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Abstract

The recently developed theory of the symmetrical T-matrix is extended here to the relativistic description of proton-nucleus scattering. The 4×4 symmetric T-matrix is reduced to an effective symmetric 2×2 matrix using projection technique. The resulting non-relativistic-looking T-matrix contains relativistic (virtual pair) effects to all orders. Glauber theory is then applied to develop a reasonable approximation which could be valid even at large angles.

During the last several years a great amount of effort has been devoted towards the construction of a relativistic theory of nuclear structure and reactions[1]. In such a theory nucleons and mesons appear explicitly, with the Dirac equation used to describe the former's motion. Though several problems still remain to be solved, many major advantages over the conventional non-relativistic theory with effective two-body interactions, have been unambiguously established. A major test of the relativistic theory has been its predictive power of spin observables in elastic and inelastic

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proton nucleus scattering at intermediate energies, where the basic vector and scalar interactions which arise from w- and σ-meson exchange, respectively, can be constructed using the impulse approximation. For a more detailed and stringest comparison of the theory with experiment data at relatively large angles are required.

Quite recently data of 200MeV proton scattering from ^{208}Pb were measured at the Indiana University Cyclotron up to $\theta = 90^o$ where the cross-section drops to about 10^{-15}mb/sr.[2] To perform an exact relativistic calculation of the cross-section up to these angles, a quite costly and lengthy numerical effort must be a allocated. On the other hand, the much simpler Glauber (eikonal) approximation is obviously inadequate at the large angles involved. In this contribution we develop a new modified Dirac eikonal amplitude which avoids the partial wave sum and yet may work well at large angles.

The starting point of our discussion is a relativistic generalization of the recently proposed symmetrical T-matrix, developed in great details by Hussein and Marques[3].

$$\langle k' | T | k \rangle = \int_0^1 d\lambda \, \langle \Psi_{k'}^{(-)}(\lambda U^\dagger) | U | \Psi_{k}^{(+)}(\lambda U) \rangle \qquad (1)$$

where $|\Psi_{\vec{k}}^{(+)}(\lambda U)\rangle$ is a four-component scattering wave function which satisfies Dirac's equation

$$|\Psi_{\vec{k}}^{(+)}(\lambda U)\rangle = |\vec{k}+\rangle + (\not{p} - m + i\eta)^{-1} \lambda U \, |\Psi_{\vec{k}}^{(+)}(\lambda U)\rangle \, , \tag{2}$$

where $|\vec{k}+\rangle$ is a positive-energy plane wave solution of the free Dirac equation and the scaled interaction λU is left here as general as possible, consistent with relativistic covariance. The ingoing wave function $\langle \Psi_{\vec{k}}^{(-)}(\lambda U^{\dagger})|$ is just the time reversed version of Eq. (2). Of course the 4×4 T-matrix itself, satisfies the Lippmann-Schwinger Dirac equation with the unscaled potential

$$T = U + U\,(\not{p} - m + i\eta)^{-1}\, T \tag{3}$$

Equation (1) is an alternative <u>exact</u> representation of the T-matrix.

Since what interests us here is just a 2×2 sub-matrix of the T-matrix, Eq. (3), which describes the upper component only, we have to first projects out from (3) the lower component. Introducing the notation T^{++} (upper), T^{--} (lower), T^{+-} (mixed upper-lower) etc., and the following spectral representation of the free Dirac Green function

$$(\not{p} - m + i\eta)^{-1} = \int d\vec{p}\,' \left\{ \frac{|\vec{p}\,'+\rangle \overline{\langle \vec{p}\,'+|}}{E_p - E_{p'} + i\eta} + \frac{|\vec{p}\,'-\rangle \overline{\langle \vec{p}\,'-|}}{E_p + E_{p'} - i\eta} \right\} \tag{4}$$

$$\equiv G_0^{(+)+} + G_0^{(+)-}$$

where $\overline{\langle \vec{p}\pm|} = \langle \vec{p}\pm|\gamma^0$

we can write down two coupled matrix integral equations for T^{++} and T^{+-}. Eliminating T^{+-} in favor of T^{++} we find finally

$$T^{++} = \mathcal{V}^{++} + \mathcal{V}^{++} G_0^{(+)+} T^{++} \tag{5}$$

with the 2×2 matrix interaction $\mathcal{V}^{++}$ given by

$$\mathcal{V}^{++} = U^{++} + U^{+-} (G_0^{(+)-} - U^{--})^{-1} U^{-+} \tag{6}$$

The second term in Eq. (6) takes into account the virtual nucleon-antinucleon pair creation. This is the term which introduces genuinely relativistic corrections to the now apparently non-relativistic T^{++} - matrix which satisfies the L-S equation (5).

