UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

IFUSP/P-597

CORRELATIONS IN THE Sp(1,R) MODEL FOR THE MONOPOLE OSCILLATIONS

by

E.J.V. de Passos and **M.M.B.M. de Oliveira**
Instituto de Física, Universidade de São Paulo

Julho/1986

CORRELATIONS IN THE Sp(1,R) MODEL FOR THE MONOPOLE OSCILLATIONS

E.J.V. de Passos and M.M.B.M. de Oliveira

Instituto de Física da Universidade de São Paulo
Caixa Postal 20516, 01498 São Paulo, SP, Brazil

ABSTRACT

In the framework of the Sp(1,R) model for the monopole oscillations, we examine how the ground state correlations affect the monopole operator sum rules m_ℓ, $0 \leq \ell \leq 3$, in ^{16}O and ^{40}Ca. Our way of probing the correlations indicates their importance for the even sum rules, whereas the odd ones practically are not affected by it. Since the scaling incompressibility is proportional to m_3, we also conclude that the scaling incompressibility is not sensitive to ground state correlations. The above conclusions are in agreement with the RPA results.

PACS NUMBERS - 21.60 Ev, 21.10 Re

Keyword Abstract - [Nuclear Structure - Sp(1,R) symplectic model applied to the monopole operator sum rules in ^{16}O and ^{40}Ca] .

I. INTRODUCTION

Recently, much effort has been devoted to the construction of microscopic collective theories, with the use of symplectic groups [1-4]. The basic idea of these approaches is to identify a symplectic algebra of collective observables. The collective subspace is then identified with an irreducible representation space of this symplectic algebra. The collective observables are realized in microscopic terms and the irreducible representation space is a subspace of the oscillator shell model space Wave functions and energies are calculated by diagonalizing a microscopic Hamiltonian in the collective subspace.

In spherical nuclei we view the monopole oscillations as compressional vibrations of the ground state density. The generator of these deformations is the scaling operator which, together with the monopole operator and the kinetic energy, span the algebra of the group Sp(1,R)

$$\hat{D} = \frac{1}{2\hbar} \sum_{n=1}^{3} \sum_{j=1}^{A-1} (\hat{x}_\mu(j)\hat{p}_\mu(j) + \hat{p}_\mu(j)\hat{x}_\mu(j)) \qquad \text{(scaling operator)}$$

$$\hat{M} = \frac{1}{2} \sum_{\mu=1}^{3} \sum_{j=1}^{A-1} \hat{x}_\mu^2(j) \qquad \text{(monopole operator)} \qquad (1)$$

$$\hat{T} = \frac{1}{2\hbar^2} \sum_{\mu=1}^{3} \sum_{j=1}^{A-1} \hat{p}_\mu^2(j) \qquad \text{(kinetic energy)} \quad .$$

In eqs. (1) the $\hat{x}_\mu(j)$ are a set of Jacobi coordinates and $\hat{p}_\mu(j)$ its associated canonical momenta, where μ indicates spatial directions and j is a Jacobi "particle" index.

The construction of an irreducible representation space proceeds along familiar lines. The raising, lowering and weight operators are linear combinations of $\hat{D}$, $\hat{M}$ and $\hat{T}$. They are, respectively, equal to:

$$\hat{A}_+ = \frac{1}{2} \sum_{\mu=1}^{3} \sum_{j=1}^{A-1} \hat{C}^+_\mu(j)\hat{C}^+_\mu(j) \quad , \quad \hat{A}_- = (\hat{A}_+)^+$$

$$\hat{A}_0 = \frac{1}{4} \sum_{\mu=1}^{3} \sum_{j=1}^{A-1} (\hat{C}^+_\mu(j)\hat{C}_\mu(j) + \hat{C}_\mu(j)\hat{C}^+_\mu(j)) = \frac{1}{2\hbar\omega_0} \hat{H}_0$$

The $\hat{C}_\mu(j)$ and $\hat{C}^+_\mu(j)$ are oscillator boson annihilation and creation operators,

$$\hat{C}_\mu(j) = \frac{1}{\sqrt{2}} \left(\frac{\hat{x}_\mu(j)}{b_0} + i \, \frac{\hat{p}_j(\mu) b_0}{\hbar} \right) \quad , \quad \hat{C}^+_\mu(j) = (\hat{C}_\mu(j))^+ \quad ,$$

and $\hat{H}_0$ is the many-body harmonic oscillator Hamiltonian in the center of mass frame. The size of the oscillator b_0, is a scaling parameter to be determined. It is related to the energy of the oscillator boson, $\hbar\omega_0$, by the equation, $\hbar\omega_0 = \frac{\hbar^2}{m b_0^2}$.

