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A QED DESCRIPTION OF RELATIVISTIC COULOMB EXCITATION IN HEAVY ION COLLISIONS

D. Galetti
Instituto de Física Teórica
Rua Pamplona 145, 01405 São Paulo, SP, Brasil

T. Kodama
Centro Brasileiro de Pesquisas Físicas
Rua Dr. Xavier Sigaud, 150
22290 Rio de Janeiro, RJ, Brasil

M.C. Nemes
Instituto de Física, Universidade de São Paulo

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D. Galetti[†]
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22290 Rio de Janeiro, RJ, Brasil

M.C. Nemes*
Universidade de São Paulo
Instituto de Física
C.P. 20516, 01498 São Paulo, SP, Brasil

ABSTRACT

A quantum electrodynamical treatment of Coulomb excitation in relativistic heavy ion collisions is presented. It is shown that a subtle interplay between quantum and relativistic kinematical effects induced by the nuclear recoil due to the excitation generates a qualitatively different prediction (in certain kinematical conditions) from the corresponding prediction of conventional theories. The present formalism is applied to the clean fission problem and the results seem to solve the puzzle associated to this process for some time.

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I. INTRODUCTION

The current image of the Coulomb excitation process is based on a semiclassical picture of a distant collision between two heavy nuclei. This is usually justified by arguments based on the smallness of de Broglie wavelength of nuclei, and the long distance nature of the Coulomb interaction. In general the excitation energies involved in such processes are small as compared to the incident energy. Therefore usual descriptions of the Coulomb excitation mechanism introduce the unperturbed Rutherford trajectory (and sometimes perturbative corrections to it). In particular this type of approximation completely neglects recoil effects due to the excitation and in this case the Coulomb excitation cross section has a factorized form[1],

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right)_{Ruth} \cdot P_{if} \tag{I.1}$$

where P_{if} is the transition probability between two nuclear states (i and f). There are several quantum treatments of the Coulomb excitation mechanism and to our knowledge all of them neglect recoil effects due to the excitation[2].

Although these ideas are very intuitive, appealing and give a correct description of Coulomb excitation at low incident energies ($E \sim$ several hundred MeV) their straightforward extension to the relativistic domain is dangerous due to the interplay between quantum and relativistic kinematical effects: it has recently been shown[3] that such effects can quantitatively alter the total Coulomb excitation cross section and qualitatively alter the angular distribution of Coulomb induced fission fragments as compared to the semiclassical result. Experiment seems to indicate that a consistent description should incorporate these combined effects[4].

From this point of view, the natural formulation of the Coulomb excitation problem should be found in Quantum Electrodynamics (QED).

The purpose of this paper is the derivation of relativistic Coulomb excitation cross section for heavy ions strictly within the framework of Quantum Electrodynamics (QED).

In section II we derive the first-order contribution to the Coulomb excitation cross section and discuss the details of the interplay between quantum and relativistic kinematical effects, which give rise to the differences pointed out above. In section III we investigate the inclusion of some higher order corrections to the cross section. Section IV contains an application of our formalism to the clean fission problem[4] and section V the conclusions.

II. QED CALCULATION OF THE COULOMB EXCITATION CROSS SECTION IN FIRST ORDER

II.1. GENERAL FORMULATION

We assume that the nucleus is described by the four component nucleon field operator $\psi(x)$. The electromagnetic interaction of nuclei is then described by the Hamiltonian density

$$H_I = j^\mu(x)\, A_\mu(x) \tag{II.1}$$

where

$$j^\mu(x) = ie\,\bar{\Psi}(x)\gamma^\mu\left(\frac{1+\tau_3}{2}\right)\Psi(x) \tag{II.2}$$

γ^μ are the Dirac matrices, τ the isospin Pauli matrices and $A_\mu(x)$ is the usual electromagnetic field operator.

Now let us consider the process $A+B \rightarrow A'+B'$ via Coulomb interaction. The lowest order contribution to this process comes from the Feynman diagram shown in Fig. 1 and the corresponding S-matrix element is given by[5]

$$S_{A+B\rightarrow A'+B'} = i\int d^4x \int d^4y \langle A';p'_A | j^\mu(x) | A;p_A\rangle D_F(x-y) \langle B';p'_B | j_\mu(y) | B;p_B\rangle \tag{II.3}$$

where $|A;P_A\rangle$ stands for the nuclear state vector of nucleus

A with total momentum P_A. $D_F(x-y)$ stands for the Feynman propagator for the electromagnetic field.

In general it is a rather difficult and delicate problem to define the nuclear matrix element $\langle A';P_A'|j^\mu(x)|A;P_A\rangle$ covariantly, when momenta are relativistic. This is due to the extended nature of the nucleus and one cannot simply separate the center of mass motion from the internal response of the system. Fortunately for the Coulomb scattering process we may assume that the momentum transfer $\Delta P \equiv P'-P$ is always non-relativistic. In such a case the Lorentz invariance of the theory can be exploited to express the current matrix elements in eq. (II.3) in terms of well established non-relativistic nuclear physics terminology in the following way: Let $\Lambda(P_A)$ be the Lorentz transformation matrix from the rest frame of A to the system moving with momentum P_A. Then

$$\langle A';P_A'|j^\mu(x)|A;P_A\rangle = \Lambda^\mu_\nu(P_A)\langle A';\bar{P}_A'|j^\nu[\Lambda^{-1}(P_A)x]|A;\bar{P}_A\rangle \quad \text{(II.4)}$$

where, for example,

$$\bar{P}_A = \Lambda^{-1}(P_A)P_A \quad \text{(II.5)}$$

From now on the notation $\bar{X}_C$ stands for the value of X in the rest frame of C. We have also

$$\langle B';P_B'|j_\mu(y)|B;P_B\rangle = \Lambda^\sigma_\mu(P_B)\langle B';\bar{P}_B'|j_\sigma[\Lambda^{-1}(P_B)y]|B;\bar{P}_B\rangle \quad \text{(II.6)}$$

