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ABSTRACT

Large scale nuclear shell-model calculations for several nuclear systems are discussed. In particular, the statistical behaviour of the energy eigenvalues and eigenstates, are discussed. The chaotic behaviour of the NSMH is then shown to be quite useful in calculating the spreading width of the highly collective multipole giant resonances.

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Quantum chaos (Q.C.) has been the subject of intensive investigation in the last several years. A particularly convenient and well studied system which exhibits QC is the atomic nucleus[1]. Nuclear reactions involving long time delays invariably show chaotic behaviour exemplified through what is known Ericson fluctuations. This is seen both in neutron-induced and heavy nucleus-induced reactions on a variety of targets. In fact, as has been recently discussed by Weidenmüller[2], the S-matrix for compound nucleus scattering can be simulated and evaluated in a universal fashion, owing precisely to the fact that its fluctuation properties are generic, and quite independent of the specific dynamics of the system, a pre-requisite of a chaotic motion.

Statistical features of nuclear structure have also been widely discussed[3]. In fact at high enough excitation energies, it is expected that a theoretical description of nuclear spectra based on random matrix theory can be contemplated. Even the coherent quadrupole oscillations in nuclei is expected to undergo a transition to a chaotic behaviour at high excitation energies[4]. Since the underlying microscopic picture of nuclear structure is the shell model, one is naturally led to investigate its statistical properties. Put slightly differently, to what extent a realistic shell-model Hamiltonian can be viewed as a representation of a random ensemble? The answer to this question is two-fold important. First, at a fundamental physics level, it would supply one more important example of QC in a finite system. Secondly, from the practical nuclear physics point of view, it is important insofar as it leads to a great amount of simplification in the theoretical description of several nuclear observables. In particular, the calculation of the detailed structure of giant resonances can be greatly simplified[5].

Quite recently, Brown and Bertsch[6] (BB) have looked into the degree of statisticality of realistic nuclear shell model calculations in s-d nuclei. They found that the states in the region of high level density show characteristic Porter--Thomas (PT) behaviour. The criterion they used to establish the degree of randomness in the output of their calculation is based on the quantity

$$\lambda = \frac{\langle V \rangle_{off.}}{D} \tag{1}$$

with $\langle V \rangle_{off}$ denoting the average rms value of the off-diagonal matrix element and D the average level spacing. It was found by BB[6] that when $\lambda > 1$, the individual level positions lose memory of their location in the diagonal part of the Hamiltonian and accordingly the distribution is closest to a PT.

In the present contribution we extend the study of BB to larger dimensions and for a wider class of nuclear states. In table I we list the cases, with the corresponding dimensions.

The SM calculation was performed with the Wildenthal interaction. This interaction includes a mass dependence of the form $\langle V \rangle (A) = \langle V \rangle (A=18)(A/18)^{-0.3}$. The value of $\langle V \rangle_{off.}(A)$ for the cases studied are listed in the last column of Table I. These values are relevant for the determination of the λ's (Eq. 1) and subsequently testing the PT criterion of BB.

We have verified that the criterion used by BB, $\lambda \geq 1$, Eq. (1), to establish the closeness of the nuclear spectrum to a PT distribution is not sufficient. To demonstrate this we have performed a full shell-model calculation with the Oak-Ridge Code. In Fig. (1) we present the distribution of the eigenvalues obtained for $N = 109$ and $N = 517$. The first case in the one discussed by BB. As we see they both show Gaussian distributions, indicating that the central-limit theorem is operative here. We have checked that all other cases presented in Table I exhibit this behaviour. Armed with this conclusion we are now in a position to analyze the amplitude distribution. We present in Table II the result of this analysis. In the last column we tabulate a measure of the deviation of the calculated spectra from a PT distribution,

$$\Delta = \sum_{r=1}^{M} \left| P^{(r)}_{Calc.} - P^{(r)}_{PT} \right| \tag{2}$$