We use Eq. (5), instead of Eq. (3), to derive the symmetrical form of the 2×2 matrix T^{++}, using the operator manipulations of Ref. (3).

$$\langle \vec{k}'|T^{++}|\vec{k}\rangle = \int_0^1 d\lambda \, \langle \Psi^{(-)}_{\vec{k}'}(\lambda v^{++})^{+}|\, v^{++} \,|\Psi^{(+)}_{\vec{k}}(\lambda v^{++})\rangle \quad (7)$$

The wave functions $|\Psi^{(+)}_{\vec{k}}(\lambda v^{++})\rangle$ and $|\Psi^{(-)}_{\vec{k}}(\lambda v^{++})\rangle$ satisfy the mone. familiar Lippmann-Schwinger equations

$$|\Psi^{(+)}_{\vec{k}}(\lambda v^{++})\rangle = |\vec{k}+\rangle + G_0^{(+)+}(E_k)\lambda v^{++}\,|\Psi^{(+)}_{\vec{k}}(\lambda v^{++})\rangle \quad (8)$$

$$|\Psi^{(-)}_{\vec{k}}(\lambda v^{++})\rangle = |\vec{k}+\rangle + G_0^{(-)+}(E_k)\lambda v^{++}\,|\Psi^{(-)}_{\vec{k}}(\lambda v^{++})\rangle$$

In what follows we shall develop modified eikonal expressions for the T-matrix both in the original Dirac form, Eq. (1) and its Schrodinger equivalent form, Eq. (7). To proceed, we write the following form for the wave functions[4] $\psi^{(\pm)}(\lambda v^{++})$

$$\psi^{(+)}_{\vec{k}}(\lambda v^{++}, \vec{r}) = e^{i\vec{k}\cdot\vec{r}}\, e^{iS_{\lambda,\vec{k}}(\vec{r}, v^{++})}\, \chi_s$$

$$\psi^{(-)*}_{\vec{k}'}(\lambda v^{++}, \vec{r}) = e^{-i\vec{k}'\cdot\vec{r}}\, e^{iS_{\lambda,\vec{k}'}(\vec{r}, v^{++})}\, \chi_{s'} \quad (9)$$

where χ_s and $\chi_{s'}$, are the Pauli spinors for the ingoing and outgoing solutions (of course both depend on the directions of the momenta, and thus the prime on s in the second spinor).

We are now in a position to derive an expression ~~o~~for the integrand $\langle \psi^{(-)}_{\vec{k}'}(\lambda u)|u|\psi^{(+)}_{\vec{k}}(\lambda u)\rangle$ in Eq. (1)

$$\langle \psi^{(-)}_{\vec{k}'}(\lambda u)|u|\psi^{(+)}_{\vec{k}}(\lambda u)\rangle = \int d\vec{r}\left(\frac{\epsilon+m}{2m}\right) e^{iS_{\lambda,\vec{k}'}(\vec{r})}\, e^{-i\vec{k}'\cdot\vec{r}} \cdot$$

$$\cdot\left\{(V_s+V_v) + \vec{\sigma}\cdot\vec{p}\,\frac{1}{\epsilon+m-\lambda(V_v-V_s)}(V_v-V_s)\frac{1}{\epsilon+m-\lambda(V_v-V_s)}\,\vec{\sigma}\cdot\vec{p}\right\}\cdot$$

$$\cdot\, e^{i\vec{k}\cdot\vec{r}}\, e^{iS_{\lambda,\vec{k}}(\vec{r})} \quad (10)$$