With the help of the Fourier decomposition of the propagator $D_F(x-y)$

$$D_F(x-y) = \frac{1}{(2\pi)^4}\int d^4q \; \frac{1}{q^2-i\epsilon} \; e^{-iq(x-y)} \quad \text{(II.7)}$$

we get

$$S_{A+B\to A'+B'} = \frac{i}{(2\pi)^4}\int d^4x \int d^4y \int d^4q \; \frac{\Lambda^\mu_\nu(P_A)\Lambda^\sigma_\mu(P_B)}{q^2-i\epsilon} \; e^{-iq(x-y)}$$

$$\langle A';P_A'|j^\nu[\Lambda^{-1}(P_A)x]|A;P_A\rangle\langle B';P_B'|j_\sigma[\Lambda^{-1}(P_B)y]|B;P_B\rangle \quad \text{(II.8)}$$

$$= \frac{i}{(2\pi)^4}\int d^4q \; \frac{\Lambda^\mu_\nu(P_A\to P_B)}{q^2-i\epsilon}\int d^4\bar{x}_A \, e^{-i\bar{q}_A\cdot\bar{x}_A}\langle A';P_A'|j^\nu(\bar{x}_A)|A;\bar{P}_A\rangle .$$

$$\int d^4\bar{y} \; e^{i\bar{q}_B\cdot\bar{y}_B}\langle B';\bar{P}_B'|j_\mu(\bar{y}_B)|B;\bar{P}_B\rangle \quad \text{(II.9)}$$

In this expression, nuclear matrix elements are defined in the reference systems where momenta P are non-relativistic.

Now let us separate the nuclear center of mass motion in the matrix element

$$I_A = \int d^4\bar{x}_A \, e^{-i\bar{q}\cdot\bar{x}_A} \langle A'; \bar{P}_c' | j^\mu(\bar{x}_c) | A; \bar{P}_c \rangle \tag{II.10}$$

In terms of coordinate representation, we have

$$I_A = e^2 \int d^4\bar{x}_A \prod_{i=1}^{N_c} \int d^3 r_i \, e^{-i\bar{q}\cdot\bar{x}_A} \langle A'; \bar{P}_c' | r_1 \dots r_{N_A} \rangle$$

$$\sum_{i=1}^{Z_A} \gamma^\mu_{(i)} \, \delta(r - r_i) \langle r_1 \dots r_{N_c} | A; \bar{P}_A \rangle \tag{II.11}$$

where we have used the fact that the current density is a local operator so that it can be expressed as

$$j^\mu(\bar{x}) = e \sum_{i=1}^{Z_A} \gamma^\mu_{(i)} \, \delta(\bar{r} - \hat{r}_i) \tag{II.12}$$

in first quantized form. Z_A is the proton number of the nucleus, $\hat{r}_i$ is the position operator for i-th nucleon and $\gamma^\mu_{(i)}$ is the usual Dirac's γ matrices for i-th nucleon spinor wavefunction.

Since $\bar{P}_A$ is non-relativistic, the nuclear wavefunction can be factorized as

$$\langle r_1 \dots r_{N_c} | A; \bar{P}_A \rangle = \frac{1}{\sqrt{V}} e^{-i(\bar{P}_A^0 t - P_A \cdot R)} \langle \xi_1 \dots \xi_{N_c} | A \rangle$$

where

$$\xi_i = r_i - R \tag{II.13}$$

is the intrinsic coordinate of i-th nucleon, ($\sum_{i=1}^{N_A} \xi_i = 0$), $|A\rangle$ is the intrinsic nuclear state, and V is the normalization volume. R is the nuclear center of mass coordinate

$$R = \frac{1}{N_c} \sum_{i=1}^{N_A} r_i \tag{II.14}$$

so that

$$\sum_i \xi_i = 0$$

Now using the identity

$$\prod_{i=1}^{N_c} \int d^3 r_i = \int d^3R \prod_{i=1}^{N_c} \int d^3 r_i \, \delta\Big(R - \frac{1}{N_c}\sum_{i=1}^{N_c} r_i\Big)$$

$$= \int d^3R \prod_{i=1}^{N_c} \int d^3\xi_i \, \delta\Big(\sum_{i=1}^{N_c} \xi_i\Big) \tag{II.15}$$

and the completeness for the nuclear intrinsic state

$$\prod_{i=1}^{N_c} \int d^3\xi_i \, \delta\Big(\sum_{i=1}^{N_c} \xi_i\Big) |\xi_1 \dots \xi_{N_c}\rangle\langle \xi_1 \dots \xi_{N_c}| = \mathbb{1} , \tag{II.16}$$

we find that Eq. (II.11) takes the form

$$I_A = \frac{e Z_A}{V} (2\pi)^4 \delta^4(P_A' - P_A - q) F_A^\mu(q_A) \tag{II.17}$$

where

$$F_A^{\mu}(\bar{q}_A) = \frac{1}{Z_A e}\int d\xi \langle E_A^* | e^{i\bar{q}\cdot\xi} j^{\mu}(\xi) | A_{gs}\rangle \tag{II.18}$$

and $j^{\mu}(\xi)$ has the same form as before eq. (II.12),

$$j^{\mu}(\xi) = e\sum_{i=1}^{Z_A} \gamma^{\mu}_{(i)}\,\delta(\xi - \hat{\xi}_i) \tag{II.19}$$

but here it represents the intrinsic current operator. In eq. (II.18), we have identified $|A\rangle = |A_{gs}\rangle$ and $|A'\rangle = |E_A^*\rangle$ to specify the initial nuclear ground state and final excited state, respectively.

