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E. Abdalla, M. Gomes
Instituto de Física, Universidade de São Paulo

M.C.B. Abdalla
Instituto de Física Teórica
Rua Pamplona, 145 - 01405 São Paulo, SP

A. Lima-Santos
Instituto de Geociências e Ciências Exatas - UNESP
Depto. de Física - Rua 10, nº 2527
C.P. 178 - 13500 Rio Claro, SP

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ANALYTIC STOCHASTIC REGULARIZATION

AND GAUGE THEORIES

E. Abdalla, M. Gomes[1]
Instituto de Física da USP - Departamento de Física Matemática
C. P. 20516 - São Paulo, S.P.

M. C. B. Abdalla[1][2][3]
Instituto de Física Teórica - rua Pamplona 145
C.E.P. 01405 - Sao Paulo, S.P.

A. Lima-Santos[4]
Instituto de Geociências e Ciências Exatas - UNESP
Departamento de Física - rua 10, n. 2527
C.P. 178 - Rio Claro, S.P.

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Abstract

We prove that analytic stochastic regularization breaks gauge invariance. This is done by an explicit one loop calculation of the two, three and four point vertex functions of the gluon field in scalar chromodynamics, which turns out not to be gauge invariant. We analyse the counterterm structure, Langevin equations and the construction of composite operators in the general framework of stochastic quantization.

I. Introduction

Due to many technical aspects, non abelian gauge theories are very difficult to deal with. Non perturbatively, there are the Gribov ambiguities which prevent a clear gauge especification[1]. On the other hand, computer simulations using Monte Carlo methods have unvealed a lot about the structure of the models on a lattice[2]. In spite of this sucess, crucial problems still persist whenever fermions are included. Even at the perturbative level the situation is not much better since, in many instances, it is not easy to find a gauge preserving regularization scheme[3]. Case examples are the supersymmetric gauge theories where most of the popular scheme fail.

With such pletora of troubles we found very fortunate Parisi and Wu proposal of stochastic quantization as a mean to circunvemt some of the above problems[4]. To begin with gauge especification is not necessary or, better saying, it is automatically incorporated thus evading the Gribov ambiguities. Moreover, concerning gauge theories on a lattice, the introduction of a fifth variable (the Langevin time) permits a unique updating of the whole lattice data in each step, saving a huge amount of computer time.

After Parisi and Wu, some authors have proposed new regularization approaches based on the Langevin equation with a non white noise.[5,6,7] Since apparently such procedures do not affect the physical space time coordinates, it was expected that the new regularization schemes would preserve most of the symmetries. Indeed, a number of results pointed in that direction, indicating that gauge invariance holds in the regularized theory. For example, in the abelian case it was shown that, as it happens in dimensional regularization, vertex functions with at least one gauge field carrying zero momenta vanish[8]. This implies that the highest divergence in the corresponding diagrams always cancel. As a consequence there is no induced mass counterterm. This result has been substantiated by an explicit calculation of the one loop polarization tensor in scalar QED, which turned out to be transversal. A similar reasoning can be applied to the two dimensional Yang Mills model in which the regularized polarization tensor is also transversal[6].

The absence of mass counterterms does not preclude the induction of gauge breaking terms, containing derivatives of the gauge field. In fact, it

is one of the purposes of this work to present a detailed calculation of the polarization tensor of scalar QED, using the so called stochastic analytic regularization. Besides the usual transversal terms we found a divergent part of the form $A_\mu \partial^2 A^\mu$. Although innocuous in this abelian situation, such term is potentially dangereous in the non abelian case. We have confirmed this suspicion, calculating the divergence of graphs with three and four non abelian external fields. Our calculation also shows how one could overcome this problem and gives some insight on the higher order corrections.

A possible failure of current conservation in the framework of stochastic regularization has already been noticed [5]. Nevertheless, it was not clear its consequences for the gauge symmetry since this symmetry is apparently preserved in the Langevin equation. In this paper we discuss the appearence of infinite terms and the possible implications for gauge theories. In section II we review the general rules of analytic stochastic regularization and analyse the polarization tensor for two dimensional QED. This is done to expose the mechanism by which it is transversal, contrarily to what happens with ordinary analytic regularization of field theory. In section III we study gauge invariance of four dimensional non abelian scalar QCD, in the external field aproximation. The standard renormalization methods are dissected on the light of stochastic quantization, in section IV. In particular, there we discuss the construction of composite operators and the formal derivation of renormalization group equations. Finally, section V presents our conclusions and an overview of our work. For the reader's convenience, we have included two appendices, one to compare our calculations with those of dimensionally regularised field theory and the other containing some technical details ommited in the text.

II. Stochastic Quantization and Regularization

a. Feynman rules.

The basic element in stochastic quantization is the Langevin equation

$$\frac{\partial\varphi}{\partial t}(x,t) = -\frac{\delta S}{\delta\varphi(x,t)} + \eta(x,t) \qquad (II.1)$$

where t is a fifth time variable and x represents a four dimensional space time coordinate. S is the classical action and η a random field with gaussian probability, defining a Marcovian process. The two point correlation function of the η field is given by

$$< \eta_i(x,t)\eta_j(y,t') >= 2\delta_{ij}\delta(x-y)\delta(t-t') \qquad (II.2)$$

and higher point Green functions are obtained with the help of Wick's decomposition. Using (II.1) and (II.2) averages can be computed through

$$< F[\varphi(t)] >_\eta = \int D\eta F[\varphi_\eta(t)] Exp[-\int_0^\infty dt \int dx \eta_j \eta_j] \qquad (II.3)$$

The above Marcovian process can be related to the field theory specified by the action S as follows. The Green functions of the quantum field are given by the stationary limit of the equal (fifth) time averages of the random field φ, namely

$$< T\phi_{i_1}(x_1)....\phi_{i_N}(x_N) >= \lim_{t\to\infty} < \varphi_{i_1}(x_1,t)....\varphi_{i_N}(x_N,t) >_\eta \qquad (II.4)$$

For perturbative purpose, it is convenient to split the action in two terms, a gaussian, quadratic in the fields, and an interaction term

$$S[\varphi] = \int dx [\frac{1}{2}\varphi_i D_{ij}\varphi_j + V(\varphi)] \qquad (II.5)$$

so that the Langevin equations can be rewritten as

$$\frac{\partial\varphi_i}{\partial t} + D_{ij}\varphi_j = -\frac{\partial V}{\partial\varphi_i} + \eta_i \qquad (II.6)$$

where

$$D_{ij} = \delta_{ij}(-\partial^2 + m^2), \text{ for a scalar field} \qquad (II.7a)$$

and

$$D_{\mu\nu} = -\partial^2\delta_{\mu\nu} + \partial_\mu\partial_\nu \text{ for a gauge field.} \qquad (II.7b)$$

As noted elsewhere, due to the time derivative on the left hand side of (II.6), the propagator

$$G_{ij} = [\partial_t + D]^{-1}_{ij} \qquad (II.8)$$

exists even for a gauge theory. Indeed, we have

$$\tilde{G}_{ij}(k,t) = \delta_{ij}\,\theta(t)\; exp[-t(k^2+m^2)] \qquad (II.9)$$

for a scalar propagator and

$$\tilde{G}_{\mu\nu}(k,t) = \{(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\, e^{-tk^2} + \frac{k_\mu k_\nu}{k^2}\}\theta(t) \qquad (II.10)$$

for a propagator of a gauge field (the tildes denote Fourier transformation).

