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HADRONS, THE SIMPLEST GENTILIONIC SYSTEMS?

M. Cattani
Instituto de Física, Universidade de São Paulo

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M. Cattani

Instituto de Física da Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brasil

SUMMARY

Basic quantum mechanical properties of systems constituted by two and three gentileons are deduced in this paper. By using Pauli's theorem and symmetry properties of the intermediate states it is shown that, in some cases, gentileons must have half-odd-integral spin. As an immediate and natural result of our theoretical analysis, we show how fundamental observed properties of composed hadrons can be predicted from first principles assuming quarks as spin 1/2 gentileons.

1. INTRODUCTION

In a recent paper(1) we have shown, according to the postulates of quantum mechanics and to the principles of indistinguishability, that three kinds of particles could exist in nature: bosons, fermions and gentileons. These results can be synthesized in terms of the following statement (Statistical Principle): "Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively".

Bosonic and fermionic systems are described by one-dimensional totally symmetric (ψ_S) and totally anti-symmetric (ψ_A) wavefunctions, respectively. For bosons and fermions the creation and annihilation operators obey bi-linear commutation relations.

Gentilionic systems are described by multi-dimensional (spinorial-like) wavefunctions (Y) with mixed symmetries. Since they are represented by intermediate Young shapes, only three or more identical gentileons can form a system of indistinguishable particles. This means that two identical gentileons are prohibited to constitute a system of indistinguishable particles. This suggests that gentileons cannot appear freely. Indeed, if this were possible, two free identical gentileons could constitute a two-particle system in an occasional collision. For gentileons the creation and annihilation operators obey multi-linear matricial commutation relations. Finally, due to very peculiar geometric properties[1] of the intermediate states, there appear selection rules confining the gentileons and prohibiting the coalescence of gentilionic systems. Two systems like [ggg] and [gggg], for instance, cannot coalesce into a composite system of indistinguishable particles [ggggggg]. Only bound states [ggg] - [gggg] could be possible. The gentileon confinement appears as a consequence of the selection rule which prohibits the decomposition of a system [ggg...gg] into [ggg...g] and [g].

In our above quoted paper[1] only systems of identical gentileons have been considered. Let us now consider systems composed of two different kinds of gentileons, g and G. Taking into account the statistical principle we must expect that systems like [gG], [gggG], [gggGGG] and so on, are allowed. On the other hand, systems like [ggG], [ggGG], [gggGG]... are prohibited because [gg] and [GG] are not allowed[1]. Of course, the coalescence of mixed systems is also forbidden, as can be easily verified.

As well known, half-odd-integral and integral spin particles are described, from the point of view of the Lorentz group, by spinorial and tensorial irreducible representations, respectively. From the point of view of the permutation group S_n, particles are described by bosonic, fermionic and gentilionic irreducible representations. According to the celebrated Pauli's theorem[2-4], if creation and annihilation particle operators obey bi-linear commutative (anti-commutative) relations

these particles have integral (half-odd-integral) spin. By using bi-linear bosonic and fermionic relations, consistent local, Lorentz-invariant quantum field theories are developed. In section 3 we show that, for a system composed by three identical gentileons, the tri-linear matricial relations can be reduced to bi-linear fermionic relations.

The confinement and non-coalescence are intrinsic properties of gentileons as the total symmetrization (anti-symmetrization) is intrinsic to bosons (fermions), not depending on their physical interpretation. Thus, they could be assimilated to real particles or to dynamical entities as quantum collective excitations.

The total anti-symmetry of the fermionic state function is responsible for the exclusion effect and, consequently, responsible for the stability of matter on a large scale. Similarly, the spinorial character of the intermediate states would be responsible for confinement of gentileons and non-coalescence of gentilionic systems. As well known, the range of the exclusion effect mechanism depends on the fermionic system. Analogously, the dimension of the gentilionic system, which would be governed by the confinement and non-coalescence mechanisms, would depend on the gentilionic system. Gentileons being, for instance, noninteracting collective excitations in crystals or in complex molecules would have very different confinement volumes. On the other hand, if quarks are gentileons[5,6] they would be interacting entities confined within hadronic dimensions.

In section 2 we present a detailed study of the symmetry properties of the three gentileons state vector Y(123). We have emphasized the simplest non-trivial case of three particles aiming to apply the theory to the description of SU(3) model of strong interactions[5,6].

In section 3, commutation relations of gentilionic systems composed by three identical gentileons are analysed in order to establish a connection between spin and statistics[2-4]. When two gentileons cannot occupy the same quantum state it is shown, by using symmetry properties of the intermediate states and Pauli's theorem, that gentileons must have half-odd-integral spin.

In section 4 we show that the $SU(3)_{color}$ representation can be naturally incorporated into the gentilionic symmetry.

