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CAN THE CAUSAL PATHOLOGIES OF GÖDEL-TYPE UNIVERSES BE AVOIDED IN HIGHER-DERIVATIVE GRAVITY?

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ABSTRACT. A completely causal rotating Gödel-type universe is obtained in the context of higher-derivative gravity. The solution is such that it has no similar in the framework of standard general relativity. The aforementioned solution presents the interesting feature of relating the mass of the nontachyonic spin-0 particle, concerning the linearized higher-derivative theory, with the velocity of rigid rotation of matter.

PACS. 04.60.+n Quantum theory of gravitation

PACS. 04.20.Jb Solutions to equations

PACS. 98.80.Dr Relativistic cosmo ogy

I. INTRODUCTION AND SUMMARY

As it is well known, Einstein's equations admit unphysical solutions whose global behaviour violates some requirements (strong causality, time orientability, etc.)[1]. The most outstanding example of such a kind of solution is provided by Gödel's metric[2]. The former is a solution of Einstein's equations (with a non-null cosmological constant Λ) of the type

$$ds^2 = [dt + H(r)d\Phi]^2 - D^2(r)d\Phi^2 - dr^2 - dz^2, \qquad (1.1)$$

which, following Rebouças and Tiomno[3], we shall call Gödel - type metric. Gödel solution is obtained when we set $H(r) = \frac{4\Omega}{m^2} \sinh^2\left(\frac{mr}{2}\right)$ and $D(r) = \frac{\sinh\,(mr)}{m}$, in case $m^2 = 2\Omega^2$. The corresponding energy-momentum tensor is that of a perfect fluid with a constant density of matter ρ and vanishing pressure p. The congruence of curves comoving with the fluid is shear-free, has no expansion, but has a constant non-null rotation Ω, which is just the achille's heel of this model. In fact, the existence of such non-null vorticity is responsible for many unusual features of Gödel's space-time, among which, the most notorious one is the existence of closed time-like curves.

In recent years, quite a lot of metrics of Gödel-type[4-11], with all sort of matter content, has a peared in the physical literature. In a sense, the leitmotiv of these investigations is the search for a causal Gödel-type universe. In spite of this consid erable effort towards this goal, as far as we know, the solution of Rebouças and Tiomno[11] is the only known Gödel-type solution of Einstein's equations describing a completely causal space-time (ST)-homogeneous rotating universe. In this remarkable solution, the introduction of a massless scalar field is the price for not having a breakdown of causality.

On the other hand, the so called higher-derivative gravity has been considered as a nice candidate for a theory of quantum gravity[12-15] since the last decade. The theory is defined by the action

$$I = \int d^4x\sqrt{-g}\,[\frac{R}{\varkappa} + \frac{\Lambda}{\varkappa} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + L_m] \ , \qquad (1.2)$$

where α and β are dimensionless coupling constants (in natural units), $\varkappa$ is the Einstein constant, and L_m is the matter Lagrangian density. The corresponding field equations are given by

$$\tilde{G}_{\mu\nu} = - T_{\mu\nu} \ , \qquad (1.3)$$

$$\tilde{G}_{\mu\nu} = \frac{1}{\varkappa}(R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}) + \frac{\Lambda}{\varkappa} g_{\mu\nu} + \alpha(-R^2 g_{\mu\nu} + 4RR_{\mu\nu} - 4g_{\mu\nu}\Box R + 4\nabla_\nu\nabla_\mu R) + \beta(-2\Box R_{\mu\nu} - R_{\rho\theta}R^{\rho\theta}g_{\mu\nu} + 4R_{\mu\rho\theta\nu}R^{\rho\theta} - g_{\mu\nu}\Box R + 2\nabla_\nu\nabla_\mu R) \ , \qquad (1.4)$$

with trace

$$T = \frac{R}{\varkappa} + \frac{4\Lambda}{\varkappa} + 4(3\alpha + \beta)\Box R. \qquad (1.5)$$

