UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-663

ISOVECTOR EXCITATIONS OF $N \neq Z$ NUCLEI

E.J.V. de Passos, D.P. Menezes
Instituto de Física, Universidade de São Paulo

A.P.N.R. Galeão
Instituto de Geociências e Ciências Exatas,
Universidade Estadual Paulista
C.P. 178, 13500 Rio Claro, SP, Brazil

Setembro/1987

ISOVECTOR EXCITATIONS OF $N \neq Z$ NUCLEI

E.J.V. de Passos*, D.P. Menezes

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brazil

A.P.N.R. Galeão

Instituto de Geociências e Ciências Exatas,
Universidade Estadual Paulista
C.P. 178, 13500 Rio Claro, SP, Brazil

ABSTRACT

We show that the method based on the tensor coupling of an appropriate family of isovector excitation operators to the parent isospin multiplet can be used, to advantage, for the correct treatment of the isospin degree of freedom in non-isoscalar nuclei. This method is applicable to any isovector excitation operator and for parent states which need not to be of the closed subshells type. As an illustration we apply it to the study of the Gamow-Teller transition strength in ^{90}Zr.

---

*CNPq fellow.

KEYWORD ABSTRACT

[Nuclear structure-isospin, isovector resonances, Gamow-Teller resonance in ^{90}Zr.]

PACS NUMBER: 21.60Cs

I. INTRODUCTION

The dynamical violation of isospin invariance in nuclei is associated with the Coulomb interaction between protons. However, since the strong nucleon-nucleon interaction conserves isospin, this violation can be disregarded in a first approximation. In fact, Bohr and Mottelson demonstrated that the isospin impurities in ground states of nuclei, even the heavy ones, where the Coulomb interaction plays a relatively important role, do not exceed a few tenths of a percent[1]. As a consequence, the nuclear spectrum tends to split into isospin multiplets. A well known manifestation of this effect is the fragmentation of the dipole resonances in nuclei with a neutron excess and, consequently, non-zero ground state isospin[2]. This phenomenon, though hard to detect experimentally, is predicted to be a general feature of isovector resonances.

Most of the theoretical investigation of the distribution of the transition strength of giant resonances is done by diagonalizing the hamiltonian in the subspace of 1 particle-1 hole (1p-1h) excitations of nuclei with a closed subshells ground state, chosen for the parent state. This is the so called Tamm-Dancoff approximation (TDA). However, for $N \neq Z$ nuclei and isovector excitations, the resulting states do not have, in general, good isospin. In this framework, the isospin invariance can be restored by taking into account

appropriate 2p-2h and 3p-3h excitations, as done in Refs. 3 and 4.

Taking advantage of the symplifying features of these configurations, those authors first determine, in each case, which isospin multiplets will appear. When all multiplets exist, standard techniques of Racah algebra are used to construct the good isospin states. In the remaining cases, the authors make use of excitation operators which do not have good isospin, leading to the necessity of alternative techniques to construct the good isospin states.

The main purpose of this paper is to show that the technique of Racah coupling can be used, to advantage, in all cases. We not only show how to construct good isospin states but we also give criteria to decide, a priori, which excited isospin multiplets exist in each case. We find this method simpler and more transparent than the previous ones[3,4], with the additional advantage that it can be used for any isovector excitation operator and for parent states which need not be of closed subshells type. The same point of view was adopted by Rowe and Ngo-Trong in the general context of their tensor equations of motion technique[5].

Our paper is organized as follows: in section II we present the method to construct good isospin states, based on the tensor coupling of an isovector excitation operator to the parent isospin multiplet. As in Refs. 3 and 4, we give criteria to decide a priori which excited isospin multiplets exist in each case and show how to relate the reduced matrix elements of an isotensor operator between good isospin states to its matrix elements between the doorway states, i.e., the states resulting from the action of each component of the isovector excitation operator on the parent state. We also present TDA-type equations conserving isospin but effectively involving only the excitations of the parent state.

As an illustration of the method, in section III, we apply it to the study of the Gamow-Teller transition strength in ^{90}Zr, taking for parent state not just the closed subshells configuration but also the 2p-2h $(\pi 1g9/2)^2(\pi 2p1/2^{-1})^2$ configuration.

II. THEORY

A. Construction of good isospin basis

In order to keep the discussion in the most general terms possible, we shall assume about the parent state $|P;T_0 T_0\rangle$ only that it has good isospin $T_0 \equiv \frac{1}{2}(N-Z)$ and, furthermore, is a state with maximum alignment in isospin, that is,

$$T_+|P;T_0 T_0\rangle = 0 \quad , \tag{1}$$

where T_+ is the isospin raising operator. By successive applications of the isospin lowering operator T_- on the parent state, followed by normalization, we can construct the whole set of analog states $|P;T_0 M_{T_0}\rangle$, with $M_{T_0} = T_0, T_0-1, \ldots -T_0$, which form the parent isospin multiplet. We are interested in the excited states resulting from an isovector excitation of this multiplet, represented by a certain family of excitation operators O_{1m_τ}, with $m_\tau = +1, 0$ and -1, which form an isospin standard tensor of rank 1.

The doorway states, $O_{11}|P;T_0 T_0\rangle$, $O_{10}|P;T_0 T_0\rangle$ and $O_{1-1}|P;T_0 T_0\rangle$, are the main components of the excited states of interest. However, they are not sufficient, in general, to build states with good isospin. In fact, those are given by

$$|T M_T\rangle = \sum_{m_\tau} (1\, T_0\, m_\tau\, M_T-m_\tau | T\, M_T)\, O_{1m_\tau}|P;T_0, M_T-m_\tau\rangle \quad , \tag{2}$$

where $(1\, T_0\, m_\tau\, M_T-m_\tau|T\, M_T)$ are Clebsch-Gordon (C.G.) coefficients. The isospin T of the excited states can in general take the values $T = T_0+1$, T_0 and T_0-1. It may, however, happen, depending on the structure of the parent state and/or the nature of the excitation considered, that, some of these isospin values do not occur, in which case the corresponding states defined in Eq. (2) are identical to zero. The equation, however, has general validity and can be inverted, by making use of the orthonormality properties of the C.G. coefficients, to give

$$O_{1m_\tau}|P;T_0 M_{T_0}\rangle = \sum_T (1\, T_0\, m_\tau\, M_{T_0}|T\, M_{T_0}+m_\tau)\,|T, M_{T_0}+m_\tau\rangle \quad . \qquad (3)$$

