tag{3.25}$$

whereas the substitution of (3.4) into (3.24) implies

$$F_M(T) \sim M - \frac{T^2}{8}\left[5g^2 + \frac{1}{3}(26a + \frac{282}{15}b)\right]4\pi\int_0^\infty r^2\,dr\,\varphi_V^2(1-\eta^2(r)) \tag{3.26}$$

IV. CONCLUSIONS

This paper deals with topological defects at finite temperatures. In the first part we were particularly concerned with the evaluation of free energies of the defects. Since, as shown in appendix A, the computation of this thermodynamical parameter is equivalent to computing the effective action for the configuration associated to the defect, it is possible to develop a formal expansion for this parameter In this formal expansion one can separate a zero temperature contribution and an explicitly temperature dependent part. Very simple arguments involving dimensional analysis allow us to infer the leading power behaviour in T of the temperature dependent piece.

We have exemplified how the method works by computing the free energy of domain walls and magnetic monopoles, in the high temperature limit, for the minimal SU(5) model. The temperatures in which these free energies vanish indicate the occurrence of phase transitions. These critical temperatures, for the minimal SU(5) model, are

$$T_W^2 = \frac{60\mu^2}{\frac{225}{2}g^2 + 13(15a + 7b) + 50b} \tag{4.1}$$

$$T_M^2 = \frac{8M}{\left[5g^2 + \frac{1}{3}(26a + \frac{282}{15}b)\right]4\pi\varphi_V^2\int_0^\infty dr\,r^2(1-\eta^2(r))} \tag{4.2}$$

where we have taken for $\Delta\varepsilon_w$ in (3.25) its classical value $\Delta\varepsilon_w \simeq \Delta\varepsilon_0 = \frac{2\sqrt{2}}{3}\frac{\mu^2}{\lambda}$.

The temperature T_W computed by us has a simple interpretation, as pointed out in ref. (4). The point is, since the minima of the effective potential at T_W enter into the region in which the effective potential develops an imaginary part (see appendix B for this), this temperature is just the highest one for which the description of the system in terms of perturbative constant field configuration makes sense (and consequently the perturbative effective potential). In appendix B we show further that if the phase transition is of second order, then one cannot avoid the perturbative effective potential becoming complex at the minimum for temperatures sufficiently high.

Therefore the topological defects free energies approach gives a richer description of the phase diagram of the SU(5) model at higher temperatures than the traditional one based on the effective potential.

Once the temperature is greater than T_W (or T_M) we expect the system to be in a new phase which is characterized by a condensate of domain walls (and/or magnetic monopoles). It would be particularly interesting to know if T_W and T_M are equal or which one is the smallest. If the two temperatures are not equal it is important to know how to evaluate the free energy of a given topological defect in the presence of a condensate of the other. (We met this kind of problem in the determination

of the phase transition for the Z_n-symmetric spin and gauge theories[11]).

The existence of a new phase, which is a condensate of topological defects, may have far-reaching consequences to cosmology. For instance, the existence of a condensate of magnetic monopoles can drive the system to a superconducting phase. Another application would be to verify whether the contrast density produced by the topological defects satisfies the Zeldovich spectrum[12] and consequently see if the condensation of defects is responsible for the large structure of the universe. These questions are now under investigation by us[13].

ACKNOWLEDGEMENTS

We would like to thank C. Schwartz for a critical reading of the manuscript. O.J.P. Éboli is grateful to the Departamento de Física Matemática - IFUSP, for their kind hospitality.

APPENDIX A

In this appendix we will justify expressions (2.9)-(2.11) that give the (Gibbs) free energies with respect to three different backgrounds. Although, within the one-loop approximation, our expressions give results that are by now standard and can be found in text books[14], we present this derivation due to the fact that it is fairly general and is just an extension, to finite temperature, of the background field method[15].

Assume that φ_0 is a generic field configuration and let us compute the thermodynamical properties of the system in the presence of such a background field. This should be inferred from the functional $Z[J,\varphi_0]$ defined by

$$Z^0[J,\varphi_0] = \int D[\varphi]\, e^{-S[\varphi-\varphi_0]+\int_0^\beta d\tau \int d^3\vec{x}\, J(x)\varphi(x)} \qquad (A.1)$$

By means of a change of variables one can write

$$Z^0[J,\varphi_0] = Z[J]\, e^{\int_0^\beta d\tau \int d^3\vec{x}\, J(x)\varphi_0(x)} \qquad (A.2)$$

From (A.2) it follows that

$$W^0[J,\varphi_0] = W[J] - \beta^{-1} \int_0^\beta d\tau \int d^3\vec{x}\, J(x)\varphi_0(x) \qquad (A.3)$$

where $W^0(W)$, $Z^0(Z)$ stands for the thermodynamical functions

evaluated with (without) the background field.