In (II.10), the presence of a longitudinal part is to be noted. It has been remarked that such term does not contribute to the Green functions of gauge invariant objects.[4] However, as we shall see later, such term becomes particularly dangereous if a non invariant regularization scheme is employed.

Equation (II.6) can be solved iteratively, giving

$$\varphi_i(x,t) = G_{ij} * (-\frac{\partial V}{\partial\varphi_j}[G * (-\frac{\partial V}{\partial\varphi} + \eta)] + \eta_j) \qquad (II.11)$$

where the asterisk is to remember that the products must be taken in the convolution sense. Using (II.11) and (II.2) the N point Green functions of the φ field can be computed. In a given order of perturbation the following results obtain[5]:

1. Draw all topologically distinct diagrams.

2. Use a cross, +, to represent the contraction of two η's. Thus a line connecting a pair of vertices can be either a crossed or an uncrossed line. The crossed lines are distributed in the graphs so that

2.1 Every loop has at least one crossed line.

2.2 Two external vertices can not be connected by a continuous path of lines without crosses.

2.3 Any crossed line can be connected with an external line by a path without crosses.

The number of crossed lines in a graph is

$$\text{number of } + = \text{number of loops} + \text{number of external lines} - 1 \quad (II.12)$$

Observe also that the vertices at the ends of an uncrosssed line are naturally ordered according the values of their fifth times. On the other hand, if all lines linking a pair of vertices are crossed lines, then the amplitude for the graph decomposes into a sum corresponding to the two possible (fifth) time orderings of the vertices.

3. To the lines are associated the propagators

Uncrossed line $\rightarrow$ $G(x,t)$ (as given by (II.9-10))

$$\text{Crossed line} \rightarrow D(x-x';t,t') = \int_0^t d\tau \int dy\, G(x-y;t-\tau)G(x'-y;t'-\tau) \quad (II.13)$$

A look at the amplitudes constructed with the above rules shows that they in general diverge. It is true that graphs with more internal crossed lines have a better ultraviolet behaviour but it is always possible to find graphs as divergent as those in the usual formulation of field theory. Following refs. (5,6) we introduce a non white noise

$$< \eta(x,t)\,\eta(x',t') >= \delta(x-x')\,f_\epsilon(t-t') \quad (II.14)$$

where

$$\lim_{\epsilon\to 0} f_\epsilon(t) = 2\,\delta(t) \quad (II.15)$$

To be concrete, we choose a particular form for f_ϵ

$$f_\epsilon(t) = \epsilon|t|^{\epsilon-1} \quad (II.16)$$

The meaning of the above process is the introduction of a non Marcovian element in the process described by (II.2).

The Green functions regulated by the use of (II.14-16) are meromorphic functions of ϵ with poles on the real axis. As in the case of analytic

regularization of field theory we could adopt different $\epsilon's$ for each η contraction [10]. Although arbitrary this has some advantages over the use of a unique ϵ.

Using (II.14-16) the crossed propagator must be replaced by

$$D^\epsilon(x-x';t,t') = \int_0^t d\tau \int_0^{t'} d\tau' \int dy G(x-y;t-\tau)\,G(x'-y;t'-\tau')\,f_\epsilon(\tau-\tau') \quad (II.17)$$

which for a scalar field gives

$$\tilde{D}^\epsilon(p) = \int_{-\infty}^{\infty} \frac{dw}{\pi} \frac{e^{-iw(t_1-t_2)}}{(p^2+m^2)^2+w^2} \tilde{f}_\epsilon(w) \quad (II.19)$$

In the special case defined by (II.16), we obtain

$$\tilde{f}_\epsilon(w) = \epsilon\Gamma(\epsilon)\,|w|^{-\epsilon}\sin[\frac{\pi}{2}(1-\epsilon)] = |w|^{-\epsilon}\,\hat{f}_\epsilon \quad (II.20)$$

such that

$$\begin{aligned}\tilde{D}^\epsilon(p) =& \frac{\hat{f}_\epsilon}{(p^2+m^2)^{1+\epsilon}}[e^{-(p^2+m^2)|t_1-t_2|} \\ &- \epsilon\int_{-\infty}^{\infty}\frac{dw}{\pi}\frac{\ln w}{1+w^2}e^{-iw(p^2+m^2)(t_1-t_2)} + O(\epsilon^2)]\end{aligned} \quad (II.21)$$

Note the similarity of the first term on the right hand side of this equation with the usual analytically regularized Feynman propagator.Since, as mentioned before, the divergences of the regularized amplitudes appear as simple poles, the second term will only contribute to their finite parts. For the same reason the amplitudes containing the $O(\epsilon^2)$ term vanish as the regulator is removed.

b. A sample calculation.

To illustrate the use of the regularization method introduced before, we will calculate the lowest order contributions to the polarization tensor

of two dimensional scalar QED. The photon polarization tensor, $\Pi_{\mu\nu}$, is a convenient objectto focus our attention as it must be transversal if gauge symmetry holds. This is also a good test on the advantages of the new regularization method because, as the reader probably knows, the usual analytic regularization method of field theory induces a mass counterterm, breaking gauge invariance.

The model is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \varphi^* D^\mu D_\mu \varphi + m^2 \varphi^* \varphi \qquad (II.22)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + i e A_\mu$ is the covariant derivative. The Langevin equations governing the evolution of the fields A_μ, φ and φ^* are[8,11]

$$\begin{aligned} \dot{A}_\mu &= -\frac{\delta S}{\delta A_\mu} + \eta_\mu = \partial_\rho F^\rho{}_\mu - ie\varphi^* D_\mu \varphi + \eta_\mu \\ \dot{\varphi} &= -\frac{\delta S}{\delta \varphi^*} + \eta = D^2\varphi - m^2\varphi + \eta \\ \dot{\varphi}^* &= -\frac{\delta S}{\delta \varphi} + \eta^* = D^2\varphi^* - m^2\varphi^* + \eta^* \end{aligned} \qquad (II.23)$$

with the random fields η_μ, η and η^* satisfying

$$\begin{aligned} &< \eta_\mu(x,t)\, \eta_\nu(x',t') >= 2\,\delta_{\mu\nu}\, f_\epsilon(t-t')\,\delta(x-x') \\ &< \eta(x,t)\,\eta^*(x',t') >= f_\epsilon(t-t')\,\delta(x-x') \\ &< \eta(x,t)\,\eta(x',t') >= 0 \end{aligned} \qquad (II.24)$$