Before proceeding, we write down, for future use, the explicit form of Eqs. (2) and (3) for the states of interest. These are:

$$|T_0+1, T_0+1\rangle = O_{11}|P;T_0 T_0\rangle \quad , \qquad (4)$$

$$|T_0+1, T_0\rangle = \sqrt{\frac{T_0}{T_0+1}}\, O_{11}|P;T_0, T_0-1\rangle + \sqrt{\frac{1}{T_0+1}}\, O_{10}|P;T_0 T_0\rangle \quad , \qquad (5)$$

$$|T_0 T_0\rangle = \sqrt{\frac{1}{T_0+1}}\, O_{11}|P;T_0, T_0-1\rangle - \sqrt{\frac{T_0}{T_0+1}}\, O_{10}|P;T_0 T_0\rangle \quad , \qquad (6)$$

$$|T_0+1, T_0-1\rangle = \sqrt{\frac{T_0(2T_0-1)}{(T_0+1)(2T_0+1)}}\, O_{11}|P;T_0, T_0-2\rangle + \sqrt{\frac{4T_0}{(T_0+1)(2T_0+1)}}\, O_{10}|P;T_0, T_0-1\rangle + \sqrt{\frac{1}{(T_0+1)(2T_0+1)}}\, O_{1-1}|P;T_0 T_0\rangle \quad , \qquad (7)$$

$$|T_0, T_0-1\rangle = \sqrt{\frac{2T_0-1}{T_0(T_0+1)}}\, O_{11}|P;T_0, T_0-2\rangle - \frac{T_0-1}{\sqrt{T_0(T_0+1)}}\, O_{10}|P;T_0, T_0-1\rangle - \sqrt{\frac{1}{T_0+1}}\, O_{1-1}|P;T_0 T_0\rangle \qquad (8)$$

and

$$|T_0-1, T_0-1\rangle = \sqrt{\frac{1}{T_0(2T_0+1)}}\, O_{11}|P;T_0, T_0-2\rangle - \sqrt{\frac{2T_0-1}{T_0(2T_0+1)}}\, O_{10}|P;T_0, T_0-1\rangle + \sqrt{\frac{2T_0-1}{2T_0+1}}\, O_{1-1}|P;T_0 T_0\rangle \quad , \qquad (9)$$

for the excited basis states with good isospin,

$$O_{11}|P;T_0 T_0\rangle = |T_0+1, T_0+1\rangle \quad , \qquad (10)$$

$$O_{10}|P;T_0 T_0\rangle = \sqrt{\frac{1}{T_0+1}}\,|T_0+1, T_0\rangle - \sqrt{\frac{T_0}{T_0+1}}\,|T_0 T_0\rangle \qquad (11)$$

and

$$O_{1-1}|P;T_0 T_0\rangle = \sqrt{\frac{1}{(T_0+1)(2T_0+1)}}\;|T_0+1, T_0-1\rangle$$

$$-\sqrt{\frac{1}{T_0+1}}\;|T_0, T_0-1\rangle + \sqrt{\frac{2T_0-1}{2T_0+1}}\;|T_0-1, T_0-1\rangle \quad , \qquad (12)$$

for the doorway states, and

$$O_{11}|P;T_0, T_0-1\rangle = \sqrt{\frac{T_0}{T_0+1}}\;|T_0+1, T_0\rangle + \sqrt{\frac{1}{T_0+1}}\;|T_0 T_0\rangle \quad , \qquad (13)$$

$$O_{10}|P;T_0, T_0-1\rangle = \sqrt{\frac{4T_0}{(T_0+1)(2T_0+1)}}\;|T_0+1, T_0-1\rangle$$

$$-\frac{T_0-1}{\sqrt{T_0(T_0+1)}}\;|T_0, T_0-1\rangle - \sqrt{\frac{2T_0-1}{T_0(2T_0+1)}}\;|T_0-1, T_0-1\rangle \qquad (14)$$

and

$$O_{11}|P;T_0, T_0-2\rangle = \sqrt{\frac{T_0(2T_0-1)}{(T_0+1)(2T_0+1)}}\;|T_0+1, T_0-1\rangle$$

$$+\sqrt{\frac{2T_0-1}{T_0(T_0+1)}}\;|T_0, T_0-1\rangle + \sqrt{\frac{1}{T_0(2T_0+1)}}\;|T_0-1, T_0-1\rangle \quad , \qquad (15)$$

for the remaining basis states. These equations show clearly

that the doorway states do not, in general, have good isospin and, in order to construct pure isospin states, we need to expand the set of basis states to include, besides the doorway states, the auxiliary states $O_{11}|P;T_0,T_0-1\rangle$, $O_{10}|P;T_0,T_0-1\rangle$ and $O_{11}|P;T_0,T_0-2\rangle$.

We want to establish now simple criteria to decide which values of the isospin T of the excited multiplets do effectively occur in each possible case. We note first of all that, if the doorway state $O_{1-1}|P;T_0 T_0\rangle$ were zero, then all the three excited isospin multiplets would be zero, as follows from Eq. (12), if one remembers that the three states on the right hand side of this equation are orthogonal to each other. This would imply that all the basis states given in Eqs. (10) to (15), including the doorway states, would also be zero and we would have no excited state of the kind at all. We shall, therefore, assume that

$$O_{1-1}|P;T_0 T_0\rangle \neq 0 \quad . \qquad (16)$$

There are, then, three separate cases to consider[3]:

<u>1rst case</u>: $T_+ O_{1-1}|P;T_0 T_0\rangle = 0$

It follows immediately from this equation that the doorway state $O_{1-1}|P;T_0 T_0\rangle$ has $T = T_0-1$. Furthermore, commuting the two operators acting on the parent state in the above equation one sees that it is equivalent to

$$O_{10}|P;T_0\,T_0\rangle = 0 \quad , \tag{17}$$

where use has been made of Eq. (1). This in turn implies, through Eq. (11), that the excited multiplets with $T = T_0+1$ and $T = T_0$ are zero. Thus, the only possible isospin value is $T = T_0-1$ and the corresponding state, constructed in the usual way as indicated in Eq. (2), is given, from Eq. (12), by

$$|T_0-1, T_0-1\rangle = \sqrt{\frac{2T_0+1}{2T_0-1}}\; O_{1-1}|P;T_0\,T_0\rangle \quad . \tag{18}$$

In the present case, therefore, the isovector excitation can only reach states in the nuclide (N-1, Z+1), where N and Z are, respectively, the neutron and proton numbers in the parent nuclide. The subspace of excited states of the kind considered here is one-dimensional, the state vectors given in Eqs. (10), (11) and (13) are zero and those given in Eqs. (12), (14) and (15) form an overcomplete basis, being related to each other as follows:

$$O_{10}|P;T_0,T_0-1\rangle = -\sqrt{\frac{1}{T_0}}\; O_{1-1}|P;T_0\,T_0\rangle \tag{19}$$

and

$$O_{11}|P;T_0,T_0-2\rangle = \sqrt{\frac{1}{T_0\,(2T_0-1)}}\; O_{1-1}|P;T_0\,T_0\rangle \quad . \tag{20}$$

<u>2^{nd} case</u>: $T_+ O_{1-1}|P;\,T_0\,T_0\rangle \neq 0$ and $T_+^2 O_{1-1}|P;\,T_0\,T_0\rangle = 0$

The first condition is equivalent to

$$O_{10}|P;\,T_0\,T_0\rangle \neq 0 \quad . \tag{21}$$

The second one implies that the above state has $T = T_0$, and is equivalent to

$$O_{11}|P;\,T_0\,T_0\rangle = 0 \quad . \tag{22}$$

This means, in view of Eq. (10), that $|T_0+1,\,T_0+1\rangle$ is zero. Consequently the multiplet with $T = T_0+1$ does not appear in this case and the excitation cannot reach states in the nuclide (N+1, Z-1). Since $|T_0+1,\,T_0\rangle$ must also be zero, Eq. (5) shows that there is the following linear dependence between the basis vectors for the nuclide (N,Z):

$$O_{11}|P;\,T_0,\,T_0-1\rangle = -\sqrt{\frac{1}{T_0}}\; O_{10}|P;\,T_0\,T_0\rangle \quad . \tag{23}$$

