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Abstract

Starting from the classical description of a left moving massless relativistic particle the quantization of chiral bosons is discussed. Using the Batalin-Fradkin-Vilkowski formalism the particle propagator is derived and the result is extended to a hidden soliton sector.

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Two dimensional chiral boson fields are very important objects. Indeed, they are basic ingredients to the formulation of the Hetherotic string and they are also the elementary building blocks to the construction of interesting theoretical models (the Thirring model illustrates this observation, [1]). However, in spite of many sucesses which recently have been achieved in the program of quantization of chiral bosons, [2,3], some relevant issues still deserve further discussions. One among these is the lack of a Lorentz scalar Lagrangian describing the chiral boson what prevents the application of systematic procedures to treat interacting fields. Even the non interacting case is not completely understood. Implementing the chiral constraint in a linear way leads to two chiral fields (instead of just one) and, moreover, the quantization space turns out not to be positive definite. The Lagrangian proposed by Siegel, [4], poses new problems coming from the fact that the constraint is quadratic.

The above remarks have motivated us to reconsider the problem of the quantization of chiral bosons starting from another point of view, namely, from the classical description of a left moving massless relativistic particle and then proceeding its quantization by summing up over all particle's histories, reobtaining the results of [2,1]. In the past such procedure has produced some interesting results. In the case of a relativistic spinning particle, for example, it unvealed an unsuspected local supersymmetry of the action [5].

Consider a left moving massless relativistic particle. More precisely, the particle world line is the left part of the light cone so that its energy and momentum are equal ($p^o = p^1$). A guiding principle to find the particle's action is its reparametrization invariance since it is physically irrelevant the way one choses to parametrize the time flow along the particle trajectory. This implies that the canonical Hamiltonian vanishes, [6].

Therefore the classical Lagrangian is given by

$$\mathcal{L} = p_\mu \dot{x}^\mu - H_{tot} = p^o \dot{x}^o - p^1 \dot{x}^1 + \lambda T \tag{1}$$

where λ is a Lagrange multiplier and T is the constraint to enforce the condition that the particle moves only to the left. Besides T, the Lagrangian (1) has the primary constraint $\pi = \frac{\delta \mathcal{L}}{\delta \dot{\lambda}} = 0$.

Let us focus our attention on T. T cannot be linear in the momenta since then it would be impossible to relate $\dot{x}^\mu$ with the $p's$; moreover, it can be checked that the resulting model is not invariant under reparametrizations. The next simplest possibility for T is it to be quadratic in the momenta. $T = (p^o - p^1)(p^o + p^1)$ is just the usual constraint for a massless relativistic particle,[5], and does not imply that the particle is chiral. The square of the linear constraint, $T = (p^o - p^1)^2$, is a first class constraint plagued with many difficulties in fixing the gauge (it is usual to replace it by the linear one, but, as said above, this does not work in our case). It remains to consider the cases $T = p^o(p^o - p^1)$ and $T = p^1(p^o - p^1)$ which are essentially equivalent. For definiteness, let us take

$$T = p^1(p^o - p^1) \tag{2}$$

Using (1) we can now verify that

$$\begin{aligned} &\dot{x}^o + \lambda p^1 = 0 \\ &\lambda(p^o - p^1) - \dot{x}^1 - \lambda p^1 = 0 \end{aligned} \tag{3}$$

so that the Lagrangian can be rewritten as

$$\mathcal{L} = \frac{1}{\lambda} \dot{x}^o (\dot{x}^1 - \dot{x}^o) \tag{4}$$

and it is easily checked that under the reparametrization

$$\delta x^o = \xi \dot{x}^o, \quad \delta x^1 = \xi \dot{x}^1, \quad \delta \lambda = (\xi \lambda)^{\cdot} \tag{5}$$

it changes by a total derivative term

$$\delta \mathcal{L} = (\xi \mathcal{L})^{\cdot} \tag{6}$$

In view of the above symmetry, a systematic way to quantize the theory is the Batalin-Fradkin-Vilkowski (BFV) formalism,[7], as we shall see now. Firstly, we enlarge the phase space by introducing two pairs of canonical ghosts η, $\bar{\eta}$, $\mathcal{P}$, $\bar{\mathcal{P}}$ satisfying

$$\{\eta, \bar{\mathcal{P}}\} = \{\bar{\eta}, \mathcal{P}\} = 1 \tag{7}$$

In this enlarged space there is a BRST symmetry generated by the charge

$$Q = \eta p^1(p^1 - p^o) + \mathcal{P}\pi \tag{8}$$

In the BFV formalism the propagator, defined by

$$K(X, Y) = \int_{x^\mu = X^\mu}^{x^\mu = Y^\mu} Dx^o\, Dx^1\, Dp^o\, Dp^1\, D\lambda\, D\pi\, D\mathcal{P} D\bar{\eta} d\eta D\bar{\mathcal{P}}\, \Theta(\lambda) \exp iS \tag{9}$$

where the action S is

$$S = \int_0^T (p^o \dot{x}^o - p^1 \dot{x}^1 + \pi \dot{\lambda} - \mathcal{P}\dot{\bar{\eta}} - \bar{\mathcal{P}}\dot{\eta} - \{Q, \psi\}) d\tau \tag{10}$$

is independent of the choice of ψ. Notice that the integrals in x^μ have their endpoints fixed at the initial, $x_\mu(0) = X_\mu$, and final, $x_\mu(T) = Y_\mu$, values of the particle's position. All the other integrals are unconstrained except for the restriction $\lambda > 0$, implemented by $\Theta(\lambda)$. This Heaviside step function in the integrand of (9) insures that we do not overcount the particle's histories. Indeed, the solutions of the classical equations of motion, satisfying the above boundary conditions, occur in pairs (x^μ, p^μ, λ) and $(x^\mu, -p^\mu, -\lambda)$; this degeneracy is eliminated by imposing that λ be positive.