By using the definition (2.4) it follows that the Gibbs free energy in the presence of the background field (φ_0) is given by

$$\Gamma^0[\bar{\varphi}_0,\varphi_0] \equiv W^0[J,\bar{\varphi}_0] - \beta^{-1}\int_0^\beta d\tau \int d^3\vec{x}\ \frac{\delta(\beta W^0)}{\delta J(x)}\, J(x) \qquad (A.4)$$

where $\bar{\varphi}_0 = \dfrac{\delta(\beta W_0)}{\delta J}$.

By substituting (A.3) into (A.4) it follows that

$$\Gamma^0(\bar{\varphi}_0,\varphi_0) = W[J] - \beta^{-1} \int_0^\beta d\tau \int d^3\vec{x}\, J(x)[\bar{\varphi}_0(x) + \varphi_0] \qquad (A.5)$$

consequently if one derives (A.5) with regard to J one obtains

$$\frac{\delta W}{\delta J} = \bar{\varphi}_0 + \varphi_0 \qquad (A.6)$$

Being $W[J]$ and $\Gamma[\bar{\varphi}]$ the generating functionals in the absence of the background field one gets, from (A.6), the following relationship

$$\bar{\varphi}_0 = \bar{\varphi} - \varphi_0 \qquad (A.7)$$

If one substitutes (A.7) into (A.5) one then obtains

$$\Gamma^0(\bar{\varphi}_0,\varphi_0) = W[J] - \beta^{-1}\int_0^\beta d\tau \int d^3\vec{x}\, J(x)\bar{\varphi}(x) \equiv \Gamma(\bar{\varphi}) \equiv$$

$$\equiv \Gamma(\bar{\varphi}_0 + \varphi_0) \qquad (A.8)$$

Expression (A.8) is well known within the context of the background field method - that is, the generating function for the theory in the presence of the background can be obtained from the generating functional without the background field computed just making the replacement $\bar{\varphi} \to \bar{\varphi}_0 + \varphi_0$.

The free energy in the presence of the background field is

$$F^0(\beta) \equiv \lim_{J\to 0} W^0[J,\varphi_V] \equiv \lim_{J\to 0} \left(\Gamma^0(\bar{\varphi}^0,\varphi^0) + \beta^{-1} \int_0^\beta d\tau \int d^3\vec{x}\, J(x)\, \bar{\varphi}_0 \right) . \tag{A.9}$$

Finally, one notes that if φ_0 is a particular solution of the classical equation

$$\left.\frac{\delta\Gamma}{\delta\varphi}\right|_{\varphi=\varphi_c} = 0 \tag{A.10}$$

- that is,

$$\varphi_0 = \varphi_c = \bar{\varphi} \tag{A.11}$$

then in the limit $J \to 0$ (A.11) leads to $\bar{\varphi}_0 = 0$. Under this circumstance it follows from (A.9) and (A.8) that

$$F(\beta,\varphi_c) = \Gamma[\varphi_c] \quad . \tag{A.12}$$

i.e., the free energy of the system in the presence of the background field φ_c satisfying the classical equation (A.10) is given by

effective action computed at this configuration. If Γ is computed at the zero loop level, (A.10) corresponds to the classical Euler-Lagrange equations. This is precisely the situation that we are interested in the semiclassical approximation.

APPENDIX B

Usually one evaluates perturbatively the effective potential in order to know the different phases of the model. However, the perturbative effective potential exhibits some problems like non-convexity (and imaginary parts)[4,16]. Although these problems can be solved at zero temperature by means of a Maxwell construction[4,16], this is not always true when one works at finite temperature, as we shall show. If the phase transition is expected to be of second order or very weak first order, then one cannot avoid the effective potential becoming complex at the minimum for sufficiently high temperatures.

Our starting point is

$$Z(J) = \int D[\varphi] \exp\left\{ -\int_0^\beta d\tau \int d^3\vec{x}(L - J\varphi) \right\} \qquad (B.1)$$

where L is the effective Lagrangean for the field φ - that is, we have already integrated all the other degrees of freedom. For high temperatures L can be written as[17]

$$L = \frac{1}{2} (\partial\mu\varphi)^2 + V(\varphi) \qquad (B.2)$$

Lets, initially, analyze the case in which the phase transition is expected to be of second order. In this situation, $V(\varphi)$ is well described by $V_{classical}(\varphi)$. Without loss of generality we are going to consider the minimal $SU(5)$ model and evaluate the effective potential for fields $\hat{\Phi} = \varphi\, diag[1,1,1,-\frac{3}{2},-\frac{3}{2}]$.