The S-matrix, eq. (II-9) can now be written as

$$S_{A+B\to A'+B'} = \frac{(2\pi)^4}{V^2}\,\delta^4(P_A'+P_B'-P_A-P_B)\,\frac{e^2 Z_A Z_B}{(P_A'-P_A)^2+i\epsilon}\,F_{B\mu}(\bar{q}_B)$$

$$\Lambda^{\mu}_{\nu}(P_A \to P_B)\,F_A^{\nu}(\bar{q}_A) \tag{II.20}$$

The differential cross section is then given by

$$\left.\frac{d\sigma}{d\Omega}\right)_{CM} = 4(\alpha Z_A Z_B)^2 \int dE_A^* \int dE_B^*\; n(E_A^*)\, n(E_B^*)$$

$$\frac{M_A' M_B' M_A M_B}{(\sqrt{s})^2}\,\frac{1}{q^4}\,\frac{\lambda^{1/2}(\sqrt{s}, M_A', M_B')}{\lambda^{1/2}(\sqrt{s}, M_A, M_B)}\,|\mathbb{F}_B \wedge \mathbb{F}_A|^2 \tag{II.21}$$

where $n(E^*)$ denotes the level density of the nuclei, α is the fine structure constant, M' is the final state mass. The four-momentum transfer q is written as

$$q = (P_A - P_A') \tag{II.22}$$

Also

$$\lambda^{\frac{1}{2}}(x,y,z) = \sqrt{(x-y-z)(x+y+z)(x-y+z)(x+y-z)} \tag{II.23}$$

and $\sqrt{s}$ is the total center of mass energy of the system.

Since we consider small momentum transfers we can safely neglect the excitation energy dependence in the integrand of eq. (II.21) except for q and $\mathbb{F}$'s, where this dependence is crucial. Thus

$$\left.\frac{d\sigma}{d\Omega}\right)_{CM} = 4\alpha^2\left(\frac{M_A M_B}{\sqrt{s}}\right)^2 \int dE_A^* \int dE_B^*\, n(E_A^*)\, n(E_B^*)\,\frac{|\mathbb{F}_A \wedge \mathbb{F}_B|^2}{q^4} \tag{II.24}$$

This is the general expression for the first order relativistic Coulomb excitation cross section within the QED formalism. Note that eq. (II.24) contains only conventional nuclear matrix elements.

II.2. PURE PROJECTILE EXCITATION - QUANTAL AND KINEMATICAL EFFECTS

If the target nucleus remains in its ground state one can safely neglect the vector components of $F_{B\mu}(q_B)$ since they are of order $\frac{v}{c}$, v being the nucleon velocity inside the nucleus. With this approximation and using an explicit expression for $\Lambda(P_A \to P_B)$

$$\Lambda(P_A \to P_B) = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \qquad \text{(II.25)}$$

where β and γ are the usual Lorentz factors corresponding to the laboratory incident energy, we get

$$|F_A \Lambda F_B|^2 = |\gamma F_A^0(\bar{q}_A) - \beta\gamma F_A^z(\bar{q}_A)|^2 \, |F_B^0(q_B)|^2 \qquad \text{(II.26)}$$

It is found to be convenient to split $F_A^Z(\bar{q}_A)$ into the longitudinal and transverse contributions as

$$F_A^z(\bar{q}_A) = F_\parallel^z(\bar{q}_A) + F_\perp^z(\bar{q}_A) \qquad \text{(II.27)}$$

where

$$F_\parallel^z(\bar{q}_A) = \int d\xi \, e^{i\bar{q}_A\cdot\xi} \, \hat{e}_z \cdot \hat{P}_\parallel(\bar{q}_A) \langle E_A^* | j(\xi) | A_{GS} \rangle \qquad \text{(II.28)}$$

$$F_\perp^z(\bar{q}_A) = \int d\xi \, e^{i\bar{q}_A\cdot\xi} \, \hat{e}_z \cdot (1 - \hat{P}_\parallel(\bar{q}_A)) \langle E_A^* | j(\xi) | A_{GS} \rangle \qquad \text{(II.29)}$$

with $\hat{P}_\parallel(\bar{q}_A)$ being a projection operator in the direction of $\bar{q}_A$, and $\hat{e}_Z$ the unit vector in Z direction. The first component of $F_A^Z(\bar{q}_A)$, $F_\parallel^Z(\bar{q}_A)$ can be cast into the form:

$$\begin{aligned} F_\parallel^z(\bar{q}_A) &= \frac{\hat{e}_z\cdot\bar{q}_A}{|\bar{q}_A|^2} \int d\xi \, e^{i\bar{q}_A\cdot\xi} \langle E_A^* | \bar{q}_A \cdot j | A_{GS} \rangle \\ &= \frac{\hat{e}_z\cdot\bar{q}_A}{|\bar{q}_A|^2} \int d\xi \, \frac{\nabla_\xi e^{i\bar{q}_A\cdot\xi}}{i} \cdot \langle E_A^* | j | A_{GS} \rangle \\ &= \frac{\hat{e}_z\cdot\bar{q}_A}{|\bar{q}_A|^2} \int d\xi \, (E_f - E_i) e^{i\bar{q}_A\cdot\xi} \langle E_A^* | \rho(\xi) | A_{GS} \rangle \end{aligned} \qquad \text{(II.30)}$$

where we have used the continuity equation after integrating by parts in last step of eq. (II.30). Using this result, the cross section for pure projectile excitation reads,

$$\left.\frac{d\sigma}{d\Omega}\right)_{CM} = 4(\alpha Z_A Z_B)^2 \left(\frac{M_A M_B}{\sqrt{s}}\right)^2 \int dE_A^* \, n(E_A^*) \frac{|\mathcal{M}|^2}{q^4} |F_B^0(\bar{q}_B)|^2 \qquad \text{(II.31)}$$

where

$$\begin{aligned} \mathcal{M} = &\left(\gamma - \frac{\beta\gamma(\hat{e}_z\cdot\bar{q}_A)E^*}{|\bar{q}_A|^2}\right) \int d\xi \langle E_A^* | \rho(\xi) | A_{GS} \rangle e^{i\bar{q}_A\cdot\xi} \\ &- \beta\gamma \, \hat{e}_z \cdot \int d\xi \langle E_A^* | j_\perp(\xi) | A_{GS} \rangle e^{i\bar{q}_A\cdot\xi} \end{aligned} \qquad \text{(II.32)}$$

The kinematical factor q^4 in eq. (II.31) is a function of the excitation energy,

$$q^4 = \left[\delta E_A^2 - \delta P_A^2 - 2P_A(P_A+\delta P_A)(1-\cos\theta)\right]^2 \qquad (II.33)$$

where

$$\delta E = -\frac{M_A}{\sqrt{s}} E^* \qquad (II.34)$$

and

$$\delta P_A = \frac{2E_B\,\delta E}{P_A + \sqrt{P_A^2 - E_B\,\delta E}} \approx -\frac{M_A E_B}{\sqrt{s}\;P_A} E^* \qquad (II.35)$$

The expression (II.33) is valid in general, equations (II.34) and (II.35) are valid for small excitation energies only. On the other hand

$$|\bar{q}_{\parallel A}|^2 = \frac{(1-\eta^2)E^{*2} + 2P_A(P_A+\delta P_A)(1-\cos\theta)}{1 - E^*/M_A} \qquad (II.36)$$

with

$$\eta = \frac{1}{\beta_B\gamma_B}\;\frac{M_A}{\sqrt{s}} \qquad (II.37)$$

where $\beta_B\gamma_B$ denotes de Lorentz factor of target nucleus B

in the CM system.

In the limit of no excitation

$$\delta P_A = \delta E_A = 0$$

and it is simple to check that one recovers the well known Rutherford cross section from eq. (II.31).

It is worthwhile to discuss at this point some essential differences with the semiclassical expressions for the Coulomb excitation mechanism at relativistic energies[(1),(2)]:

a) Even in the cases where excitations are important, one can still recover the factorized semiclassical expression from eq. (II.31) provided

$$1-\cos\theta \gg \left|\frac{\delta E^2 - \delta P^2}{2P_A(P_A+\delta P)}\right| \simeq \frac{1}{2}\eta^2\left(\frac{E^*}{P_A}\right)^2 \qquad (II.38)$$

In nonrelativistic Coulomb excitation processes this condition is easily satisfied, since most of the contribution to the cross section will come from finite deflection angles. However, in the relativistic limit, this will not be the case since forward scattering is dominant. Thus the factorization assumption is not valid. In particular for forward angles which satisfy the condition

$$\theta \ll \eta\,\frac{E^*}{P_A} \qquad (II.39)$$

the semiclassical expression for the cross section completely breaks down.

Apart from the analytical expression for the cross section within QED, a very important point in our quantum treatment, resides in its prediction that the collective nuclear excitation should be quite different from those expected from semiclassical theories in this kinematical region. For forward angles, (Eq. (II.39)), the momentum transfer seen by the nucleus A is almost parallel to the incident beam axis,

$$\bar{q}_A \cdot \hat{e}_z \approx -|q_A| \tag{II.40}$$

since $\bar{q}_\perp \simeq P_a . \theta << |q_A|$ (see Fig. 2.a). In this case the contribution to the cross section will come from the first term on the r.h.s. of Eq. (II.32), which contains the nuclear transition operator

$$\int d\xi \; \rho(\xi) \, e^{i \hat{q}_A \cdot \xi} \tag{II.41}$$

This operator obviously causes the charge polarization to be parallel to $\hat{q}_A$. Since $\hat{q}_A$ is parallel to the beam direction, we conclude that the charge polarization of nuclear final states are populated in the longitudinal direction for forward angles.