Solving iteratively these equations we found in lowest oder of perturbation the graphs shown in fig I. Note that there is one graph contributing to fig I.a, four graphs contributing to fig I.b - they correspond to different graphs with the same topology having two crossed lines, one external and the other internal - and two graphs for the fig I.c. To get a better idea of the details of the calculation, we divide it in two parts. In the first part we calculate the amplitudes for the graphs, neglecting the contributions of the second term in (II.21) to the crossed propagators. Also, for simplicity, in computing correlation function, we suppose that the fifth times of the fields

are all equal and very high. We then integrate over the fifth times of the internal vertices and keep only the dominant terms (i.e., only those surviving in the infinite fifth time limit). In the appendix A, for the reader's convenience, we have included more details of the calculation. For the graph I.a we get

$$I.a = 2\hat{f}_\epsilon^2 \int_{\tau_2>\tau_1} d\tau_1\, d\tau_2 e^{-p^2(t-\tau_1)}\, e^{-p^2(t-\tau_2)} \times \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{[k^2+m^2]^{1+\epsilon}[(k+p)^2+m^2]^{1+\epsilon}} e^{-|(k+p)^2+k^2+2m^2||\tau_2-\tau_1|} \qquad (II.25)$$

As explained in the appendix, the factor 2 on the right hand side of this equation comes from the two possible orderings of the internal times ($\tau_1 > \tau_2$ or $\tau_1 < \tau_2$). Integrating in τ_1 and τ_2 we get

$$I.a = \frac{\hat{f}_\epsilon^2}{p^2} \int \frac{d^2k}{(2\pi)^2} \Big[\frac{1}{(k^2+m^2)^{1+\epsilon}} \times \frac{(2k+p)_\mu (2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}[(k+p)^2+k^2+p^2+2m^2]} \Big] \qquad (II.26)$$

This expression is very difficult (even for $\epsilon = 0$) to be evaluated. Although in two dimensions we could still obtain a closed form for it, we find more instructive to employ a different procedure which has the advantage of being generalizable to four dimensions. The basic observation is that $\Pi_{\mu\nu}$ is analytic in m for m big enough (equivalently, for small p). Then $\Pi_{\mu\nu}$ can be expanded in powers of m^{-1} (or, equivalently, in powers of the external momenta) and the transversality property of $\Pi_{\mu\nu}$ will be correct only if it is satisfied in each order of the expansion. In the forthcoming calculation we will analyse the terms of the mentioned expansion, up to the first one to be finite when the regularization is removed. With this aproximation we have

$$I.a = \frac{\hat{f}_\epsilon^2}{p^2} \int \frac{d^2k}{(2\pi)^2} \frac{2k_\mu k_\nu}{(k^2+m^2)^{3+2\epsilon}} = \frac{\delta_{\mu\nu}}{8\pi p^2 m^2} + O(\epsilon) \qquad (II.27)$$

For the graph of fig (I.b) the calculation is also straightforward but a little bit more extense. From the appendix A we get

$$I.b = \frac{\delta_{\mu\nu}}{2\pi(p^2)^2}\Big(\frac{1}{\epsilon} - 1\Big) - \frac{1}{12\pi m^2 (p^2)^2}\Big(\frac{5}{2}\delta_{\mu\nu}p^2 - p_\mu p_\nu\Big) \qquad (II.28)$$

Finally, the graph of fig (I.c) gives

$$I.c = \frac{-2\,f_e^2\delta_{\mu\nu}}{(p^2)^{2+\epsilon}} \int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+m^2)^{1+\epsilon}} = -\frac{\delta_{\mu\nu}}{2\pi\epsilon(p^2)^2} \qquad (II.29)$$

where the extra factor of two comes from the two graphs of fig (I.c).

Adding the contributions (II.27-29) we note that the divergent pieces exactly cancel, leaving the result

$$\frac{1}{12\pi p^2 m^2}(-\delta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}) - \frac{\delta_{\mu\nu}}{2\pi(p^2)^2} \qquad (II.30)$$

which is, evidently, non transversal. This expression should be compared with the one employing the usual analytical regularization of field theory. In that case the regularised integrand is obtained by replacing the free propagator (p^2+m^2) by $(p^2+m^2)^\lambda$. With this substitution we get a polarization tensor differing from (II.30) just by a term which vanishes after a judicious choice of the participating lambdas. Thus, up to this point there is no great advantage in using stochastic analytic regularization instead of the more usual one. However we still have to compute the corrections coming from the neglected terms in (II.21). These terms are very important because, as shall see right now, they will make the final result gauge invariant. Firstly notice that there is no correction coming from the graph of Fig. I.a since it is finite without the regularization. The contribution of the remaining graphs are not difficult to be evaluated (see appendix A) and we get the following additional terms

$$\frac{\delta_{\mu\nu}}{\pi(p^2)^2} \qquad \text{from Fig. I.b}$$

$$\frac{-\delta_{\mu\nu}}{2\pi(p^2)^2} \qquad \text{from Fig. I.c}$$

Adding all these contributions we get a miraculous cancellation of the non transversal parts, leaving the net result

$$\frac{1}{12\pi m^2}(\frac{p_\mu p_\nu}{p^2} - \delta_{\mu\nu}) \qquad (II.31)$$

The cancellation of the non transversal terms is a consequence of a general theorem proven in ref. 7, asserting the non existence of mass corrections to the photon field. However, the mentioned theorem does not

preclude the induction of non gauge invariant terms, containing derivatives of the A_μ field.

III. Breaking of Gauge Invariance in the non Abelian Case.

The calculations of previous section indicate that analytic stochastic regularization can be a very efficient tool for the study of properties of gauge theories, at least the abelian ones. Unfortunately this result, as we will show now, does not extend to the non abelian situation. Our discussion will be based in the non abelian version of (II.22)

$$\mathcal{L} = Tr[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \varphi^* D_\mu D^\mu \varphi + m^2\varphi^*\varphi] \qquad (III.1)$$

where now

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu] \qquad (III.1a)$$

$$D_\mu = \partial_\mu + ieA_\mu, \qquad A_\mu = A^a_\mu \lambda^a \qquad (III.1b)$$

and the λ^a are the generators of the algebra of the gauge group.

The Langevin equations are given by (II.23) with the replacements (III.1a,b). They can be formally iterated producing terms that can be represented by stochastic diagrams, as before. If dimensional regularization is employed one finds that they distribute themselves into classes of gauge invariant amplitudes. In particular, the set consisting of the diagrams of figures I till IV is gauge invariant and in the large t limit generates a counterterm

$$Z_1 F^r_{\mu\nu} F^{r\,\mu\nu} \qquad (III.2)$$

where $F^r_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ieZ_2[A_\mu, A_\nu]$.