Furthermore, Eq. (11) gives directly

$$|T_0\,T_0\rangle = -\sqrt{\frac{T_0+1}{T_0}}\; O_{10}|P;\,T_0\,T_0\rangle \quad , \tag{24}$$

guaranteeing the occurrence of the multiplet with $T = T_0$ in view of Eq. (21). Making use, now, of the fact that the state vector given in Eq. (7) is zero one gets

$$O_{11}|P; T_0 \cdot T_0-2\rangle = -\sqrt{\frac{4}{2T_0-1}}\, O_{10}|P; T_0, T_0-1\rangle$$

$$-\sqrt{\frac{1}{T_0(2T_0-1)}}\, O_{1-1}|P; T_0\, T_0\rangle \quad , \tag{25}$$

showing that the basis for the nuclide (N-1, Z+1) is overcomplete. Since the multiplet with $T=T_0$ has been shown to occur in the present case, the state $|T_0, T_0-1\rangle$ is necessarily non-zero and, introducing Eq. (25) into Eq. (8), one gets for it

$$|T_0, T_0-1\rangle = -\sqrt{\frac{T_0+1}{T_0}}\, O_{10}|P; T_0, T_0-1\rangle - \frac{\sqrt{T_0+1}}{T_0}\, O_{1-1}|P; T_0\, T_0\rangle \tag{26}$$

which, as it should, is the analog of the state (24). There is, however, no guarantee that the multiplet of isospin T_0-1 will also occur. The condition for this to be the case is, as can be seen in Eq. (12), that the projector onto the subspace of isospin T_0-1 applied to $O_{1-1}|P; T_0\, T_0\rangle$ gives a non-zero state. This is equivalent, as can be seen from Eqs. (11) and (12), to saying that $O_{1-1}|P; T_0\, T_0\rangle$ should not be merely the analog of $O_{10}|P; T_0\, T_0\rangle$. Assuming that this supplementary condition holds, $|T_0-1, T_0-1\rangle$ will be non-zero and, introducing Eq. (25) into Eq. (9) one gets

$$|T_0-1, T_0-1\rangle = -\sqrt{\frac{2T_0+1}{T_0(2T_0-1)}}\, O_{10}|P; T_0, T_0-1\rangle$$

$$+\frac{T_0+1}{T_0}\sqrt{\frac{2T_0+1}{2T_0-1}}\, O_{1-1}|P; T_0\, T_0\rangle \quad . \tag{27}$$

In summary we conclude that in the present case the multiplet with $T = T_0+1$ does not occur, the excited multiplet with $T=T_0$ necessarily occurs and the one with $T = T_0-1$ will occur provided the above mentioned supplementary condition is satisfied.

3rd case: $T_+ O_{1-1}|P; T_0\, T_0\rangle \neq 0$ and $T_+^2 O_{1-1}|P; T_0\, T_0\rangle \neq 0$

The above conditions are equivalent to

$$O_{10}|P; T_0\, T_0\rangle \neq 0 \tag{28}$$

and

$$O_{11}|P; T_0\, T_0\rangle \neq 0 \quad , \tag{29}$$

meaning, together with (16), that all three doorway states are non-zero. Condition (29) guarantees, in view of Eq. (4), that the multiplet with $T = T_0+1$ will appear in this case, and we will have an excited state in the nuclide (N+1, Z-1). Coming to the nuclide (N,Z), the state $|T_0+1, T_0\rangle$, given in Eq. (5), is necessarily non-zero. As in the previous case one needs an extra condition to guarantee the occurrence of the other multiplet. Thus, state $|T_0\, T_0\rangle$ will be non-zero only if the projection of $O_{10}|P;\, T_0\, T_0\rangle$ onto the subspace of isospin T_0 is non-zero. Similarly to the previous case this is equivalent to the statement that $O_{10}|P;\, T_0\, T_0\rangle$ should not be merely the analog of $O_{11}|P;\, T_0\, T_0\rangle$. Finally, getting to the nuclide (N-1, Z+1), the states $|T_0+1,\, T_0-1\rangle$ and $|T_0,\, T_0-1\rangle$ are non-zero, assuming for the latter that the supplementary condition above is satisfied. Again, to guarantee the occurrence of the $T = T_0-1$ multiplet one has to impose the same supplementary condition as in the 2nd case. Now, however, as can be seen from Eqs. (10) to (12) this means that $O_{1-1}|P;\, T_0\, T_0\rangle$ cannot be written as a linear combination of the analog of $O_{10}|P;\, T_0\, T_0\rangle$ and the double-analog of $O_{11}|P;\, T_0\, T_0\rangle$.

The above scheme for the determination of the non-vanishing excited isospin multiplets resulting from a given isovector excitation and parent state, satisfying Eqs. (1) and (16), is summarized in Table 1.

B. Norms and matrix elements

The states we have been dealing with are not necessarily normalized. Their norms, however, can all be written in terms of those of the three doorway states, whose squares we shall denote by $\xi(m_\tau)$ in the following, i.e.,

$$\xi(m_\tau) \equiv \langle P; T_0\, T_0 | O^\dagger_{1m_\tau} O_{1m_\tau} | P; T_0\, T_0 \rangle \quad . \tag{30}$$

To see this, we start by noticing that the Wigner-Eckart theorem assures us that we can write the following expressions for the overlaps of the states defined in Eq. (2):

$$\langle T\, M_T | T'\, M'_T \rangle = \delta_{TT'}\, \delta_{M_T M'_T}\, \eta(T) \quad . \tag{31}$$

We, then, make use of this result to compute the norms of the states defined in Eqs. (10) to (12) and invert the resulting equations to get, for the normalization constants,

$$\eta(T_0+1) = \xi(1) \quad , \tag{32}$$

$$\eta(T_0) = -\frac{1}{T_0}\, \xi(1) + \frac{T_0+1}{T_0}\, \xi(0) \tag{33}$$

and

$$\eta(T_0-1) = \frac{1}{T_0(2T_0-1)}\, \xi(1) - \frac{2T_0+1}{T_0(2T_0-1)}\, \xi(0) + \frac{2T_0+1}{2T_0-1}\, \xi(-1) \quad . \tag{34}$$

Since the norm of any state of interest can be written in terms of the normalization constants $\eta(T)$, they can also be written in terms of $\xi(m_\tau)$.

We turn our attention now to the calculation of the reduced matrix element of an isotensor operator W_{tm_t}, of isospin rank t, between the parent and the good isospin basis states defined in Eq. (2). Making use of standard procedures of Racah algebra one gets

$$\langle T \| W_t \| P;\, T_0 \rangle =$$

$$= \sqrt{2T+1} \sum_{\Theta} (-)^{t+\Theta-1} \sqrt{2\Theta+1}\; W(1tT_0T_0;\Theta T) \langle P;\, T_0 \| (\bar{O}_1 \times W_t)_{\Theta} \| P;\, T_0 \rangle \; , \tag{35}$$

where $W(1tT_0T_0;\Theta T)$ is a Racah recoupling coefficient, $\bar{O}_{1m_\tau}$ is the tensor adjoint operator defined by

$$\bar{O}_{1m_\tau} = (-)^{1+m_\tau} (O_{1-m_\tau})^{\dagger} \tag{36}$$

and the cross (×) indicates tensor coupling, i.e.,

$$(\bar{O}_1 \times W_t)_{\Theta m_\Theta} \equiv \sum_{m_\tau} (1t\, m_\tau\; m_\Theta - m_\tau | \Theta m_\Theta)\, \bar{O}_{1m_\tau}\; W_{t,m_\Theta - m_\tau} \; . \tag{37}$$