where M represents the total number of intervals used in the construction of the histograms, and the Porter-Thomas distribution is given as usual by

$$P_{PT} = \sqrt{\frac{2N}{\pi}}\, exp\left[-N\, |a_{j,i}|^2/2\right] \tag{3}$$

In the above expression, N is the dimension of the basis $|i\rangle$ used in the construction of the nuclear state $|j\rangle$, and $a_{j,i}$'s are the expansion amplitudes,

$$|j\rangle = \sum_i a_{j,i}\, |i\rangle \tag{4}$$

It is quite clear from Table II that a good measure of the adequacy of the PT description involves a close correlation between λ, Eq. (1), and Δ, Eq. (2). The good case presented by BB is that presented in the third line of Table II, with $\lambda = 2.63$ and $\Delta = 0.16$. We have verified that this situation involves an energy interval which corresponds to the maximum in the Gaussian distribution of the eigenvalues. On the other hand, other cases shown in Table II indicate that the correlation between <u>large λ</u> and <u>small Δ</u> is not always automatically guaranteed, and there is a strong dependence on the dimension of the basis. The important variable seems to be the energy--range. As long as the energy range is situated around the region of the maximum in the Gaussian distribution of the

eigenvalues (Fig. 1) then, in this range, no levels attain special characteristics, rendering the amplitudes quite random.

It would be interesting to discover the right combination of the parameters λ, Δ and the energy range which encompasses the region of maximum in the level distribution, such that the PT distribution approximation can be unambiguously and economically verified.

The above findings are relevant to the discussion of the decay of giant multipole resonances in nuclei, as has been recently demonstrated by Hussein[5]. In a recent theoretical development, Dias et al[8], have introduced a GR mixing parameter, μ, which measures the relative importance of "direct" and compound contributions to the decay of the GR. This parameter can be written as

$$\mu = \frac{\Gamma^{\downarrow}}{\Gamma^{\downarrow} + \Gamma^{\uparrow}} \tag{19}$$

with $\Gamma^{\uparrow}$ being the escape width and $\Gamma^{\downarrow}$ the damping width of the GR arising from the coupling between the 1p-1h, subspace on the one hand, and the more complex configurations (2p-2h, 3p-3h, etc.) on the other. Since $\Gamma^{\uparrow}$ can be calculated from continuum RPA[9], we concentrate our attention here on the evaluation of $\Gamma^{\downarrow}$, which is connected with the preceeding statistical considerations.

The simplest possible expression for $\Gamma^{\downarrow}$ is Fermi's Golden Rule's

$$\Gamma^{(J)\downarrow} \simeq 2\pi \sum_j |\langle GR^{(J)} | V | (2p-2h)_j^{(J)} \rangle|^2 \tag{5}$$

If we identify the $|(2p-2h)_j^{(J)}\rangle$ states with those of Eq. (4), we can then approximate the sum in Eq. (5) with

$$\Gamma^{(J)\downarrow} \simeq 2\pi \overline{|\langle GR^{(J)} | V | (2p-2h)^{(J)} \rangle|^2} \rho^{(J)}_{(2p-2h)} \tag{6}$$

where

$$\overline{|\langle GR^{(J)} | V | (2p-2h)^{(J)} \rangle|^2} = \sum \overline{|a_{j,i}|^2}\, \overline{|\langle GR | V | i \rangle|^2} \tag{7}$$

In the above equations, V represents the coupling interaction responsible for the GR fragmentation $\overline{|a_{j,i}|^2}$, the width of the PT distribution for the amplitudes, and $\rho^{(J)}_{2p-2h}$ is the 2p-2h density of states (with spin J). In obtaining Eq. (7), we have assumed that

$$\overline{a_{j,i}\, a^*_{j',i}} = \overline{|a_{j,i}|^2}\, \delta_{jj'} \tag{8}$$

consistent with a chaotic behaviour.