This higher-derivative theory has the great advantage of being renormalizable by power counting[12], whereas classical general relativity is clearly perturbatively nonrenormalizable by power counting in four dimensions[16,17]. From the classical point of view the theory may be considered as a possible generalization of Einstein's general relativity, in the sense that it respects the geometrical nature of gravity as well as its gauge symmetry (invariance under general coordinate transformations). Recent work has shown that the arguments traditionally used to reject this fourth-order gravity theory as a viable possibility, i. e., its pseudo-nonunitarity does not proceed[15,18-23]. The reason is that the ghost contained in the theory, which has haunted the quantum field theorist's mind for a long time, is an unstable one. We note for future reference that the theory has eight total degrees of freedom[12-14]. Two degrees represent a massless spin-2 particle, a graviton. Another degree is a spin-0 particle with mass

$$m_0{}^2 = 1/2\varkappa(3\alpha + \beta) \ . \qquad (1.6)$$

The former has significance even in the nonlinear sector[24]. The remaining five degrees of freedom are related to a spin-2 particle with mass

$$m_2{}^2 = -1/\varkappa\beta \ . \qquad (1.7)$$

So, in order to have nontachyonic spin-0 and spin-2 particles, we shall require $(3\alpha + \beta)$ to be positive and β to be negative, respectively.

Thus, we may wonder what will happen to the causal pathologies of ST-homogeneous Gödel-type universes when quantum corrections are introduced in the standard general relativity theory. To answer this question we proceed as follows. In section 2 we exhibit a class of exact solutions of the higher-derivative gravity field equations, concerning ST-homogeneous Gödel-type universes, whose source of the geometry is a perfect fluid. These solutions are characterized by two parameters, one of which is related to the rate of rotation of matter. The next section is devoted to the discussion of the breakdown of causality in the universes under consideration. A first consequence of such analysis is the fact that among our solutions there is one which is completely causal and has the remarkable property of relating the mass of the nontachyonic spin-0 particle (microphysics) to the constant velocity of rotation of matter (macrophysics). We conclude the paper with some interesting observations about the results obtained.

II. MATTER CONTENT, FIELD EQUATIONS AND SOLUTIONS

We choose a tetrad field $e^{(A)}{}_{\alpha}(x)$ such that the line element (1.1) can be expressed as

$$ds^2 = \eta_{AB}\theta^A\theta^B$$

$$= (\theta^0)^2 - (\theta^1)^2 - (\theta^2)^2 - (\theta^3)^2 , \qquad (2.1)$$

where the $\theta^A = e^{(A)}{}_{\alpha}dx$ are given by

$$\theta^0 = dt + H(r)d\Phi,\ \theta^1 = dr,\ \theta^2 = D(r)d\Phi,\ \theta^3 = dz . \qquad (2.2)$$

Here capital latin indices are tetrad indices and run from 0 to 3, whereas greek indices are tensor indices.

The matter content of the model is a perfect fluid. In the local inertial frame considered, an observer comoving with the fluid is assumed to have the four-velocity

$$v^A = \delta^A{}_0, \qquad (2.3)$$

which corresponds to a matter velocity field $e^{(A)}{}_0$. Denoting by ρ and p the density of matter and the pressure of the fluid, respectively, as measured locally by the comoving observer (2.3), the energy-momentum of the fluid can be written as

$$T_{AB} = (\rho + p)\, v_A v_B - p\eta_{AB}. \qquad (2.4)$$

On the other hand, the non-null components of $\tilde{G}_{AB}$ for the metric (1.1) are

$$\tilde{G}_{00} = \frac{1}{\varkappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 - \frac{R}{2}\right] + \frac{\Lambda}{\varkappa} + \alpha[-R^2 - 2R(H'/D)^2 + (4D'R')/D$$
$$+ 4R''] + \beta\left\{-\frac{15}{4}\left(\frac{H'}{D}\right)^4 - 3\frac{H'}{D}\left(\frac{H'}{D}\right)'' - \frac{3}{2}\left[\left(\frac{H'}{D}\right)'\right]^2\right.$$
$$- 3\frac{D'}{D}\frac{H'}{D}\left(\frac{H'}{D}\right)' + 6\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - 2\left(\frac{D''}{D}\right)^2 - 2\left(\frac{D''}{D}\right)''$$
$$\left.- 2\frac{D'}{D}\left(\frac{D''}{D}\right)'\right\},$$