One has, furthermore,

$$\langle P;\, T_0 \| (\bar{O}_1 \times W_t)_{\Theta} | P;\, T_0 \rangle =$$

$$= \frac{\sqrt{2T_0+1}}{(T_0\,\Theta\, T_0\, 0 | T_0\, T_0)} \sum_{m_t} (-)^{1+m_t} (1t - m_t\, m_t | \Theta 0) \langle P; T_0 T_0 | O^{\dagger}_{1m_t}\, W_{tm_t} | P;\, T_0\, T_0 \rangle \; , \tag{38}$$

which, inserted into Eq.(35) leads to the result

$$\langle T \| W_t \| P;\, T_0 \rangle =$$

$$= (-)^{t+T_0-T} \sqrt{2T+1} \sum_{m_t} P_T(1\,T_0;\, t\,T_0;\, m_t) \langle P;\, T_0\, T_0 | O^{\dagger}_{1m_t}\, W_{tm_t} | P;\, T_0\, T_0 \rangle \; , \tag{39}$$

where

$$P_T(1\,T_0;\, t\,T_0;\, m_t) \equiv \sum_{\Theta} (-)^{T+\Theta+m_t-T_0} \sqrt{(2T_0+1)(2\Theta+1)} \; .$$

$$W(1\,t\,T_0\,T_0;\Theta\, T)\; \frac{(1\,t - m_t\, m_t | \Theta\, 0)}{(T_0\, \Theta\, T_0\, 0 | T_0\, T_0)} \tag{40}$$

The above equations are equivalent to the analogous ones obtained in Refs. (4) and (5). In particular, the transformation coefficients defined in Eq. (40) are identical to the ones introduced in Ref. (4). However, we give in the next section a simple technique to obtain their numerical values without having to perform the tedious calculations indicated in that equation.

An important point to notice in Eqs. (30) to (34) and (39) is that, even though the correct treatment of isospin requires, in general, the extension of the basis to include, besides the doorway states, the auxiliary states given in Eqs. (13) to (15), all calculations (norms and matrix elements) can be performed in a way that involves only the parent and the doorway states. This is of great practical value because, in a specific calculation in which the doorway states involved at most n particle-n hole configurations, the auxiliary basis states would involve, in general, up to (n+2) particle-(n+2) hole configurations (cf. Eqs. (13) to (15)), which, were it not for the point just mentioned, would considerably increase the computational task. This point has already been made in Refs. (4) and (5) in the special case of parent states with closed subshells and excitation operators of particle-hole type. What we have shown here is that this result depends only on the fact that the parent state has good isospin and on the isotensor nature of the excitation operator.

C. TDA equations

In a TDA type calculation, the excited states of interest can be reached through several excitation operators, $O_{\tau m_\tau}(\alpha)$, which in general can have different isotensor ranks. For instance, in a 1 particle - 1 hole calculation, isovector and isoscalar operators would appear. From the point of view of isospin, the isoscalar case is trivial. On the other hand, other isotensor ranks can be treated similarly to the isovector case. With these remarks in mind, we shall include explicitly, in the following, isovector excitations only, since in our paper the emphasis is on the technique for constructing good isospin states. However, this might be appropriate in a realistic situation depending on the value of the final isospin.

We write, then, for the excited basis with good isospin

$$|\alpha; T M_T\rangle = \sum_{m_\tau} (1\, T_0\, m_\tau\, M_T - m_\tau | T M_T) O_{1 m_\tau}(\alpha) |P; T_0\, M_T - m_\tau\rangle \quad , \qquad (41)$$

in analogy to Eq. (2). The rest of the preceding discussion can be applied to each excitation separately.

In the Tamm-Dancoff approximation, the excited states, labelled by n,

$$|n; T M_T\rangle = \sum_\alpha X(\alpha; nT) |\alpha; T M_T\rangle \quad , \qquad (42)$$

and the corresponding excitation energies $E(nT)$ are determined by diagonalizing the (isoscalar) nuclear hamiltonian H in the subspace spanned by the excited basis given in Eq. (41), that is, by solving the TDA equation

$$\sum_{\alpha'} \langle\alpha; T \| H \| \alpha'; T\rangle X(\alpha'; nT) = E(nT) \sum_{\alpha'} \langle\alpha; T \| \alpha'; T\rangle X(\alpha'; nT) \quad . \qquad (43)$$

The summation over α should run only over the transitions which can lead to the given isospin, T, according to the discussion of section A. We are assuming that the excited basis states are orthogonal to the parent isospin multiplet and are not coupled to it by the nuclear hamiltonian.

The reduced matrix elements appearing above are in turn given by

$$\langle \alpha;T\|\alpha';T\rangle = (-)^{1+T_0-T} \langle \alpha;T\|O_1(\alpha')\|P;T_0\rangle =$$

$$= \sqrt{2T+1} \sum_{m_\tau} P_T(1T_0;1T_0;m_\tau) \langle P;T_0\,T_0|O_{1m_\tau}(\alpha)^\dagger O_{1m_\tau}(\alpha')|P;T_0\,T_0\rangle \tag{44}$$

and

$$\langle \alpha;T\|H\|\alpha';T\rangle = (-)^{1+T_0-T} \langle \alpha;T\|(H\,O_1(\alpha'))\|P;T_0\rangle =$$

$$= \sqrt{2T+1} \sum_{m_\tau} P_T(1T_0;1T_0;m_\tau) \langle P;T_0\,T_0|O_{1m_\tau}(\alpha)^\dagger H O_{1m_\tau}(\alpha')|P;T_0\,T_0\rangle \quad , \tag{45}$$

where, in both cases, we have made use of Eq. (39).

As a byproduct we notice that, comparing Eq. (44) for $\alpha \equiv \alpha'$ with Eqs. (32) to (34), one can read off the explicit expressions for the transformation coefficients. Those are listed in Table 2.

III. APPLICATION

As an illustration of our method we are going to discuss the unperturbed Gamow-Teller transitions in ^{90}Zr.

In an unperturbed calculation each many-particle configuracion is an eigenstate of the unperturbed Hamiltonian.

Therefore, in this case, it is meaningful to include only those configurations which carry Gamow-Teller strength.

The Gamow-Teller transition operator transforms as a tensor of rank 1 both with respect to rotations in position-spin as in isospin spaces, and is given by

$$GT^{1m_\sigma}_{1m_\tau} = \sum_{k=1}^{A} \hat{\sigma}_{1m_\sigma}(k)\,\hat{t}_{1m_\tau}(k) \quad , \tag{46}$$

where $\hat{\sigma}_{1m_\sigma}(k)$ and $\hat{t}_{1m_\tau}(k)$ are the standard spherical components of the Pauli spin-matrices and the isospin operators, respectively, for the $k^{\underline{th}}$ nucleon. Alternatively, one has, in second quantized form,

$$GT^{1m_\sigma}_{1m_\tau} = \sum_{rs} \delta_{n_r n_s}\, \delta_{\ell_r \ell_s} \sqrt{(2j_r+1)(2j_s+1)}\; W(\tfrac{1}{2}\,1\,\ell_r\,j_r;\tfrac{1}{2}\,j_s) \left(a_r^\dagger \times b_s^\dagger\right)^{1m_\sigma}_{1m_\tau} \quad , \tag{47}$$

where the cross indicates, here, tensor coupling with respect to both position-spin (upper indices) and isospin (lower indices). The operator

$$a_r^\dagger \equiv a^\dagger_{q_r n_r \ell_r j_r m_r} \tag{48}$$

creates a neutron ($q_r = +\frac{1}{2}$) or proton ($q_r = -\frac{1}{2}$) in state r and $b_s^\dagger$ is related to the destruction operator by

$$b^{\dagger}_{q_s n_s \ell_s j_s m_s} = (-)^{\frac{1}{2}+q_s+j_s+m_s} a_{-q_s n_s \ell_s j_s -m_s} , \qquad (49)$$

so that it have the transformation properties of a standard tensor in position-spin and isospin spaces.