From the PT distribution, we have immediately

$$\overline{|a_{j,i}|^2} = \frac{1}{N} \tag{9}$$

Therefore we have for $\Gamma^{\downarrow(J)}$, the following simple estimate

$$\Gamma^{\downarrow(J)} \simeq 2\pi \frac{\sum_i |\langle GR^{(J)} | V | i^{(J)} \rangle|^2}{N} \rho^{(J)}_{2p-2h} \tag{10}$$

Eq. (10) should supply a convenient and simple way of obtaining an estimate for $\Gamma^{\downarrow}$, once the dimension of the basis is decided upon. In a future work we shall discuss the evaluation of Eq. (10). Here, we simply estimate $\Gamma^{\downarrow(J)}$, having in mind the A-dependence of the nondiagonal interaction $\langle V(A)\rangle_{off.} \sim A^{-0.3}$. Knowing $\overline{\langle GR|V|i\rangle}$ (A = 208) ≃ 0.02 MeV[10,11], we have for A = 18, $\overline{\langle GR|V|i\rangle}$ (A = 18) ~ 0.1 MeV. Thus $\sum_i |\langle GR|V|i\rangle|^2 \simeq N \frac{0.01}{A^{0.6}}$ MeV and, accordingly

$$\Gamma^{\downarrow(2)}_{(A)} \simeq \frac{0.6}{A^{0.6}} \rho^{(2)}_{2p-2h}(A) \times 10^{-1} \ \text{MeV} \tag{11}$$

The above formula is very close to the empirically derived formula of $\Gamma \sim 85\, A^{-2/3}$ [12] if the density of states $\rho^{(2)}_{2p-2h}(A)$ is taken to be $\sim 1.14 \times 10^3$ MeV^{-1}. In fact this seems to be quite reasonable in, e.g., ^{24}Mg, where at E* = 23 MeV, the experimentally determined density of states is about 1,000 MeV^{-1} [13].

In conclusion, we have analysed in this paper the statistical properties of the realistic nuclear shell model Hamiltonian. We have verified that a Porter-Thomas distribution of the basis amplitudes can be assigned as long as the energy interval considered encompasses the region of the maximum in the spectrum distribution. The results of our analyse is then applied to estimate the spreading width of giant multipole resonances. In particular, for the giant quadrupole resonance, our expression for $\Gamma^{\downarrow}$ comes out very close to the empirically determined formula $\Gamma \sim 85\ A^{-2/3}$ MeV.

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TABLE CAPTIONS

Table I - Cases studied in the present contribution. The column $\langle V \rangle_{off}$ denotes the average rms value of the off-diagonal matrix element.

Table II - Results of our analysis for different cases in differents energy intervals. The parameters λ and Δ are explained fully in the text.

TABLE I

Number of Nucleons in sd Shell	Spin (J^{π})	Isospin	Dimension	Number of States	$\langle V \rangle_{off}$ [Mev]
5	$1/2^+$	1/2	109	109	0.64
7	$1/2^+$	1/2	517	200	0.62
7	$3/2^+$	1/2	923	200	0.62
7	$5/2^+$	1/2	1142	200	0.62
8	2^+	0	1206	200	0.61
10	1^+	0	1753	200	0.60
10	1^+	1	3011	200	0.60

TABLE II

Number of Nucleons	Energy Interval [Mev]	Number of States	λ	Δ [%]
5	0 - 12.87	10	0.50	44
5	14.17 - 17.47	10	1.93	34
5	20.17 - 27.47	30	2.63	16
5	38.77 - 54.57	10	0.40	44
7	0 - 15.32	24	0.97	50
7	15.32 - 20.53	39	4.64	34
7	20.53 - 25.48	55	6.88	23
7	25.48 - 30.62	82	9.89	15
7	0 - 10	11	0.68	62
7	10 - 15	29	3.60	46
7	25 - 20	61	7.56	35
7	20 - 25	99	12.28	24
7	0 - 7.4	7	0.58	65
7	7.4 - 12.4	20	2.48	51
7	12.4 - 17.4	52	6.45	42
7	17.4 - 23.14	121	13.07	29
8	0 - 11.51	7	0.37	65
8	11.51 - 16.51	20	2.44	54
8	16.51 - 21.51	45	5.49	44
8	21.51 - 28.32	128	11.46	30
10	0 - 7.81	13	0.99	63
10	7.81 - 12.82	24	2.87	50
10	12.82 - 17.94	57	6.69	44
10	17.94 - 23	106	12.57	34
10	0 - 13.91	27	1.16	66
10	13.91 - 16.82	32	6.59	54
10	16.82 - 19.76	56	11.42	48
10	19.76 - 23.23	84	14.52	41

