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GLAUBER CALCULATION OF HEAVY-ION AND LIGHT-ION INCLUSIVE BREAK-UP CROSS SECTIONS

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ABSTRACT

The small-angle, single spectra of α-particles in the reaction $^{159}Tb(^{14}N,\alpha)$ at E_{Lab} = 95 MeV are qualitatively well reproduced with a simple hybrid model including break up of the ^{14}N nucleus and the subsequent strong interaction of the ^{10}B with the target. The Glauber approximation was employed for the purpose. The model is also tested in the reaction $^{62}Ni(\alpha,{}^3He)$ at E_α^{Lab} = 172.5 MeV.

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I. INTRODUCTION

During the last few years, nuclear inclusive break-up processes have received a great amount of experimental attentions[1]. The processes, which are also named incomplete fusion, massive transfer, etc., become important at intermediate energies both light- and heavy-ion induced reactions. A quantal reaction theory of these processes is available[2,3,4] and has been applied for several cases.

The cross-section which describes a typical IB process

$$A + a \equiv (x+b) + A \longrightarrow b + (x+A)^* \tag{1}$$

is invariably written as

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{2}{\hbar v_a}\,\rho(E_b)\,\langle \hat{\psi}_x^{(+)} | -W_{xA}(E_i + B_a - B_b) | \hat{\psi}_x^{(+)} \rangle \tag{2}$$

where $\rho(E_b)$ is the density of states of the $x+A$ system with x and A denoting participant and target nuclei, respectively. $-W_{xA}$ is the imaginary part of the $x+A$ system and $\psi_x^{(+)}$ is the "negative energy" x-particle wave function[4]

$$\hat{\psi}_x^{(+)}(\vec{r}_x) = \langle \chi_b^{(-)}(\vec{r}_b) | \phi_a(\vec{r}_b - \vec{r}_x)\, \chi_a^{(+)}(\vec{r}_b, \vec{r}_x) \rangle \quad . \tag{3}$$

where the χ's are distorted waves and φ is intrinsic wave function.

The purpose of this paper is to supply a deeper formal analysis of the matrix element appearing in Eq. (2). Further, a numerical test of the theory that underlies Eq. (3) as developed in Ref. , is also done here within Glauber's approximation. Though a debate is still going on concerning the foundation of the theory[5], we shall, in the following, take Eq. (2) as the correct formula for $\frac{d^2\sigma}{d\Omega_b dE_b}$ as is done in Refs. 3) and 4).

The paper is organized as follows. In Section II we supply a formal discussion of the inclusive break up cross section and its behaviour as a function of angle and energy. In Section III we evaluate the IB cross-section for two nuclear systems within Glauber's approximation. Finally, in Section IV we present several concluding remarks.

II. THE STRUCTURE OF THE IB CROSS-SECTION: THE OFF-ENERGY AND OFF-ANGLE SHELL TOTAL REACTION CROSS-SECTION

The purpose of this Section is to analyse the matrix element $\langle \hat{\psi}_x^{(+)} | W_x | \hat{\psi}_x^{(+)} \rangle$ which appears in Eq. (2). We shall identify this matrix element with a more fundamental quantity which appears naturally in any development of formal scattering

theory of absorptive systems. The basis of this development has recently been achieved in a couple of publications by one of the authors[6].

Using the notation of Ref. 4), we write the following general decomposition of the break-up cross section

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{d^2\sigma^{DIR}}{d\Omega_b\, dE_b} + \frac{d^2\sigma^{fl.}}{d\Omega_b\, dE_b} \tag{4}$$

the direct term describes what is called the direct inelastic break-up process while the fluctuation part accounts for incomplete fusion . To derive expressions for the direct and fluctuation parts of $\frac{d^2\sigma}{d\Omega_b dE_b}$, we recall some general results of reaction theory. We introduce the projection operator P which projects out the elastic and open inelastic channels. The Q-space $(Q = 1 - P)$ is assumed to contain break-up channels. The total reaction cross-section may then be written as

$$\sigma_R = \sigma_{INEL} + \frac{k}{E_k} \langle \Psi_P^{(+)} | -\mathrm{Im}\, H_{PQ} (E - H_{QQ} + i\epsilon)^{-1} H_{QP} | \Psi_P^{(+)} \rangle \tag{5}$$