It is now a simple matter to derive the equations that would determine the ingoing and outgoing eikonals, with the aid of Eq. (3) (or, more precisely, the corresponding Dirac equations)

$$\frac{1}{m}\vec{k}\cdot\vec{\nabla} S_{\lambda,\vec{k}}(\vec{r}) + V_c(\lambda) + V_{so}(\lambda)\left[\vec{\sigma}\cdot\vec{r}\wedge\vec{k} - i\vec{r}\cdot\vec{k}\right] +$$

$$+ V_{so}(\lambda)\left[\vec{\sigma}\cdot(\vec{r}\wedge\vec{\nabla}) - i\vec{r}\cdot\vec{\nabla}\right] S_{\lambda,\vec{k}}(\vec{r}) = 0 \quad (11)$$

and similarly for $S_{\lambda,\vec{k}'}(\vec{r})$. Eq. (11) was obtained, as usual, after dropping second order derivative terms and assuming the usual scalar vector form for U. In Eq. (11), the scaled central and spin-orbit potentials are given by

$$V_c(\lambda) = \lambda V_s + \lambda \frac{\epsilon}{m} V_v + \frac{\lambda^2}{2m} (V_s^2 - V_v^2)$$

$$V_{so}(\lambda) = \frac{1}{2m} \frac{1}{E+m-\lambda(V_v - V_s)} \frac{1}{r}\frac{d}{dr}\left[\lambda(V_v - V_s)\right] \tag{12}$$

We note here that the last term in V_c (quadratic in the V,s) is scaled by λ^2. Secondly the scaled spin-orbit interaction depends on λ only throughs the potential term λ ($V_v - V_s$).

The "eikonal" form of $\langle \psi^{(-)}_{\vec{k}'}(\lambda u) | u | \psi^{(+)}_{\vec{k}}(\lambda u) \rangle$ is now easily obtained by employing Eq. (11) in (10). We find

$$\int d\vec{r} \; \frac{E+m}{2m} \; e^{i S_{\lambda,\vec{k}'}(\vec{r})} e^{-i\vec{k}'.\vec{r}} \Big\{ V_s + V_v - $$

$$- \vec{\sigma}.\vec{p} \; \frac{\vec{\sigma}.\vec{r}}{r} \; \frac{1}{E+m-\lambda(V_v - V_s)} (V_v - V_s) \frac{1}{\frac{d}{dr}\left[\lambda(V_v - V_s)\right]} \cdot$$

$$\cdot \left(2i\vec{k}.\vec{\nabla} S_{\lambda,\vec{k}} + 2mi V_c \right) \Big\} e^{i\vec{k}.\vec{r}} e^{i S_{\lambda,\vec{k}}(\vec{r})} \tag{13}$$

We observe that a mixture of λ - scaled and unscaled quantities appear in the integrand. It would be interesting to compare Eq. (13) with the corresponding (and formally identical) one for $\langle \psi^{(-)}(\lambda v^{++}) | v^{++} | \psi^{(+)}(\lambda v^{++}) \rangle$ of Eq. (7). Where we find

$$\langle \psi^{(-)}(\lambda v^{++}) | v^{++} | \psi^{(+)}(\lambda v^{++}) \rangle =$$

$$= \int d\vec{r} \left(\frac{E+m}{2m} \right) e^{i S_{\lambda,\vec{k}'}(\vec{r})} e^{-i\vec{k}'.\vec{r}} \cdot$$

$$\cdot \left\{ V_c - i V_{so} \left[\vec{\sigma}.\vec{r}\wedge\vec{\nabla} - i\vec{r}.\vec{\nabla} \right] \right\} e^{i S_{\lambda,\vec{k}}(\vec{r})} e^{i\vec{k}.\vec{r}} \tag{14}$$

$$= \int d\vec{r} \left(\frac{E+m}{2m} \right) e^{i\vec{q}.\vec{r}} e^{i S_{\lambda,\vec{k}'}(\vec{r})} \cdot$$

$$\cdot \left\{ - \frac{1}{\lambda m} \vec{\nabla}(\vec{k}.\vec{r}) \cdot \vec{\nabla} S_{\lambda,\vec{k}}(\vec{r}) \right\} e^{i S_{\lambda,\vec{k}}(\vec{r})} ; \tag{15}$$

$$\vec{q} = \vec{k} - \vec{k}'$$

The equation that determines S_λ in this case is

$$\frac{1}{m}\vec{\nabla}(\vec{R}.\vec{r}).\vec{\nabla}S_{\lambda,\vec{R}}+\lambda V_c+\lambda V_{so}\vec{\sigma}.(\vec{r}\wedge\vec{\nabla})(S_{\lambda,\vec{R}})$$
$$-i\lambda V_{so}\vec{r}.\vec{\nabla}(S_{\lambda,\vec{R}})+\lambda V_{so}\vec{\sigma}.(\vec{r}\wedge\vec{k})-$$
$$-i\lambda V_{so}\vec{r}.\vec{k}=0 \tag{16}$$