It is clear from Eq. (47) that the appropriate set of excitation operators for this problem is

$$O_{1m_\tau}(s \to r : 1m_\sigma) \equiv \left(a^{\dagger}_r \times b^{\dagger}_s\right)^{1m_\sigma}_{1m_\tau} , \qquad (50)$$

which can also be written in the form

$$\left(a^{\dagger}_r \times b^{\dagger}_s\right)^{1m_\sigma}_{11} = -\left(a^{\dagger}_{\nu r} \times b^{\dagger}_{\pi s}\right)^{1m_\sigma} , \qquad (51)$$

$$\left(a^{\dagger}_r \times b^{\dagger}_s\right)^{1m_\sigma}_{10} = \frac{1}{\sqrt{2}}\left[\left(a^{\dagger}_{\nu r} \times b^{\dagger}_{\nu s}\right)^{1m_\sigma} - \left(a^{\dagger}_{\pi r} \times b^{\dagger}_{\pi s}\right)^{1m_\sigma}\right] \qquad (52)$$

and

$$\left(a^{\dagger}_r \times b^{\dagger}_s\right)^{1m_\sigma}_{1-1} = \left(a^{\dagger}_{\pi r} \times b^{\dagger}_{\nu s}\right)^{1m_\sigma} , \qquad (53)$$

where we have explicitly performed the tensor coupling with respect to isospin. The operators labelled by π (protons) or ν (neutrons) appearing on the right hand side of these equations are directly related to the ones defined in Eqs. (48) and (49),

except that they are tensors in position-spin space only.

The experimental evidence indicates that the ground state of ^{90}Zr is well represented by a linear combination of the closed subshells configuration, $|P;0\rangle$ (Fig. 1a), and the 2p-2h configuration $(\pi 1g_{9/2})^2(\pi 2p_{1/2}^{-1})^2$, $|P;2p\text{-}2h\rangle$ (Fig. 1b), with comparable probabilities[6]. Choosing the former as the vacuum, the latter is given, with the correct normalization, by

$$|P;2p\text{-}2h\rangle = \frac{1}{2}\left(a^{\dagger}_{\pi g_{9/2}} \times a^{\dagger}_{\pi g_{9/2}}\right)^{00}\left(b^{\dagger}_{\pi p_{1/2}} \times b^{\dagger}_{\pi p_{1/2}}\right)^{00} |P;0\rangle . \qquad (54)$$

Of course, both configurations have $J^\pi = 0^+$ and $T_0 = M_{T_0} = 5$. Since this is an unperturbed calculation, we will consider separately the action of the excitation operators on each one of these two configurations, taken as parent states, and determine, for each case, the possible isospin values according to the discussion of Section II.A. We will also give explicit expressions for the doorway states.

For the transitions based on $|P;0\rangle$ the results of this discussion are summarized in Table 3 and we shall give no further detail here, since in this case, our method is equivalent to the one of references 3 and 4[(7)]. The non-zero doorway states are: $\left(a^{\dagger}_{g_{9/2}} \times b^{\dagger}_{b_{9/2}}\right)^{1m_\sigma}_{1-1}|P;0\rangle$, $\left(a^{\dagger}_{g_{7/2}} \times b^{\dagger}_{g_{9/2}}\right)^{1m_\sigma}_{1-1}|P;0\rangle$ and $\left(a^{\dagger}_{g_{7/2}} \times b^{\dagger}_{g_{9/2}}\right)^{1m_\sigma}_{10}|P;0\rangle$. A pictorial representation of these states is shown in Fig. 2.

For the transitions based on $|P;2p\text{-}2h\rangle$, we notice first of all that the relevant ones, i.e., those whose excitation operators satisfy (16) , are: (i) $2p_{1/2} \rightarrow 2p_{1/2}$; (ii) $1g_{9/2} \rightarrow 1g_{9/2}$; (iii) $2p_{3/2} \rightarrow 2p_{1/2}$; (iv) $1g_{9/2} \rightarrow 1g_{7/2}$. We proceed to discuss each one of them in detail.

i) Transition $2p_{1/2} \rightarrow 2p_{1/2}$

It is easily seen from Fig. 1b that

$$\left(a^{\dagger}_{p_{1/2}} \times b^{\dagger}_{p_{1/2}}\right)^{1m_\sigma}_{10} |P;2p\text{-}2h\rangle = 0 \quad , \qquad (55)$$

since this $J = 1$ excitation is possible neither for neutrons ($2p_{1/2}$ level is totally filled) nor for protons ($2p_{1/2}$ level is empty). Therefore it belongs to the $1^{\underline{rst}}$ case of Section II.A and the only possible isospin multiplet is the one with $T = T_0 - 1 = 4$. The only non-zero doorway state is $\left(a^{\dagger}_{p_{1/2}} \times b^{\dagger}_{p_{1/2}}\right)^{1m_\sigma}_{1-1} |P;2p\text{-}2h\rangle$, which is pictorially represented in Fig. 3a. Its norm, $\sqrt{\xi(-1)}$, can be directly computed from the above expression and that of the good isospin state, $\sqrt{\eta(4)}$, can be obtained from Eq. (34). The results are summarized in Table 4.

(ii) Transition $1g_{9/2} \rightarrow 1g_{9/2}$

Similarly to the previous case one has

$$\left(a^{\dagger}_{g_{9/2}} \times b^{\dagger}_{g_{9/2}}\right)^{1m_\sigma}_{10} |P;2p\text{-}2h\rangle = 0 \quad , \qquad (56)$$

since two protons in the same level cannot have $J=1$. The transition belongs to the $1^{\underline{rst}}$ case and only the $T=4$ multiplet occurs. The only non-zero doorway state is $\left(a^{\dagger}_{g_{9/2}} \times b^{\dagger}_{g_{9/2}}\right)^{1m_\sigma}_{1-1} |P;2p\text{-}2h\rangle$, which is represented in Fig. 3b. Its norm, which must be directly computed, and that of the good isospin state, obtained from Eq. (34), are given in Table 4.

(iii) Transition $2p_{3/2} \rightarrow 2p_{1/2}$

We notice, first of all, that

$$\left(a^{\dagger}_{p_{1/2}} \times b^{\dagger}_{p_{3/2}}\right)^{1m_\sigma}_{11} |P;2p\text{-}2h\rangle = 0 \quad , \qquad (57)$$

since the $2p_{1/2}$ level is completely filled with neutrons (Fig. 1b). On the other hand, the doorway state with $m_\tau = 0$ is different from zero, since the neutron transition is possible. We are dealing therefore with a transition belonging to the $2^{\underline{nd}}$ case studied in Section II.A. The supplementary condition is satisfied, since the doorway state with $m_\tau = -1$ is clearly seen to be different from the analog of the one with $m_\tau = 0$. Therefore, the isospin multiplets are the ones with $T = T_0 = 5$ and $T = T_0 - 1 = 4$. The non-zero doorway states $\left(a^{\dagger}_{p_{1/2}} \times b^{\dagger}_{p_{3/2}}\right)^{1m_\sigma}_{10} |P;2p\text{-}2h\rangle$ and $\left(a^{\dagger}_{p_{1/2}} \times b^{\dagger}_{p_{3/2}}\right)^{1m_\sigma}_{1-1} |P;2p\text{-}2h\rangle$ are shown in Figs. 3c and 3d, respectively. All norms of interest are given in Table 4.

