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by

V.L.M. Franzin and **M.S. Hussein**
Instituto de Física, Universidade de São Paulo

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V.L.M. Franzin and M.S. Hussein
Instituto de Física da Universidade de São Paulo,
C.P. 20516, 01498 - São Paulo, S.P. - BRASIL

ABSTRACT

With the aid of an inverse dispersion relation, which gives the imaginary part of the fusion inclusive polarization potential (IPP) in terms of the principal part integral involving the real part of the IPP, the sub-barrier fusion of heavy inos is discussed. The system $^{16}O+^{A}Sm$ is taken as an example. The reactive content of the extracted IPP is analysed within the coupled channels theory.

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A natural framework within which the well-established phenomenon of the enhancement of heavy-ion sub-barrier fusion over the prediction of one-dimensional barrier penetration calculation[1], can be tackeled is the coupled channels theory. In fact most of the early models developed for this purpose such as that of Esbensen[2], Dasso et al.[3] and Lindsay and Rowley[4], are based on this theory, though in a schematic way. Recently, several authors have addressed the heavy-ion compound nucleus formation cross section in the presence of directly coupled channels in the entrance stage. This entails using for the inclusive fusion cross section an expression which explicitly shows the action of the non-elastic channels. In fact one finds[4-6]

$$\sigma_F = \frac{k_o}{E_o}\left[\langle \psi_o^{(+)} | W_o^F | \psi_o^{(+)} \rangle + \sum_{i \neq o} \langle \psi_i^{(+)} | W_i^F | \psi_i^{(+)} \rangle\right] \tag{1}$$

where $\psi_i^{(+)}$ represents the exact wave function of the HI system in channel i and W_i^F the imaginary part of the piece of the corresponding optical potential which represents absorption due to fusion in channel i.

Clearly a numerical coupled channels calculation of σ_F one should obtain the same value as that extracte directly from Eq. (1). However Eq. (1) serves as a useful guideline to develope approximate expressions for σ_F which exhibit, in a transparent way the coupled channels effects. Udagawa et al.[7] and Nagarajan and Satchler[8], have recently used Eq. (1) to discuss the sub-barrier fusion of several HI systems. Whereas Udagawa et al.[7] developed a purely geometrical optical model for σ_F based on Eq. (1), N-S[8] on the other hand employed arguments based on the recently, rather extensively, studied dispersion relation (DR) technique, usually discussed in the context of elastic scattering. The very nice work of Nagarajan, Mahaux and Satchler[9] on the near-barrier elastic scattering of $^{16}O+^{208}Pb$[10], demonstrated the usefulness of the DR in explaining the optical potential anomalias. As far as the fusion work of N-S[8], however, their discussion was restricted to qualitative arguments concerning the nature of the energy-dependence of the effective potential used in an effective one-channel description of fusion. No attempt was made towards the elucidation of the reactive content of this potential.

The purpose of this letter is to fully explore the dispersion relation technique in the context of sub-barrier heavy-ion fusion reactions. By extracting the energy-dependent correction to the "frozen" proximity potential used in a one-channel description of fusion, we are then able to construct the corresponding imaginary component of this inclusive fusion polarization potential (IFPP), through the use of an inverse dispersion relation, and subsequently analyze its reactive content. We take the rather extensively studied $^{16}O+^{A}Sm$ system as our study case. The extracted IFPP is then compared with an appropriate local and angular-momentum-inde-

pendent inelastic polarization potential.

To simplify the discussion, we take a two-coupled channels model for σ_F. We call these channels 0 and 1. Eq. (1) becomes

$$\sigma_F = \frac{k_0}{E_0}\left[\langle\psi_0^{(+)}|W_0^F|\psi_0^{(+)}\rangle + \langle\psi_1^{(+)}|W_1^F|\psi_1^{(+)}\rangle\right] \tag{2}$$

