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ON THE NON-EQUIVALENCE OF RANDOM-FIELD FERRO-MAGNETS AND SITE-DISORDERED ANTIFERROMAGNETS: FLUCTUATIONS

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ABSTRACT

It is shown that the fluctuations of the order parameter in the Curie-Weiss versions of the dilute antiferromagnet and of the Ising model with random magnetic field are not equivalent under the mapping which establishes their thermodynamical equivalence.

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I. INTRODUCTION

Considerable theoretical effort has been made in recent years to understand the Ising model in the presence of a random magnetic field (RMF)[1-4]. However, random fields cannot be directly produced in laboratories. After the original paper by Fishman and Aharony[5] and the later one by Wong et al.[6], there is a generalized belief that this model is related to site-dilute antiferromagnet Ising models in the presence of an applied uniform magnetic field (DAF), which are experimentally accessible systems[7]. Particularly, the degree of dilution and the intensity of the field, which are supposed to be related to the RMF parameters, can be well controlled.

With a few exceptions[8] the works on this equivalence have been centered in the usual mean field approximation[5,6,9]. A complete mapping between the parameters and phase diagrams has been obtained[10] for Curie-Weiss versions of both models, which were solved[2,10] by a method due to van Hemmen[11]. In spite of being mean field models, the latter are somewhat subtler from the probabilistic point of view. Rigorous work by Ellis and Newman[12] studying large deviations in classical Ising-like Curie-Weiss models has shown that they display non-trivial fluctuations of the order parameter at criticality. These results have been extended to disordered models such as RMF[4] and van Hemmen's spin glass[11,14].

In this note we reconsider the question of equivalence between the Curie-Weiss versions of the two models as to show that it does not hold at the more delicate level of fluctuations.

The mean field DAF model we use is described in a finite volume $\Lambda \subset \mathbb{Z}^d$ by the Hamiltonian

$$H_{DAF} = \frac{2J}{n} \sum_{\substack{i \in \Lambda_e \\ j \in \Lambda_0}} \varepsilon_i \varepsilon_j \sigma_i \sigma_j + H \sum_{i \in \Lambda} \varepsilon_i \sigma_i \qquad (1)$$

where $\Lambda_e = \Lambda \cap \mathbb{Z}_e^d$, $\Lambda_0 = \Lambda \cap \mathbb{Z}_0^d$ with $\mathbb{Z}_e^d$ $(\mathbb{Z}_0^d)$ being the sublattice of $\mathbb{Z}^d$ for which the sum of coordinates of each site are even (odd) integers. The interaction is antiferromagnetic $(J > 0)$ between sites in different sublattices. The random variables $\varepsilon_i \in \{0,1\}$ describe the site dilution and they are taken to be independent and identically distributed, with

$$\begin{aligned} \varepsilon_i &= 1 \quad , \quad \text{probability} \quad p \\ &= 0 \quad , \quad \text{probability} \quad 1-p \ . \end{aligned}$$

The spin variables σ_i are, for simplicity, taken to be of Ising type: $\sigma_i = \pm 1$. The external magnetic field H is uniform and deterministic, and n denotes the number of points in Λ.

The Hamiltonian (1) is slightly different and more natural than that used in a previous work[10] as no explicit ferromagnetic interaction inside the sublattices is necessary.

The RMF model to be compared to the model given by (1) is described by the Hamiltonian

$$H_{RMF} = -\frac{J}{2n} \sum_{i,j \in \Lambda} \sigma_i \sigma_j + \sum_{i \in \Lambda} h_i \sigma_i \qquad (2)$$

where h_i, $i \in \Lambda$, are independent, identically distributed random variables, being equal to $\pm h$ with probability $\frac{1}{2}$.

In the next section we show the equivalence of the thermodynamics of the two models, and in the last one we make an analysis of their fluctuations and show them to be non-equivalent.