A lowest weight state, $|0\rangle$, is annihilated by $\hat{A}_-$ and an eigenstate of $\hat{A}_0$

$$\hat{A}_-|0\rangle = 0 \qquad \hat{A}_0|0\rangle = k|0\rangle \quad .$$

The eigenvalue k of $\hat{A}_0$, for the lowest weight state $|0\rangle$, labels the irreducible representations of $Sp(1,R)$.

Given such a state, we can construct a basis in a

irreducible representation space by repeated action of $\hat{A}_+$ on $|0\rangle$,

$$|n\rangle = \sqrt{\frac{\Gamma(2k)}{\Gamma(n+1)\Gamma(2k+n)}} \, (A_+)^n |0\rangle \quad .$$

The action of the operators of the algebra in this basis is determined by

$$\hat{A}_+ |n\rangle = ((2k+n)(n+1))^{\frac{1}{2}} |n+1\rangle$$

$$\hat{A}_- |n\rangle = ((2k+n-1)n)^{\frac{1}{2}} |n-1\rangle \qquad (2)$$

$$\hat{A}_0 |n\rangle = (k+n)|n\rangle \quad .$$

It is evident that the lowest energy configuration of a doubly closed shell nucleus in the translationally invariant oscillator shell model space is non-degenerate and is a lowest weight state of an irreducible representation whose label $2k$ is the oscillator energy in units of $\hbar\omega_0$.

The symplectic basis states $|n\rangle$ has a very simple interpretation in terms of the $Sp(1,R)$ Holstein-Primakoff bosons, $\hat{S}^+$ and $\hat{S}$ (1):

$$\hat{A}_+ = \hat{S}^+(2k + \hat{S}^+\hat{S})^{\frac{1}{2}} \quad , \quad \hat{A}_- = (\hat{A}_+)^+ \quad , \quad \hat{A}_0 = k + \hat{S}^+\hat{S} \quad .$$

From these expressions and equations (2) one deduces that $\hat{S}^+$ and $\hat{S}$ act on the basis states in canonical fashion;

$$\hat{S}^+ |n\rangle = (n+1)^{\frac{1}{2}} |n+1\rangle \quad , \quad \hat{S}|n\rangle = n^{\frac{1}{2}} |n-1\rangle$$

$$[\hat{P}\hat{S} \, , \hat{P}S^+] = \hat{P} \quad ,$$

where $\hat{P}$ is the projection operator in the collective subspace (the irreducible representation space). As a consequence, one has for the symplectic basis that

$$\hat{S}\,|0\rangle = 0 \quad , \quad |n\rangle = \frac{(\hat{S}^+)^n}{\sqrt{n!}}\,|0\rangle \quad ,$$

i.e, the lowest weight state is the boson vacuum and $|n\rangle$ are n-boson states. Therefore the symplectic basis constitutes the state space of a bosonic excitation whose harmonic oscillator energy is $2\hbar\omega_0$. Once we have constructed a basis in the collective subspace, wave functions and energies are calculated by diagonalizing a microscopic hamiltonian in the collective subspace. In the calculations of the matrix elements of the Hamiltonian, it is used the generating function method and the parameter b_0 is such that, it minimizes $\langle 0(b)|\hat{H}|0(b)\rangle$ as a function of b. This condition implies that the Hamiltonian does not couple the vacuum and the one boson state,

$$\langle 0|\hat{H}|1\rangle = 0 \quad . \tag{3}$$

In references 1 and 2 they use a number of Skyrme type Hamiltonians to calculate the energy of the giant monopole resonance and the incompressibility, in a series of spherical nuclei. The calculated values of these quantities agree very well with the experimental values. Besides, they are very close to the RPA values. In an analysis of the results of the calculations in terms of the Sp(1,R) boson states, it is shown that the ground state is dominated by the vacuum state. The first excited state, which nearly exhausts the EWSR, is dominated by the one

boson state [1,2]. However, in these references they claim that the presence of a small admixture of a two-boson state in the ground state has a large effect in the calculations of the incompressibility. Typically an admixture of the order of 1% has an effect of the order of (20-30)% in the value of the incompressibility. As will be shown in section II this conclusion depends crucially on the way that they probe the correlations. We will see, based on a calculation in ^{16}O and ^{40}Ca, that there is an alternative way of probing the correlations which leads to a different conclusion, compatible with the RPA results.