In section 5 our theoretical results are applied to investigate some aspects of the hadronic physics. Assuming quarks as spin 1/2 gentileons, basic observed properties of composed hadrons are predicted from first principles. It will be seen that the color quantum number plays a fundamental role in the gentilionic theory of hadrons. In terms of the color quantum number, hadrons can be divided into two classes: colored (quarks) and uncolored (baryons and mesons). Gentileons are colored particles, bosons and fermions are uncolored particles.

In section 6, taking quarks as spin 1/2 gentileons, a quantum chromodynamics is proposed where, instead of fermions, gentileons interact with gluons.

2. SYMMETRY PROPERTIES OF THE GENTILIONIC STATE VECTOR Y(123)

We present in this section a detailed study of the symmetry properties of the state vector Y(123) of a system composed by three indistinguishable gentileons. This simplest three particles case (N=3) has been emphasized in order to apply the theory to the description of SU(3) models for strong interactions. Of course, it is possible to extend our results, concerning the structure of Y, for $N > 3$, at the expenses of unnecessary labour and non essential complications for our immediate purposes. Thus, according to our general results[1], the symmetry properties of Y(123) is completely described in terms of the three quantum states α, β and γ. In analogy with the electromagnetic color theory these states will be named "primary colors". In terms of the colors α, β and γ, $Y(123) = Y(\alpha\beta\gamma)$ is written as[1]:

$$Y(\alpha\beta\gamma) = Y(123) = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_1(123) \\ Y_2(123) \\ Y_3(123) \\ Y_4(123) \end{pmatrix} = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \tag{2.1}$$

where,

$Y_1(123) = (|\alpha\beta\gamma\rangle + |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle - |\gamma\beta\alpha\rangle)/\sqrt{4}$,

$Y_2(123) = (|\alpha\beta\gamma\rangle + 2|\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle + |\gamma\alpha\beta\rangle - 2|\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{12}$,

$Y_3(123) = (-|\alpha\beta\gamma\rangle + 2|\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + 2|\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{12}$,

and $Y_4(123) = (|\alpha\beta\gamma\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + |\gamma\beta\alpha\rangle)/\sqrt{4}$. The state Y is decomposed into two parts, $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$, where $Y_+ = \frac{1}{\sqrt{4}}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ and $Y_- = \frac{1}{\sqrt{4}}\begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}$, corresponding to the duplication of the states implied by the reducibility of our representation in the intermediate gentilionic states[1,5,6]. We shall show, in what follows, that Y_+ and Y_- have a spinorial character[5,6], resulting a "bi-spinorial" character in Dirac's sense for Y(123)[7]. The probability density function for Y(123) is given by the permutation invariant function[1] $Y^+Y = |Y|^2 = (|Y_1|^2 + |Y_2|^2 + |Y_3|^2 + |Y_4|^2)/4$. The bi-spinorial character of Y(123) is responsible for selection rules[1] predicting: (1) gentileon confinement and (2) non-coalescence of gentilionic systems. It is worthwhile to note that, in this context, our theory differs drastically from parastatistics[8-12] and fermionic theories. In the case of fermions, the three particles state function $\psi(123)$ would be given by $\psi_A(123) = (|\alpha\beta\gamma\rangle - |\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle + |\gamma\alpha\beta\rangle + |\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{6}$ and for paraparticles $\psi(123)$ would be written as, $\psi_P(123) = a\,Y_1 + b\,Y_2 + c\,Y_3 + d\,Y_4$, where a, b, c and d are arbitrary constants. For these theories the wavefunction $\psi(123)$ is one-dimensional,

from which the selection rules (1) and (2), above mentioned, cannot be deduced.

Our intention, in this section, is to show explicitely the spinorial character of Y(123) and to establish fundamental properties of the gentilionic system that can be deduced from this spinorial character. In this way we must remember that, due to the six permutation operators of the symmetric group S_3, the Y(123) is transformed to[1]:

$$\begin{pmatrix} Y'_+ \\ Y'_- \end{pmatrix} = \begin{pmatrix} \eta_j & 0 \\ 0 & \eta_j \end{pmatrix} \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \qquad (2.2)$$

where η_j $(j = 1, 2, \ldots, 6)$ are 2×2 matrices given by:

$$\eta_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \quad ; \quad \eta_2 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \quad ;$$

$$\eta_3 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} \quad ; \quad \eta_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \qquad (2.3)$$

$$\eta_5 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \quad \text{and} \quad \eta_6 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} .$$

From the point of view of group representation theory, Eq. (2.2) immediately suggests the reducibility of the intermediate representation. Due to the separation of Y into two components, Y_+ and Y_-, an interpretation of these objects is claimed.