On the other hand, in the semiclassical approximation

where recoil effects due to nuclear excitation are neglected, we always have

$$\bar{q}_A \cdot \hat{e}_z \approx 0 \tag{II.42}$$

for forward angles (see Fig. 2.b), namely $\hat{q}_A$ is perpendicular to the incident beam direction. Therefore, the semiclassical treatment leads to the conclusion that the final charge polarization state is transversal, contrary to our present result.

b) In the semiclassical approaches a given scattering angle corresponds to a well defined value for the momentum transfer. In this way the scattering angle limits the excitation energy which will be available. On the other hand our quantum treatment shows that the scattering angle specifies <u>only</u> the transverse component of the momentum transfer and even for zero scattering angle high excitations are kinematically allowed.

III. <u>DISCUSSION OF SOME HIGHER ORDER CORRECTIONS</u>

In heavy ion reactions Coulomb distortion effects are known to be important. Usually these effects are treated by DWBA (Distorted Wave Born Approximation) where the two nuclei are treated non relativistically and recoil effects neglected. In order to treat this problem covariantly one has to sum up all higher order contributions arising from the exchange of internal photons (see Fig. 3). One of the well suited approaches to this problem within QED is the eikonal approximation in quantum field theory developed by M. Levy and J. Sucher[6]. They study the Feynman amplitude $M(s,t)$ describing the scattering of two spin-0 elementary particles, a and b, interacting by the exchange of spin-0 mesons. They show that if $M_n(s,t)$ (the contribution to $M(s,t)$ arising from all n-th order Feynman diagrams in which exactly n mesons are exchanged between a and b) is written in an appropriately symmetrized way, and if the terms in any a or b particle propagator which are quadratic in the internal momenta are dropped, the resulting expression for the amplitude may be carried out in closed form. To adapt their formulation to the nucleus-nucleus Coulomb excitation processes, we have to introduce, instead of the spin-0 particle propagator $\Delta_F(p)$, the nuclear propagator $\hat{G}(p)$ for the intrinsic nuclear state. For a nucleus whose total four momentum is p, the intrinsic nuclear propagator can be expressed as

$$G(P) = \frac{1}{\sqrt{P^2} - H_N + i\epsilon} \qquad \text{(III.1)}$$

in its rest frame. H_N is the nuclear Hamiltonian operator

(including the rest mass). We also define the vertex operators $\mathbb{F}_\mu(q)$ which take the nuclear transitions into account. To simplify our derivation here we consider only one step excitation, where the nuclear excitation takes place at once in one of the vertices. In addition, we assume that

$$\langle E^* | \mathbb{F}_\mu(q) | E^* \rangle \approx \langle A_{GS} | \mathbb{F}_\mu(q) | A_{GS} \rangle \qquad \text{(III.2)}$$

Let $S_{A+B\to A'+B'}$ be the scattering amplitude corresponding to the sum of all diagrams of the form indicated in Fig. 3. Using the same technique as in Section II to express nuclear matrix elements non-relativistically, and after separating the nuclear CM motion, we can write

$$S_{A+B\to A'+B'} = (2\pi)^4 \delta^4(P_A + P_B - P_A' - P_B')\,\mathcal{M} \qquad \text{(III.3)}$$

where

$$\mathcal{M} = \sum_{n=1}^{\infty} M_n \qquad \text{(III.4)}$$

and

$$M_n = \sum_{r=1}^{n} \int \prod_{i=1}^{n} \frac{dk_i}{(2\pi)^4} (-i D_F(k_i)) (2\pi)^4 \delta^4(q - \sum_{i=1}^{n} k_i)$$

$$\Big[\prod_{i\neq r}^{n} (-i F_{\mu i}(\bar{k}_i^A))(-i F_{\mu r}^{(*)}(\bar{k}_r^A)) \prod_{\ell=1}^{n-1} (i G^A(P_A - \sum_{i=1}^{\ell} k_i))$$

$$\sum_{D} \Big[\prod_{j=1}^{n} (-i F^{\mu j'}(\bar{k}_j^{B'})) \prod_{m=1}^{n} (i G^B(P_B + \sum_{j=1}^{m} k_j'))\Big] \qquad \text{(III.5)}$$

where $k_1, \ldots, k_n$ denote the momenta of exchanged photons in the order of emission along the world line of nucleus A and the nuclear excitation is supposed to occur at $i = r$. $D_F(k_i)$ stands for the photon propagator

$$D_F(k_i) = \frac{1}{k_i^2 + i\epsilon} \tag{III.6}$$

as before. The factor $F_{\mu_i}(\bar{k}_i^A)$ is the Fourier transform of the nuclear current, as defined by

$$F_{\mu\ell}(\bar{k}_i^A) = \Lambda^{\sigma}_{\mu\ell}(A)\int d\xi \langle A_{GS}| e^{i\bar{k}_i^A\cdot\xi} j_\sigma(\xi)|A_{GS}\rangle \tag{III.7}$$

$$F^{\mu\ell}(\bar{k}_j^B) = \Lambda^{\mu\ell}_{\sigma}(B)\int d\xi \langle B_{GS}| e^{i\bar{k}_\ell^B\cdot\xi} j^\sigma(\xi)|B_{GS}\rangle \tag{III.8}$$

and $F^{(*)}_{\mu r}(\bar{k}_r^A)$ given by

$$F^{(x)}_{\mu r}(\bar{k}_r^A) = \Lambda^{\sigma}_{\mu r}(A)\int d\xi \langle E^*| e^{i\bar{k}_r^A\cdot\xi} j_\sigma(\xi)|A_{GS}\rangle \tag{III.9}$$

where $\Lambda(A)$ is the Lorentz transformation matrix from the rest from of A to the observational system. The function $G^A(k)$ corresponds to the expectation value of $\hat{G}(k)$ in eq. (III.1) in the appropriate intermediate nuclear state of the indicated nucleus. The summation $\sum_D$ is to be performed over all distinct diagrams in which the momenta $k_1, \ldots, k_n$ may be absorbed

along the world line of b. The primed variables refer to suitable permutations of the corresponding momenta.