However, had we used the analytic stochastic regularization method the induced counterterm would be

$$Z_1 F^r_{\mu\nu} F^{r\,\mu\nu} + Z_3 A_\mu \partial^2 A^\mu \qquad (III.3)$$

which is not gauge invariant. To verify (III.3) we consider explicitly the contribution of each relevant diagram. This calculation is rather simplified by noting that, since we are interested in divergent pieces, the $O(\epsilon)$ terms in (II.21) can be freely disregarded. To be systematic, we divide the calculation in three parts, accordingly the number of external gluon lines.

1. Graphs with two external gluon lines. As in the two dimensional example, there are three diagrams, shown in fig. I. Differently from two

dimensions the diagram I.a is now logarithimically divergent. Actually this diagram is the responsible for the breaking of gauge invariance, as it will become clear shortly. Analogously to (II.27), we have

$$Fig.I.a = \frac{\hat{f}_\epsilon}{p^2}\int [\frac{d^4k}{(2\pi)^4}\Big[\frac{1}{(k^2+m^2)^{(1+\epsilon)}}\times \frac{(2k+p)_\mu(2k+p)_\nu}{[(k+p)^2+m^2)]^{(1+\epsilon)}[(k+p)^2+k^2+p^2+2m^2]}\Big] = \qquad (III.4)$$

$$= \frac{\hat{f}^2_\epsilon}{p^2}\int \frac{d^4k}{(2\pi)^4}\frac{2k_\mu k_\nu}{(k^2+m^2)^{3+2\epsilon}} = \frac{\delta_{\mu\nu}}{64\pi^2 p^2 \epsilon} + \text{ finite terms}$$

We would like to call the reader's attention to the exponent 2ϵ in the denominator of the last line of the above expression. The factor 2 comes in because there are two crossed lines in the graph. This implies that (III.4) is only half of the corresponding result for the dimensionally regulated amplitude. The disastrous consequence of this is that (III.4) will not "match" the contributions from the other graphs.

The computation of the graphs of Fig I.b and I.c is also very simple and gives (up to finite terms).

$$Fig\ I.b = \frac{2}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\frac{(2k+p)_\mu(2k+p)_\nu}{(k^2+m^2)^{1+\epsilon}[p^2+k^2+(p+k)^2+2m^2]} =$$

$$= \frac{1}{(4\pi)^2}[-\frac{2m^2\delta_{\mu\nu}}{(p^2)^2} - \frac{5\delta_{\mu\nu}}{6p^2} + \frac{p_\mu p_\nu}{3(p^2)^2}]\frac{1}{\epsilon} \qquad (III.5)$$

$$Fig\ I.c = -\frac{2\delta_{\mu\nu}}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+m^2)^{1+\epsilon}} = \frac{2m^2\delta_{\mu\nu}}{(4\pi)^2(p^2)^2\epsilon} \qquad (III.6)$$

2.Now come the graphs with three external gluon lines. These diagrams decompose into two sets of topologically similars graphs. In the first set, fig II.a, there are 18 (linearly) divergent diagrams. However the divergent part of these diagrams cancell in groups of two. This is to be expected since each graph generates a counterterm of the type $Tr\ (\partial^\mu A_\mu)A_\nu A^\nu$ which violates charge conjugation. The other set of topologically similar diagrams with 3 external gluon lines contains 18 diagrams. To illustrate the calculation we

write the amplitude for the graph of fig II.b

$$FigII.b = -e^3 p_1^2 \frac{Tr[A^\mu A^\nu A^\rho]}{(p_1^2+p_2^2+p_3^2)} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+m^2)^{1+\epsilon}} \frac{(2k+p_1)_\mu (2k+2p_1+p_2)_\nu (2k-p_3)_\rho}{[p_3^2+k^2+(k-p_3)^2+2m^2][p_2^2+p_3^2+(k+p_1)^2+k^2+2m^2]} \quad (III.7)$$

where the external field propagators have been eliminated. Expanding at $p_i = 0$ we find the result

$$-\frac{e^3 p_1^2 Tr(A_\mu A_\nu A_\rho)}{p_1^2+p_2^2+p_3^2}\left[(p_1^\mu \delta^{\nu\rho} - p_3^\rho \delta^{\mu\nu} + (p_1-p_3)^\nu \delta^{\mu\rho}\right] \frac{1}{(4\pi)^2\epsilon} \quad (III.8)$$

This result can be used to get the amplitudes for the other graph, just changing the momenta and Lorentz indices adequately. Adding all these contributions we get a final result for graphs with three external gluon lines.

$$-\frac{2e^3}{(4\pi)^2\epsilon} Tr[(\partial_\mu A_\nu - \partial_\nu A_\mu) A^\mu A^\nu] \quad (III.9)$$

3. Graphs with four external gluon lines. These diagrams can be grouped into three sets of topologically similar graphs. The first set is constituted by graphs with two internal lines. There are 48 such diagrams. They can further divided into six subsets of 8 diagrams each. In each subset the indices and momenta of the external lines are fixed. A typical graph, fig III.a, let us say, has an analytic expression like (for the rest of this section the propagators of the external fields have been eliminated)

$$\frac{2e^4 p_1^2 Tr[(A_\mu A^\mu)^2]}{(p_1^2+p_2^2+p_3^2+p_4^2)} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+m^2)^{1+\epsilon}((p_2^2+p_3^2+k^2+(k+p_2+p_3)^2+2m^2)} =$$
$$= \frac{2e^4 p_1^2 Tr[(A_\mu A^\mu)^2]}{(2\pi)^2\epsilon(p_1^2+p_2^2+p_3^2+p_4^2)} + \text{ finite terms} \quad (III.10)$$

Note that a quadrilinear vertex gives a factor of 2 or 1 depending if there are one or two crossed gluon lines ending at it. Adding these 48 diagrams we obtain

$$\frac{3e^4}{4\pi^2\epsilon} Tr(A_\mu A^\mu)^2 \quad (III.11)$$

The next group of diagrams consist of triangle graphs, having three internal lines. There are 144 diagrams of this type and, as in the previous case they can be grouped into 12 groups of twelve diagrams so that in each subset the labels of the external lines are fixed. A typical diagram, fig. III.b, has the expression

$$\frac{2e^4(p_1)^2\, Tr[A^{\mu_1} A^{\mu_2}(A^\mu A_\mu)]}{(p_1^2+p_2^2+p_3^2+p_4^2)} \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2+m^2)^{1+\epsilon}} \frac{(2k+p_1)_{\mu_1}(2k+2p_1+p_2)_{\mu_2}}{[k^2+(k+p_1+p_2)^2+p_4^2+p_3^2+2m^2][(k+p_1)^2+k^2+p_2^2+p_3^2+p_4^2+2m^2]}\right]$$
$$= \frac{e^4(p_1)^2 Tr(A_\mu A^\mu)^2}{(p_1^2+p_2^2+p_3^2+p_4^2)} \frac{1}{2(4\pi)^2\epsilon} + \text{ finite terms} \quad (III.12)$$

