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STOCHASTIC ELECTRODYNAMICS AND THE COMPTON EFFECT+

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ABSTRACT

In this paper we describe some of the main ***qualitative*** features of the Compton effect within the realm of classical Stochastic Electrodynamics (SED). We found very clear indications that the combined action of the incident wave (frequency ω), the radiation reaction force and the ***zero point fluctuating electromagnetic fields*** of SED, are able to give a high average recoil velocity $v = c\alpha/(1+\alpha)$ to the charged particle. Our estimate of the parameter α gives $\alpha = \hbar\omega/mc^2$ where $2\pi\hbar$ is the Planck constant and mc^2 is the rest energy of the particle. We have verified that this recoil is just that necessary to explain the frequency shift, observed in the scattered radiation, as due to a classical double Doppler shift. We have also calculated the differential cross section for the radiation scattered by the recoiling charge using classical electromagnetism. We found the same expression as obtained by Compton in his fundamental work in 1923.

I. INTRODUCTION

Certainly the two greatest revolutions in the XX century Physics are directly connected with the electromagnetic phenomena. One of them, the Theory of Relativity, generated profound conceptual achievements that contributed to harmonize Newton's mechanics with Maxwell's electromagnetism. The other revolution was Quantum Theory, which was born with the problem of blackbody radiation, and, gradually, penetrated the domain of microscopic phenomena. After three decades of development it has become the most powerful theory conceived up to now.

However, during this period (and also later on) several distinct interpretations of Quantum Theory were proposed attempting to clarify conceptual problems(1). Despite the efforts of De Broglie, Schrödinger, Einstein and others, the Copenhagen interpretation of Bohr, Born and Heisenberg has prevail over other interpretations. With the appearance of the so called Relativistic Quantum Electrodynamics, with a quite impressive predictive power, the attempts to find other theories and interpretations of microscopic phenomena has almost disappeared. In the same period we have observed an almost complete absence of attempts to understand microscopic phenomena through Classical Physics.

Despite the predictive power of Quantum Electrodynamics some important conceptual problems of this theory remain unsolved, for instance the renormalization problem and the more delicate questions concerning the violation of causality in the phenomena involving the so called wave function collapse. Because of this a growing number of physicists are more and more involved in the debate concerning the interpretation of microscopic phenomena(2).

One of the many attempts, developed in order to clarify at least a few points of those complicated questions, is the so called Stochastic Electrodynamics (SED). This theory is simply Classical Electrodynamics with new boundary conditions, that is the existence of fluctuating electromagnetic fields in free space even at zero temperature(1,3,4).

In this view, SED is an attempt to extend the frontier of Classical Physics up to the domain of microscopic "stochastic" phenomena(1,3).

This is done by postulating that the zero–point electromagnetic fields have a Lorentz invariant(1,3,4) spectral distribution $\rho_0(\omega)$, which is uniquely given by

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3}, \tag{1.1}$$

where ω is the frequency, c is the velocity of light and $\hbar$ is the only free parameter of the theory. This parameter can be identified with $h/2\pi$ where h is the Planck constant. In this way the theory is able to explain, within an entirely classical context, many phenomena before considered to belong to the exclusive domain of Quantum Theory. As examples we have the blackbody radiation(1), the microscopic properties of the harmonic oscillator(4), the diagmagnetic(4) behavior of free and harmonically bound charges, the paramagnetic(5) behavior of a rigid magnetic dipole, the Casimir forces(3) between macroscopic objects and polarizable particles and a few other phenomena(1,3,4). These achievements of SED and also the historic development of this theory, are very well presented in many interesting papers by Boyer(3), de la Peña(1), Santos(4), Milonni(4) and others(5–7). We address the reader to these references and also to the 1963 paper by Marshall(8) which is one of the first in SED.

If we accept the zero point electromagnetic field as real but random, we must look for more indirect observations of its effects, because direct detection is prevented due to isotropy and the Lorentz invariance(9) of the spectrum. However, a *formal* expression for the zero–point electromagnetic density, which can be shown to be equivalent to (1.1), is quite suggestive as we shall see in a while.