where V_c and V_{so} are given by Eq. (12)with λ equal to 1. The difference between the symmetrical Dirac representation and the symmetrical Schrodinger representation can be more easily seen by comparing the way λ modifies the central and spin-orbit interaction. We exhibit in Fig. 1 a plot of $V_c(\lambda)/V_c(\lambda=1)$ vs. λ, with V_v and V_s taken from $p + {}^{40}Ca$ elastic scattering Dirac fit at E = 500MeV. ($V_v \simeq + 270$MeV and $V_s \simeq - 400$MeV) and using for the radial density shape a constant value of 1 (nuclear mather). We see clearly that the Dirac scaled central interaction is attractive in the interval $0 < \lambda < 0.5$ and repulsive in the interval $0.5 < \lambda \leqslant 1$. This is to be compared with the Schrodinger equivalent scaled potential which is purely repulsive in the whole λ -interval.

The imaginary part of $V_c(\lambda)$, however is always negative in the whole λ -interval, guaranteing thus the absorptive nature of the scattering process. In fact, we have for the same system,

$$\mathrm{Im}\, V_c^{\mathrm{DIRAC}}(\lambda) = -14\lambda + 3.75\lambda^2 \ (\mathrm{MeV})$$
$$\mathrm{Im}\, V_c^{\mathrm{SCHRÖDINGER}}(\lambda) = -10.25\lambda \qquad (\mathrm{MeV}) \tag{17}$$

The above findings imply clearly that a near-far decomposition of the type used by Carlson et al.,[6] would result the Dirac case, Eq. (1), in a far-side dominance in the λ -integrand in the range $0 < \lambda < 0.5$ followed by a near-side dominance. In contrast, the Schrodinger version of the λ -integrand, Eq. (7), is near-dominated in the whole λ — interval.

As a final remark, the modified Glauber expression for $\langle \vec{k}' | T | \vec{k} \rangle$ developed above should be a more adequate large angle high-energy approximation. Which one of the two representations, Eq. (13) or Eq. (15) is more convenient can only be setteled through a detailed numerical comparison, which will be carried out soon.

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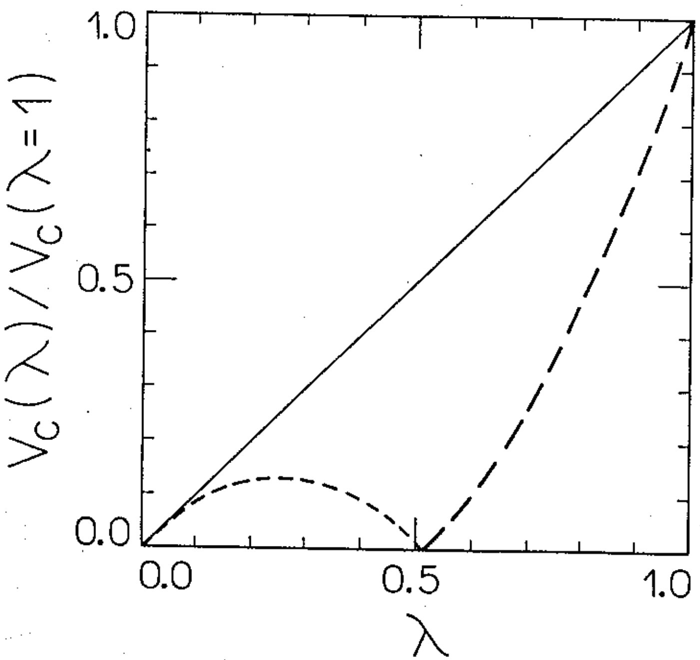
FIGURE CAPTION

FIG. 1: A plot of $V_c(\lambda)/V_c(\lambda=1)$ vs. λ. Short - dashed lim represent negative values. Ful line the scaled Schrödinger V_c. See text for details.



1.0
0.5
0.0
0.0
0.5
1.0
$V_c(\lambda) / V_c(\lambda=1)$
λ