Let us show that it is possible to interpret the transformations of Y_+ and Y_- in terms of rotations of an equilateral triangle in an particular Euclidean space E_3. That is, we assume E_3 as a space where the color states are defined by three orthogonal coordinates (X,Y,Z). Due to this assumption, this space will be named "color space". It is also assumed that, in this color space, the colors α, β and γ occupy the vertices of an equilateral triangle taken in the (X,Y) plane, as seen in Fig. 1. The unit vectors along the X, Y and Z axes are indicated, as usually, by $\vec{i}$, $\vec{j}$ and $\vec{k}$. In Fig. 1, the unit vectors $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ are given by, $\vec{m}_4 = -\vec{k}$, $\vec{m}_5 = -(\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$ and $\vec{m}_6 = (\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$, respectively.

(INSERT FIGURE 1)

We represent by $Y(123)$ the state whose particles 1, 2 and 3 occupy the vertices α, β and γ, respectively. Thus, we see that the true permutations, (312) and (231), are obtained from (123) under rotations by angles $\vartheta = \pm 2\pi/3$ around the unit vector $\vec{j}$. As one can easily verify, the matrices η_2 and η_3, that correspond to these permutations are represented by:

$$\eta_2 = -I/2 + i(\sqrt{3}/2)\sigma_y = \exp[i\vec{j}.\vec{\sigma}(\vartheta/2)] \quad \text{and}$$
$$\eta_3 = -I/2 - i(\sqrt{3}/2)\sigma_y = \exp[i\vec{j}.\vec{\sigma}(\vartheta/2)] \tag{2.4}$$

where the σ_x, σ_y and σ_z are Pauli matrices.

Similarly, the transpositions (213), (132) and (321) are obtained under rotations by angles $\Phi = \pm\pi$ around the axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. The corresponding matrices are given by:

$$\eta_4 = \sigma_z = i\exp[i\vec{m}_4 \cdot \vec{\sigma}(\Phi/2)] \quad ,$$
$$\eta_5 = (\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_5 \cdot \vec{\sigma}(\Phi/2)] \quad \text{and} \tag{2.5}$$
$$\eta_6 = -(\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_6 \cdot \vec{\sigma}(\Phi/2)] \quad .$$

According to our preceding paper[5], there is an algebraic invariant, $K^{[2,1]}_{(2,1)}$, with a zero eigenvalue, associated with the S_3 gentilionic states. In analogy with continuous groups, this invariant will be named "color Casimir"[5]. For permutations, that are represented by matrices with $\det = +1$, the invariant is given by $K_{rot} = \eta_1 + \eta_2 + \eta_3$. For transpositions, which matrices have $\det = -1$, it is defined by $K_{inv} = \eta_4 + \eta_5 + \eta_6$. Taking into account $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ and Eqs. (2.5) we see that, $K_{inv} = \eta_4 + \eta_5 + \eta_6 = (\vec{m}_4 + \vec{m}_5 + \vec{m}_6) \cdot \vec{\sigma} = 0$. This means that the invariant K_{inv} can be represented geometrically, in the plane (X,Y) of the color space, by $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$, and that the equilateral triangle symmetry of the S_3 representation is an intrinsic property of $K_{inv} = 0$.

Eqs. (2.4) and (2.5) suggest a spinorial interpretation for Y_+ and Y_-. Here, starting from a general standpoint, we show the correctness of this contention. It is well known that the non-relativistic spinor can be introduced in several ways[13]. The interrelation of the various approaches is not ovbious and can lead to misconceptions. In order to overcome the necessity of enumerating several approaches, let us stick on a geometrical image, recalling the very fundamental result ou group isomorphism[14]: $S_3 \sim PSL_2(F_2)$, where $PSL_2(F_2)$ is the projective group associated with the special group SL_2 defined over a field F_2 with only two elements. Obviously, $PSL_2(F_2) \sim SL_2(F_2)/SL_2(F_2) \cap Z_2$, where the group in the denominator is the centre of SL_2 and corresponds to the central homotheties, since Z_2 is the intersection of the collineation group with SL_2.

If we consider the matrices (2.3) as representing transformations in a two-dimensional complex space characterized by homogeneous coordinates Y_1 and Y_2,

$$\begin{pmatrix} Y_1' \\ Y_2' \end{pmatrix} = \frac{1}{\rho}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \qquad (2.6)$$

where ρ is an arbitrary complex constant and the latin letters substitute the coefficients taken from (2.3), it is clear that (2.3) constitute a homographic (or projective) group.

Making use of definition (2.6) we can see from (2.3) that, apart from the identity η_1, the two matrices η_2 and η_3, which have $\det = +1$, are elliptic homographies with fixed points $\pm i$. If we translate these values for the variables of E_3, we see that η_2 and η_3 correspond to finite rotations around de $\vec{j}$ axis by an angle $\vartheta = \pm 2\pi/3$, agreeing thus with Eqs. (2.4). The remaining matrices η_4, η_5 and η_6 are elliptic involutions, with $\det = -1$. They correspond to space inversions in E_3, considered as rotations of $\pm\pi$ around the three axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. These matrices completely define the axis of inversion and the angle $\pm\pi$, as is seen from Eqs. (2.5). It is an elementary task to establish the correspondence, via stereographic projection, between the transformations in the two spaces, $Y_+(Y_-)$ and E_3.