It turns out to be convenient to use the energy-momentum conservation δ-function in order to eliminate the r-th momenta k_r at the r-th vertex, where the nuclear excitation takes place. In this case, the products of nuclear propagators will be written as

$$\prod_{\ell=1}^{n-1} i\, G^A(P_A - \sum_{i=1}^{\ell} k_i) = \prod_{\ell=1}^{r-1} i \langle A_{GS}|\hat{G}^A(P_A - \sum_{i=1}^{\ell} k_i)|A_{GS}\rangle \times$$

$$\prod_{\ell=r}^{n-1} i\langle E^*|\hat{G}^A(P_A' + \sum_{i=\ell}^{n-1} k_i)|E^*\rangle \tag{III.10}$$

A typical member in the first group of factors can be written as

$$\langle A_{GS}|\hat{G}^A(P_A - K)|A_{GS}\rangle = \langle A_{GS}|\frac{1}{\sqrt{(P_A-K)^2} - H_N + i\epsilon}|A_{GS}\rangle =$$

$$= \frac{1}{\sqrt{P_A^2 + K^2 - 2P_AK} - M_A + i\epsilon} \simeq \frac{M_A}{P_AK + i\epsilon} \tag{III.11}$$

where we have used the eikonal approximation[5] and

$$P_A^2 = M_A^2 \tag{III.12}$$

A typical member in the second group of factors can be written as

$$\langle E^* | \hat{G}^A(P_A'+K) | E^* \rangle = \langle E^* | \frac{1}{\sqrt{(P_A'+K)^2} - H_N + i\epsilon} | E^* \rangle .$$

$$= \frac{1}{\sqrt{P_A'^2 + K^2 - 2P_A'K} - M_A' + i\epsilon} \simeq \frac{M_A'}{P_A'K + i\epsilon} \qquad \text{(III.13)}$$

where we have used

$$P_A'^2 = M_A'^2 \qquad \text{(III.14)}$$

From this point on it is a straightforward matter to follow the steps in the derivation of the scattering amplitude given in (6) to get

$$\mathcal{M} = \int d^4x \; e^{iqx} \, D(x) \, e^{i\chi(x)} \qquad \text{(III.15)}$$

where

$$D(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{k^2 + i\epsilon} \, F_\mu^*(\bar{k}_A) F^\mu(\bar{k}_B) \qquad \text{(III.16)}$$

and

$$\chi = -i \left(U_1 + U_2 + U_3 + U_4 \right) \qquad \text{(III.17)}$$

with

$$\begin{aligned} U_1 &= U(x, P_A, P_B) \\ U_2 &= U(x, P_A, -P_B') \\ U_3 &= U(x, -P_A', P_B) \\ U_4 &= U(x, -P_A', -P_B') \end{aligned} \qquad \text{(III.18)}$$

and

$$U(x, P, P') = -i \int \frac{d^4k}{(2\pi)^4} \frac{M_A M_B F_\mu(\bar{k}_A) F^\mu(\bar{k}_B) \, e^{ikx}}{(PK + i\epsilon)(-P'k + i\epsilon)(k^2 + i\eta)} \qquad \text{(III.19)}$$

In this form, the studied higher order corrections are contained in the factor $e^{i\chi}$ (eq. (III.15)) and correspond to a properly symmetrized DWBA phase. It is relatively simple to show, again following the steps of ref. (6) (section B), that the eikonal phase can be expressed in the static limit as a convolution integral of the nuclear charge densities with the Coulomb potential. If we neglect the finite size of the charge distribution, we obtain

$$\chi \longrightarrow \alpha^2 \int_0^\infty \left[\frac{1}{|\mathbf{r} + \mathbf{u}'\varsigma|} + \frac{1}{|\mathbf{r} - \mathbf{u}\cdot\varsigma|} \right] d\varsigma$$

$$= \alpha^2 \lim_{\varsigma_{max} \to \infty} \Big[\ln \left| \varsigma + \mathbf{r}\cdot\mathbf{u}' + \sqrt{\varsigma^2 + 2(\mathbf{r}\cdot\mathbf{u}')\varsigma + r^2} \right| + \ln \left| \varsigma - \mathbf{r}\cdot\mathbf{u} + \sqrt{\varsigma^2 - 2(\mathbf{r}\cdot\mathbf{u})\varsigma + r^2} \right| \Big]_{\varsigma=0}^{\varsigma=\varsigma_{max}}$$

which contains a divergent term as $\zeta_{max} \to \infty$. However the eikonal function χ enters in the matrix element as $e^{i\chi(x)}$ so that the constant divergent factor can be dropped since it does not affect the final form of the cross section. Thus the effective eikonal function in the static limit reads

$$\chi^{static} \longrightarrow \frac{e^2 Z_A Z_B}{\hbar v}\left[\ln \sin^2\left(\frac{\theta_1}{2}\right) + \ln \cos^2\left(\frac{\theta_2}{2}\right) - 2\ln 2pr\right]$$

which is exactly the properly symmetrized DWBA phase factor[7], where θ_1 and θ_2 are, respectively, angles between $\mathbb{r}$ and $\mathbb{P}$ and $\mathbb{r}$ and $\mathbb{P}'$. In this limit and for small scattering angles this DWBA-type correction to the cross section is known to be very small.