$$\tilde{G}_{11} = \frac{1}{\varkappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 + \frac{D''}{D} + \frac{R}{2}\right] - \frac{\Lambda}{\varkappa} + \alpha\left[-R^2 - R(H'/D)^2\right.$$
$$\left.- (4D'R')/D\right] + \beta\left\{-\frac{9}{4}\left(\frac{H'}{D}\right)^4 - 5\frac{D'}{D}\frac{H'}{D}\left(\frac{H'}{D}\right)' + 4\frac{D'}{D}\left(\frac{D''}{D}\right)'\right.$$
$$\left.+ \frac{1}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 - \frac{H'}{D}\left(\frac{H'}{D}\right)'' + 4\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - 2\left(\frac{D''}{D}\right)^2\right\},$$

$$\tilde{G}_{22} = \frac{1}{\varkappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 + \frac{D''}{D} + \frac{R}{2}\right] - \frac{\Lambda}{\varkappa} + \alpha\left[-R^2 - 4R'' - R(H'/D)^2\right]$$
$$+ \beta\left\{-\frac{9}{4}\left(\frac{H'}{D}\right)^4 - \frac{9}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 + 4\left(\frac{D''}{D}\right)'' - 2\left(\frac{D''}{D}\right)^2\right.$$
$$\left.- \frac{H'}{D}\frac{D'}{D}\left(\frac{H'}{D}\right)' - 5\frac{H'}{D}\left(\frac{H'}{D}\right)'' + 4\left(\frac{H'}{D}\right)^2\frac{D''}{D}\right\},$$

$$\tilde{G}_{33} = \frac{1}{\varkappa}\frac{R}{2} - \frac{\Lambda}{\varkappa} + \alpha\left[R^2 - 4R'' - (4D'R')/D\right] + \beta\left\{\frac{3}{4}\left(\frac{H'}{D}\right)^4\right.$$
$$- \frac{3}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 + 2\left(\frac{D''}{D}\right)^2 - 2\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - \frac{H'}{D}\left(\frac{H'}{D}\right)'' + 2\left(\frac{D''}{D}\right)''$$
$$\left.- \frac{D'}{D}\frac{H'}{D}\left(\frac{H'}{D}\right)' + 2\frac{D'}{D}\left(\frac{D''}{D}\right)'\right\},$$

$$\tilde{G}_{02} = -\frac{1}{2\varkappa}\left(\frac{H'}{D}\right)' + \alpha\left[-2R(H'/D)' - (2H'R')/D\right] + \beta\left\{-\left(\frac{H'}{D}\right)'''\right.$$
$$- 9\left(\frac{H'}{D}\right)^2\left(\frac{H'}{D}\right)' + 3\left(\frac{H'}{D}\right)'\frac{D''}{D} + 4\frac{H'}{D}\left(\frac{D''}{D}\right)' + \left(\frac{H'}{D}\right)'\left(\frac{D'}{D}\right)^2$$
$$\left.- \frac{D'}{D}\left(\frac{H'}{D}\right)''\right\} \quad ; \qquad (2.5)$$

where

$$R = \frac{1}{2}(H'/D)^2 - (2D'')/D \,, \qquad (2.6)$$

and the primes denote differentiation with respect to r.

Following Gödel steps[2], we restrict our analysis to the case of a perfect fluid with constant density of matter and constant pressure (Gödel original work is concerned with a perfect fluid with vanishing pressure). Consequently, T_{AB} is constant. However, T_{AB} constant implies that H_{AB} is constant too. Now, the preceding equations show us that H_{AB} is constant provided that

$$\frac{H'}{D} = \text{const} \equiv 2\Omega, \quad \frac{D''}{D} = \text{const} \equiv m^2. \qquad (2.7)$$

Taking into account that Eqs. (2.7) are the necessary and sufficient conditions to a Gödel-type metric be space-time-homogeneous [10,11], we conclude that our models are homogeneous in space and time. We remark, en passant, that any Gödel-type solution of higher-derivative gravity field equations $H_{AB} = -T_{AB}$, with T_{AB} constant, is ST-homogeneous up to a local Lorentz transformation.