A topological image can help us to see the 4π invariance of Y_+ and Y_-. If we consider the rotation angle $\vartheta(\Phi)$ as the variable describing an Euclidean disc, the covering space associated to this disc is a Moebius strip[15]. Adjusting correctly the position of the triangles we can have a vivid picture of the rotation properties for each axis. This construction allow us to visualize the double covering of the transformation in E_3 and is a convincing demonstration of the spinorial link between E_3 and $Y_{\pm}$.

From this analysis we conclude that Y_+ and Y_- are spinors. As one can easily verify(7) by using the projective geometry, the four-dimensional state function $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$ is a "bi-spinor" in Dirac's sense. Since E_3 is a "color space", Y_+ and Y_-, in analogy with the isospinor in the isospace, will be named "colorspinor".

We observe that the same transformation properties of Y_+ and Y_- can be obtained if, instead of the equilateral triangle shown in Fig. 1, we consider the triangle drawn in Fig. 2.

(INSERT FIGURE 2)

In the vertices of the equilateral triangle of the Fig. 2 we have the colors $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. The unit vectors $\vec{m}_4^*$, $\vec{m}_5^*$ and $\vec{m}_6^*$ are given by, $\vec{m}_4^* = -\vec{m}_4$, $\vec{m}_5^* = -\vec{m}_5$ and $\vec{m}_6^* = -\vec{m}_6^*$. This means that, in this case, K_{inv} is represented geometrically by $\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0$. This two fold possibilities for depicting the triangle will be physically interpreted, in the next sections, in terms of the existence of colors and anti-colors.

3. SPIN AND STATISTICS

In this section the commutation relations of gentilionic systems composed by three identical gentileons are analysed in order to establish a connection between spin and statistics(2-4). We show that when two gentileons cannot occupy the same quantum state the matricial relations(1) are reduced to bi-linear fermionic relations. Indeed, when two gentileons cannot occupy the same quantum state, that is, when $\alpha \neq \beta \neq \gamma \neq \alpha$, we see that the gentilionic commutation relations are given by(1):

$$[a_i^*, a_j]_+ = \delta_{ij} \quad , \quad a_i a_i = a_i^* a_i^* = 0$$

$$a_i a_j a_k = a_\alpha a_\beta a_\gamma \, G\binom{kji}{\gamma\beta\alpha} \quad \text{and} \qquad (3.1)$$

$$a_i^* a_j^* a_k^* = G\binom{\alpha\beta\gamma}{ijk} a_\alpha^* a_\beta^* a_\gamma^* \quad ,$$

where the indices i, j and k can assume the values α, β and γ and $G(\ldots)$ are 4×4 matrices given elsewhere(4). From the above tri-linear relations one can deduce the bi-linear relations, seen below applied on gentilionic states Y,

$$
\begin{aligned}
a_\beta\, a_\alpha\, Y(\alpha\beta\gamma) &= Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\alpha\beta\gamma) &= G\binom{\alpha\beta\gamma}{\beta\alpha\gamma}\, Y(00\gamma)\\
a_\beta\, a_\alpha\, Y(\alpha\gamma\beta) &= G\binom{\alpha\gamma\beta}{\alpha\beta\gamma}\, Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\alpha\gamma\beta) &= G\binom{\alpha\gamma\beta}{\beta\alpha\gamma}\, Y(00\gamma)\\
a_\beta\, a_\alpha\, Y(\beta\alpha\gamma) &= G\binom{\beta\alpha\gamma}{\alpha\beta\gamma}\, Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\beta\alpha\gamma) &= Y(00\gamma)\\
a_\beta\, a_\alpha\, Y(\gamma\alpha\beta) &= G\binom{\gamma\alpha\beta}{\alpha\beta\gamma}\, Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\gamma\alpha\beta) &= G\binom{\gamma\alpha\beta}{\beta\alpha\gamma}\, Y(00\gamma)\\
a_\beta\, a_\alpha\, Y(\beta\gamma\alpha) &= G\binom{\beta\gamma\alpha}{\alpha\beta\gamma}\, Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\beta\gamma\alpha) &= G\binom{\beta\gamma\alpha}{\beta\alpha\gamma}\, Y(00\gamma) \qquad (3.2)\\
a_\beta\, a_\alpha\, Y(\gamma\beta\alpha) &= G\binom{\gamma\beta\alpha}{\alpha\beta\gamma}\, Y(00\gamma)\\
a_\alpha\, a_\beta\, Y(\gamma\beta\alpha) &= G\binom{\gamma\beta\alpha}{\beta\gamma\alpha}\, Y(00\gamma)\\
a^*_\alpha\, a^*_\beta\, Y(00\gamma) &= Y(\alpha\beta\gamma)\\
a^*_\beta\, a^*_\alpha\, Y(00\gamma) &= G\binom{\beta\alpha\gamma}{\alpha\beta\gamma}\, Y(\alpha\beta\gamma)\\
a^*_\alpha\, a^*_\beta\, Y(0\gamma 0) &= G\binom{\beta\gamma\alpha}{\alpha\beta\gamma}\, Y(\alpha\beta\gamma)\\
a^*_\beta\, a^*_\alpha\, Y(0\gamma 0) &= G\binom{\alpha\gamma\beta}{\alpha\beta\gamma}\, Y(\alpha\beta\gamma)\\
a^*_\alpha\, a^*_\beta\, Y(\gamma 00) &= G\binom{\gamma\beta\alpha}{\alpha\beta\gamma}\, Y(\alpha\beta\gamma) \qquad \text{and}\\
a^*_\beta\, a^*_\alpha\, Y(\gamma 00) &= G\binom{\gamma\alpha\beta}{\alpha\beta\gamma}\, Y(\alpha\beta\gamma) \quad ,
\end{aligned}
$$