Let us consider the electromagnetic energy in an infinitesimal volume arround an arbitrary point **r** of free space. This is a rapidly fluctuating quantity because the electric

$\mathbf{E}(\mathbf{r},t)$ and magnetic $\mathbf{B}(\mathbf{r},t)$ fields are random functions in SED. The average electromagnetic energy density can be written as

$$\frac{<\mathbf{E}^2(\mathbf{r},t)>}{4\pi} = \frac{<\mathbf{B}^2(\mathbf{r},t)>}{4\pi} = \int_0^\infty d\omega\, \rho_0(\omega) \tag{1.2}$$

with all frequencies contributing to the energy present in the infinitesimal volume because $\rho_0(\omega)$ is given by (1.1).

If we consider a box with volume V, and write $\mathbf{E}(\mathbf{r},t)$ as a superposition of plane waves with frequencies $\omega_{\mathbf{k}} = ck$, where $\mathbf{k}$ is the wave vector, then it is not difficult to show that[3]

$$\frac{<\mathbf{E}^2(\mathbf{r},t)>}{4\pi} = \frac{1}{V} \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \tag{1.3}$$

is formally equivalent to (1.2) if $\rho_0(\omega)$ is given by (1.1).

The above expressions (1.2) and (1.3) deserve some comments. Both are divergent if $\rho_0(\omega)$ is extended to the full range of frequencies $0 \le \omega \le \infty$. The questions related to this ultraviolet divergence will not be discussed here. We simply assume that (1.1) is valid up to a very high frequency that we cannot estimate. On the other hand (1.3) is very suggestive. First of all we see that there is an average energy $\hbar\omega_{\mathbf{k}}$, associated to the waves with frequency $\omega_{\mathbf{k}}$, inside the volume V. If V is the volume of a charged particle[10], and for some reasons to be explained later, the particle is induced to absorb energy from a wave with frequency $\omega_{\mathbf{k}}$ from the background radiation, then an energy $\hbar\omega_{\mathbf{k}}$ and a momentum $\hbar\mathbf{k}$ is imparted to the charge. This resembles very much the kinematics used by Compton[11] in his corpuscular theory of light proposed in 1923 in order to explain the wavelength shift observed in the scattered radiation.

Having the above observations in mind it is quite easy to explain the purpose of our

present paper: we want to see, by using the simplest calculation, if it is possible to obtain a semiquantitative description of some of the main features of Compton scattering, within the realm of classical SED.

In order to reach our goal this paper is presented as follows.

In the next section we give a brief discussion of the historic development of the phenomena related to the Compton effect[12]. We start with the first propositions which appear a few years after Roentgen's (1895) discovey of X rays[12] and end the section with some comments about the Klein–Nishina[13] formula. However the main purpose of this section is to review Compton's efforts, both experimental and theoretical, in his attempts to explain the observed physical properties of X and γ–rays. We stress in this section the hybrid nature (*classical and quantum*) of Compton's 1923 paper.

In section III we give our qualitative description of the wavelength shift and also discuss the departure from the Thomson theory observed in the scattering radiation cross section. In order to do this we have invoked the possibility that a *resonance*, that is constructive interference, between the X–ray pulse (from the primary beam) and the waves, with the same frequencies, from the zero point radiation, may occur. In such a case it is possible to show that the radiation reaction force is able to impart a high recoil velocity $v = c\beta$ to the electron. Within our qualitative calculation we were able to show that $\beta = \alpha/(1+\alpha)$ with $\alpha = \hbar\omega/mc^2$ where m is the mass of the charge. This high recoil velocity generates a wavelength shift by double Doppler effect exactly as was proposed by Compton in his hybrid (*classical and quantum*) 1923 paper.

For completeness we discuss in the appendix Einstein–Ehrenfest's model[14–16], for the equilibrium between matter and cavity radiation at temperature T, adapted to the realm of classical SED. With simple assumptions and a nonrelativistic calculation we derive the kinematics of the Compton effect, which we show to be necessary to maintain the equilibrium between radiation and matter. We also try to identify the hypothesis (made by Einstein and Ehrenfest) which introduces the "corpuscular" properties of the random classical electromagnetic fields of SED.

Finally we present in section IV a summary of our conclusions and we also comment the connections between this work and a related work by Marshall and Santos[17,18] within the realm of "Stochastic Optics". A little discussion about future research is also presented.