II. THE THERMODYNAMICAL EQUIVALENCE

We compute, for both models, their free energy f, given by

$$\beta f = \lim_{n \to \infty} -\frac{1}{n} \ln \Big(\sum_{\{\sigma\}} e^{-\beta H} \Big)$$

where β is the inverse of the temperature, $\{\sigma\}$ denotes all the possible spins configurations, and H is the Hamiltonian. Taking $H = H_{DAF}$ as in (1), one may write

$$Z^{(n)}_{DAF} = \sum_{\{\sigma\}} e^{-\beta H_{DAF}} = \frac{2^n n}{2\pi} \int dm\, dq \; \exp\left\{ \frac{m^2+q^2}{2} - \varphi_n(m,q) \right\}$$

where

$$\varphi_n(m,q) = \frac{1}{n} \sum_{j\in\Lambda_e}' \ln\cosh\left[\left(\sqrt{\beta J}\, m - i\sqrt{\beta J}\, q - \beta H \right) \epsilon_j \right] +$$

$$+ \frac{1}{n} \sum_{j\in\Lambda_o} \ln\cosh\left[\left(\sqrt{\beta J}\, m + i\sqrt{\beta J}\, q + \beta H \right) \epsilon_j \right]$$

Here we have, twice, made use of the identity

$$\exp\left(a^2/2\right) = \frac{1}{\sqrt{2\pi}} \int dx \; \exp\left(-\frac{x^2}{2} + a x \right)$$

with $a = \sqrt{\frac{2J}{n}} \cdot \frac{1}{2} \cdot \left[\sum_{i\in\Lambda_e} \varepsilon_i \sigma_i + \sum_{i\in\Lambda_o} \varepsilon_i \sigma_i \right]$ in one case and

$a = i\sqrt{\frac{2J}{n}} \cdot \frac{1}{2} \cdot \left[\sum_{i\in\Lambda_e} \varepsilon_i \sigma_i - \sum_{i\in\Lambda_o} \varepsilon_i \sigma_i \right]$ in the other, together

with a suitable change of the integration variables.

Since $Z^{(n)}_{DAF}$ is real we have

$$Z^{(n)}_{DAF} = \frac{2^n n}{2\pi} \int dm\, dq \; \cos\left(n \varphi_n^I \right) \exp\left[-n\, \Phi^{(n)}_{DAF}(m,q) \right]$$

where

$$\varphi_n^R(m,q) = \frac{1}{2n} \left[\!\left[\sum_{j\in\Lambda_e}' \ln\left\{ \cosh^2\left[\left(\sqrt{\beta J}\, m - \beta H \right) \varepsilon_j \right] + \cos^2\left(\sqrt{\beta J}\, q\, \varepsilon_j \right) - 1 \right\} + \right.\right.$$

$$\left.\left. + \sum_{j\in\Lambda_o}' \ln\left\{ \cosh^2\left[\left(\sqrt{\beta J}\, m + \beta H \right) \varepsilon_j \right] + \cos^2\left(\sqrt{\beta J}\, q\, \varepsilon_j \right) - 1 \right\} \right]\!\right] ,$$

$$\varphi_n^I(m,q) = \frac{1}{n} \left[\!\left[\sum_{j\in\Lambda_e}' \mathrm{tg}^{-1} \left\{ \mathrm{tgh}\left[\left(\sqrt{\beta J}\, m - \beta H \right) \varepsilon_j \right] \mathrm{tg}\left(-\sqrt{\beta J}\, q\, \varepsilon_j \right) \right\} + \right.\right.$$

$$\left.\left. + \sum_{j\in\Lambda_o}' \mathrm{tg}^{-1} \left\{ \mathrm{tgh}\left[\left(\sqrt{\beta J}\, m + \beta H \right) \varepsilon_j \right] \mathrm{tg}\left(\sqrt{\beta J}\, q\, \varepsilon_j \right) \right\} \right]\!\right]$$

and

$$\Phi^{(n)}_{DAF}(m,q) = \frac{m^2+q^2}{2} - \varphi_n^R(m,q)$$

The asymptotic behaviour[13] of $Z^{(n)}_{DAF}$ is then given by

$$Z^{(n)}_{DAF} \sim \int dm \; \exp\left[-n\, \Phi^{(n)}_{DAF}(m,0) \right]$$

since the maximum of the integrand is attained at $q = 0$ for any fixed m and n. This can be seen from the fact that

$$\left. \frac{\partial \Phi^{(n)}_{DAF}}{\partial q} \right|_{q=0} = 0 \; , \quad \left. \frac{\partial^2 \Phi^{(n)}_{DAF}}{\partial q^2} \right|_{q=0} > 1 \quad \text{and} \quad \cos\left[n\, \varphi_n^I(m,0) \right] = 1 .$$