IV. APPLICATION: THE CLEAN FISSION PROBLEM

The Coulomb induced fission represents a well suited problem to test our results. In fact, recent studies of relativistic (1 GeV/A) Uranium beams shows a beautiful example of such a process[5]. These experiments reveal that approximately 700 mb of the total cross section (averaged over the emulsion components) corresponds to the so called clean fission events in which only two heavy fragments are observed. Recent calculations using the conventional semiclassical theories under-

estimate the experimental cross section by a factor of seven. However, the most intriguing fact with respect to these data is the angular distribution of the fission fragments, which exhitibts a peak at zero degree in the Uranium rest frame. This also cannot be reproduced by the available theories[8].

In order to evaluate the total cross section for the clean fission events we assume that the projectile excitation mechanism is a collective dipole or quadrupole transition. Furthermore we assume the target to remain in its ground state. In this case the excitation energy is of the order of 10 MeV and if the scattering angle is small, the second term on the r.h.s. of eq. (II.32) can be safely neglected. The basic ingredient in the evolution of eq. (II.32) is then the transition density

$$\langle E_A^*|\rho(\bar{q}_A)|A_{GS}\rangle = \langle D|\rho(\bar{q}_A)|A_{GS}\rangle \quad \text{or} \quad \langle Q|\rho(\bar{q}_A)|A_{GS}\rangle \tag{IV.1}$$

where $|D\rangle$ and $|Q\rangle$ correspond to the dipole and quadrupole state respectively. It is well known that the macroscopic Tassie's model[9] gives reasonable estimates of these quantities, and we shall use it in what follows. We get

$$\langle D|\rho(\bar{q}_A)|A_{GS}\rangle \propto \int \frac{d\rho}{d\xi} Y_1(\Omega)\, e^{i\bar{q}_A\cdot\xi}\, d\xi$$

$$\simeq N_D\, e^{-\frac{a^2}{4}q_A^2}\left[j_1(\bar{q}_A R_A) + \frac{1}{2}\left(\frac{a}{R_A}\right)^2 \sin(\bar{q}_A R_A)\right] \tag{IV.2}$$

where N_D is a normalization constant determined by the sum rule for low momentum transfers. Also de derivative $\frac{d\rho_o}{d\xi}$ is approximated by a gaussian centered at the nuclear radius R_A and has a width a. Analogously we have

$$\langle Q|\rho(\bar q_A)|A_{GS}\rangle \cong N_Q\, e^{-\frac{a^2}{4}\bar q_A^2}\left[j_2(\bar q_A R_A) + \frac{3}{2}\left(\frac{a}{R_A}\right)^2(\bar q_A R_A)\, j_1(\bar q_A R_A) + \bar q_A \frac{a}{4}\left(\frac{a}{R_A}\right)^3 \sin(\bar q_A R_A) + \frac{1}{2}\left(\frac{a}{R_A}\right)^2 \cos(\bar q_A R_A)\right] \qquad \text{(IV.3)}$$

Now, a word about the angular integration is in order: The so called fission events are experimentally characterized by the apparent lack of target recoil (or fragmentation). This can be verified by checking the coplanarity of the fissioning fragments with the incoming beam. However, in the most favourable experimental conditions this check cannot rule out transverse momentum transfers $\lesssim 100$ MeV/c. The transverse momentum transfer is related to the scattering by

$$\Delta x = 1 - \cos\theta_{max} = \frac{1}{2}\left(\frac{\bar q_{T\,max}}{p_A}\right)^2 \qquad \text{(IV.4)}$$

Finally, integrating eq. (II.31) up to θ_{max} we get

$$\sigma^D = 8\pi(\alpha Z_A Z_B)^2 \frac{1}{Z_A^2}\frac{3}{R_A^2}\, B(E1)\left(\frac{M_A M_B}{\sqrt{s}}\right)^2 \gamma^2_{A\to B}$$

$$\int_{1-\Delta x}^{1} dx \left[\frac{1}{\eta + \varsigma(1-x)}\right]^2 e^{-\frac{a^2}{2}\bar q_A^2}\left| j_1(\bar q_A R_A) + \frac{1}{2}\left(\frac{a}{R_A}\right)^2 \sin(\bar q_A R_A)\right|^2 \qquad \text{(IV.5)}$$

and

$$\sigma^Q = 8\pi(\alpha Z_A Z_B)^2 \frac{1}{Z_A^2}\frac{45}{4R_A^4} B(E2)\left(\frac{M_A M_B}{\sqrt{s}}\right)^2 \gamma^2_{A\to B}\, .$$

$$\int_{1-\Delta x}^{1} dx \left[\frac{1}{\eta + \varsigma(1-x)}\right]^2 e^{-\frac{a^2}{4}\bar q_A^2}\left| j_2(\bar q_A R_A) + \frac{3}{2}\bar q_A R_A\left(\frac{a}{R_A}\right)^2 j_1(\bar q_A R_A) + \frac{1}{4}\bar q_A a\left(\frac{a}{R_A}\right)^3 \sin(\bar q_A R_A) + \frac{1}{2}\left(\frac{a}{R_A}\right)^2 \cos(\bar q_A R_A)\right|^2 \qquad \text{(IV.6)}$$

with

$$B(E1) = 66.4\ \text{fm}^2$$

$$B(E2) = 2.54\times 10^4\ \text{fm}^4$$

$$\gamma_{A\to B} = \frac{M_A + T_L}{M_A}$$

and T_L is the laboratory kinetic energy of the projectile.