II. BRIEF HISTORY OF THE COMPTON EFFECT

Near the end of the last century, doing experiments with catode rays, Roentgen (1895) discovered what he called X–rays[12]. Their nature was then discussed for approximately three decades, generating many different interpretations and theories. The clarification of the subject only started with the presentation of a corpuscular theory of radiation by Compton[11] in 1923. Later on, in 1929 with the work of Klein and Nishina[13], the phenomena involving the scattering of radiation by electrons were incorporated into the recently developed Relativistic Quantum Mechanics.

In what follows we shall give a brief exposition of some of the attempts to explain the Compton effect as well as the experiments which gradually contributed to the comprehension of the phenomena.

After his discovery of X–rays, Roentgen was not able to observe either reflection, refraction or polarization of these rays, and therefore made the proposition that they were longitudinal oscillations of the aether. Two years later, Stokes and independently Wiechert, put forward a theory based on transverse electromagnetic pulses. Latter on, in 1903, J.J. Thomson improved this theory[12].

In 1905 one piece of experimental evidence was obtained favouring the Stokes–Thomson theory, namely the detection of the X–ray polarization by Barkla[19,20]. At approximately the same time, however, the first controversy appears. It was noticed

that the incidence of X–rays on matter (and also the γ–rays just discovered) was followed by the ejection of electrons. This behavior was difficult to explain by the theory of electromagnetic pulses and Bragg[21,22] (1907) was compelled to suggest that the X–rays were made by "neutral pairs of particles travelling with some unknown velocity". A division of the physicists around the corpuscular and ondulatory theories was again starting. Very important names such Planck and Sommerfeld were resisting the corpuscular interpretation of X–rays, while Stark[23,24] was defending the identification of X–rays with the energy quanta introduced by Einstein (in 1905) in order to explain the photoelectric effect.

Those discussions stimulated many experimental works, mainly between 1908 and 1914, with very interesting results. Firstly there was observed (by doing experiments with γ–rays mainly) a deviation from the angular distribution predicted by Thomson (based on the wave theory of light). The experimental observations could not be explained by the simple expression (valid for unpolarized beams or for circular polarization)

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Thomson}} = \left[\frac{1+\cos^2\theta}{2}\right]\left[\frac{e^2}{mc^2}\right]^2 \tag{2.1}$$

where θ is the angle between the direction of primary and secondary terms. The radiation scattered in the direction of the primary beam ($\theta = 0$) seems to be more intense than that scattered in the opposite direction ($\theta = \pi$) and this fact is not predicted by the expression (2.1) which is symmetric in $\theta = 0$ and $\theta = \pi$. Another observation was that the secondary beam was less penetrating than the primary beam. Afterwards it was verified that the scattered radiation frequency deviates from the frequency of the original beam and is a function of the scattering angle

In 1912 Laue discovered X–ray diffraction which reinforced the experimental evidences favorable to the theory of electromagnetic pulses. Many physicists were convinced that Maxwell's electromagnetic theory should be applied to X–rays. The next step was, therefore, to define more clearly its behaviour when in interaction with matter.

One of the physicists who initiated careful experiments involving X–rays was Compton, in 1916, and he was a supporter of the classical wave theory rather than the corpuscular theory. Because of this Compton made many attempts, based on Classical Electrodynamics, to explain results apparently strange to the theory. He conceived, in 1917, a model(25) in which the electron was extended enough so that interference effects should be able to explain the asymmetry in the intensity of the scattered radiation. Neverthless, this model presented some difficulties, for instance the mass of the extended electron. According to Compton's calculations the electron must have a radius like 1/10 of the diameter of hydrogen atom and therefore with an electromagnetic mass 2000 times less than that observed experimentally. Later on the conceived an electron physically more acceptable, that is, with a bigger mass, by proposing the ring electron model(12) in 1918. At the same time he was developing and realizing experiments in order to test his theories.

In 1919 Compton travelled to England and there he performed a series of experiments with γ–rays. Due to the results of these experiments he decided to abandon the ring electron model. Despite the buoyant state of Physics in Europe at that time, Compton decided to continue his experiments insisting on the ideas of the theory of electromagnetic pulses. So returning to America, he prepared more experiments and, in 1921, he was sure that the scattered radiation had a lower frequency than the radiation from the primary beam(12).