The free energy can therefore be obtained from

$$\beta f_{DAF} = \min_{m\in\mathbb{R}} \phi_{DAF}(m,c)$$

where

$$\phi_{DAF}(m,c) = \lim_{n\to\infty} \phi^{(n)}_{DAF}(m,c) = \frac{m^2}{2} - \frac{p}{2}\left[\ln\cosh(\sqrt{\beta J}\, m - H) + \ln\cosh(\sqrt{\beta J}\, m + H)\right] \tag{3}$$

However, it is known[4] that the same method as applied to the RMF hamiltonian (2), yields for the free energy

$$\beta f_{RMF} = \min_{m\in\mathbb{R}} \phi_{RMF}(m)$$

where

$$\phi_{RMF}(m) = \frac{m^2}{2} - \frac{1}{2}\left[\ln\cosh(\sqrt{\beta J}\, m - \beta h) + \ln\cosh(\sqrt{\beta J}\, m + \beta h)\right] \tag{4}$$

From (3) and (4) follows that

$$\phi_{DAF}(\beta, J, m) = p\, \phi_{RMF}(\beta, Jp, m/\sqrt{p}) \tag{5},$$

and this establishes what we call the thermodynamical equivalence of the two models. The corresponding phase diagrams are known[2,4].

There is a second order critical line defined by the equation

$$p\beta_c J = \cosh^2 \beta_c H$$

for $\beta_c \le \beta_t$, where $p\beta_t J = \frac{3}{2}$ defines a tricritical point, and for $\beta_c \ge \beta_t$ there is a first-order critical line.

III. FLUCTUATIONS

We shall now study the fluctuations of the order parameter of the DAF, by discussing the large n asymptotics of the probability distribution of the random variable

$$y^{(n)}_{DAF} = \frac{\frac{1}{2}(S_e - S_o) - n\, m^*}{n^{1-\gamma}}$$

where

$$S_{e(o)} = \sum_{i\in\Lambda_{e(o)}}' \varepsilon_i \sigma_i \quad ,$$

m^* is the equilibrium value of $\frac{S_e - S_o}{2n}$, and γ is to be chosen such that $y^{(n)}_{DAF}$ has a non-trivial limiting distribution.

The probability distribution of $y^{(n)}_{DAF}$ for large n is related to $\phi^{(n)}_{DAF}$ in the following way[12]. Let w be a random variable independent of S_e and S_o with a Gaussian distribution with mean zero and variance 1 (what we denote by $w \sim N(0,1)$), then for any real a and $0 < \gamma \leq \frac{1}{2}$

$$\frac{W}{n^{1/2-\gamma}} + \frac{\frac{1}{2}(s_e - s_0) - na}{n^{1-\gamma}} \sim \exp\left[-n\,\phi^{(n)}_{DAF}\left(s/n^{\gamma} + a, 0\right)\right] ds \tag{6}$$

Expanding $\phi^{(n)}_{DAF}(\cdot\,, 0)$ around its minimum, say m_n^* — which is in fact the only point to contribute in the computation of expectation values when $n \to \infty$[14] — one obtains

$$\exp\left[-n\,\phi^{(n)}_{DAF}\left(s/n^{\gamma} + a, 0\right)\right] \sim \exp\left\{-\sum_{j=0}^{\infty} \frac{n^{1-\gamma j}}{j!}\,\phi^{(n)}_{DAF,j}(m_n^*, 0)\,(s-\bar{s})^j\right\} \tag{7}$$

where $\phi^{(n)}_{,j}$ is the j-th order derivative of $\phi^{(n)}$ and

$$\bar{s} = n^{\gamma}(m_n^* - a) \tag{8}$$

To make the distribution (6) non-trivial in the limit $n \to \infty$, we must chose $\gamma = \frac{1}{k}$ where k is the order of the first non-zero derivative of ϕ_{DAF} at $m^* = \lim_{n\to\infty} m_n^*$. This choice of γ is closely related to the determination of the critical exponents[12].