The amplitudes for the other diagrams can be get similarly. The final result for the sum of the 144 diagrams is

$$\frac{-6e^4 Tr[(A_\mu A^\mu)^2]}{4\pi^2\epsilon} \quad (III.13)$$

Let us finally consider the box diagrams shwon in fig. III.c. There are 96 diagrams which can be separated into six groups of 16 diagrams, fixing the labels pf the external legs. As an example, we write the expression for the graph III.c.

$$\frac{e^4(p_1)^2 Tr[A^{\mu_1}...A^{\mu_4}]}{(p_1^2+p_2^2+p_3^2+p_4^2)} \int \frac{d^4k}{(2\pi)^2} \frac{(2k+p_1)_{\mu_1}}{(k^2+m^2)^{1+\epsilon}[k^2+(k-p_4)^2+p_4^2+2m^2]} \times$$
$$\frac{(2k+2p_1+p_2)_{\mu_2}(2k+2p_1+2p_2+p_3)_{\mu_3}(2k-p_4)_{\mu_4}}{[k^2+(k+p_1+p_2)^2+p_4^2+p_3^2+2m^2][k^2+(k+p_1)^2+p_2^2+p_3^2+p_4^2+2m^2]} =$$
$$= \frac{p_1^2 e^4}{12(4\pi)^2\epsilon} \frac{Tr\left\{3(A_\mu A^\mu)^2 + \frac{1}{2}[A_{\mu_1}, A^{\mu_2}]^2\right\}}{p_1^2+p_2^2+p_3^2+p_4^2} \quad (III.14)$$

The other diagrams can be computed similarly. However, we must not forget that some diagrams have additional combinatoric factors due to possibly

different time orderings for the vertices of the diagrams. With these cautions the remaining diagrams are calculated giving

$$\frac{3e^4}{4\pi^2\epsilon}Tr(A_\mu A^\mu)^2 + \frac{e^4}{8\pi^2\epsilon}[A_\mu, A_\nu]^2 \qquad (III.15)$$

Collecting now the results (III. 4, 5, 6, 9, 11, 13, 15) we obtain the counterterm for the graphs considered in the form (III.3) with

$$\begin{aligned} Z_1 &= \frac{1}{(4\pi)^2 12\epsilon} \\ Z_3 &= -\frac{1}{(4\pi)^2 4\epsilon} \end{aligned} \qquad (III.16)$$

From the above computation it is clear why our result is not gauge invariant. This is so because diagram I.b has two internal crossed lines to be regulated and this produces an overall factor of 1/2. The solution for this problem is very simple and consists in using $\frac{\epsilon}{2}$ for each line of the graph I.b. In higher orders an analogous prescription can lead to the correct result. Nevertheless our computation puts serious doubts in the consistency of the stochastic analytic regulator employed as a nonperturbative regularization scheme.

In the usual (i. e. non stochastic) formulation of gauge theories we have to add a gauge fixing terms (and possibly Fadeev-Popov ghosts) to the Lagrangian density.In a Lorentz gauge, this term is given by $\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$ and in general the Green functions are α dependent. Observables (including the S matrix), on the other hand, must be α independent. If a non gauge regularization scheme is employed then the α independence can be achieved only at the expense of adding gauge dependent counterterms to the original Lagrangian. In the abelian situation, the putative α dependence of the Green functions comes from graphs pictorially represented in fig.IV. The contributions of this type of diagrams vanish on shell only if the gauge field is coupled to a conserved current. *

In the framework of stochastic quantization, the above problem can be particularly dangereous since the longitudinal part of the A_μ field has a piece proportional to the fifth time. Therefore it is mandatory that the gauge field be coupled to a conserved current.

* For a rigorous discussion of gauge invariance in QED, see ref. 12

IV. Renormalization

In this section we shall discuss some peculiarities of the renormalization in the formalism of stochastic quantization. In particular we will show that the counterterms necessary to render the field theory finite can also be interpreted as counterterms in the stochastic theory, i.e., at finite fifth time.In spite of that, there may be differences (finite renormalizations) since the degree of divergence depends not only on the topology, but also on the number of internal crossed lines. For simplicity, but without loss of generality we we choose the ϕ^4 model and scalar electrodynamics to base our considerations.

First of all, we notice that for finite non-zero times there will be a strong convergence factor, provided by the exponential in the stochastic propagators. As an especific example, consider the case of the photon self-energy in scalar electrodynamics

$$\int \frac{d^4k}{(2\pi)^4} \frac{exp\{-\tau[(k+p)^2 + k^2 + 2m^2]\}}{k^2+m^2} \frac{exp\{-(t-\tau)p^2\}}{p^2} \qquad (IV.1)$$

which is finite, insofar τ is non vanishing. If we now integrate over the fictitious time then a divergence shows up. Indeed we have

$$\int_0^t e^{-t\tau} dx = \frac{1 - e^{-tx}}{x} \qquad (IV.2)$$

and the first term gives origin to a (logarithmic) divergent integral. To exhibit this divergence in a more natural way, it is convenient also to Fourier transform with respect to the fifth time. Our notational convention is that the Fourier transform, $f(\omega)$, of f(t) is given by

$$f(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\omega t} f(t) \qquad (IV.3)$$

and we observe that ω has dimension two in mass units

With the above definition, the Fourier transform of the propagators are

$$Uncrossed: \; G(p,\omega) = \frac{1}{p^2+m^2+i\omega} \qquad (IV.4)$$

$$Crossed: \; D(p,\omega) = \frac{1}{p^2+m^2+i\omega}\frac{1}{p^2+m^2-i\omega} \qquad (IV.5)$$

As an ilustration, in the following we will be specific to the four dimensional Φ^4 theory. Using (IV.4,5) and noting also that each loop contributes with 6 to the power counting, we get the degree of superficial of a graph

$$\delta(\gamma) = 6m - 2n - 2x \qquad (IV.6)$$

where

$$m = \#\ of\ loops$$

$$n = \#\ of\ internal\ lines$$

$$X = \#\ of\ internal\ crossed\ lines$$

Using now the relations

$$X = m + N_{unc} - 1 \qquad (IV.6a)$$

$$m = n - V + 1 \qquad (IV.6b)$$

and

$$4V = 2n + N \qquad (IV.6c)$$

with N, N_{unc}, and V denoting the numbers of external lines, uncrossed external lines and vertices, we obtain

$$\delta(\gamma) = 6 - N_c - 3N_{unc} \qquad (IV.7)$$

where N_c is the number of external crossed lines. We see that graphs with $N_{unc} > 2$ are superficially convergent. Let us now examine each case of possible divergence.