(iv) Transition $1g_{9/2} \rightarrow 1g_{7/2}$

In this case the doorway state with $m_\tau = +1$ is non-zero, since there are protons in the $g_{9/2}$ level and no neutrons in the $g_{7/2}$ level (Fig. 1b). The transition belongs, therefore, to the 3rd case of Section II.A. The supplementary conditions are fulfilled, since: i) the doorway state with $m_\tau = 0$ is obviously different from the analog of the one with $m_\tau = +1$ and ii) as can be easily seen, the doorway state with $m_\tau = -1$ cannot be written as a linear combination of the double analog of the doorway state with $m_\tau = +1$ and the analog of the one with $m_\tau = 0$. As a consequence, the three isospin multiplets, i.e., those with $T = 4$, 5 and 6, appear. The doorway states $\left(a^\dagger_{g_{7/2}} \times b^\dagger_{g_{9/2}}\right)^{1m_\sigma}_{11} |P;2p\text{-}2h\rangle$, $\left(a^\dagger_{g_{7/2}} \times b^\dagger_{g_{9/2}}\right)^{1m_\sigma}_{10} |P;2p\text{-}2h\rangle$ and $\left(a^\dagger_{g_{7/2}} \times b^\dagger_{g_{9/2}}\right)^{1m_\sigma}_{1-1} |P;2p\text{-}2h\rangle$ are shown in Figs. 3e-3g. Norms are given in Table 4.

To end this section we discuss the computation of the unperturbed distribution of Gamow-Teller strength based on each of the two parent configurations in ^{90}Zr mentioned above. Quite generally, the transition strength from the parent state $|P;T_0T_0\rangle$ to the final state $|f;T_0,T_0+m_\tau\rangle$, in the residual nucleus, is defined as

$$S_{m_\tau}(f;T) = |\langle f;T,T_0+m_\tau \| GT^1_{1m_\tau} \| P;T_0T_0\rangle|^2 \quad , \tag{58}$$

where the matrix element is reduced with respect to angular

momentum only. We are considering the case of a parent nucleus with angular momentum $J_0 = 0$. Otherwise, for unpolarized targets, the above expression should be divided by $2J_0+1$. The total transition strength is then given by

$$S_{m_\tau} = \sum_{fT} S_{m_\tau}(f;T) \quad . \tag{59}$$

It can be easily shown that it satisfies the following non-energy-weighted sum rule

$$S_{-1} - S_{+1} = \frac{3}{2}(N-Z) \quad . \tag{60}$$

This is exact and should hold even in an approximate calculation as long as the model space for the residual nucleus contains all the doorway states as in the present case. More especifically, it must contain the state resulting from the action of the Gamow-Teller operator, Eq. (47), on the parent state. Since our interest here is merely to illustrate our method we shall not worry about the well-known problem of the quenching of the Gamow-Teller strength.

In an unperturbed calculation we take as final states of the residual nucleus their good isospin states obtained as discussed in Section II.A. That is, we write for each transition $s \rightarrow r$

$$|f;T,T_0+m_\tau\rangle = \frac{1}{\sqrt{\eta(s\to r;T)}}\,|s\to r;T,T_0+m_\tau\rangle \quad , \qquad (61)$$

where the state vector on the right is constructed as shown in Eq. (41) and the factor in front of it takes care of normalization. One gets, for the unperturbed transition strengths,

$$S^{(0)}_{m_\tau}(s\to r;T) = \delta_{n_r n_s}\,\delta_{\ell_r \ell_s}\,3(2j_r+1)(2j_s+1)W(\tfrac{1}{2}\,1\,\ell_r\,j_r;\tfrac{1}{2}\,j_s)^2\,\cdot$$

$$\cdot\,(1\,T_0\,m_\tau\,T_0|T,T_0+m_\tau)^2\,\eta(s\to r;T) \quad . \qquad (62)$$

The results of this calculation, taking the closed subshells and the 2p-2h configurations in ^{90}Zr as parent states, are listed in Tables 5 and 6, respectively. One can check that the sum-rule, Eq. (60), is satisfied in both cases. For a parent state of the general form

$$|P\rangle = a|P;0\rangle + b|P;2p\text{-}2h\rangle \quad , \qquad (63)$$

the unperturbed transition strengths would be given by the averages of the corresponding results for each of the two parent-configurations with weights $|a|^2$ and $|b|^2$.

IV. SUMMARY AND CONCLUSIONS

We applied the method based on the tensor coupling

of the excitation operators to the parent isospin multiplet[5] to the study of isovector excitations in $N \neq Z$ nuclei. This approach is completely general, since no restriction is made on the nature either of the parent state or of the excitation operator. We have shown that this method applies even when some of the possible excited isospin multiplets do not occur. What happens in such cases is that some of the states $O_{1m_\tau}|P;T_0M_{T_0}\rangle$, Eq. (3), are either linearly dependent or have zero norm. (See discussion of the 1st and 2nd cases in Section II.A). However, the good isospin states are correctly given by Eq. (2) in all cases. We give general criteria, which depend only on the properties of the doorway states with respect to isospin, that make it possible to decide, in each specific case, which excited isospin multiplets will effectively appear. We also show how to express the relevant matrix elements of operators of physical interest in terms of excitations of the parent state only, and give explicit expressions for the geometrical coefficients needed for the computation of these matrix elements.

Other methods for the treatment of isovector excitations, which adopt a different point of view, have already appeared in the literature[3,4]. However, they can be used only for parent states with closed subshells and excitation operators of particle-hole type, in which case the results are identical to ours[7]. Nonetheless, despite the general character of our method, we find it conceptually more transparent and computationally simpler to use.

In the interest of clarity, we limited our considerations here to isovector excitations. However, the treatment of the isospin degree of freedom for isoscalar excitations is trivial, as is clear from our discussion. Therefore, excitations which are a mixture of isovector and isoscalar components would present no problem to the application of our method. It is also clear how to generalize it to include isotensor excitations of higher order, which would be of relevance, for instance, to the study of Δ-hole excitations(3).

As a simple illustration of the power of our method we applied it to the discussion of the Gamow-Teller transition strength in ^{90}Zr, including in the parent state a configuration which is not of the closed subshells type.

This paper is based on a master thesis submitted by DPM to the University of São Paulo in 1986.

REFERENCES AND NOTES

1. A. Bohr and B. Mottelson, Nuclear Structure (Benjamin, New York, 1968), Vol. I.
2. W.A. Sterrenburg, Phys. Rev. Lett. 45, 1839 (1980) and references therein.
3. H. Toki, Phys. Rev. C26, 1256 (1982).
4. F. Krmpotić, C.P. Malta, K. Nakayama and V. Klemt, Phys. Lett. 149B, 1 (1984). We compare our results with the TDA limit of this paper.
5. D.J. Rowe and C. Ngo-Trong, Rev. Mod. Phys. 47, 471 (1975).
6. F. Krmpotić and F. Oesterfeld, Phys. Lett. 93B, 218 (1980).
7. D.P. Menezes, Master thesis, Universidade de São Paulo, Brazil, 1986.

FIGURE CAPTIONS

Fig. 1 - Configurations for the ground of ^{90}Zr. Particles are represented by crosses (x) and holes by open circles (o). A bar connecting two particles or two holes indicates that the pair is coupled to $J^{\pi} = 0^{+}$. For each such pair a normalization factor equal to $1/\sqrt{2}$ is understood.

Fig. 2 - Doorway states for the Gamow-Teller and related transitions based on the closed subshells parent configuration, $|P;0\rangle$. The single particle levels are the same as those of Fig. 1. Each figure corresponds to a state written down in second quantized form with particle and hole creation operators appearing in the same order as the corresponding crosses and open circles are read from the figure, i.e., from left to right and from top to bottom.