II. SUM RULES AND CORRELATIONS

To justify our statement at the end of the Introduction, let us consider the monopole operator non-negative sum rules which are quantities particularly sensitive to ground state correlations [5]. These sum rules are moments of the monopole strength function and, as shown in reference 5, they are equal to the expectation value of certain operators in the ground state (except in the case of m_0). In the framework of the Sp(1,R) model, the monopole operator non-negative sum rules are the moments of the model strength function

$$\bar{S}(E) = \sum_{n\neq 0} |\langle\bar{\varphi}_n|\hat{M}|\bar{\varphi}_0\rangle|^2\, \delta(\bar{E}_n - E) \quad ,$$

$$\bar{m}_\ell = \int_0^\infty E^\ell\, \bar{S}(E)\, dE = \sum_{n\neq 0} \langle\bar{\varphi}_n|\hat{M}|\bar{\varphi}_0\rangle|^2\, \bar{E}_n^\ell \quad .$$

In the above expressions $|\bar{\varphi}_o\rangle$ is the model ground state, $|\bar{\varphi}_n\rangle$ the model n-th excited state and $\bar{E}_n$ its excitation energy. (In what follows we will denote all quantities calculated according to the Sp(1,R) model by a bar on the top of their symbols, that is $\bar{m}_\ell$, $\bar{E}_n$, etc.).

As for the exact non-negative sum rules [5], we can find closed expressions for the model ones [6]

$$\bar{m}_\ell = \frac{1}{2}\,(i)^s\,(-i)^t\,\langle\bar{\varphi}_o|[\hat{\bar{M}}_s,\hat{\bar{M}}_t]|\bar{\varphi}_o\rangle \quad , \quad \ell_{odd}$$

$$= \frac{1}{2}\,(i)^s\,(-i)^t\left(\langle\bar{\varphi}_o|\{\hat{\bar{M}}_s,\hat{\bar{M}}_t\}|\bar{\varphi}_o\rangle - 2\langle\bar{\varphi}_o|\hat{\bar{M}}|\varphi_o\rangle^2\,\delta_{\ell o}\right) \quad , \quad \ell_{even}$$

where s and t are any two non-negative integers such that

$$s + t = \ell$$

and, as usual, [ ] means the commutator and { } the anti-commutator. In turn, the operators $\hat{\bar{M}}_s$ are given by:

$$\hat{\bar{M}}_s = \underbrace{[i\hat{\bar{H}},[i\hat{\bar{H}},\ldots[i\hat{\bar{H}},\hat{\bar{M}}]}_{s\text{-commutators}}\ldots]]$$

$$\hat{\bar{M}}_o = \hat{\bar{M}} \quad , \quad \hat{\bar{M}} = \hat{P}\hat{M}\hat{P} \quad , \quad \hat{\bar{H}} = \hat{P}\hat{H}\hat{P} \quad .$$

where $\hat{P}$ is the projection operator in the collective subspace (the irreducible Sp(1,R) representation space). Also, as for the exact case [5], they satisfy the inequalities

$$\bar{m}_{\ell+2}\,\bar{m}_\ell \;\geq\; \bar{m}^2_{\ell+1} \qquad (4)$$

Supposing that [5],

$$[\hat{A},\hat{M}] = \frac{\hbar^2}{m}\,[\hat{T},\hat{M}] \quad , \qquad (5)$$

and using the commutation relations between the operators of the Sp(1,R) algebra and the property that, by construction, these operators commute with the projection operator $\hat{P}$, the expressions for the sum rules $\bar{m}_\ell$,

$0 \leq \ell \leq 3$ become equal to:

$$\bar{m}_o = \langle\bar{\varphi}_o|\hat{M}^2|\bar{\varphi}_o\rangle - \langle\bar{\varphi}_o|\hat{M}|\bar{\varphi}_o\rangle^2 \qquad (6.a)$$

$$\bar{m}_1 = \frac{\hbar^2}{m}\,\langle\bar{\varphi}_o|\hat{M}|\bar{\varphi}_o\rangle = \frac{\hbar^2 A}{2m}\langle r^2\rangle \qquad (6.b)$$

$$\bar{m}_2 = \frac{\hbar^4}{m^2}\,\langle\bar{\varphi}_o|\hat{D}^2|\bar{\varphi}_o\rangle \qquad (6.c)$$

$$\bar{m}_3 = \frac{\hbar^4}{2m^2}\,\langle\bar{\varphi}_o|[\hat{D},[\hat{A},\hat{D}]]|\bar{\varphi}_o\rangle = \frac{\hbar^4 A}{2m^2}\,\bar{K} \quad . \qquad (6.d)$$

The above expressions show that, when eq. (5) hold, the Sp(1,R) model fulfils these sum rules [6] and that $\bar{m}_1$ and $\bar{m}_3$ are, respectively, proportional to the mean square radius, $\langle r^2\rangle$, and to the incompressibility,

$$\bar{K} = \left[\frac{d^2}{d\vartheta^2}\left(\langle\bar{\varphi}_o|e^{-i\vartheta\hat{D}}\hat{A}\,e^{i\vartheta\hat{D}}|\bar{\varphi}_o\rangle/A\right)\right]_{\vartheta=0} \quad .$$

In our calculations presented here we used the Skyrme III Hamiltonian without the Coulomb and spin-orbit terms. This Hamiltonian obeys eq. (5) [5], and in all cases it was sufficient to truncate the basis states at most at $n_{max} = 10$.