remembering that there are six intermediate states, $Y(\alpha\beta\gamma)$, $Y(\alpha\gamma\beta)$, $Y(\beta\alpha\gamma)$, $Y(\gamma\alpha\beta)$, $Y(\beta\gamma\alpha)$ and $Y(\gamma\beta\alpha)$. The above bi-linear

relations have been written in order to calculate the non null matrix elements of the operators $A = [a_\alpha , a_\beta]_+$ and $A^* = [a^*_\alpha , a^*_\beta]_+$.

Since the six different state vectors Y are equivalent to represent the system, only average matrix elements of A and A* will be meaningful in our approach. Let us consider that all these states are assumed by the system, one at a time in a temporal sequence. As gentileons are continuously changing of state it is not possible to say at any moment which gentileon is which state. This hypothesis, which is similar to that adopted to describe gluons exchange between quarks in baryons(16), is illustrated in Fig. 3. In this case we verify, taking into account Eqs. (3.2), the G(...) matrices and remembering that

(INSERT FIGURE 3)

Y_1, Y_2, Y_3 and Y_4 are orthonormal states(1), that the average matrix elements $\langle A\rangle$ and $\langle A^*\rangle$ are equal to zero. That is, $\langle [a_\alpha , a_\beta]_+\rangle = \langle [a^*_\alpha , a^*_\beta]_+\rangle = 0$. Thus, in this context we can write, for $\alpha \neq \beta$, that

$$[a_\alpha , a_\beta]_+ = [a^*_\alpha , a^*_\beta]_+ = 0 \quad . \qquad (3.3)$$

According to the first three bi-linear terms of Eqs. (3.1) and to Eqs. (3.3) the following commutation relations are valid for gentileons when they occupy different states,

$$[a_i^*, a_j]_+ = \delta_{ij} \quad \text{and}$$
$$[a_i, a_j]_+ = [a_i^*, a_j^*]_+ = 0 \quad , \tag{3.4}$$

where the indices i and j can assume the values α, β and γ.

Thus, we conclude from Eqs. (3.4) and by using Pauli's theorem[2-4] that, in these conditions, gentileons must be half-odd-integral spin particles. Of course, it does not mean that these gentileons are fermions; it only implies that they can be taken algebraically as fermions in the framework of a quantum field theory.

Our above results are extremely important since, as will be seen in section 5, they will permit us to study the composed hadrons assuming quarks as spin 1/2 gentileons. Moreover, as gentilionic quarks obey bi-linear commutation relations it will be possible to construct for these gentilionic quarks a consistent local, Lorentz-invariant quantum field theory (see section 6).

4. THE S_3 SYMMETRY AND THE $SU(3)_{color}$ EIGENSTATES

In section 2, we have shown that it was possible to interpret the $Y(\alpha\beta\gamma)$ transformations in terms of rotations, in a color space E_3, of only two equilateral triangles with

vertices occupied by three privileged colors $\alpha(\bar{\alpha})$, $\beta(\bar{\beta})$ and $\gamma(\bar{\gamma})$. The Y must constitute symmetry adapted kets for S_3. In other words, their disposition in the plane of the triangle must agree with the imposition made by the color Casimir. According to Fig. 1, these colors are defined by, $\alpha = \vec{m}_5 = (-\sqrt{3}/2, 1/2)$, $\beta = \vec{m}_6 = (\sqrt{3}/2, 1/2)$ and $\gamma = \vec{m}_4 = (0, -1)$, and according to Fig. 2, $\bar{\alpha} = \vec{m}_5^* = -\vec{m}_5$, $\bar{\beta} = \vec{m}_6^* = -\vec{m}_6$ and $\bar{\gamma} = \vec{m}_4^* = -\vec{m}_4$. The equilateral triangle symmetry for S_3 plays a fundamental role in E_3, allowing us to obtain a very simple and beautiful geometrical interpretation for the invariant $K_{inv} = 0$. Indeed, since the S_3 symmetry, according to section 2, implies that $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$ $(\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0)$, we conclude that the <u>total color quantity</u> of the system, pictured in E_3, is a constant of motion, which is null.