The total cross section for clean fission events in emulsion is given by an average of eq. (IV.5) and (IV.6) over the various target nuclei multiplied by a fission branching ratio which is assumed to be 0.25 in this excitation energy range. In Fig. 4 we plot the calculated total cross section of the events in the emulsion as a function of $\bar q_{T_{max}}$. For values of $\bar{q}_{T_{max}}$ up to 60 MeV/c, the total cross section is increased by a factor of 3 at least as compared to the corresponding values in ref. (8). Note the saturation of the calculated contribution at $\bar{q}_{T_{max}}$ ~ 60 MeV/c. After this value the main contribution comes from the second term. We have not estimated its contribution to the total cross section however. This is one of the reasons why our estimate is certainly conservative. Besides, even for small $\bar{q}_{T_{max}}$ (forward angle scattering) high nuclear excitation other than these Dipole and Quadrupole states may well contribute to this fission mechanism, e.g., the photon absorption by a correlated pair of nucleons and so on. Therefore we believe that the Coulomb excitation process is responsible for the largest part of the total cross section of clean fission events.

As for the fission fragments angular distribution, we have pointed out in section II that the forward angle scattering (corresponding to finite nuclear excitation) always polarizes the final state in longitudinal direction. This explains naturally the forward peaked angular distribution of fission fragments. When the scattering angle becomes relatively large the main contribution to the angular distribution comes from the second term of eq. (II.32) and in this case also the polarization will be in longitudinal direction since the transverse component of the current induces nuclear polarization perpendicular to $\bar{q}_A$, henceforth parallel to the beam direction.

V. CONCLUSIONS

We have developed a quantum electrodynamical approach to relativistic Coulomb excitation in heavy ion collisions. Due to the small momentum transfers involved in this process it is possible to derive covariant expressions for cross sections in closed form which contain usual non relativistic nuclear physics matrix elements. Our approach represents not only a formal derivation of the traditional available results, but reveals new physical aspects of such processes as a subtle consequence of the interplay between quantum and relativistic kinematical effects. In fact one of the essential differences between conventional treatments of Coulomb excitation and the present one resides in the predictions for the polarization of final states. This can be experimentally checked by the study of the angular distribution of Coulomb induced fission fragments. At this moment there are only few experimental results available and they seem to confirm our predictions.

Since the longitudinal polarization will be enhanced for high excitation energy states and forward angles the ideal experiment to test our results would be a direct measurement of charge polarization states in such conditions.

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FIGURE CAPTIONS

Fig. 1 - Feynman diagram for the first order relativistic Coulomb escitation.

Fig. 2 - Schematic illustration of the momentum transfer direction. a) The case $\delta p \geq P_T$. b) The case $\delta p \ll P_T$.

Fig. 3 - Higher order elastic correction, where the nuclear excitation takes place at only one of the vertices.

Fig. 4 - Total cross section for clean fission events in nuclear emulsion plates due to Coulomb excitation. $P_{T_{max}}$ is the maximum value for the transverse momentum transfer.

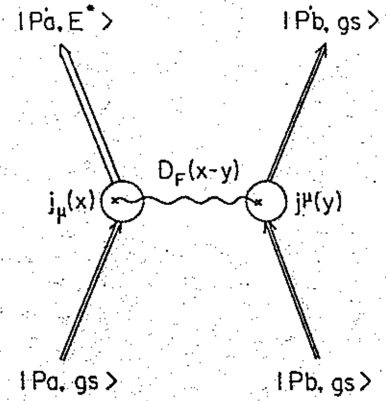

Fig. 1

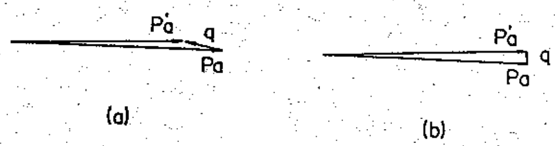

Fig. 2

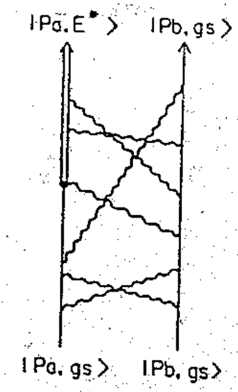

Fig. 3

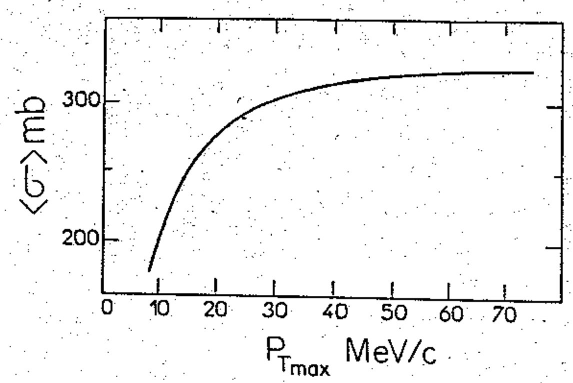

Fig. 4