Therefore, taking $a = m^*$ and being

$$\phi_{,j} = \lim_{n\to\infty} \phi^{(n)}_{,j} \,,$$

the probability distribution given in (6) will be, in the

limit $n \to \infty$, proportional to

$$\exp\left(-\frac{\phi_{DAF,k}(m^*,0)\, s^k}{k!}\right) ds \tag{9}$$

at criticality of order $k/2$, with $k > 2$ (k must be even, since $\phi_{DAF,k}(m^*,0) = 0$ for every odd k). Away from criticality or at a first order phase transition, when $k = 2$, one finds for the probability distribution of y_{DAF}, that

$$y_{DAF} \sim N\left(\alpha, \frac{1}{\phi_{DAF,2}(m^*,0)} - 1\right), \tag{10a}$$

where

$$\alpha \sim N\left(0, \frac{\beta}{2}\left[\mathrm{tgh}^2\left(\sqrt{\beta J}\, m^* + \beta H\right) + \mathrm{tgh}^2\left(\sqrt{\beta J}\, m^* - \beta H\right)\right]\right) \tag{10b}$$

from (7), (8) and the Central Limit Theorem. In this situation, the fluctuations are said to be non self averaging[4], since their mean α is sample dependent, i.e., dependent upon the dilution configuration.

It has been shown[4] that a similar expansion holds for the fluctuations of the order parameter in the RMF problem. The relevant variable in this case is

$$y^{(n)}_{RMF} = \frac{\sum_{i\in\Lambda} \sigma_i - n\mu^*}{n^{1-\gamma}}$$

where μ^* is the equilibrium value of the magnetization per spin, and similarly we have

$$\frac{\omega}{n^{1/2-\gamma}} + y_{RMF}^{(n)} \sim \exp\left[-n\,\phi_{RMF}^{(n)}\left(\frac{s}{n^{\gamma}} + \mu^*\right)\right] ds$$

In the limit $n \to \infty$, y_{RMF} will have a self-averaging probability distribution proportional to

$$\exp\left(-\frac{\phi_{RMF,k}(\mu^*)\; s^k}{k!}\right) ds \qquad (11)$$

at a criticality of order $k/2$, $k \geq 4$. Again, away from criticality and at first order criticality

$$y_{RMF} \sim N\left(\alpha, \frac{1}{\phi_{RMF,2}(\mu^*)} - 1\right), \qquad (12)$$

where α is a random variable distributed just as in (10b).

Under the mapping (5) which establishes the thermodynamical equivalence of the two models, the random variable y_{DAF} at values (β, J) of the external parameters, is mapped into $\frac{y_{RMF}}{\sqrt{p}} = \tilde{y}_{RMF}$ at (β, pJ).

Using (3) and (4), ϕ_{RMF} and ϕ_{DAF} may be rewritten as:

$$\phi_{RMF}(m) = \frac{m^2}{2} + \Psi(m\sqrt{\beta J})$$

$$\phi_{DAF}(m,0) = \frac{m^2}{2} + p\,\Psi(m\sqrt{\beta J})$$

where

$$\Psi(x) = -\frac{1}{2}\left[\ln\cosh(x+\beta H) + \ln\cosh(x-\beta H)\right],$$

so that

$$\phi_{RMF,j}(\mu^*) = \frac{\mu^{*2}}{2}\delta_{j,0} + (\beta J)^{j/2}(1-\delta_{j,1})\Psi_{,j}(\mu^*\sqrt{\beta J}) + \delta_{j,2}$$

$$\phi_{DAF,j}(m^*,0) = \frac{m^{*2}}{2}\delta_{j,0} + (\beta J)^{j/2} p\,(1-\delta_{j,1})\Psi_{,j}(m^*\sqrt{\beta J}) + \delta_{j,2}.$$

Therefore, under the mapping (5),

$$\phi_{RMF,j} \longrightarrow p^{j/4}\,\phi_{DAF,j}.$$

One then finds for the probability distribution of $\tilde{y}_{RMF}$

$$\tilde{y}_{RMF} \sim \exp\left[-\frac{\phi_{DAF,j}}{p^{j/4}}\,\frac{(s-\alpha)^j}{j!}\right] ds, \qquad (13)$$

where for $j > 2$, $\alpha = 0$ and for $j = 2$ α is a random variable with distribution given by (10b).

Comparison of (13) with (9) and (10) shows that for any $j \geq 2$ the distributions are different, fluctuating less in the RMF than in the DAF.

So, neither away from criticality (where $j = 2$), nor at a criticality of any order $j/2$ (j even) are the fluctuations equivalent; their distributions are of the same type, but their moments are different, except for the average.

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