Thus, our solution is characterized by the two parameters m^2 and Ω, where the last one is related to the vorticity. In fact, the vorticity vector

$$\omega^A = \frac{1}{2}\,\epsilon^{ABCD}\,\omega_{BC}v_D$$

is given by

$$\omega^A = (0,0,0,\Omega), \qquad (2.8)$$

where $2\Omega = H'/D$.

As a result, the higher-derivative gravity field equations for the model

$$H_{AB} = -T_{AB},$$

reduce to the following set of equations

$$\rho = \Omega^2/\kappa + 4\Omega^4(\alpha+3\beta) - 2m^4(2\alpha+\beta) - \Lambda/\kappa\ , \qquad (2.9)$$

$$p = \Omega^2/\kappa + 12\Omega^4(\alpha+3\beta) - 16\Omega^2 m^2(\alpha+\beta) + 2m^4(2\alpha+\beta) + \Lambda/\kappa\ , \qquad (2.10)$$

$$m^2/\kappa = 2\Omega^2/\kappa + 16\Omega^4(\alpha+3\beta) - 24m^2\Omega^2(\alpha+\beta) + 4m^4(2\alpha+\beta). \qquad (2.11)$$

In order to have physically significant solutions we must guarantee the positivity of energy and pressure. To accomplish this, the cosmological constant must be bounded within the interval

$$-12\Omega^4(\alpha+3\beta) - 2m^4(2\alpha+\beta) + 16m^2\Omega^2(\alpha+\beta) - \Omega^2/2 \leqq \Lambda/\kappa \leqq 4\Omega^4(\alpha+3\beta)$$
$$- 2m^4(2\alpha+\beta) + \Omega^2/\kappa\ , \qquad (2.12)$$

which implies that

$$8\Omega^4(\alpha+3\beta) + \Omega^2/\kappa - 8m^2\Omega^2(\alpha+\beta) \geqq 0, \qquad (2.13)$$

the equality having as consequence

$$\Lambda/\kappa = -2\Omega^2/\kappa - 20\Omega^4(\alpha+3\beta) + 24m^2\Omega^2(\alpha+\beta) - 2m^4(2\alpha+\beta)\ . \qquad (2.14)$$

In the integration of Eqs. (2.9)-(2.11) we have three different classes of solutions. The first of them corresponds to

$$16\Omega^4(\alpha+3\beta) + 4m^4(2\alpha+\beta) - 24\Omega^2 m^2(\alpha+\beta) + 2\Omega^2/\kappa = m^2 > 0. \qquad (2.15)$$

The metric is given by

$$ds^2 = [dt + \frac{4\Omega}{m^2}\sinh^2(\frac{mr}{2})d\phi]^2 - \frac{1}{m^2}\sinh^2(mr)d\phi^2 - dr^2 - dz^2\ , \qquad (2.16)$$

and the following relation holds

$$\Omega^2/\kappa > -8\Omega^4(\alpha+3\beta) + 10\Omega^2 m^2(\alpha+\beta) - m^4(2\alpha+\beta)\ . \qquad (2.17)$$

The next class of solutions is characterized by

$$16\Omega^4(\alpha+3\beta) + 4n^4(2\alpha+\beta) + 24n^2\Omega^2(\alpha+\beta) + \frac{2\Omega^2}{\kappa} = -n^2 < 0,$$

$$m^2 \equiv -n^2 < 0 . \qquad (2.18)$$

The corresponding metric is as follows:

$$ds^2 = [dt + \frac{4\Omega}{n^2}\sin^2\left(\frac{nr}{2}\right)d\Phi]^2 - \frac{\sin^2 nr}{n^2}\,d\Phi^2 - dr^2 - dz^2. \quad (2.19)$$

We also have that the relation

$$n^4(2\alpha+\beta) + 2n^2\Omega^2(\alpha+\beta) < 0 \qquad (2.20)$$

holds. Clearly, Eq. (2.19) is an analytical extension of Eq.(2.16) with $m \to in$.

Finally, the last class of solutions is such that

$$16\Omega^4(\alpha+3\beta) + \frac{2\Omega^2}{\kappa} = m^2 = 0, \qquad (2.21)$$

with a metric given by

$$ds^2 = [dt + \Omega r^2 d\Phi]^2 - r^2 d\Phi^2 - dr^2 - dz^2 . \qquad (2.22)$$

It may be thought as a limit of the first ($m^2 \to 0$) and the second ($n^2 \to 0$) classes of solutions, respectively.