At this point we compare our color states α, β and γ with the $SU(3)_{color}$ eigenstates[17], blue, red and green. These color states are eigenstates of the color hypercharge $\tilde{Y}$ and of the color isospin $\tilde{I}_3$, both diagonal generators of the algebra of the $SU(3)_{color}$. The eigenstates blue (b), red (r) and green (g) are written as $|b\rangle = |-1/2, 1/3\rangle$, $|r\rangle = |1/2, 1/3\rangle$ and $|g\rangle = |0, -2/3\rangle$.

Taking into account that the SU(3) and S_3 fundamental symmetries are defined by equilateral triangles[17,18], it is quite apparent that the color states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ can be represented by eigenstates of $\tilde{I}_3$ and $\tilde{Y}$. Indeed, assuming that the axes X and Z (see Fig. 1) correspond to the axes $\tilde{I}_3$ and $\tilde{Y}$, respectively, and adopting the units along these axes as the side and the height of the triangle[17], we verify that $|\alpha>$, $|\beta>$ and $|\gamma>$ would be given by, $|\alpha> = |b> = |-1/2, 1/3>$, $|\beta> = |r> = |1/2, 1/3>$ and $|\gamma> = |g> = |0, -2/3>$. If we have considered the states $|\bar{\alpha}>$, $|\bar{\beta}>$ and $|\bar{\gamma}>$, seen in Fig. 2, we should verify that these states would correpond to the anti-colors $|\bar{r}>$, $|\bar{b}>$ and $|\bar{g}>$ of the $\bar{3}$ color representation.

Thus, if we assume that the states $|\alpha>$, $|\beta>$ and $|\gamma>$ correspond to $|b>$, $|r>$ and $|g>$, respectively, each unit vector $\vec{m}_j$ ($j = 4, 5$ and 6) is represented, in the plane $(\tilde{I}_3, \tilde{Y})$ by the operator $\tilde{q} = \tilde{I}_3 + \tilde{Y}/2$. This means that the vector $\vec{M}$ will be represented by the operator $\tilde{M} = \tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3$, where the indices 1, 2 and 3 refer to the three gentileons of the system. Thus, adopting the $SU(3)_{color}$ eigenvalues we see that $\tilde{M}$ will have a zero eigenvalue only when Y is given by Y(brg). That is, the wavefunctions Y(nnm), where $n, m = b, r$ and g, with two particles occupying the same color state[1,5], are prohibited.

It is important to note that, in our previous paper[5], since the $SU(3)_{color}$ scheme was not adopted, we have assumed that two gentileons could occupy the same color state. This is a point that remains to be analysed: the existence of another kind of color representation, besides the $SU(3)_{color}$, which would be able to describe consistently the gentilionic approach.

5. <u>THE GENTILIONIC HADRONS</u>

Since gentileons are confined entities and their systems are non-coalescent it seemed natural to think quarks as spin 1/2 gentileons[1,5,6]. With this hypothesis we have shown[5] that baryons [qqq], that are formed by three indistinguishable gentileons in color space, are represented by wavefunctions $\psi = \varphi \cdot Y(\alpha\beta\gamma)$. The state $\varphi = (SU(3) \times O_3)_{symmetric}$ corresponds, according to the symmetric quark model of baryons[17], to a totally symmetric state. The state function $Y(\alpha\beta\gamma)$ corresponds to the colorspinor written explicitely in section 2.

It will be assumed in what follows that the color states α, β and γ are the $SU(3)_{color}$ eigenstates blue, red and green, respectively. According to section 4, $Y(\alpha\beta\gamma)$ must necessarily be composed by three different color states, resulting for the baryon wavefunctions, $\psi = \varphi \cdot Y(brg)$. In these conditions the quarks in [qqq] that, according to φ and Y(brg), have disponible an infinite number of quantum states, cannot assume the same color in the color space. In other words, two quarks in [qqq] cannot occupy the same quantum state. This implies, taking into account the results of section 3, that gentilionic quarks must be half-odd-integral spin particles. Thus, our initial hypothesis of assuming quarks as spin 1/2 gentileons is now justified, showing the self-consistency of our model.