1. $N_{unc} = 2, N_c = 0$. These are graphs contributing to the two point function and having no external crossed lines (see fig.V). It is easily verified that such divergences can be absorved in a multiplicative renormaization of the random field.

2. $N_{unc} = 1, N_c = 3$. This is the usual logarithmic divergence of graphs with four external lines (see example in fig.VI). It corresponds to the usual four point vertex renormalization.

3. $N_{unc} = 1, N_c = 1$. This is the usual quadratic divergence of self energy graphs (see fig. VII). It can be eliminated through a mass and wave function renormalizations.

In the stationary limit of field theory, all the external fifth time are made equal. This corresponds to join all external lines of each contributing graph in a new vertex, V_∞, and then integrate over the w variables of the additional loops. One can verify that no divergences arise in this process. Actually a simple calculation, similar to the one above gives

$$\delta(\gamma) = 4 - N_c - 3N_{unc} - 3W \qquad (IV.8)$$

where W denotes the number of lines meeting at V_∞. Since $W > 1$ then, necessarily $\delta(\gamma) < 0$. These conclusions agree with those of reference (13), where a similar power counting was done directly in the space of the fifth time coordinate.

From the above discussion, we conclude that the renormalization problem is essentially the same as in the usual formulation of the theory. The divergence parts can be removed reparametrizing the original model. This can be implemented by adopting a convenient subtraction scheme. Without commiting ourselves to any particular scheme we want to add some remarks on the properties of the resulting theory. The first issue concerns the derivation of the field equations. If for instance we consider the bilinear $N[\Phi(-\partial^2+m^2)\Phi]$ where the symbol N indicates a normal product prescription, then a basic step in the derivation is amputation of the line associated to the operator $(-\partial^2+m^2)\Phi$. This is a trivial task since in momentum space $(-\partial^2+m^2)$ is just the inverse of the propagator and one can allways arrange things so that also happens in the regulated theory. So, it is possible to define bilinear normal products formally obeying the classical

Euler-Lagrange equations. However, this result does not imply the absence of anomalies in Ward identities, since the Green functions of the current operator have an independent definition and, in principle, their divergence are unrelated to the above mentioned normal products. Indeed, Green functions of the object $\Phi^* \overleftrightarrow{\partial}^\mu \Phi$, regulated with the use of equation (II.16), do not satisfy current conservation because the regularized propagator is not the inverse of $(-\partial^2 + m^2)$. This observation is in complete accordance with the results of section II.

Another remark concerns the behaviour of the Green functions under renormalization group transformations at finite fifth time. Since as we have seen before, the elimination of all divergences can be accomplished by the usual wave, mass and charge renormalization (the η field renormalization is, as we saw before, part of the wave function renormalization), then one should expect that the Green functions $G(x_1, x_2, ..., x_N)$ of the stochastic field would obey the renormalization group equation

$$[\mu^2 \frac{\partial}{\partial \mu^2} + \beta \frac{\partial}{\partial g} + N\gamma] G^{(N)} = 0 \qquad (IV.9)$$

with the same β and γ as in the limiting field theory. Equation (IV.9) can be proved as follows. We introduce differential vertex operators[14] (DVO), corresponding to the different field monomials of the lagrangian of the model. Thus, in Φ^4 theory we consider (to be more careful we could dimensionally regularize our amplitudes)

$$\Delta_1 = \frac{1}{2} \int d^4x \Phi^2$$

$$\Delta_2 = \frac{1}{2} \int d^4x \partial^\mu \Phi \partial_\mu \Phi$$

$$\Delta_3 = \frac{1}{24} \int d^4x \Phi^4 \qquad (IV.10)$$

which are defined by the same Feynman rules specifying the Green functions. Thus, Δ_1 means the insertion of a mass vertex in the graphs contributing to the Green functions. It is also convenient to introduce a DVO, Δ_4, which counts the number of crossed lines in a graph. It is given by

$$\Delta_4 = \int d^4x \, \eta^2 \qquad (IV.11)$$

As will be clear shortly, the latter is not independent of those in (IV.9). Using these DVO's the action becomes formally

$$S[\Phi] = (1+b)\Delta_2 + (m^2 - a)\Delta_1 + (c - g)\Delta_3 + d\Delta_4 \qquad (IV.12)$$

where a, b, c and d are counterterms. We adopt intermediate renormalization so that the propagator has a pole at $p^2 = -m^2$ but its residuum and the other renormalization conditions are dependent on the value of a new parameter μ^2, a different normalization spot.

The derivatives $\frac{\partial}{\partial g}$ and $\frac{\partial}{\partial \mu^2}$ have simple expressions in terms of the DVO's (IV.10)

$$\frac{\partial}{\partial g} G^{(N)} = [-\frac{\partial a}{\partial g}\Delta_1 + \frac{\partial b}{\partial g}\Delta_2 + \frac{\partial (c-g)}{\partial g}\Delta_3 + \frac{\partial d}{\partial g}\Delta_4] G^{(N)} \qquad (IV.13.a)$$

$$\frac{\partial}{\partial \mu^2} G^{(N)} = [-\frac{\partial a}{\partial \mu^2}\Delta_1 + \frac{\partial b}{\partial \mu^2}\Delta_2 + \frac{\partial c}{\partial \mu^2}\Delta_3 + \frac{\partial d}{\partial \mu^2}\Delta_4] G^{(N)} \qquad (IV.13.b)$$

There is also a counting identity which is nothing but the integrated equation of motion

$$-N G^{(N)} = [(1+b)\Delta_2 + (m^2 - a)\Delta_1 + (c-g)\Delta_3 + d\Delta_4] G(N) \qquad (IV.14)$$

Beside the above identities, typical of the usual formulation of field theory, we have another equation which is a consequence of Δ_4 being an operation counting the number of crossed lines. Explicitly, from (IV.6a-c),

$$X = \frac{n}{2} - \frac{N}{4} + N_{unc} \qquad (IV.15)$$

Using now (IV.13-15), we can easily establish the renormalization group equation (IV.9). We replace (IV.13-15) into (IV.9) and equate to zero the coefficient of each DVO. Two of these equations (namely those associated to the coefficients of Δ_1 and Δ_2) can be used to fix β and γ. The remaining one is then shown to be identically satisfied by virtue of our on shell mass renormalization.

V. Conclusions.

In this paper we have discussed the use of analytic stochastic regularization in field theory. For abelian gauge theories there is some clear advantege in the use os such method. Indeed, in lower dimensions (< 4) the polarization tensor turns out to be transversal and gauge invariance is not broken. In four dimensions, because of quadratic divergences, there is the induction of a non gauge invariant counterterm of the form $A_\mu \partial^2 A^\mu$. Technically, this happens because the number of internal crossed lines (which are the ones affected by the regularization) varies from graph to graph. This causes unbalances in the usual weights of each individual graph, so that non invariant pieces do not cancell anymore. These observations are of course related to the fact that the gauge field is not coupled to a conserved current, as would be mandatory in a gauge preserving scheme. Nevertheless, this induction of a non gauge invariant counterterm is not really dangerous in the abelian case, since it can be traded by a renormalization of the gauge fixing term.