where $\sigma_{INEL.}$ is the total inelastic cross-section and the second term accounts for the break-up contribution to σ_R. Clearly the decomposition (5) of σ_R can be generalized if necessary to account for other processes (e.g., complete fusion of A+a). The double differential cross section (2) can then be derived from the second term of Eq. (3). Note here that ψ_p is in principle obtainable from the coupled channel problem involving the elastic and inelastic channels. The second term of (3) can be further decomposed using operator identifies. The final result is

$$\frac{d^2\sigma^{fl}}{d\Omega\, dE} = \frac{2}{\hbar v_a}\rho(E_b)\langle\hat{\psi}_x^{(+)}| -W_{xA}^{F}(E_i+B_a-B_b)|\hat{\psi}_x^{(+)}\rangle \qquad (7)$$

$$\frac{d^2\sigma^{DIR}}{d\Omega\, dE} = \frac{2}{\hbar v_a}\rho(E_b)\langle\hat{\psi}_x^{(+)}| -\hat{W}_{xA}^{D}(E_i+B_a-B_b)|\hat{\psi}_x^{(+)}\rangle \qquad (8)$$

where the absorptive piece of the x-A interaction has been split into a direct and fusion parts. We note here that $\frac{d^2\sigma D}{d\Omega dE}$ does not contain elastic break-up. It accounts for inelastic <u>direct</u> break-up processes. $\frac{d^2\sigma fl}{d\Omega dE}$ on the other hand describes break-up fusion. This last term contains also processes involving intermediate stages such as break-up, inelastic excitation, followed by fusion. In any case, the summed inclusive cross-section can be written as (without the pure elastic break-up piece)

$$\frac{d^2\sigma^{Incl.}}{d\Omega_b\, dE_b} = \frac{2}{\hbar v_a}\rho(E_b)\langle\hat{\psi}_x| -W_{xA}(E_i+B_a-B_b)|\hat{\psi}_x\rangle \equiv \rho(E_b)\,\sigma^{xA}_{Reac.}(\Omega_b, E_b) \qquad (6)$$

where we have introduced, in the above, a <u>new</u> cross section which we call off-angle- and off-energy-shell total reaction cross section of the xA subsystem.

The cross-section $\sigma^{xA}_{Reac}(\Omega_b, E_b)$ is related to the total xA reaction cross section when the angle variable is set equal to zero and the system is allowed to be on the energy shell. Several features of this cross section can be easily analysed from conventional optical-model studies once the recognition is made that a very similar quantity to $\sigma^{xA}_{Reac}(\Omega_b, E_b)$, measures the deviation from unitarity of the optical S-matrix[6].

In Ref. 6), the following equation was derived,

$$\langle \vec{k}'|S^{-1}|\vec{k}\rangle = \langle \vec{k}'|S^{\dagger}|\vec{k}\rangle + 2\pi\delta(E_k - E_{k'})\cdot \int \frac{d\vec{k}''}{(2\pi)^3}\frac{2E_{k''}}{k''}\langle \vec{k}'|S^{-1}|\vec{k}''\rangle\,\sigma_{Rea.}(\vec{k},\vec{k}'') \qquad (9)$$

When $\sigma_{Reac}(\hat{k}.\hat{k}'', |\vec{k}|, |\vec{k}''|)$ is set equal to zero one recovers the unitarity condition of the S-matrix, $S^{-1} = S^{+}$. The off-shell "cross-section" $\sigma_{Reac.}(\hat{k}.\hat{k}'', |\vec{k}|, |\vec{k}''|)$ becomes just the total reaction cross section if $|\vec{k}| = |\vec{k}''|$ (which is the case in the above equation) and $\hat{k}.\hat{k}'' = 1$.