The non-elastic channel wave function $\psi_1^{(+)}$ is related to the elastic $\psi_0^{(+)}$ through the well-known relation

$$|\psi_1^{(+)}\rangle = G_1^{(+)}(E)\,V_{10}\,|\psi_0^{(+)}\rangle \tag{3}$$

where $G_1^{(+)}$ is the unperturbed inelastic channel Green function and V_{10} is the channel coupling interaction. Whereas W_i^F is the fusion absorption potential, the optical potential that determines $\psi_0^{(+)}$ and $\psi_1^{(+)}$ in a one channel description of these wave functions, contain the effect of the coupling V_{01} to all orders. The polarization potential resulting from this coupling (e.g. $0 \rightarrow 1 \rightarrow 0$) is

$$V_{pol.} = V_{01}\,G_1^{(+)}\,V_{10} \tag{4}$$

It is an easy matter[5] to show that

$$\mathrm{Im}\,V_{pol.} = V_{01}\,G_1^{(+)\dagger}\,W_1^F\,G_1^{(+)}\,V_{10} - \pi\,V_{01}\,|\chi_1^{(-)}\rangle\,\delta(E_1 - H_1^0)\,\langle\chi_1^{(-)}|V_{10} \tag{5}$$

The secondterm above accounts for the flux lost to channel 1 (direct non-elastic process), while the second term represents flux lost to 1, followed by fusion. It is exactly this term which gives rise to the second term in Eq. (2). Clearly, as the energy is lowered, $\mathrm{Im}V_{pol}$ becomes gradually smaller, and eventually when the barrier is reached, one expects a rather sharp drop since the barrier acts as a natural threshold for absorption processes. This indicates that the first term in (5) and corresponding the second term in (2) should become very small as the energy is decreased, since they represent a genuine two-step process.

A striking phenomenon connected with the real part of V_{pol} accompanies the behaviour of $\mathrm{Im}V_{pol}(E)$. From dispersion relation based analysis of near-barrier heavy-ion elastic scattering[9] one finds that in the barrier region where $\mathrm{Im}V_{pol}$ suffers its rapid drop, $\mathrm{Re}V_{pol}$ increases as the energy is lowered further attaining a maximum value further down in the energy axis. Finally, it drops again. Since the direct inelastic effects become small as we lower the energy, as discussed above, we would expect that the inclusive fusion polarization potential, ΔV^F (namely the correction to the fusion optical potential, due to channel coupling), to exhibit a behaviour close to that of V_{pol}.

The above discussion clearly indicates that the effective barrier height in a one-channel approximation to fusion (Eq. (2) without the second term) should be a decreasing

function of decreasing center of mass. Further, since the imaginary part of the IFPP, $\mathrm{Im}\Delta V^F$ is a small correction to W_o^F, Eq. (2), as far as its influence on $\psi_o^{(+)}$ is concerned, whatever treatment of absorption due to fusion, such as the incoming wave boundary condition method which is the theoretical basis of the barrier penetration model, should be quite valid for $W_o^F + \mathrm{Im}\Delta V$ if it is valid for W_o^F.

Based on the above considerations we propose for σ_F, the following partial wave sum representation

$$\sigma_F = \frac{\pi}{k_0^2}\sum_{\ell=0}^{\infty}(2\ell+1)\, T_\ell^F(V_B + \mathrm{Re}\,\Delta V^F) \tag{6}$$

where V_B is an appropriate static, energy-independent barrier. The model we use for T_ℓ^F is simply the one-dimensional barrier penetration model of Hill and Wheeler (see preceeding discussion)

$$T_\ell^F(V_B+\Delta V^F) = \left[1+\exp\frac{2\pi}{\hbar\omega}\left(V_B+\mathrm{Re}\,\Delta V^F(E)+\frac{\hbar^2(\ell+1/2)^2}{2\mu R_B^2}-E\right)\right]^{-1} \tag{7}$$

where R_B is the position of the barrier, and the energy-dependence of the $\mathrm{Re}\Delta V$ is made explicit. The quantity $\hbar\omega$ is the barrier curvature at R_B, and is taken to be energy-independent for simplicity. Of course, if $\Delta V^F(r,E)$ is known a priori from a dynamical model, then the resulting $\hbar\omega$, which is defined as