Fig. 3 - Doorway states for the Gamow-Teller and related transitions based on the 2 particle-2 hole parent configuration $|P; 2p\text{-}2h\rangle$. The single particle levels and conventions are the same as those of Figs. 1 and 2.

Table 1 - Scheme for determination of existing excited isospin multiplets. In all cases $O_{1-1}|P\rangle \neq 0$, where $|P\rangle$ is the parent state.

Case	Defining conditions	Equivalent conditions	Existing excited isospin multiplets	Required supplementary condition
1rst	$T_{+}O_{1-1}|P\rangle = 0$	$O_{10}|P\rangle = 0$	T_0-1	-
2nd	$T_{+}O_{1-1}|P\rangle \neq 0$	$O_{10}|P\rangle \neq 0$	T_0	-
	$T_{+}^{2}O_{1-1}|P\rangle = 0$	$O_{11}|P\rangle = 0$	T_0-1	$O_{1-1}|P\rangle \neq \frac{\hat{T}_{-}}{\sqrt{2}\,T_0} O_{10}|P\rangle$
3rd	$T_{+}O_{1-1}|P\rangle \neq 0$	$O_{10}|P\rangle \neq 0$	T_0+1	-
	$T_{+}^{2}O_{1-1}|P\rangle \neq 0$	$O_{11}|P\rangle \neq 0$	T_0	$O_{10}|P\rangle \neq \frac{T_{-}}{\sqrt{2}\,(T_0+1)} O_{11}|P\rangle$
			T_0-1	$O_{1-1}|P\rangle \neq \frac{T_{-}}{\sqrt{2}\,T_0} O_{10}|P\rangle - \frac{T_{-}^{2}}{2T_0(2T_0+1)} O_{11}|P\rangle$

Table 2 - Values of the transformation coefficients $P_T(1T_0; 1T_0; m_\tau)$.

T \ m_τ	+1	0	−1
T_0+1	1	0	0
T_0	$-\frac{1}{T_0}$	$\frac{T_0+1}{T_0}$	0
T_0-1	$\frac{1}{T_0(2T_0-1)}$	$-\frac{2T_0+1}{T_0(2T_0-1)}$	$\frac{2T_0+1}{2T_0-1}$

Table 3 - Classification of Gamow-Teller transitions in ^{90}Zr based on $|P;0\rangle$ according to isospin structure following the discussion of Section II.A. Also tabulated are the squared norms of the doorway states, $\xi(m_\tau)$, and of the good isospin states, $\eta(T)$.

Transition	Values of $\xi(m_\tau)$			Values of $\eta(T)$			Case
	m_τ			T			
	−1	0	+1	4	5	6	
$1g_{9/2} \rightarrow 1g_{9/2}$	1	0	0	$\frac{11}{9}$	0	0	1rst
$1g_{9/2} \rightarrow 1g_{7/2}$	1	$\frac{1}{2}$	0	$\frac{11}{10}$	$\frac{3}{5}$	0	2nd

Table 4 - Classification of Gamow-Teller transitions in ^{90}Zr based on $|P; 2p\text{-}2h\rangle$ according to isospin structure following the discussion of Section II.A. Also tabulated are the squared norms of the doorway states, $\xi(m_\tau)$, and of the good isospin states, $\eta(T)$.

Transition	Values of $\xi(m_\tau)$			Values of $\eta(T)$			Case
	m_τ			T			
	-1	0	+1	4	5	6	
$2p_{1/2} \to 2p_{1/2}$	1	0	0	$\frac{11}{9}$	0	0	1rst
$1g_{9/2} \to 1g_{9/2}$	$\frac{4}{5}$	0	0	$\frac{44}{45}$	0	0	1rst
$2p_{3/2} \to 2p_{1/2}$	1	$\frac{1}{2}$	0	$\frac{11}{10}$	$\frac{3}{5}$	0	2nd
$1g_{9/2} \to 1g_{7/2}$	1	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{27}{25}$	$\frac{17}{25}$	$\frac{1}{5}$	3rd

Table 5 - Unperturbed distribution of strength for Gamow-Teller and related transitions starting from $|P;0\rangle$ configuration of ^{90}Zr.

Transition $(s \to r)$	T	$S^{(0)}_{m_\tau}(s \to r; T)$		
		$m_\tau = -1$	$m_\tau = 0$	$m_\tau = +1$
$1g_{9/2} \to 1g_{9/2}$	4	$\frac{55}{9}$	-	-
$1g_{9/2} \to 1g_{7/2}$	4	8	-	-
	5	$\frac{8}{9}$	$\frac{40}{9}$	-
Total strength		15	$\frac{40}{9}$	0

Table 6 - Unperturbed distribution of strength for Gamow-Teller and related transitions starting from $|P;\ 2p\text{-}2h\rangle$ configuration of ^{90}Zr .

Transition $(s \rightarrow r)$	T	$S^{(0)}_{m_\tau}(s \rightarrow r; T)$		
		$m_\tau = -1$	$m_\tau = 0$	$m_\tau = +1$
$2p_{1/2} \rightarrow 2p_{1/2}$	4	$\frac{1}{3}$	-	-
$1g_{9/2} \rightarrow 1g_{9/2}$	4	$\frac{44}{9}$	-	-
$2p_{3/2} \rightarrow 2p_{1/2}$	4	$\frac{12}{5}$	-	-
	5	$\frac{4}{15}$	$\frac{4}{3}$	-
$1g_{9/2} \rightarrow 1g_{7/2}$	4	$\frac{432}{55}$	-	-
	5	$\frac{136}{135}$	$\frac{136}{27}$	-
	6	$\frac{8}{297}$	$\frac{8}{27}$	$\frac{16}{9}$
Total strength		$\frac{151}{9}$	$\frac{20}{3}$	$\frac{16}{9}$