The overlap of the two lowest energy states with the vacuum, one and two-boson states are presented in Table I. The value of the sum rules $\bar{m}_\ell$, $0 \leq \ell \leq 3$ and the fraction exhausted by the first excited state is presented in the last column of Table II.

To investigate the effect, in the sum rules, of the admixture of n-boson states, $n \neq 0$, in the model ground state, we diagonalize the many-body Hamiltonian in a truncated basis, with n_{max} ranging from 1 to N [1,2]. The ground state calculated at each step is used in the eqs. (6) to calculate the sum rules. The results of these calculations are presented in Table II. Notice that, due to equation 3, in the diagonalization for $n_{max} = 1$, the ground state is the boson vacuum, $|0\rangle$. Therefore the first column in Table II is equal to the uncorrelated value of the sum rules. We see that, even though the ground state is dominated by the boson vacuum, the small admixture of a two-boson state in the ground state has a large effect in $\overline{m}_0$ and $\overline{m}_2$ of the order of (20-30)%. On the other hand, the effect of correlations in $\overline{m}_1$ and $\overline{m}_3$ is almost negligible, being of the order of a few percent. To have a qualitative understanding of what is happening let us calculate, following reference 1, the sum rules m_ℓ, $0 \leq \ell \leq 2$ to first order in the small coefficients C_n^0, $n \neq 0$, which are the amplitudes of finding an n-boson state in the model ground state.

Doing this one has:

$$\overline{m}_0 = k \frac{b_0^4}{2} \left(1 + 2\left(\frac{2}{k}\right)^{\frac{1}{2}} C_1^0 + 2\sqrt{2}\left(1 + \frac{1}{2k}\right)^{\frac{1}{2}} C_2^0 + 0\left(C_n^0\right)^2\right)$$

$$\overline{m}_1 = k \frac{\hbar^2 b_0^2}{m} \left(1 + \left(\frac{2}{k}\right) C_1^0 + 0\left(C_n^{o2}\right)\right)$$

$$\overline{m}_2 = k \frac{2\hbar^4}{m^2} \left(1 - 2\sqrt{2}\left(1 + \frac{1}{2k}\right)^{\frac{1}{2}} C_2^0 + 0\left(C_n^{o2}\right)\right)$$

where the terms in front of the parenthesis are the uncorrelated values of the sum rules. We see that these sum rules, to first order in C_n^0, $n \neq 0$, depend only on C_1^0 and C_2^0. However there is a quenching of the effect of the one-boson component

since k is a large number. This quenching is more pronounced when we increase the number of particles since K increases and $\left|\frac{C_1^0}{C_2^0}\right|$ diminishes as can be seen in Table I.

The effect of correlations in the incompressibility, $\overline{K}$, follows from its effect on $\overline{m}_3$, since $\overline{K}$ and $\overline{m}_3$ are related (see eq. (6.d)). So, according to our analysis, the effect of correlations in the incompressibility is small, which differs from the conclusions of references 1 and 2. The source of the discrepancy comes from the different ways of measuring the effect of correlations in these two papers. Our point of view is that, owing to the fact that the moments of the monopole operator are given by the expectation value in the ground state of certain many-body operators (except in the case of m_0), to investigate the effect of correlations in a given sum rule one has to examine how sensitive these operators are, to the presence of admixture of n-boson states, $n \neq 0$, in the model ground state.

On the other hand, in references 1 and 2 a different point of view is adopted. They write the incompressibility as

$$\overline{K} = \frac{2}{A}\left(\langle\overline{\varphi}_0|\hat{D}\hat{A}\hat{D}|\overline{\varphi}_0\rangle - \langle\overline{\varphi}_0|\hat{A}|\overline{\varphi}_0\rangle\langle\overline{\varphi}_0|\hat{D}^2|\overline{\varphi}_0\rangle\right) \qquad (7)$$

and they undertake the analysis outlined above using expression (7) for $\overline{K}$.