On the other hand, nontachyonic spin-0 and spin-2 particles, require $(3\alpha+\beta)$ to be positive and β to be negative, respectively (cf. Introduction). Consequently, these restrictions on the parameters α and β must be included in our results.

We remark that our coordinates are true cylindrical coordinates, in the sense that they satisfy Maitra's conditions[25], i.e.,

$$H = r^2 \times \text{const} \;, \; D = r . \qquad (2.23)$$

Last but not least, we call attention to the fact that all Riemannian Gödel-type ST-homogeneous metrics with the same value of m^2 and Ω are isometric[11].

III. A COMPLETELY CAUSAL UNIVERSE OF THE GÖDEL TYPE VIA HIGHER-DERIVATIVE GRAVITY

Now we are ready to investigate if the closed time-like lines usually present in the solutions of Einstein's equations related to Gödel-type universes, will remain in the framework of higher-derivative gravity. To do so, we write Eq. (1.1) in the form

$$ds^2 = dt^2 + 2H d\phi dt - dr^2 - dz^2 + C d\phi^2 , \qquad (3.1)$$

where

$$C(r) = H^2 - D^2 . \qquad (3.2)$$

Clearly, if C(r) becomes positive at $r_1 < r < r_2$, then the curve $t = z = 0$, $r = \text{const}$ is a closed time-like trajectory. As a result, the principle of causality is violated.

It is not difficult to show that in case $m \leqq 0$, we can not have completely causal solutions. Thus, our analysis must be concentrated on the models concerning $m^2 > 0$. In this case, Eq.(2.16) gives

$$C(r) = \frac{4}{m^2} \sinh^2\left(\frac{mr}{2}\right) \left[\left(\frac{4\Omega^2}{m^2} - 1\right) \sinh^2\left(\frac{mr}{2}\right) - 1\right] . \qquad (3.3)$$

Consequently, if

$$m^2 \geqq 4\Omega^2 , \qquad (3.4)$$

our solutions will be completely causal. On the other hand, the following inequality holds in case $m^2 > 0$:

$$\frac{4\Omega^2}{\kappa} \geqq \frac{m^2}{\kappa} - 32\Omega^4(\alpha+3\beta) + 40m^2\Omega^2(\alpha+\beta) - 4m^4(2\alpha+\beta). \qquad (3.5)$$

Eq. (2.13) was used here.

Undoubtedly, the solution $m^2 = 4\Omega^2$ is compatible with (3.5). It follows then from (2.11) and (2.13) that

$$\frac{m^2}{4} = \Omega^2 = \frac{1}{8(3\alpha+\beta)\kappa} . \qquad (3.6)$$

Here $(3\alpha+\beta)$ is positive in order to avoid the presence of a tachyonic spin-0 particle [cf. Eq. (1.6)].

Now, from Eqs. (2.14), (2.9) and (2.10) we get

$$\Lambda = -\frac{3\Omega^2}{2} , \ \rho = p = 0. \qquad (3.7)$$

Equations (3.6) and (3.7) completely determine our causal solution.

IV. CONCLUSION

We have thus succeeded in finding a completely causal rotating universe of the Gödel type in the framework of higher-derivative gravity. An interesting feature of this solution is that it relates the mass of the nontachyonic spin-0 particle (microphysics) to the constant rotation of matter relative to the compass of inertia (macrophysics). In fact, Eqs. (1.6) and (3.6) provide us with the remarkable result

$$\Omega^2 = m_0{}^2/4 . \qquad (4.1)$$

In a sense, it allows us a naïve estimative concerning the mass of the spin-0 particle. Indeed, from Eqs. (3.7) and (4.1) we get immediately that

$$m_0 \sim 10^{-34} \text{ MeV}. \qquad (4.2)$$

On the other hand, the above results tell us that the rate of rigid rotation of matter is very small. In this sense our model is, "grosso modo", a Machian one.

To conclude we point out that this causal solution has no analogue in the context of the standard general relativity.

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