The same quantity, $\sigma_{Reac.}(\hat{k}.\hat{k}'', |\vec{k}|, |\vec{k}''|)$ appears also in the equation which gives the orthonormality condition of the optical wave function $|\psi_{\vec{k}}^{(+)}\rangle$,

$$\langle \Psi^{(+)}_{k'} | \Psi^{(+)}_{k} \rangle = (2\pi)^3 \delta(\vec{k}'-\vec{k}) - 2i \sqrt{\frac{E_k E_{k'}}{k' k}} \, \frac{\sigma_{Rea.}(\vec{k}, \vec{k}')}{E_k - E_{k'} + i\epsilon} \quad (10)$$

When expanded in partial waves, $\sigma_{Reac.}(\hat{k}.\hat{k}', |\vec{k}|, |\vec{k}'|)$ takes the following form

$$\sigma_{Rea.}(\vec{k}, \vec{k}') = \frac{\pi}{k\,k'} \sum_{\ell=0}^{\infty} (2\ell+1)\, T_\ell(k, k')\, P_\ell(\hat{k}\cdot\hat{k}') \quad (11)$$

where

$$T_\ell(k, k') = \frac{8\mu\sqrt{k\,k'}}{\hbar^2} \int_0^\infty dr\, \psi_\ell^*(k', r)\, W(r)\, \psi_\ell(k, r) \quad (12)$$

$T_\ell(|\vec{k}|, |\vec{k}'|)$ on-the-energy $(|\vec{k}| = |\vec{k}'|)$ becomes just the optical transmission coefficients, $T_\ell(|\vec{k}|)$, which is related to the modulus of the optical partial wave, S-amplitude,

$$T_\ell(|\vec{k}|) = 1 - |S_\ell(k)|^2 \quad (13)$$

On the energy-shell $(|\vec{k}| = |\vec{k}'|)$, $\sigma_{Reac.}$, becomes, in the sharp--cut-off limit $(T_\ell(|\vec{k}|) = \Theta(\ell_g - \ell)$, where ℓ_g is the grazing

angular momentum

$$\sigma_{Rea}(k^2, \hat{k}\cdot\hat{k}') = \frac{\pi}{k^2}\left[P'_{\ell_g+1}(\hat{k}\cdot\hat{k}') + P'_{\ell_g}(\hat{k}\cdot\hat{k}')\right] \quad (14)$$

where $P_\ell(x)$ is the Legendre polynomial and $P'_\ell(x) = \frac{d}{dx} P_\ell(x)$. Eq. (4) sows that $\sigma_{Reac.}$ exhibits oscillation as a function of $\hat{k}.\hat{k}'$. These oscillations become more rapid as ℓ_g increases.

We turn now to the effects arising from the dependence of $\sigma_{Reac.}$ on the off-shell variable $\xi \equiv |\vec{k}'| - |\vec{k}|$. To simplify the discussion we take ξ to be small enough that a Taylor series expansion of $\psi_\ell^*(k'r)$ can be comtemplated. It is then clear that higher order terms in ξ bring about terms in $\sigma_{Reac.}$ which contain higher order Bessel functions. The alternating orders of these incoherently summed Bessel functions would thus bring about a damping of the θ-oscillations seen in the on-shell $\sigma_{Reac.}$. Then we may conclude that the larger ξ the more smooth $\sigma_{Reac.}(\theta)$ is expected. Similar features would be expected to be present in the inclusive break-up cross section. This property is clearly confirmed through direct numerical evaluation, which we describe fully below.

III. EVALUATION OF THE INCLUSIVE BREAK-UP CROSS-SECTION WITH GLAUBER THEORY

In this Section we turn to the calculation of $\frac{d^2\sigma}{d\Omega_b dE_b}$ using the formalism of Ref. 4). For the purpose we use the Glauber approximation which though represents a drastic approximation at the not-so-high energies involved, it allows for a transparent discussion of the role played by the different physical parameters that enter in the calculation.

First we have for the distorted wave $\chi^{(+)}_{\vec{k}}(\vec{r})$, the following

$$\chi^{(+)}_{k}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}\, e^{i\int_{-\infty}^{\infty}\Delta k(z',b)\,dz'} \tag{15}$$

where $\Delta k(z',b)$ is given by

$$\Delta k(z',b) = -\frac{k}{2E}\, U(z',b) \tag{16}$$

Here $U(z,b)$ is the complex optical potential. using the form (3) for all wave functions appearing in Eq. (2), we obtain for $\hat{\psi}^{(+)}_x$,

$$\hat{\psi}^{(+)}_x(\vec{r}_x,\vec{q}) = e^{i\vec{k}_x\cdot\vec{r}}\, e^{i\int_{-\infty}^{z}\Delta k_x(z',b_x)\,dz'} \cdot$$

$$\cdot \int d^3r_b\, e^{i\vec{q}\cdot\vec{r}_b}\, S_{bA}(b_b)\, \varphi_a(\vec{r}_b-\vec{r}_x) \quad . \tag{17}$$

where $\vec{q} = \vec{k}_b - \vec{k}_b'$, the average momentum transfer from b to A by elastic scattering.