The situation is more complicated in the non abelian case. It is not possible to absorve the counterterm $A_\mu \partial^2 A^\mu$ into a change of the gauge fixing term. The problem is that the renormalization of the gluon polarization tensor is now dependent on the renormalization of the three and four point proper vertex functions of the gluon field. At the perturbative level we could remedy this disease by attributing diferent epsilons to each crossed propagator.[10] This is not in the original spirit of the method but can produce gauge invariant amplitudes if the epsilons are chosen adequately. Non perturbatively however there is not, up to the present, any prescription which guarantees gauge invariant results.

To define composite operators care must be taken since there are at least two "natural" but inequivalent ways to introduce normal products. The first possibility, which we have adopted in the derivation of the renormalization group equations, is to treat the formal product in the same way as an ordinary Lagrangian vertex. Another possibility, inequivalent to this one, is obtained by taking the product of fields at different points and then letting the points coincide. The difference between these two object can be traced back to the flow of the fifth time. In the first case the fifth time flows into the special vertex(i.e., its time variable is higher than those of any vertex nearby) whereas in the latter case the flow goes continuously

through it.

References

1. V. N. Gribov, Nucl Phys., B139 (1978) 1.
2. K. Wilson, Proceedings of Cargese Summer School, 1979, Plenum Press.
3. E. Abdalla and M. C. B. Abdalla, Phys. Lett. B160 (1985) 101; Nucl. Phys. B266 (1986) 423.
4. V. G. Parisi and Wu Yongshi, Scientia Sinica 24 (1981) 463.
5. J. D. Breit, S. Gupta and A. Zaks, Nucl. Phys. B233 (1984) 61.
6. J. Alfaro Nucl. Phys. B253 (1985) 464.
7. Z. Bern and M.B. Halpern, Phys. Rev. D33 (1986) 1134. D. Zwanziger, Nucl. Phys B192 (1981) 483.
8. Zvi Bern, Nucl. Phys. B251 (1985) 633.
9. W. Grimus and H. Huffel, Z. Phys. C18 (1983) 129. H. Huffel and P. V. Landshoff, Nucl. Phys. B260 (1985) 545.
10. E. Speer, "Generalized Feynman Amplitudes", Ann. Math. Studies 62. Princeton Univ. Press, 1969.
11. E. Gozzi, Phys. Rev. D31 (1985) 1349.
12. J. H. Lowenstein and B. Schroer, Phys. Rev. D6 (1972) 1553.
13. J. Alfaro, R. Jengo and N. Parga, Phys. Rev. Lett. 54 (1985) 369.
14. J. H. Lowenstein, "BPHZ Renormalization", in "Renormalization Theory" G. Velo and A. S. Whightman eds., D. Reidel Pub. Comp., 1975.

Figure Captions

Fig. I One loop graphs contributing to the photon self energy in scalar electrodynamics.

Fig. II One loop contibutions to the gluon three point vertex function Graphs with two internal lines do not induce counterterms because of charge conjugation.

Fig. III Typical graphs contributing to the gluon four point vertex function.

Fig. IV The non vanishing of this graph makes the Green functions gauge dependent. The special vertex V corresponds to the insertion of the operator $(\partial_\mu A^\mu)^2$.

Fig. V Logarithmically divergent contribution to the renormalization of the random field η.

Fig. VI This graph contains part of the usual mass and wave function renormalization.

Fig. VII Lowest order graph contribution to the charge renormalization in the framework of stochastic quantization.

Appendix A

In this appendix we will show details of the calculation of the graph I.b in two dimensions. The calculation of the other graphs in both two and four dimensions is completely analogous and shoud not present any additional difficulties. Using the η field two point function (II. 14, 16) it is easy to see that the crossed propagator is given by (II. 21). As the divergences of the Feynman integrals appear as simple poles, the term of order of ϵ in (III.21) will contribute only in those integrals which become divergent as ϵ tends to zero.

Since our primary interest is in the stationary limit of quantum field theory, then the external fifth times in fig I.b are equal and very high. Moreover, because of the rules of stochastic quantization, the internal times satisfy

$$\tau_1 < \tau_2 < t_2 = t_1 = t \to \infty \tag{A.1}$$

Therefore, we have

$$\begin{aligned}
Fig I.b &= \int d\tau_1 d\tau_2 D_c(p;t_1,\tau_1) \int \frac{d^2k}{(2\pi)^2} D_c(k+p;\tau_1,\tau_2) D(k;\tau_1,\tau_2) D(p;t_2,\tau_2) \\
&= f_\epsilon^2 \int_{-\infty}^{t} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu e^{-(k^2+m^2)(\tau_2-\tau_1)-p^2(t_2-\tau_2)}}{(p^2)^{1+\epsilon}\left[(k+p)^2+m^2\right]^{1+\epsilon}} \\
&\times \int_{-\infty}^{\infty} \frac{dx_1}{\pi} \frac{(1-\epsilon \ln|x_1|)}{1+x_1^2} e^{-ix_1 p^2 (t_1-\tau_1)} \\
&\times \int_{-\infty}^{\infty} \frac{dx_2}{\pi} \frac{(1-\epsilon \ln|x_2|)}{1+x_2^2} e^{-ix_2\left[(k+p)^2+m^2\right](\tau_1-\tau_2)}
\end{aligned} \tag{A.1}$$

Integrating in τ_1 and τ_2 and incorporating also the contributions of the other three diagrams gives

$$Fig.\ I.b = A + B$$

$$A = \frac{2 f_\epsilon^2}{(p^2)^2} \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2+m^2)^{1+\epsilon}\left[(k+p)^2+k^2+p^2+2m^2\right]} \tag{A.2}$$

$$B=\frac{4\hat{f}_\epsilon^2}{(p^2)^2}\int_{-\infty}^{\infty}\frac{dx_1}{\pi}\frac{-\epsilon ln|x_1|}{(1+x_1^2)(1+ix_1)}\times$$
$$\int\frac{d^2k}{(2\pi)^2}\frac{(2k+p)_\mu(2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}(ix_1p^2+(k+p)^2+k^2+2m^2)}+$$
$$+\frac{2\hat{f}_\epsilon^2}{(p^2)^2}\int_{-\infty}^{\infty}\frac{dx_2}{\pi}\frac{-\epsilon ln|x_2|}{(1+x_2^2)}\times$$
$$\int\frac{d^2k}{(2\pi)^2}\frac{(2k+p)_\mu(2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}[p^2+k^2+m^2+ix_2((k+p^2)+m^2)]} \quad (A.3)$$

