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FLUCTUATIONS ON THE CURIE-WEISS VERSION OF THE ISING MODEL WITH RANDOM FIELD

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ABSTRACT

We show the fluctuations on the Curie-Weiss version of the Ising Model with Random Field to have either gaussian distribution with random mean away from criticality and at first order phase transition, or deterministic, non-gaussian distribution at other types of criticality.

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1. INTRODUCTION

In a beautiful series of papers[1,2,3] Ellis and Newman (EN for short from now on) discussed the statistics of the large spin-block variables in classical Curie-Weiss models of spin systems and showed their fluctuations to be of non-trivial nature at second order critical temperatures. It is therefore natural to investigate how the behaviour of these variables is affected by the presence of randomness. Of special interest is the so-called Ising Model with Random Magnetic Field whose thermodynamics and phase diagram in its Curie-Weiss version has been already computed by Salinas and Wreszinski[4].

In this paper we revisit the model with the purpose of discussing the probability distribution of its fluctuation variables in the spirit of EN's ideas. In particular we are interested in questions concerning their self-averaging properties[5]. Our findings are as follows:

Away from criticality the fluctuations are non-self-averaging. Their probability distribution is a gaussian with random mean.

At criticality two different kind of phenomena may occur: if there is a first order phase transition, fluctuations will remain non-self-averaging, just as above. If not, they will become deterministic and no longer gaussian. An eventual tricritical point would fall in this last category.

A suggestive physical picture may be drawn from these results, considering the non-self-averaging effect due to the presence of the random fields. At a second order phase transition the correlations between spins, typical of criticality, are strong enough to daub the effect of the field's fluctuations.

We describe here the main ideas leading to our results. Full mathematical details and applications to other Curie-Weiss random spin models, like Van Hemmen's "spin glass"[6] will be presented elsewhere[7].

The paper is organized as follows: in the next section we present the model and briefly review its thermodynamics within a slightly different method than that used by Salinas and Wreszinski. In the third section we develop the analysis of the fluctuations and obtain our results.

2. THE MODEL AND ITS THERMODYNAMICS

The model is described by the following hamiltonian:

$$H_n = -\frac{1}{2n}\left(\sum_{i=1}^{n}\sigma_i\right)^2 - \sum_{i=1}^{n} h_i\,\sigma_i \qquad (2.1)$$

where $\sigma_i = \pm 1$ are spin variables and the fields h_i are i.i.d. random variables with distribution $d\nu(h_i)$, what we denote by $h_i \sim d\nu(h_i)$.

Denoting by $\{\sigma\}$ all the possible configurations of spins, its partition function may be written as

$$Z_n = \sum_{\{\sigma\}} e^{-\beta H_n} = \left(\frac{n}{2\pi}\right)^{1/2} 2^n \int dx\; e^{-n\frac{x^2}{2}} \prod_{i=1}^{n} \exp\left[\ln\cosh\sqrt{\beta}\,(x+\sqrt{\beta}h_i)\right] =$$

$$= 2^n \left(\frac{n}{2\pi}\right)^{1/2} \int dx\; \exp\left[-n\,G_n(x)\right] \qquad (2.2)$$

Here we made use of the identity $e^{a^2/2} = \frac{1}{\sqrt{2\pi}}\int e^{-\frac{x^2}{2}+ax}\,dx$ in the first equality with $a = \sqrt{\beta/n}\sum_{i=1}^{n}\sigma_i$, and

$$G_n(x) = \frac{x^2}{2} - \frac{1}{n}\sum_{i=1}^{n}\ln\cosh\sqrt{\beta}(x+\sqrt{\beta}h_i) \xrightarrow[n\to\infty]{} G(x) = \frac{x^2}{2} - \overline{\ln\cosh\sqrt{\beta}(x+\sqrt{\beta}h)}$$

by the law of large numbers, where

$$\overline{\ln\cosh\sqrt{\beta}(x+\sqrt{\beta}h)} = \int d\nu(h)\,\ln\cosh\sqrt{\beta}(x+\sqrt{\beta}h)$$

Following Laplace's asymptotic method, the free energy $f = \lim_{n\to\infty} -\frac{1}{n\beta}\ln Z_n$ will be given by

$$f = \inf_{x\in\mathbb{R}} G(x) = G(x^*)$$

Two important points should be noticed. Firstly, if one computes Z_n with a fixed configuration of fiels $\{h_i\}$, one obtains a free energy independent of the choice of $\{h_i\}$, since the thermodynamic limit itself provided an average on

$dv(h)$. This is the self-averaging property of the free energy. The second point is that $G(x)$ is <u>not</u> the free energy at fixed magnetization, say $f(m)$, which is, however, related to $G(x)$ by

$$f = G(x^*) = \inf_m f(m) = f(m^*) \qquad x^* = \sqrt{\beta}\, m^*$$

Denoting by $G^{(k)}(x)$ and $G_n^{(k)}(x)$ the k^{th} derivatives of $G(x)$ and $G_n(x)$ respectively, one may write for its first derivatives:

$$G^{(1)}(x) = x - \sqrt{\beta} \int dv(h) \tanh \sqrt{\beta}(x+\sqrt{\beta}h)$$

$$G^{(2)}(x) = 1 - \beta \int dv(h) \operatorname{sech}^2 \sqrt{\beta}(x+\sqrt{\beta}h) \tag{2.3}$$