With the above form for $\hat{\psi}_x$, we have for the inclusive cross section the following simple expression

$$\frac{d^2\sigma^{inc}}{d\Omega_b dE_b} = \rho(E_b)\sum_{\ell_x} P(q,\ell_x)\, \sigma_{xA}^{Rea.}(\ell_x) \tag{18}$$

$$P(q,\ell_x) = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, \left|\hat{\varphi}_{a,b}(\vec{q},\ell_x/k_x)\right|^2 \tag{19}$$

where $\hat{\varphi}_{a,b}(q,\frac{\ell_x}{k_x})$ describes the zero-point relative motion ("Fermi motion") of x and b within the projectile, which is broadened in the transverse direction by absorption of the spectator, namely

$$\hat{\varphi}_{a,b}(\vec{q},b_x) = e^{-iq_{\parallel}z_x}\int d^3r_b\, e^{-i\vec{q}\cdot\vec{r}_b}\, S_{bA}(b_b)\,\cdot$$

$$\cdot\, \varphi_a(\vec{r}_b-\vec{r}_x) \tag{20}$$

In Eq. (20) S_{ba} is the b-A elastic scattering matrix. Finally σ^R_{xA} is the ℓ_x-partial reaction cross section of the x-A system.

We have evaluated Eq. (18) for the systems $^{159}Tb(^{14}N,\alpha)$ and $^{181}Tb(^{14}N,\alpha)$ at E_{Lab} = 95 MeV and 115 MeV, respectively. We have used for the real parts of the bA and xA optical potentials the double folding interaction[7] and merely multiplied these by factors of the form $(1+i\xi_1)$, with ξ_1 being adjustable imaginary strengths. As for the wave function ϕ_a, we used a Gaussian form, with a width σ, given by[8]

$$\phi_a(r) \sim e^{-\left(\frac{\sigma}{2}\right)^2 r^2}$$
$$\sigma = \frac{k_F}{\sqrt{5}}\sqrt{\frac{A_F(A_p-A_F)}{A_p-1}} \qquad (21)$$

where A_F and A_p are the spectator (observed fragment) and projectile mass numbers respectively and k_F is the nuclear Fermi momentum of $\sim 1.36\ fm^{-1}$. For the system $^{159}Tb(^{14}N,\alpha)$ we get $\sigma = 2.1\ fm^{-1}$.

The result of our calculation for the α-inclusive spectrum at 20° in $^{159}Tb(^{14}N,\alpha)$ at 95 MeV is shown in Figure 1. The values of the parameter ξ_A and ξ_B are, respectively 0.01 and 0.5. The agreement with the data is quite good. Also shown are the results of Udagawa et al. which greatly underestimate the cross section at the lower end of the spectrum. The reason is quite clearly the fact that Udagawa uses a model where only the brak-up fusion process is considered and in this order, whereas we include all processes with varying

orders of happening. Loosely speaking the function $\hat{\rho}_x^{(+)}$ of Udagawa contains an x-A elastic propagator $G_x^{(+)}(E_x)$ which clearly damps the cross section at low E_b,s (high E_x) as it behaves, roughly, like $\frac{1}{E_x}$. Incidentally, the total reaction cross section of the participant-target system, $^{10}B + {}^{159}Tb$, extracted from our calculation, comes out within 20% of that extracted from the data of similar systems. This is quite reasonable in view of the usually large size of the error bars in σ_R.

Of course, we do not expect our model to work as well at larger angles because of the Glabuer approximation which is valid for small deflection angles. In fact, the angular distribution we obtain, has a much steeper slope than the data. Similar results were obtained for the angular distribution of the α,s from the $^{181}Ta(^{14}N,\alpha)$ reaction at E_{Lab} = 112 MeV[3].