This remarkable fact was difficult to be incorporated in the classical electromagnetic theory, and led Compton in the direction of the corpuscular radiation theory. Initially Compton suggested(26) that the electron absorbs from the incoming radiation an energy "quantum" with momentum $h\nu/c$, which is able to impart to the electron a velocity $v = h\nu/mc$ where m is the mass of the particle and ν is the frequency of the incident radiation. The electron reemits the energy during its motion, providing a modification in

the wavelength that was calculated, up to order v/c, according to the *classical Doppler effect.* He was able to obtain, with this reasoning, a value for the wavelength of the radiation, scattered to $\pi/2$ from the incident beam, which was very close to the experimental value.

The posture of Compton during these years of theoretical and experimental investigations had two main characteristics: a great liberty in doing experiments and *theoretical concepts derived from Classical Physics.* In his first models (spherical electron and ring electron), he believed that classical electromagnetism was a good theory to explain the scattering of radiation by electrons. The deviations observed should be attributed to the structure of the electron. Gradually, however, he modified his point of view in the direction towards the theory of energy quanta (as well as the associated concepts of energy and momentum). Therefore, he published in 1923 his fundamental work about the quantum theory of the scattering of X and γ–rays by electrons(11). However, as we shall see below, his theory was hybrid since he used many classical concepts.

He assumed, as is well known, that a "photon" with frequency ν_0 (momentum $h\nu_0/c$) collydes with an electron in such a way that energy and momentum are conserved as in a game of billiards. With a simple relativistic calculation he obtained the wavelength displacement law

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda(\theta)-\lambda_0}{\lambda_0} = \frac{h\nu_0}{mc^2}(1-\cos\theta) \qquad (2.2)$$

where θ is the same as before and $\lambda_0\nu_0 = \lambda\nu = c$.

The existence of recoiling electrons helped Compton in his calculation of the cross section for the scattered radiation. To get this he assumed that the recoiling electrons behave as a system that *emits quanta* in such a way that in the rest frame the intensity is emitted according to the Thomson *classical* theory. He was also able to prove that (2.2) is due to a double (classical) Doppler effect if each electron is moving with a constant velocity in the direction of the incident radiation beam. In this way, having succeeded by means of two different methods in obtaining the same result for the wave length shift, he postulated *that the intensity of the scattered radiation, obtained by the two methods* (the first one quantum and the second one classical) *should be the same.* With this assumption he was able to calculate, by using the *classical* method, the angular distribution of "photons" emitted by an electron moving with constant velocity $v = c\beta = c\alpha/(1+\alpha)$ where $\alpha = h\nu_0/mc^2$. The result was

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Compton}} = \frac{1}{2}\left[\frac{e^2}{mc^2}\right]^2 \left[\frac{1 + \cos^2\theta + 2\alpha(1+\alpha)(1-\cos\theta)^2}{[1+\alpha(1-\cos\theta)]^5}\right] \quad (2.3)$$

after the removing[11] of an overplus factor namely $(1+2\alpha)$.

In the next section we shall explain in details why this part of the calculation is classical as was pointed out very soon by Woo[27] in 1925.

The result (2.3) was verified to be in good agreement with the experimental data and when $h\nu_0 \ll mc^2$, we have $\alpha \ll 1$ and $\beta \to 0$, so that the expression reduces to the Thomson cross section as expected.

Independently, also in 1923, Debye[28] published a paper proposing a theory which had many points in common with the Compton calculations. By using the same considerations as Compton he was able to calculate, not only the wavelength shift (expression (2.2)), but also the energy of secondary electrons and the relation between the scattering angle θ, of the emitted "photon", and the angle φ of the recoiling electron. This is shown in the figure 1.

The relation between these angles is given by

$$(1+\alpha)\tan\left(\frac{\theta}{2}\right)\tan\varphi = 1 \quad (2.4)$$

with $0 \le \theta \le \pi$. Debye therefore concluded that, in the laboratory frame, the electrons are always scattered in the forward direction $0 \le \varphi \le \pi/2$ while the "photon" can be scattered in any direction, a result that was not so evident from Compton's work.