The expressions (7) and (6.d) are equivalent when $|\overline{\varphi}_0\rangle$ is the exact model ground state. However, when instead of $|\overline{\varphi}_0\rangle$ we use an approximation to it, the two expressions differ This explains the difference between our Table II and

Table 3 of reference 2 and, by consequence, the different conclusions we reach with respect to the role of correlations in the incompressibility.

III. FINAL REMARKS

Our conclusion that the odd sum rules $\overline{m}_1$ and $\overline{m}_3$ are not sensitive to correlations whereas the even ones $\overline{m}_0$ and $\overline{m}_2$ are, gets further support from the RPA calculations [5]. It is well known that the m_3 and m_1 monopole sum rules satisfy Thouless theorem [5]. Therefore the RPA values of these sum rules are given by the expressions (6.b) and (6.d) with $|\overline{\varphi}_0>$ replaced by the Hartree-Fock ground state and the operators by their one body expressions. This result is an indication that, for these sum rules, the effect of ground state correlations are not very important. However, if we do the same replacement in the expressions for $\overline{m}_0$ and $\overline{m}_2$ we obtain inconsistent result that do not even satisfy the inequalities (4) [5]. This shows clearly the importance of correlations for these two sum rules. It is a remarkable achievement of the Sp(1,R) model that it gives almost identical values to the RPA for m_1 and m_3 and consistent results for m_0 and m_2. This is a strong indication that this model describes properly the ground state correlations associated to monopole oscillations at least in ^{16}O and ^{40}Ca.

This paper is partially based on a master thesis submitted by M.M.B.M.O. to the University of São Paulo in 1985. EJVP acknowledges the financial support of CNPq and M.M.B.M.O. of FAPESP.

REFERENCES

[1] J. Broeckhove and P. van Leuven, Phys. Rev. C29, 628 (1984).

[2] J. Broeckhove, Phys. Lett. 109B, 5 (1982).

[3] F. Arickx, J. Broeckhove and E. Deumans, Nucl. Phys. A318, 269 (1979).

[4] D.J. Rowe, Rep. Prog. Phys. 48, 1419 (1985).

[5] O. Bohigas, A.M. Lane and J. Martorell, Phys. Rep. 51, 267 (1979).

[6] E.J.V. de Passos and M.M.B.M. de Oliveira, to be published.

TABLE CAPTIONS

Table I - Overlap of the ground and first excited state with the vacuum, one and two-boson state.

Table II - The value of the sum rules for different basis truncations (see text). The last column at $n_{max} = 10$ is the converged value of the sum rule. In this column the number inside the parenthesis is the fraction of the sum rule exhausted by the first excited state.

TABLE I

	$\|\langle\bar{\varphi}_0\|0\rangle\|^2$	$\|\langle\bar{\varphi}_0\|1\rangle\|^2$	$\|\langle\bar{\varphi}_0\|2\rangle\|^2$	$\|\langle\bar{\varphi}_1\|0\rangle\|^2$	$\|\langle\bar{\varphi}_1\|1\rangle\|^2$	$\|\langle\bar{\varphi}_1\|2\rangle\|^2$
^{16}O	0.988	0.003	0.005	0.005	0.956	0.023
^{40}Ca	0.988	0.001	0.009	0.002	0.966	0.005

TABLE II

sum rules \ n		1	2	3	4	5	6	7	10
^{16}O	$\overline{m}_0$ (fm^4)	83.8	64.1	65.8	65.9	65.4	65.3	65.3	65.3 (97)
	$\overline{m}_1 \times 10^2$ ($MeV\ fm^4$)	22.3	22.1	21.9	21.9	21.9	21.9	21.9	21.9 (94)
	$\overline{m}_2 \times 10^3$ ($MeV^2\ fm^4$)	59.3	77.2	75.4	74.4	75.2	75.4	75.3	75.3 (89)
	$\overline{m}_3 \times 10^5$ ($MeV^3\ fm^4$)	27.0	26.2	27.1	27.4	27.3	27.3	27.3	27.4 (80)
^{40}Ca	$\overline{m}_0 \times 10$ (fm^4)	44.6	33.7	33.7	33.5	33.3	33.4	33.4	33.3 (99)
	$\overline{m}_1 \times 10^2$ ($MeV\ fm^4$)	95.4	95.2	94.9	94.9	94.9	94.9	94.9	94.9 (98)
	$\overline{m}_2 \times 10^4$ ($MeV^2\ fm^4$)	20.4	27.0	26.8	27.0	27.2	27.2	27.2	27.2 (97)
	$\overline{m}_3 \times 10^5$ ($MeV^3\ fm^4$)	78.9	78.1	79.2	79.1	79.1	79.1	79.1	79.1 (94)