We have also tested our model for the reaction $^{62}Ni(\alpha,{}^3He)$ at E^{Lab} = 172.5 MeV[9]. Here, the participant particle is a neutron, and thus we have chosen the Bechetti--Greenless optical potential[10] to represent its interaction with ^{62}Ni. As for the $^3He + {}^{62}Ni$ interaction we utilized a Saxon-Woods potential with the parameters V = 54.0 MeV, $R_r = 1.17\ A^{1/3}$ fm, a_r = 0.75 fm, W = 10.0 MeV, R_i = 10 fm and a_I = 0.3 fm.

The results of our evaluation of Eq. (1), with $k_F = 1.6\ fm^{-1}$, are shown in Figure 2a. The corresponding data are exhibited in Figure 2b. The shapes of the spectra are reasonable, though one needs a normalization factor (~ 10 at $= 61.5^{\circ}$) which changes slightly with angles. Again, we trace this low value of our cross section at large angles to the inadequacy of the Glauber approximation there. Of course, we did not attempt to optimize our fit. Our intention was more towards establishing the fact that by including more processes in the cross section not contained in the usual treatment, the shape of the inclusive spectrum of the spectator particle can be nicely reproduced.

In conclusion, we have shown that inclusive energy spectra of heavy ion and light ion incomplete fusion reactions can be nicely reproduced by the simple expression of Eq. (1), which is more inclusive than the usual break-up fusion model. It is hoped that more precise DWBA calculation will be forthcoming to supply a better account of the angular distributions.

IV. CONCLUSIONS

We have in this paper, carefully analysed the formal structure and properties of the inclusive break-up cross section after identifying it with what we have called the off-energy and off-angle total reaction cross section of the participant + target system. We have shown that the angle oscillations of this cross

section are completely washed out as a result of its being off-energy-shell.

The inclusive break-up cross section was then calculated, within the Glauber approximation, for the systems $^{181}Ta(^{14}N,\alpha)$ at $E_{Lab} = 95$ MeV and $^{62}Ni(\alpha,^{3}He)$ at $E_{\alpha}^{Lab} = 172.5$ MeV. The magnitude of the calculated cross section was found smaller than the data, however the general trend is well reproduced.

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FIGURE CAPTIONS

Fig. 1 - Calculated (full line) inclusive spectrum of α particles at $_b = 20^{o}$ in the reaction $^{159}Tb(^{14}N,\alpha)$ at $E_{Lab} = 95$ MeV. The dashed curve is the result of the calculation of Ref. 2b). The data points were also taken from Ref. 2b).

Fig. 2 - a) Calculated inclusive ^{3}He spectra in the reaction $^{62}Ni(\alpha,{}^{3}He)$ at $E_{\alpha}^{Lab} = 172.5$ MeV.

b) The data points of $^{62}Ni(\alpha,{}^{3}He)$ at $E_{\alpha}^{Lab} = 172.5$ MeV from Ref. 9).