In this case $V_{classical}(\varphi)$ is shown in figure 1.

The Maxwell construction for $Veff(\varphi)$ is obtained by considering the contribution of all the local minima of $V_{cl} - J\varphi$ to $Z(J)$[16]. This procedure yields

$$[Veff]_M = \begin{cases} Veff(\varphi) & \text{for} \quad |\varphi| \geq \varphi_M \\ Veff(\varphi_M) & \text{for} \quad |\varphi| \leq \varphi_M \end{cases} \qquad (B.3)$$

where $[Veff]_M$ is the Maxwell construction for the effective potential, $Veff$ is the result that one obtains when just the global minimum of $V_{cl} - J\varphi$ is considered, and φ_M is the positive of the minimum of $Veff$ (see figure 2). At temperatures low enough φ_M is outside the region where $V''_{cl} < 0$ and $[Veff]_M$ is real. However, for sufficiently high temperatures, φ_M lies in the region of $V''_{cl} < 0$ and $[Veff]_M$ is complex. Since φ_M goes to zero continuously, $[Veff]_M$ becomes complex before φ_M vanishes. Thus, one cannot trust the perturbative effective potential when the phase transition is expected to be of second order.

For strong first order phase transitions, the situation is completely different since V differs a lot of V_{cl}. In this situation $\varphi_2 = 0$ is a local minimum (if T is high enough) as shown in figure 3. The Maxwell construction for the effective potential yields[16]

$$[\mathrm{Veff}]_M = \begin{cases} \mathrm{Veff}(\varphi) & \text{for} \quad |\varphi| \geq \varphi_M \\ \mathrm{Veff}(\varphi_m) & \text{for} \quad |\varphi| \leq \varphi_M \end{cases} . \tag{B.4}$$

As the temperature is raised, φ_M jumps to zero without passing the region of $V'' < 0$. Therefore $[\mathrm{Veff}]_M$ is always real, and so, reliable.

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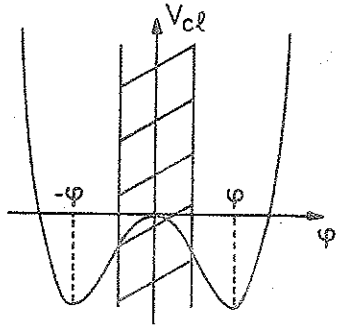
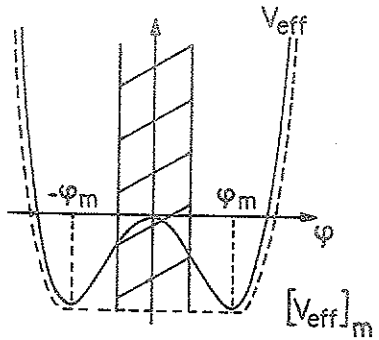
FIGURE CAPTIONS

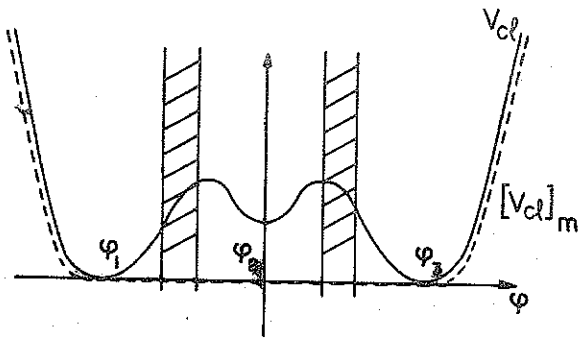
Fig. 1 - $V_{classical}(\varphi)$. The region between the dotted line is the one where $V''_{cl} < 0$.

Fig. 2 - The solid (dot-dashed) line stands for Veff ($[Veff]_M$). The region between the vertical lines is the one for which $V''_{cl} < 0$.

Fig. 3 - The solid (dot-dashed) line stands for V ($[Veff]_M$). The regions between the vertical lines are the ones for which $V'' < 0$.


V_{cl}
$-\varphi$
φ
φ


V_{eff}
$-\varphi_m$
φ_m
φ
$[V_{eff}]_m$


V_{cl}
$[V_{cl}]_m$
φ_1
φ_2
φ_3
φ