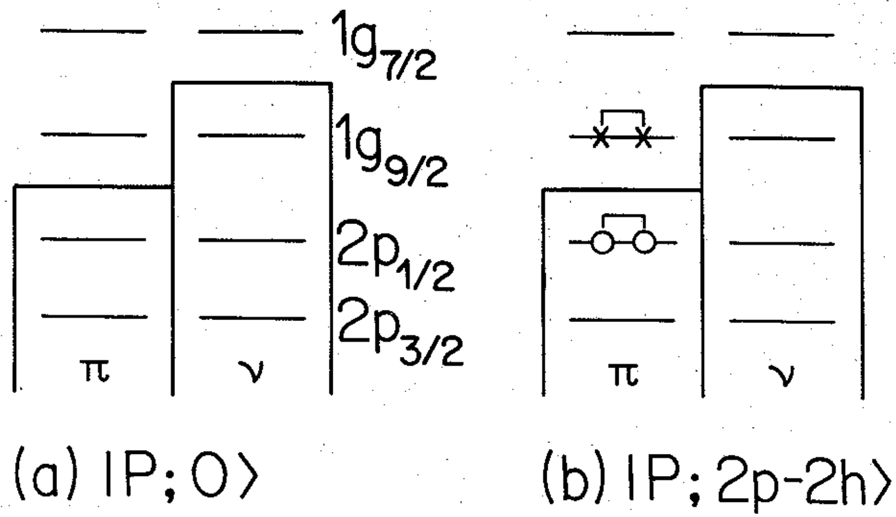

1g7/2
1g9/2
2p1/2
2p3/2
π
ν
(a) |P; 0⟩
(b) |P; 2p-2h⟩

Fig. 1

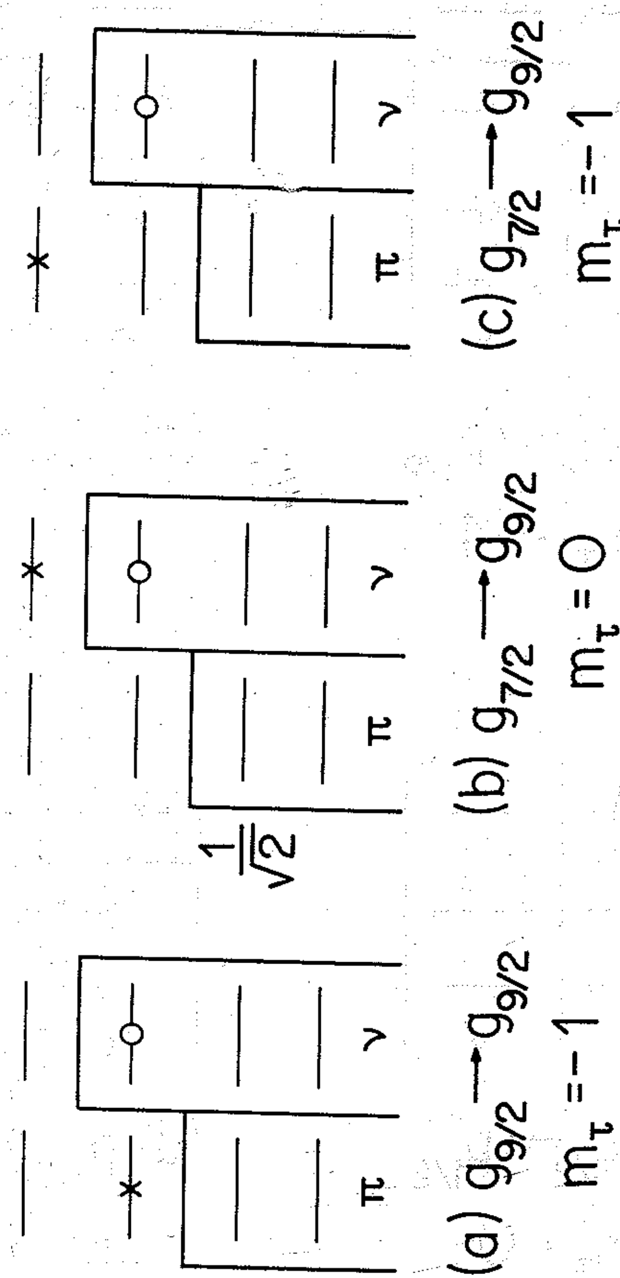
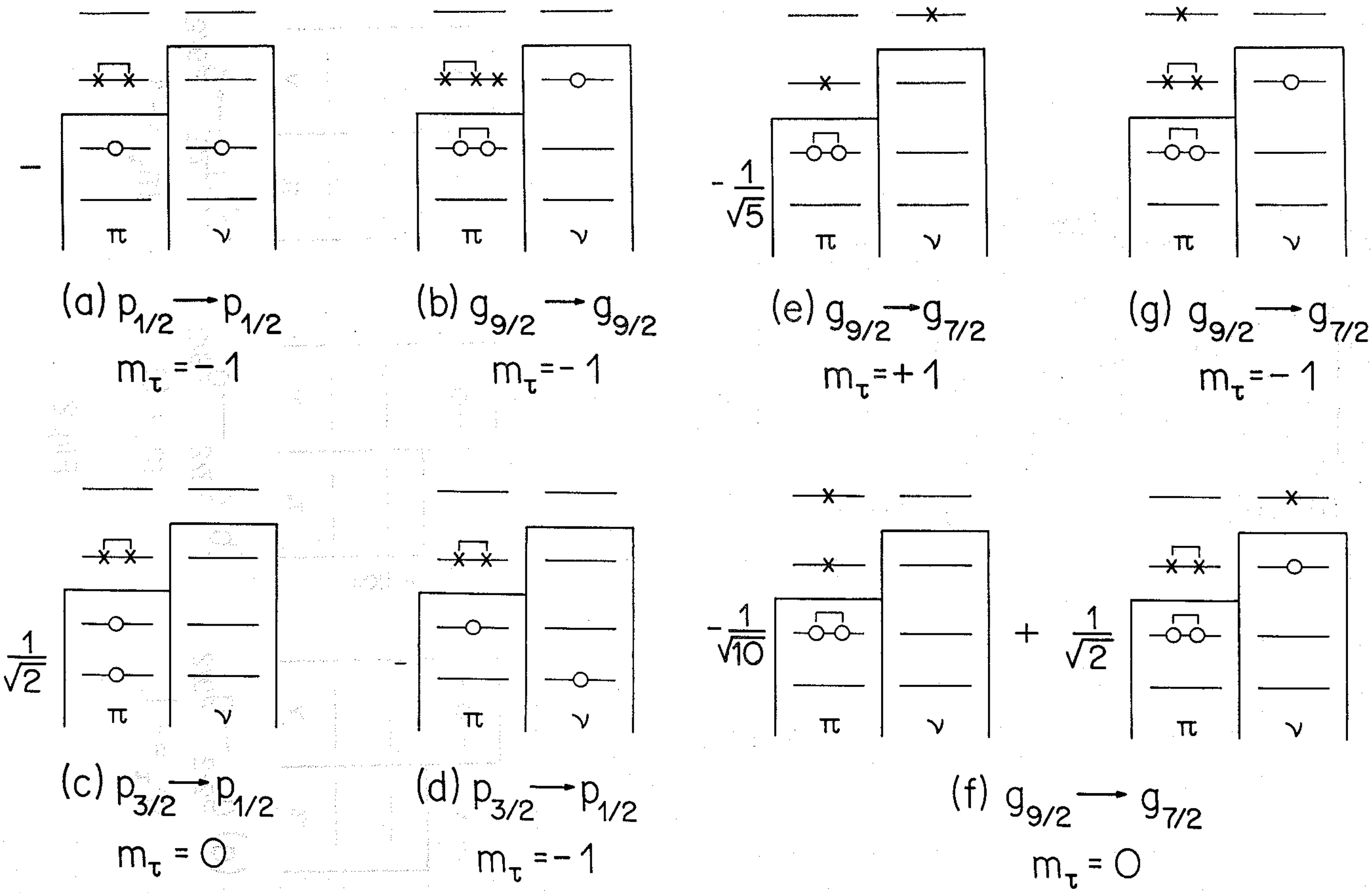

π
ν
1/√2
(a) g9/2 → g9/2
mτ = -1
(b) g7/2 → g9/2
mτ = 0
(c) g7/2 → g9/2
mτ = -1

Fig. 2



π
ν
(a) $p_{1/2} \rightarrow p_{1/2}$
$m_\tau = -1$
(b) $g_{9/2} \rightarrow g_{9/2}$
$m_\tau = -1$
$-\frac{1}{\sqrt{5}}$
(e) $g_{9/2} \rightarrow g_{7/2}$
$m_\tau = +1$
(g) $g_{9/2} \rightarrow g_{7/2}$
$m_\tau = -1$
$\frac{1}{\sqrt{2}}$
(c) $p_{3/2} \rightarrow p_{1/2}$
$m_\tau = 0$
(d) $p_{3/2} \rightarrow p_{1/2}$
$m_\tau = -1$
$-\frac{1}{\sqrt{10}}$
$+ \frac{1}{\sqrt{2}}$
(f) $g_{9/2} \rightarrow g_{7/2}$
$m_\tau = 0$

Fig. 3