FIGURE CAPTIONS

Fig. 1 - Calculated spectral histogram for $(sd)^5$ case (a) and $(sd)^7$ case (b). See text for details.

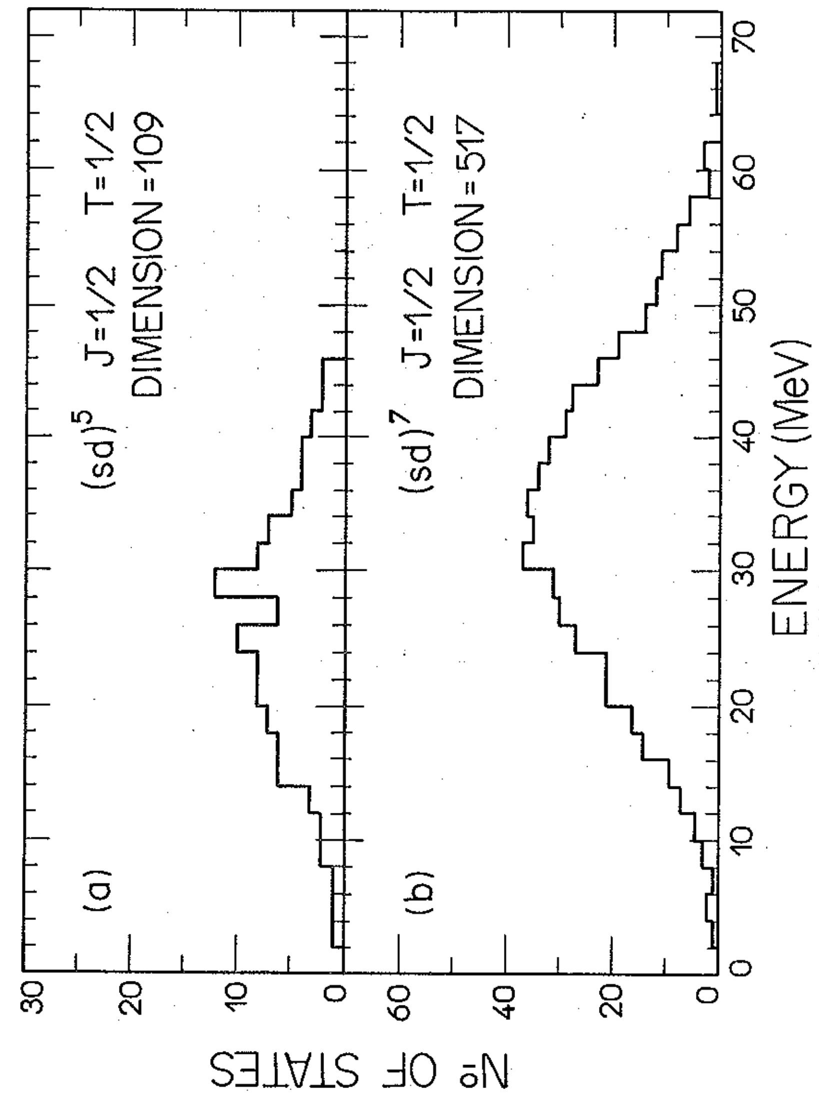

Fig. 1