Once adopted the color eigenstates blue, red and green we see, based in sections 2 and 4, that the $SU(3)_{flavor} \times SU(3)_{color}$ representation, both 3 and $\overline{3}$, is naturally incorporated in our scheme. With this in mind and observing section 2 we see that in the gentilionic formalism one possibility is to define the individual quark charge as:

$$q = q_f + \tilde{q}_c = (I_3 + Y/2) + \lambda(\tilde{I}_3 + \tilde{Y}/2) \qquad (4.1)$$

where $q_f = I_3 + Y/2$ refers to flavour charge, $\tilde{q}_c = (\tilde{I}_3 + \tilde{Y}/2)$ refers to color charge and λ is an arbitrary constant that cannot be determined in the framework of the theory. With this definition, the total color baryon charge $\tilde{Q}$ is given by $\tilde{Q} = \lambda\langle\tilde{M}\rangle$. Remembering that the expected value $\langle\tilde{M}\rangle$ is a constant of motion equal to zero, that is, $\langle\tilde{M}\rangle$ = constant = 0, for the states $Y(brg)$, as shown in section 3, we see that the generalized Gell-Mann-Nishijima relation is automatically satisfied[5] independently of the λ value. Putting $\lambda = -1$ we obtain integer quark charges, according to Han-Nambu, and if $\lambda = 0$ we have the fractional charges adopted by Gell-Mann[17]. Note that the result $\tilde{Q}$ = constant = 0 can be interpreted as a selection rule for quark confinement in baryons.

In our approach[1,5] mesons are composed by a quark-antiquark pair $[q\bar{q}]$. According to the statistical princile, systems like $[q]$, $[qq]$, $[qq\bar{q}]$ and $[qq\bar{q}\bar{q}]$, for instance, are prohibited (it could exist only bound states $[q\bar{q}] - [q\bar{q}]$ of the

mesons $[q\bar{q}]$). Since q and $\bar{q}$ are different particles in color space we can conclude, in agreement with our general results[1], that mesons are represented by one-dimensional state functions. This implies, remembering that q and $\bar{q}$ are spin 1/2 particles, that the system $[q\bar{q}]$ is represented, in fermionic and gentilionic theories, by the same state vector.

We are now in condition to make a summary of the fundamental properties that must be observed for hadrons composed by gentilionic quarks: (1) quarks are confined, (2) hadrons cannot coalesce, (3) baryonic number is conserved, (4) the hadron color charge is a constant of motion equal to zero and (5) only color singlet hadrons can exist.

The above mentioned hadronic properties have been predicted independently of the intrinsic nature of the gentileons; they could be particles, quantum collective excitations or something else. Consequently, no dynamical hypothesis, phenomenological or approximate arguments have been used to prove them. They have been deduced from first principles: from the statistical principle or by using the symmetries of the S_3 intermediate representation. Thus, if quarks are gentileons, there may be hidden or explicit a confining mechanism in the dynamical laws. The confining mechanism could be produced by a very peculiar interaction between quarks, by an impermeable bag as proposed by the bag model or something else. At the moment these mechanisms are unknown. It is not our intention, in this paper, to study this problem.

In spite of our stimulating general results, there remains the crucial problem of determining the intrinsic nature of the quarks and their dynamical properties. According to the current theoretical ideas, quarks are fermionic elementary particles. The mathematical formulation of the fermionic model, the QCD, is a successful modern field theory since it is able to explain many properties of the hadrons. In next section, taking quarks as spin 1/2 gentileons, a quantum chromodynamics is proposed where, instead of fermions, gentileons interact with gluons.

6. A QUANTUM CHROMODYNAMICS FOR GENTILIONIC HADRONS

To construct a quantum field theory for hadrons composed by spin 1/2 gentilionic quarks we must take into account the SU(3) symmetry, flavor and color, and remember, according to section 3, that the creation and annihilation quark operators obey fermionic commutation relations. Furthermore, the gentilionic quantum field approach must be formulated incorporating the symmetry properties of the intermediate states in order to predict, as conservation laws or selection rules, the hadronic properties deduced in section 5: (a) quark-confinement, (b) non-coalescence of hadrons, (c) baryon number conservation, (d) only

color singlet hadrons can exist and (e) the hadron color charge is a constant of motion equal to zero. This is a very ambitious and extremely difficult task. Since we were not able, up to now, to develop this "intermediate S_3 gauge invariant theory", an alternative one will be proposed. In this way, let us suggest the following Lagrangian density for gentilionic quarks interacting with gluons,

$$L = \sum_f \left[i\, q_a^+ \gamma^\mu \frac{\partial}{\partial x^\mu} q_a + g\, q_a^+ \gamma^\mu \left(\frac{\lambda_i}{2}\right)_{ab} A_\mu^i q_b - m_f q_a^+ q_a \right] + $$

$$- \frac{1}{4} \left(\frac{\partial A_\nu^i}{\partial x^\mu} - \frac{\partial A_\mu^i}{\partial x^\nu} + g\, f_{ijk} A_\mu^i A_\nu^k \right)^2$$

where the summation is over the flavors $f = u,d,s,c\ldots$. The summation over repeated indices $a,b,\ldots,$ referring to color is understood. The $\lambda_i/2$ are the 3×3 matrix representation of the $SU(3)_{color}$ algebra generators, satisfying the commutation relations $[\lambda_i,\lambda_j] = i\, f_{ikj}\, \lambda_k/2$, where f_{ijk} are the SU(3) structure constants. The flavor symmetry is only broken by the lack of degeneracy in the quark masses. Finally, the quark free fields $q(x)$ are expanded in terms of positive and negative frequency solutions, $\varphi_{k+}(x)$ and $\varphi_{k-}(x)$, of Dirac's equation,

$$q(x) = \sum_k \left\{ a_{k+}\, \varphi_{k+}(x) + a_{k-}^*\, \varphi_{k-}(x) \right\} ,$$

where a_i and a_i^* obey fermionic commutation relations.