$$(\hbar\omega)^2 = \left\{ \frac{\hbar^2}{\mu} \left[\frac{d^2}{dr^2} \left(U_N(r) + U_C(r) + Re\, \Delta V^F(r,E) \right) \right]_{r=R_B} \right\} \tag{8}$$

would necessarily come out energy-dependent. Such an energy dependence of the barrier curvature is crucial to the understanding of the sub-barrier energy dependence of the first and second moments of the fusion ℓ-distribution. These moments, which furnish information about the angular momentum content and distribution of the compound nucleus are usually determined from neutron multiplicities and compound nucleus fission angular distribution[11]. In this paper we concentrate on σ_F and ignore the energy dependence of $\hbar\omega$.

Eq. (6) was used for the analysis of the $^{16}O+^{A}Sm$ system[12]. We have taken for the nuclear potential $U_N(r)$, the proximity form, which gave the following values for the barrier height V_B, $V_B(^{16}O+^{144}Sm)=60.04$ MeV, $V_B(^{16}O+^{148}Sm)=59.75$ MeV, $V_B(^{16}O+^{150}Sm)=59.62$ MeV, $V_B(^{16}O+^{152}Sm)=59.48$ MeV, and $V_B(^{16}O+^{154}Sm)=59.35$ MeV. Clearly, it is not expected that the calculation of σ_F for the $^{16}O+^{144}Sm$ system with the above value of V_B. using Eq. (7) for T_ℓ^F with the $Re\Delta V^F=0$, will reproduce the experimental data, since even in this system one expects, on general grounds, important deviations from a static one-dimensional barrier penetration fusion. Therefore a renoramalization of the interaction in $^{16}O+^{144}Sm$ will be needed in a meaningful analysis of the other O+Sm systems. This means that we take

$V_B(O+^{144}Sm)+Re\Delta V(O+^{144}Sm)$ to be the bare barrier height V_B for the other samarium isotopes.

In figure 1 we show (solid curves) the extracted $Re\Delta V(R_B,E)$ vs center of mass energy for the systems $^{16}O+^{148}Sm$, $^{16}O+^{150}SM$, $^{16}O+^{152}Sm$ and $^{16}O+^{154}Sm$. These energy-dependent corrections to V_B seem to exhibit a universal trend just as the corresponding ones found in the elastic scattering case discussed by Nagarajan et al.[9]. We construct the imaginary part of ΔV from the above results for $Re\Delta V$, using a dispersion relation. General requirements of causality and analyticity impose an important constraint on the physical, energy-dependent elastic scattering optical potential which arises from channel coupling. In general, we have for the elastic channel polarization potential, the following

$$\Delta V(E;r,r') = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\Delta V(E';r,r')}{E' - E + i\eta}\, dE' \tag{9}$$

We propose the same relation for the inclusive fusion polarization potential under discussion here. Thus we can write immediately the following

$$Im\, \Delta V^F(E) = -\frac{P}{\pi} \int \frac{Re\, \Delta V^F(E')}{E' - E}\, dE' \tag{10}$$

where we have taken ΔV^F to be local, and have fixed r to be R_B (see earlier discussion). We have solved Eq. (10) for $Im\Delta\, V^F(E)$, after representing $Re\Delta V^F(E')$ with an appropriate polynomial expansion parametrization. Our results for $\mathrm{Im}\Delta V^F(E)$ are shown as the dashed curves in Fig. (1). The resulting $\mathrm{Im}\Delta V^F$ at $E_{LAB} < 64.0$ MeV comes out unphysical (positive) so we set it equal to zero.

An interesting feature of our results for $\mathrm{Im}\Delta V^F(E)$ and $\mathrm{Re}\Delta V^F(E)$ is that they resemble to some extent the behaviour of the corresponding inclusive elastic polarization potential[9]. This shows that our procedure of analysis is consistent since the difference between the inclusive elastic and fusion polarization potentials at low energies is expected to be small.