We now rescale the loop momentum, $k \to mk$, and expand the integrand in powers of p/m. Keeping again only the contributions surviving in the $\epsilon \to 0$, we note that the two terms in (A.3) give identical results. We get

$$A=\frac{\delta_{\mu\nu}}{2\pi(p^2)^2}(\frac{1}{\epsilon}-1)-\frac{5\delta_{\mu\nu}}{24\pi p^2m^2}+\frac{p_\mu p_\nu}{12\pi(p^2)^2m^2} \quad (A.4)$$

For the B term the only divergence comes from $p=0$, in which case the integral over x can be performed

$$\int_{-\infty}^{\infty}\frac{dx}{\pi}\frac{ln|x|}{(1+x^2)(1+ix)}=-\frac{1}{2} \quad (A.5)$$

Using also

$$\int\frac{d^2k}{(2\pi)^2}\frac{4k_\mu k_\nu}{(k^2+m^2)^{2+\epsilon}}=\frac{\delta_{\mu\nu}}{2\pi(p^2)^2\epsilon}+\text{ finite terms} \quad (A.6)$$

we are left with

$$B=\frac{\delta_{\mu\nu}}{\pi(p^2)^2} \quad (A.7)$$

Observe that the contributions of the term of order of ϵ are necessarily finite. In four dimensions there is no such cancellation and the computation is simpler since it is not necessary to look at the finite contributions.

A very important point in the whole calculation is the possibility of more than one contribution arising from the same graph, due to different time orderreing for the internal vertices. In general time flows from outside to inside a graph. However if there is no uncrossed line linking a pair of adjacent vertices then all possible orderings must be taken into account.

Appendix B

Using dimensional regularization, we will reproduce here the results for the induced counterterms associated to graphs with two, three and four external gluon fields in scalar chromodynamics. These results are ($D=4-\frac{\epsilon}{2}$)

1. Graphs with two external (amputated) gluon lines. The graphs are similar to those shown in fig I (but, of course, all propagators are the usual ones of field theory). They give

$$e^2\int\frac{d^Dk}{(2\pi)^D}\frac{-2\delta\mu\nu}{(k^2-m^2)}+$$
$$e^2\int\frac{d^Dk}{(2\pi)^D}\frac{(2k+p)_\mu(2k+p)_\nu}{(k^2+m^2)[(k+p)^2+m^2]}= \quad (B.1)$$
$$=\frac{e^2}{3\epsilon}(-\delta_{\mu\nu}p^2+p_\mu p_\nu)$$

2. Graphs with three external gluon lines. Only the triangle graphs, similar to those of fig II.b, contribute. The divergent part is

$$\frac{-2e^3}{(4\pi)^2\epsilon}Tr[(\partial_\mu A_\nu-\partial_\nu A_\mu)A^\mu A^\nu] \quad (B.2)$$

3. Graphs with four external lines. The calculation is straightforward and the final result is

$$\frac{e^4}{8\pi^2\epsilon}[A_\mu,A_\nu]^2 \quad (B.3)$$

which coincides with the one obtained by the use of analytic stochastic regularization.

We see that the breaking of gauge invariance in the analytic stochastic regularization method is entirely due to the graphs with two external gluon lines.

Fig. I

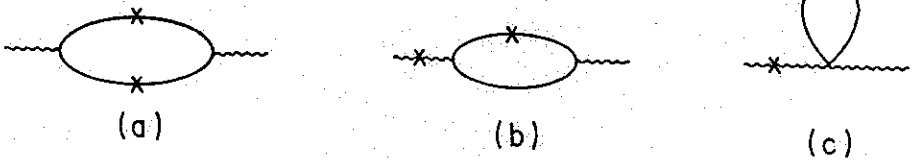


Fig. II

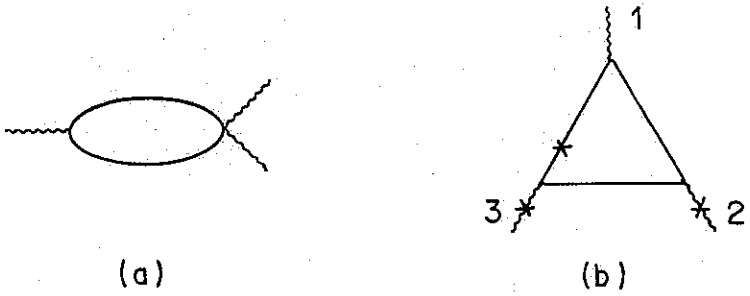
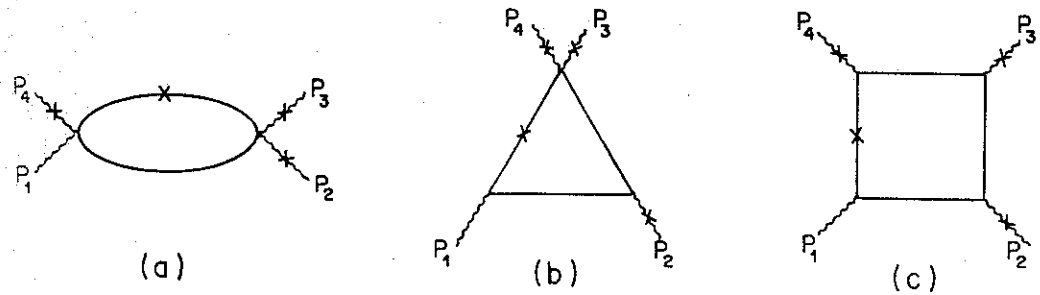


Fig. III



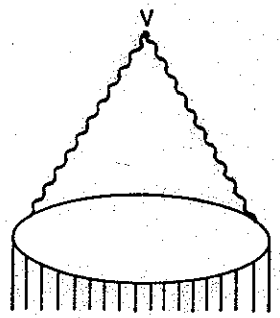


Fig. IV

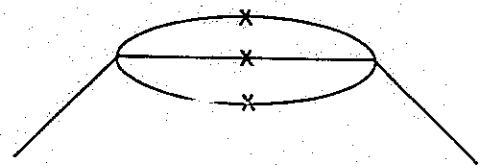


Fig. V

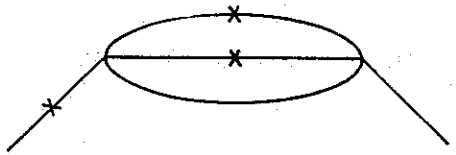


Fig. VI

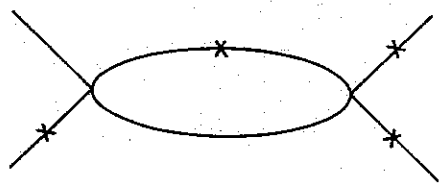


Fig. VII