With the above assumptions, both theories, the usual QCD and the gentilionic QCD, that will be indicated by QCDG, will have the same gluons and the same Lagrangian density. In both approaches the previously mentioned properties (a), (b) ... and (e) appear now as additional conditions. In these circumstances, both theories will give identical predictions for hadronic properties. In spite of this we note that they are not equivalent. Indeed, in QCDG the five conditions cited above appear naturally, deduced from first principles, whereas in QCD they are imposed "ad hoc".

Finally, as pointed out in section 3, the time evolution process illustrated in Fig. 3, describes in quantum chromodynamics (QCD and QCDG) the exchange of gluons between quarks in baryons[16]. In gentilionic approach this process can be interpreted, in a topological scheme[19], by rotations of the equilateral color triangle inside the T^2 torus generated by two angular variables ϑ and Φ that appear in discrete rotations, $R(\vartheta) . R(\Phi) = \exp(i\vec{j}.\vec{\sigma}\frac{\vartheta}{2}) . i\exp(i\vec{m}.\vec{\sigma}\frac{\Phi}{2})$, given by Eqs. (2.4) and (2.5).

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FIGURE CAPTIONS

Fig. 1 - The equilateral triangle in the color space (X,Y,Z) with vertices occupied by the colors α, β and γ.

Fig. 2 - The equilateral triangle in the color space (X,Y,Z) with vertices occupied by the colors α, β and γ.

Fig. 3 - The vertical dimension represents the spatial separation between gentileons and the horizontal dimension represents time. The gentileons 1, 2 and 3 are continuously changing of state. In practice it is not possible to determine the states of the gentileons; only the probability of each state can be calculated.

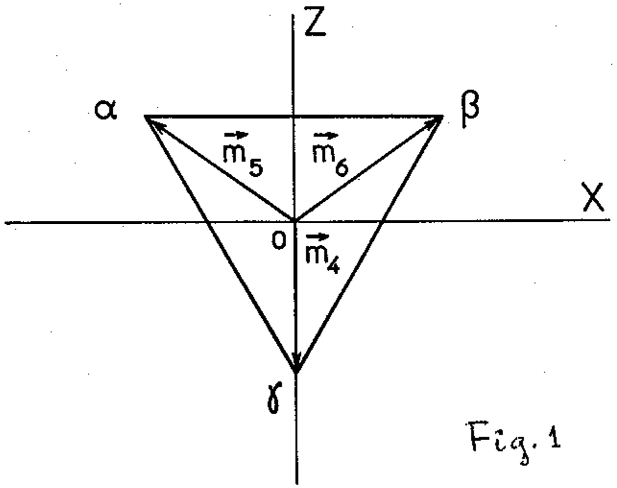

Fig. 1

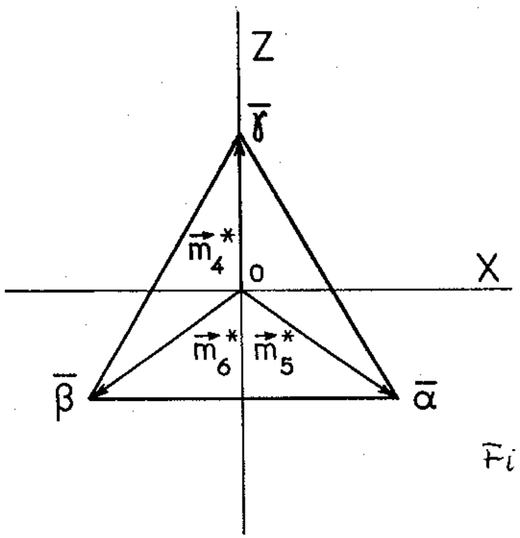

Fig. 2

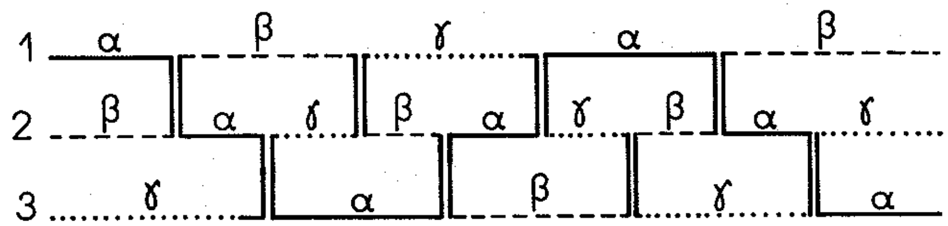

Fig. 3