We turn now to the reactive content of our inclusive fusion polarization potential. Owing to the fact that the quadrupole deformation parameter β_2 gradually increases from 0.0 to 0.27 in going from ^{144}Sm to ^{154}Sm (including ^{148}Sm, ^{150}Sm and ^{152}Sm), the obvious candidate for the causing element in the enhancement of σ_F as one goes from A=144 to A=154 is multiple Coulomb and nuclear inelastic scattering. In fact several coupled channels calculations have shown that the inclusion of the 2^+ and 4^+ states in the Cc calculation of σ_F does cause a great deal of enhancement at sub-barrier energies, especially when the excitation energies are small (such as in ^{154}Sm)[13,14]. Thus in the following we analyse the inelastic component of ΔV^F.

To simplify the discussion we take only the 2^+

state and consider the corresponding dynamic polarization potential. In general this potential is non-local and ℓ and E-dependent. Clearly an equivalent local and ℓ-independent potential is required in order to directly compare with ΔV^F extracted from σ_F earlier. El-Itaoui et al.[15] have recently calculated such an interaction for the nuclear excitation using the local plane wave approximation of Love et al.[1]. We take only the imaginary part of this potential, which has the form

$$\mathrm{Im}\,V_{inel} = -\frac{k_o}{64\pi E_o a}(\beta_2 R_o V_o)^2 \frac{R_o}{a} \cosh^{-1}\left(\frac{r-R_o}{a}\right) \cdot \left[Ci\left(\frac{|r-R_o|}{\pi a}\right) - Ci(k_-|r-R_o|) + \frac{2}{e}\frac{\pi^2}{\pi^2+\left(\frac{r-R_o}{a}\right)^2}\right] \quad (11)$$

where V_o is the strength of the nuclear (proximity) potential and R_o and a its radius and diffusivitiy, respectively. In the above $k_- = k_{0^+} - k_{2^+} \cong k_o \frac{E^*_{2^+}}{2E_o}$, the adiabaticity parameter, and Ci is the cosine integral[17].

We have evaluated $\mathrm{Im}V_{inel}$, Eq. (11) for the $^{16}O+^{A}Sm$ systems, taking for the deformation parameters the following values $\beta_2(^{148}Sm)=0.12$, $\beta_2(^{150}Sm)=0.15$, $\beta_2(^{152}Sm)=0.24$ and $\beta_2(^{154}Sm)=0.27$. To be consistent with our analysis of σ_F, we take $\beta_2(^{144}Sm)=0$. With the excitation energies E^*_2 taken from the nuclear data sheets, we have found that Eq. (11) gives large values for $\mathrm{Im}V_{inel}$, reaching order of magnitudes proportions when compared with our $\mathrm{Im}\Delta V^F$. The reasons are several. Multiple Coulomb excitations, which is not taken into account above, tend to decrease the value of σ_F[18], thus effectively reduces that of $\mathrm{Im}V_{pol}$. Further the approximations employed by El-Itaoui et al.[15] are questionable at low energies. Clearly, an inelastic polarization potential valid at low energies is missing. It would seem that the effect of barrier penetration, not taken into account by El-Itaoui et al.[15] should drastically reduce the value of $\mathrm{Im}V_{pol}$ from that obtained from Eq. (11).

In conclusion, a new method of analysis of sub barrier heavy-ion fusion reactions based on the use of an inverse dispersion relation is developed in this paper and applied to the system $^{16}O+^{A}Sm$. It is found that the extracted inclusive fusion polarization potential varies with energy in close correspondence with the energy variation of the inclusive elastic polarization potential. The reactive content of ΔV^F is discussed and the need for a theoretical development of low-energy polarization potential is pointed out.

To end, we should mention that our present work, is in a way, a natural continuation and refinement of the work of Balantekin et al.[19].

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FIGURE CAPTIONS

Figure 1. The extracted real part of the inclusive fusion polarization potential $\mathrm{Re}\Delta V^F$, vs $E_{C.M.}$ for $O+^{148}Sm$ (a), $O+^{150}Sm$ (b), $O+^{152}Sm$ (c) and $O+^{154}Sm$ (d), (solid curves). The dashed curves represent $\mathrm{Im}\Delta V^F$ as obtained from the inverse dispersion relation, Eq. (10), see text for details.