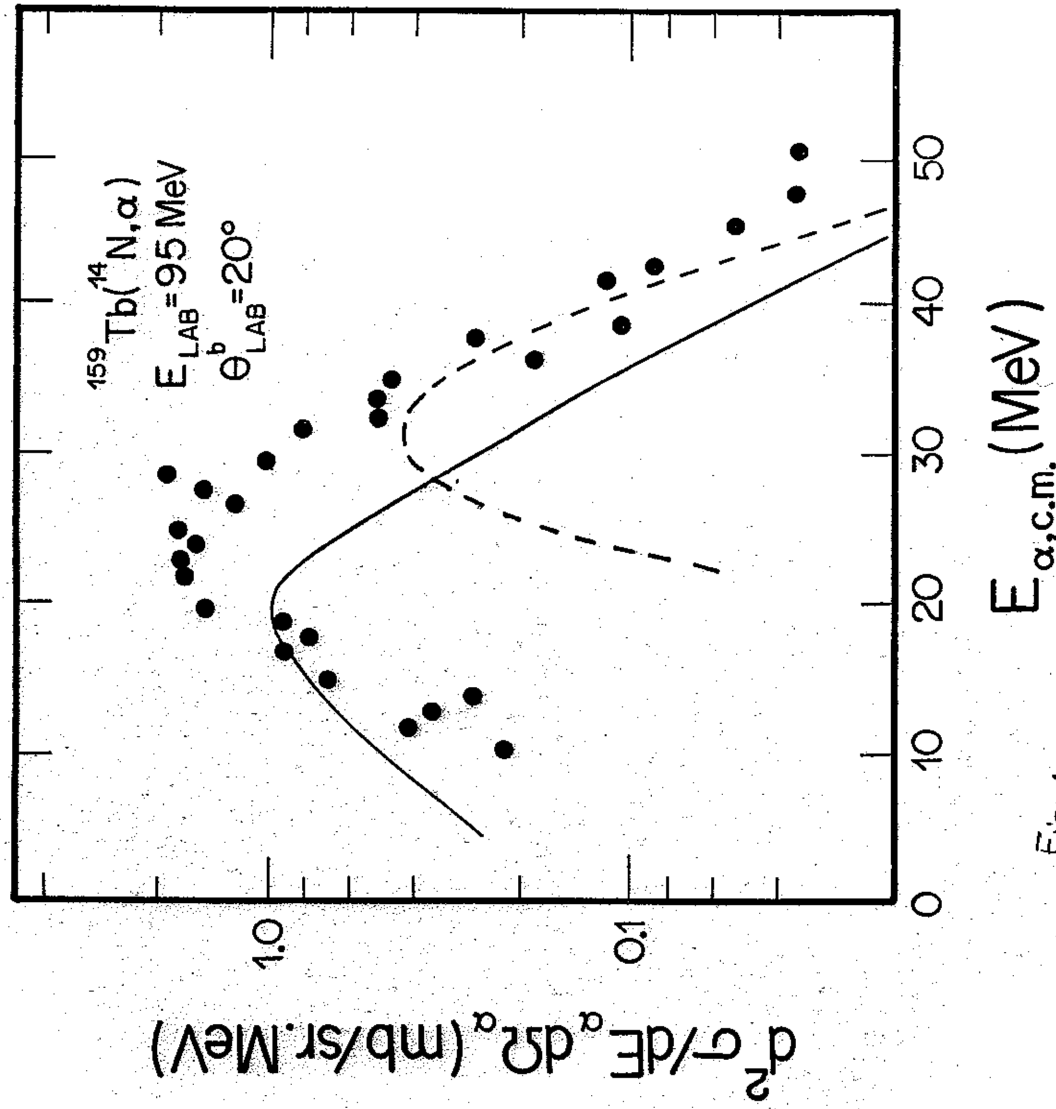

$^{159}Tb(^{14}N,\alpha)$
$E_{LAB}=95$ MeV
$\theta^{b}_{LAB}=20°$
$d^2\sigma/dE_\alpha d\Omega_\alpha$ (mb/sr.MeV)
1.0
0.1
$E_{\alpha,c.m.}$ (MeV)
0
10
20
30
40
50