A convenient choice for ψ is

$$\psi = \bar{\mathcal{P}}\lambda \tag{11}$$

giving rise to the proper time gauge $\dot{\lambda} = 0$. We then get

$$\{Q, \psi\} = -\lambda p^1(p^1 - p^o) - \mathcal{P}\bar{\mathcal{P}} \tag{12}$$

Integrating over π, $\mathcal{P}$, $\bar{\mathcal{P}}$, η, $\bar{\eta}$ we get

$$K(X, Y) = \int_{x^\mu = X^\mu}^{x^\mu = Y^\mu} Dx^o Dx^1 Dp^o Dp^1 D\lambda\, \delta(\dot{\lambda})\Theta(\lambda) \exp i \int_0^T [p^o \dot{x}^o - p^1 \dot{x}^1 + \lambda p^1(p^1 - p^o)] \tag{13}$$

Similarly, the boundary conditions in the $x's$ integrals are taken into account by introducing a new variable $\bar{x}^\mu$ through

$$x^\mu = X^\mu + \frac{\tau}{T}(Y^\mu - X^\mu) + \bar{x}^\mu \tag{14}$$

satisfying $\bar{x}^\mu(0) = \bar{x}^\mu(T) = 0$. We have

$$\begin{aligned} K(X, Y) &= \int Dp^o Dp^1 D\lambda\, \delta(\dot{\lambda})\, \delta(\dot{p}^o)\delta(\dot{p}^1)\, \Theta(\lambda)\, \times \\ &\exp i \textstyle\int_0^T [\frac{p^\mu}{T}(Y_\mu - X_\mu) + \lambda(p^1(p^1 - p^o) + i\epsilon)] \end{aligned} \tag{15}$$

where a damping exponential has be introduced to make the λ integral well defined. The integrals involving the functional delta functions can be easily performed since only the constant modes survive, turning the functional integrals into ordinary integrals. After integrating in λ we get (up to a multiplicative constant)

$$\begin{aligned} K(X, Y) &= \textstyle\int \frac{d^2p}{(2\pi)^2} e^{ip^\mu(Y_\mu - X_\mu)} \frac{i}{p^1(p^1 - p^o) + i\epsilon} = \\ &= \textstyle\int_0^\infty \frac{dk}{2\pi k} e^{ik(Y_o - X_o + Y_1 - X_1)} \epsilon(Y_o - X_o) \end{aligned} \tag{16}$$

It is instructive to compare this result with the propagators for chiral bosons fields [2]. From the above expressions it is clear that (16) agrees with the propagator for a field ϕ, satisfying the equation of motion

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial t}\right)\phi = 0 \tag{17}$$

(of course this is just the field theoretical version of the constraint (2))

Besides de field ϕ there is another field χ in ref. [2] which, from the beginning, is chiral ($\dot{\chi} = \chi'$) but satisfies a non local field equation

$$\chi(x) = \frac{1}{2} \int dy \epsilon(x - y) \dot{\chi}(y) \tag{18}$$

Associated to this field χ there is a classical description of a left moving relativistic particle with the constraint T taken as

$$T = \frac{p^o}{p^1} - 1 \tag{19}$$

In fact we can proceed exactly as we did for the constraint (2) to find that the propagator is now given by

$$\begin{aligned} K(X, Y) &= \textstyle\int \frac{d^2p}{(2\pi)^2} e^{ip^\mu(Y_\mu - X_\mu)} \frac{ip^1}{p^1 - p^o + i\epsilon} = \\ &= \textstyle\int_0^\infty \frac{kdk}{2\pi} e^{ik(Y_o - X_o + Y_1 - X_1)} \epsilon(Y_o - X_o) \end{aligned} \tag{20}$$

In the reference [8] chiral soliton excitations were introduced through a charge creating field, u, by the the equal times commutation relation

$$[\chi(y), u(x)] = \gamma\delta(x - y)u(x) \tag{21}$$

γ is a numerical parameter, related to the spin $s = \frac{\gamma^2}{4\pi}$ of the particle. Interesting enough, these soliton excitations can also be generated by a slight modification in the form of the constraint T. If we choose ($\rho = 1 - \frac{\gamma^2}{2\pi}$)

$$T = (p^1)^\rho (p^o - p^1) \tag{22}$$

then the particle's Lagrangian is

$$\mathcal{L} = \left(-\frac{\dot{x}_o}{\lambda}\right)^{\frac{1}{\rho}} (\dot{x}^o - \dot{x}^1) \tag{23}$$

The action is reparametrization invariant and the model can be quantized along the same lines as before. We just quote the final result for the propagator

$$K(X,Y)=\int \frac{d^2p}{(2\pi)^2}e^{ip^\mu(Y_\mu-X_\mu)}\frac{i(p^1)^{-\rho}}{p^1-p^o+i\epsilon}=$$
$$=\int_0^\infty \frac{(k)^{-\rho}dk}{2\pi}e^{ik(Y_o-X_o+Y_1-X_1)\epsilon(Y_o-X_o)} \tag{24}$$

Although this final formula is well defined for all values of ρ its derivation is not rigorously correct for the spin 1/2 case, corresponding to $\rho=0$. In that case two alternatives procedures can be adopted. We mantain $\rho\neq 0$ in the intermediary steps and then let it tend to 0 only at the end. Unfortunately this does not lead to a Lorentz invariant Lagrangian, the limit being rather singular as we see from (23).

The other possibility makes use of the supersymmetry properties of spin 1/2 relativistic particles to which we will address ourselves in a forthcoming paper.

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