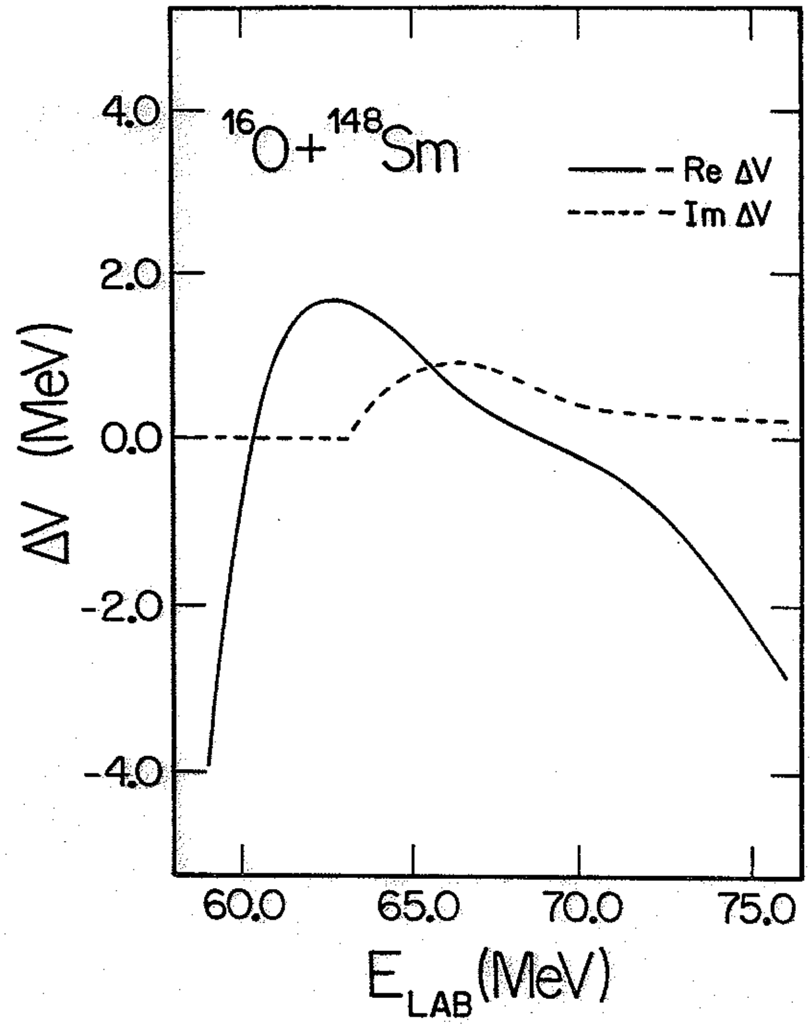

$^{16}O+^{148}Sm$
Re ΔV
Im ΔV
4.0
2.0
0.0
-2.0
-4.0
ΔV (MeV)
60.0
65.0
70.0
75.0
E_{LAB}(MeV)