Fig. 1

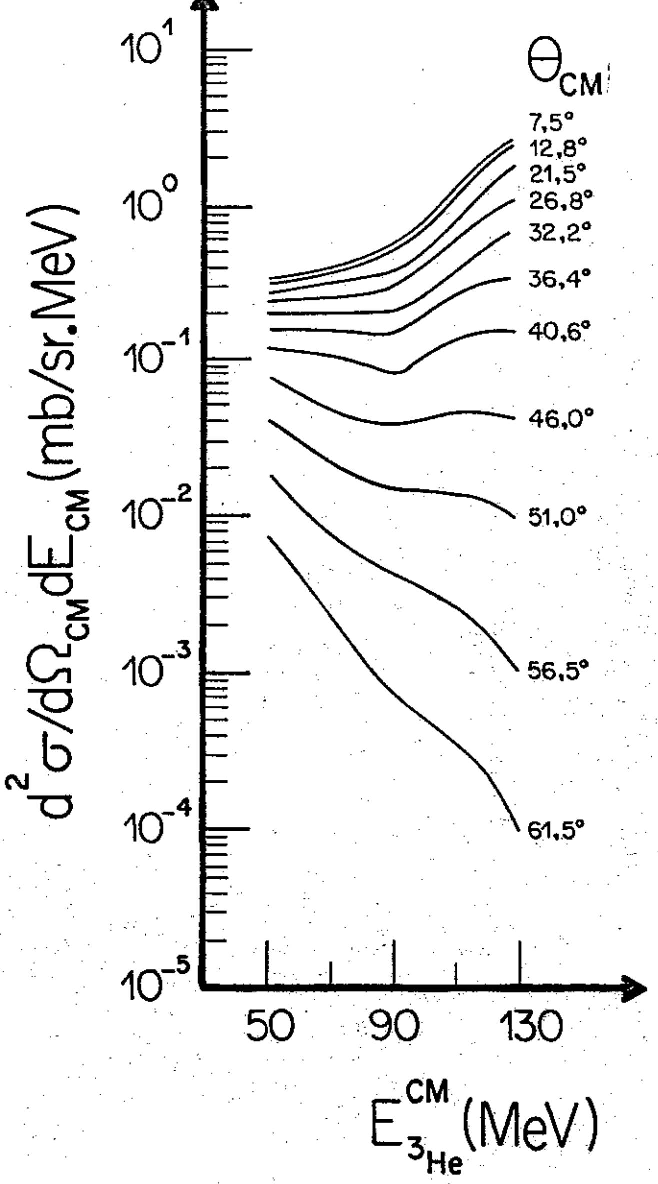
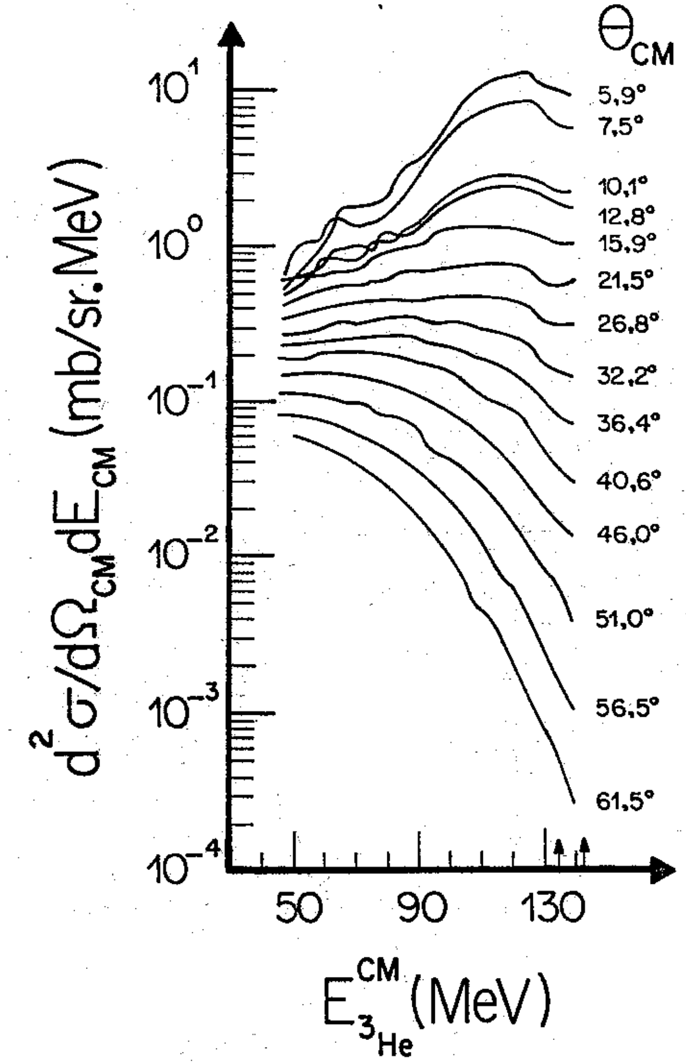

Θ_{CM}
7,5°
12,8°
21,5°
26,8°
32,2°
36,4°
40,6°
46,0°
51,0°
56,5°
61,5°
$d^2\sigma/d\Omega_{CM}dE_{CM}$ (mb/sr.MeV)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
50
90
130
$E^{CM}_{^3He}$ (MeV)

Fig. 2 a



Θ_{CM}
5,9°
7,5°
10,1°
12,8°
15,9°
21,5°
26,8°
32,2°
36,4°
40,6°
46,0°
51,0°
56,5°
61,5°
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
50
90
130
$d^2\sigma/d\Omega_{CM}dE_{CM}$ (mb/sr.MeV)
$E^{CM}_{^3He}$ (MeV)

Fig. 2b