The derivatives of G_n will be given by similar expressions, just replacing $\int dv(h)$ by $\frac{1}{n}\sum_i$ and h by h_i. The search for the minima of $G(x)$ is related with the study of these functions. Clearly $G^{(1)}(x^*) = 0$ and for any even distribution $dv(h)$ all the odd derivatives of $G(x)$ vanish at $x = 0$. So, in this case $x^* = 0$ will be a (at least local) minimum as long as $G^{(2)}(0) > 0$. For $\beta = 0$ one can see that it is the only global minimum. As β increases, if there is no first order phase transition, the condition $G^{(2)}(0) = 0$ defines a second order phase transition at $\beta = \beta_c$ given by (2.3):

$$\beta_c = \left[\int dv(h) \operatorname{sech}^2 \sqrt{\beta}(x+\sqrt{\beta}h)\right]^{-1}$$

We will consider two types of distribution:

type I: even, absolutely continuous with density $p(h)$ decreasing in $[0,\infty]$

type II: density of the form $p(h) = C_0\delta(h) + \sum_i C_i[\delta(h-h_i) + \delta(h+h_i)]$ with $C_i \geq 0$ for all i.

It may be shown[4] that for dv of type I there will be no first order phase transition and neither for type II if $C_0 - \frac{1}{3}\sum_i C_i \geq 0$. If on the other hand, one takes

$$p(h) = \frac{1}{2}\left[\delta(h-H) + \delta(h+H)\right]$$

one finds a phase diagram as depicted in Fig. 1[4], where (H_t, T_t) is a tricritical point.

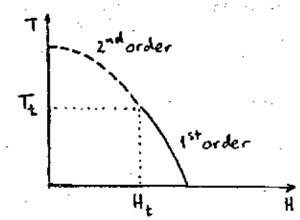

Fig. 1

3. FLUCTUATIONS

The study of fluctuations should be regarded as a finite size statistical correction to the evaluation of the equilibrium magnetization. We are then interested in the asymptotic distribution of the r.v. y_n defined as

$$y_n = \frac{\frac{\sum_{i=1}^{n} \sigma_i}{n} - m^*}{n^\gamma}$$

with $\gamma > 0$ and to be chosen such that $y = \lim_{n\to\infty} y_n$ has a stable distribution. One should take very carefuly into account that we are taking y_n as a block-spin variable of the size of the system with n (finite) fixed. This crucial point is in the heart of the whole approach, for had one considered a block-spin variable of fixed size (say N) in the thermodynamic limit $(n \to \infty)$, its statistics would have been trivial (gaussian distribution), since the spin variables are asymptoticaly independent. All non-triviality of our results to be exposed hereafter comes from this very fact. Here, the role of the function $G(x)$ becomes clearer. Its origin is an integral representation for Z_n where the spins are uncoupled already for finite n. The cost of such an operation is the introduction of the real, continuous variable x in (2.2). It appears as an extra field in a modified hamiltonian of independent spins, with a gaussian weight in the measure under integration. In fact, as one easily sees[2], the function $\exp[-n\,G(x)]$ admits a decomposition as a convolution of a gaussian distribution with that of y_n:

$$\frac{W}{n^{1/2-\gamma}} + y_n \sim \exp\left[-n\, G_n\left(s/n^\gamma + m^*\right)\right] ds \qquad (3.1)$$

where W is a r.v. independent of y_n and $W \sim N(0,1)$, with $N(0,1)$ being a gaussian distribution with mean zero and variance 1. This fact allows one to obtain the asymptotic distribution in (3.1) by expanding $G_n\left(\frac{s}{n^\gamma} + m^*\right)$ around its minimum and then taking the thermodynamic limit. If G_n has several minima, say p minima given by $\left\{x_n^{(i)}\right\}$ $i = 1, 2 \ldots p$, the distribution obtained will correspond to condition the magnetization to the point around which one expanded G.

For the class of measures we considered, we can state our results in the following form

$$y \sim \begin{cases} \exp\left[-G^{(6)}(0)\,\frac{s^6}{6!}\right] ds & \text{at the tricritical point if it exists} \\ \exp\left[-G^{(4)}(0)\,\frac{s^4}{4!}\right] ds & \text{at the critical point of a } 2^{\text{nd}} \text{ order phase transition} \\ N\left(\alpha, \frac{1}{G^{(2)}(0)} - 1\right) & \text{at any other temperature where } \alpha \sim N(0,1)\,. \end{cases}$$

This can be seen from the expansion of $G_n(x)$ in the form:

$$n\, G_n\left(s/n^\gamma + x^*\right) = \sum_{k=0}^{\infty} G_n^{(k)}\left(x_n^*\right)\left[s - n^\gamma\left(x_n^* - x^*\right)\right]^k \frac{n^{1-k\gamma}}{k!} \quad \text{as} \quad s \longrightarrow n^\gamma\left(x_n^* - x^*\right)$$

where $G_n^{(0)}(x) = G_n(x)$. At the tricritical point all the derivatives before the sixth vanish, so one must take $\gamma = \frac{1}{6}$ in order to save the expansion as $n \to \infty$. This gives our first result. The second comes from a similar argument, since the criticality of a 2nd order phase transition is characterized by $G^{(2)}(0) = 0$. In that case one must take $\gamma = \frac{1}{4}$. Finally, if $G^{(2)}(0) > 0$ one must take $\gamma = \frac{1}{2}$ and the r.v. $\alpha = \sqrt{n}\,(x_n^* - x^*)$ has the distribution given above by the Central Limit Theorem.

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