Fig. 1a

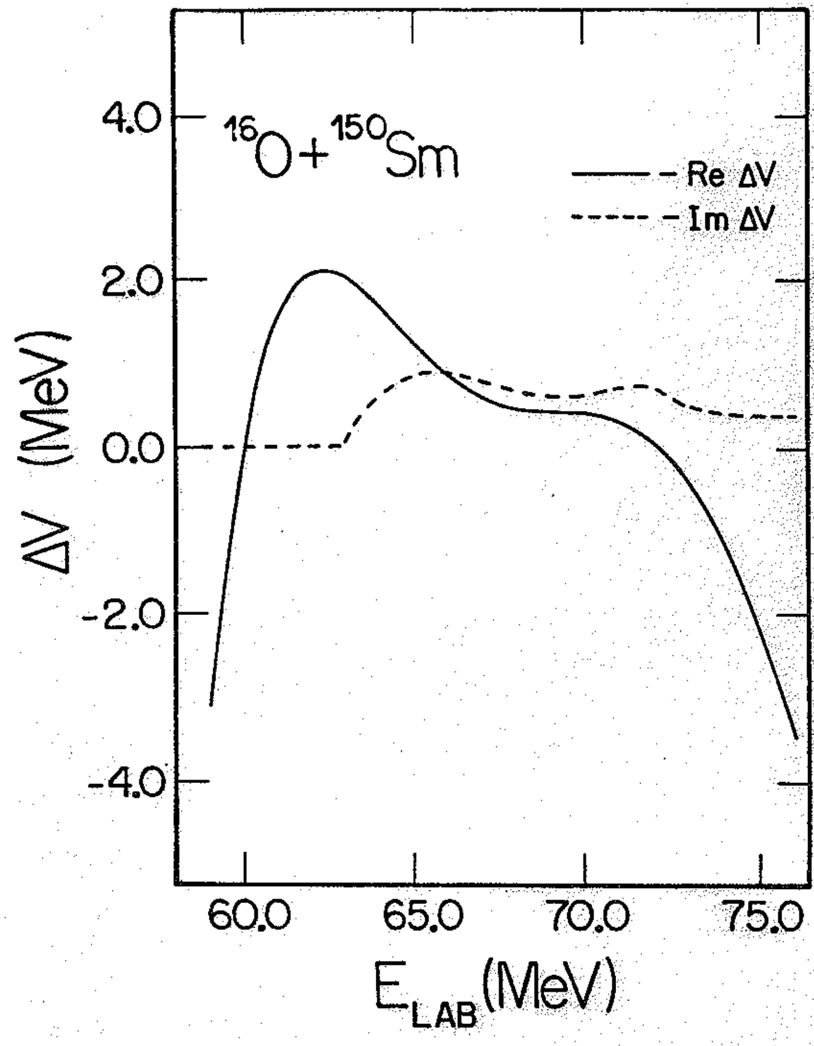
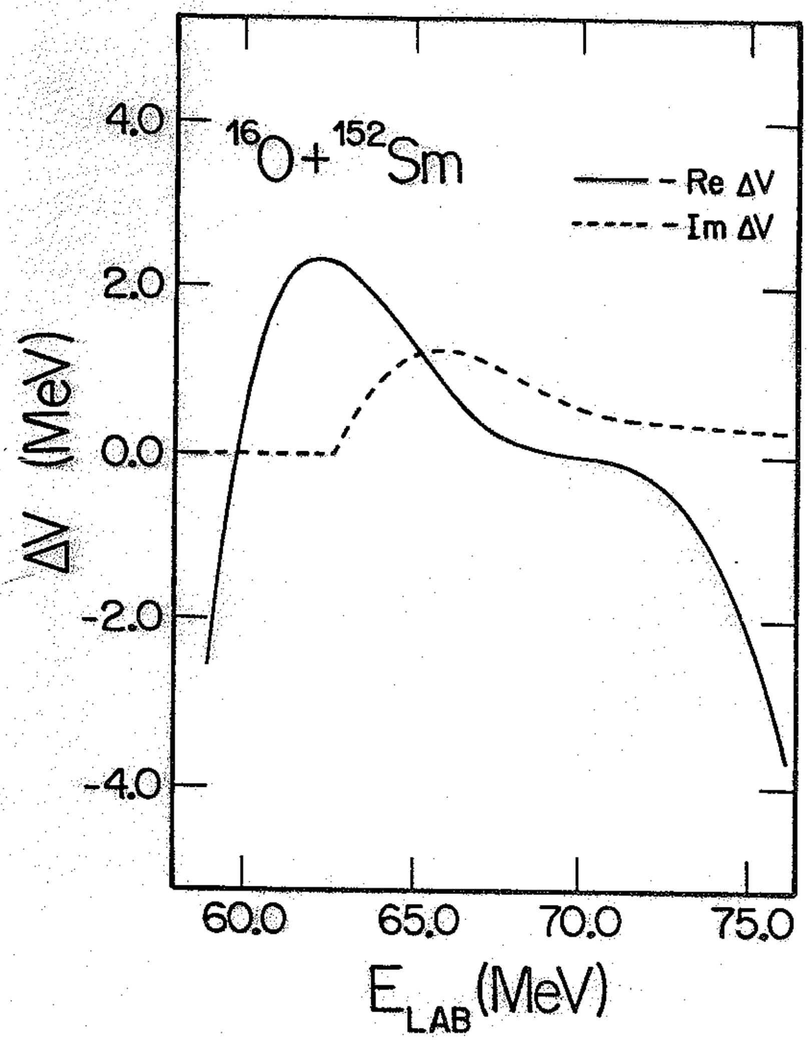
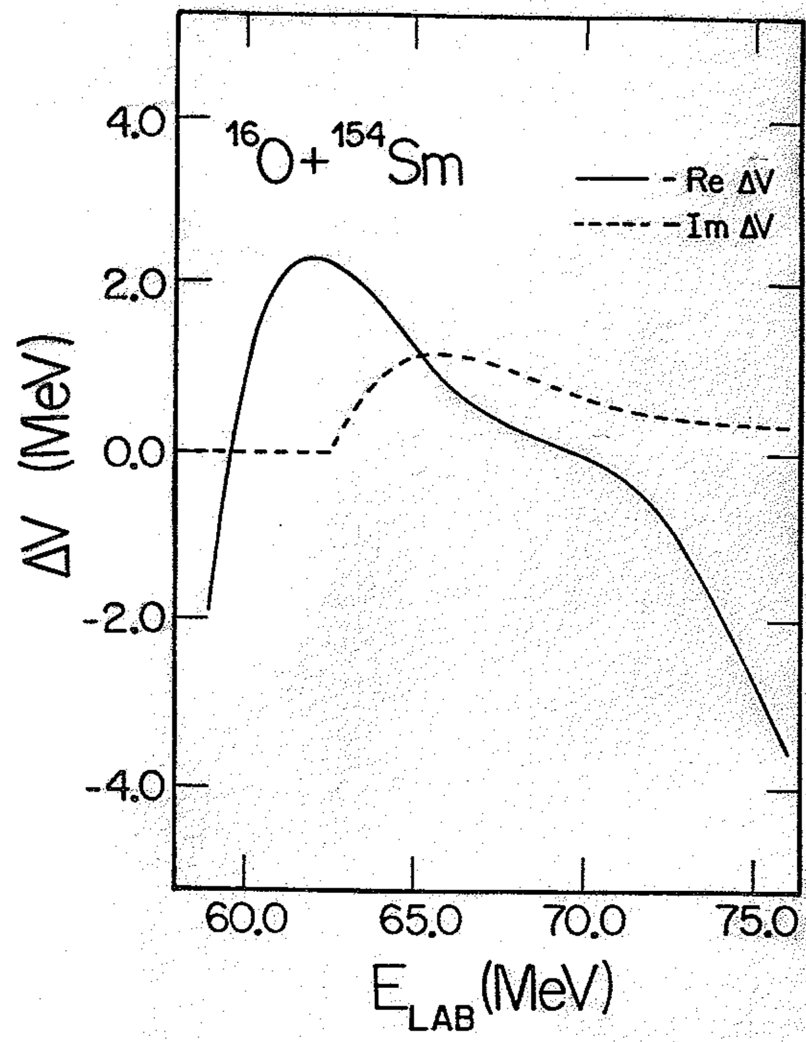

$^{16}O+^{150}Sm$
Re ΔV
Im ΔV
4.0
2.0
0.0
-2.0
-4.0
ΔV (MeV)
60.0
65.0
70.0
75.0
E_{LAB}(MeV)

Fig. 1b



$^{16}O+^{152}Sm$
Re ΔV
Im ΔV
4.0
2.0
0.0
-2.0
-4.0
ΔV (MeV)
60.0
65.0
70.0
75.0
E_{LAB}(MeV)

Fig. 1c



$^{16}O+^{154}Sm$
Re ΔV
Im ΔV
4.0
2.0
0.0
-2.0
-4.0
ΔV (MeV)
60.0
65.0
70.0
75.0
E_{LAB}(MeV)

Fig. 1d