In order to calculate the cross section, Debye modified the Thomson result by multiplying the cross section by the factor $\nu(\theta)/\nu_0$ according to the correspondence principle. With this he obtained a result qualitatively similar to the Compton case, but with a worse quantitative agreement with the experimental data.

Immediately after these works, a series of attempts by more conservative physicists were made, trying to incorporate the Compton effect to Classical Electrodynamics through semi–classical theories. All these attempts started from the fact, pointed out firstly by Compton, that the radiation emitted by an electron which is moving in the direction of the incident beam, suffers double Doppler effect in such a way that the wavelength change is given by (2.2). As we said before a good example of such theories is the calculation by Woo (1925), by means of which it is possible to get the cross section (2.3) by using Classical Electrodynamics[27]. In order to do this Woo assumed that the incident classical wave is scattered by an electron which is moving with constant velocity $v = c\alpha/(1+\alpha)$ (here again $\alpha = h\nu_0/mc^2$) just necessary to get (2.2) through Doppler effect. We also mention the work by Breit (1926) in which he tried to accomplish Compton's theory utilizing the correspondence principle but without the concept of the "photon"[29].

An interesting and controversial[30] work is due to Bohr, Kramers and Slater (1921). It was a qualitative work in which the main goal was an attempt to conciliate two apparently contradictory situations, that is, how classical electromagnetic radiation (with continuous energy variation) can interact with a system that can only occupy discrete energy levels (an atom), in such a way that the conservation of energy is verified. The authors reasoning was that the atom is in interaction with a "virtual" radiation field which contains all the frequencies necessary to make all the possible transitions, and that energy conservation is valid only statistically. These ideas generated many arguments that were resolved by the experiments of Bothe and Geiger (1925) concerning the electron recoil[31,32]. The predictions about the ejected electrons made by all semiclassical theories were refuted by these experiments. Everything pointed towards the way proposed by Compton.

At the same time, the efforts of De Broglie and Schrödinger generated the "wave mechanics" that become popular very quickly due to its simplicity and the power of its predictions. This motivated Schrödinger (1927) to publish a paper[33] (almost unknown) with a different approach to the Compton effect. He considered that the electrons are characterized by a wave function which is a solution of a Klein–Gordon type equation, that is, "quantum" electrons (latter on it was verified this equation is not quite appropriate to describe electrons). To Schrödinger, however, the radiation was made of classical electromagnetic fields which are diffracted by the "wave matter" pattern of the incoming and outgoing electrons. This semi–classical treatment is quite different from those of Compton and Debye, mainly because Schrödinger did not mention the concept of the "photon". However he did not calculate the scattering cross sections. The only result obtained by him was the wavelength displacement given by (2.2). This formula was also derived more recently by Dodd[34]. His calculation is based on a classical model (without the concept of the "photon") for the absorption and emission or radiation.

Later on, with the proposition of a covariant equation for the electron by Dirac, Klein and Nishina (1929) obtained the famous expression for the cross section describing the scattering of radiation by electrons[13]. The treatment includes effects due to the magnetic dipole of the electron, and the results are in very good agreement with the experimental data. The Klein–Nishina formula for unpolarized beams is given by

$$\left[\frac{d\sigma}{d\Omega}\right]_{K.N} = \frac{1}{2}\left[\frac{e^2}{mc^2}\right]^2 \left[\frac{1+\cos^2\theta}{1+\alpha(1-\cos\theta)}\right]\left\{1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1-\cos\theta)]}\right\} \tag{2.5}$$

It is interesting to note that the analyses by Klein and Nishina was done ***without an explicit quantization*** of the electromagnetic field. Only Tamm (1930) realized the calculations within the realm of Quantum Electrodynamics[35] for the first time, that is, 35 years after Roentgen's discovery of the mysterious X–rays.

III. QUALITATIVE DESCRIPTION OF THE COMPTON EFFECT WITHIN THE REALM OF SED

In the last section we have shown the reasons why the theory of electromagnetic pulses, proposed by Stokes and Thomson, did not explain the Compton effect. The theory was not able to explain the wavelength displacement, the asymmetry observed in the radiation scattering and also the recoil of the electrons.

However, as far as we know, there is no classical treatment (or even semiclassical) that takes into account the possible effects generated by the zeropoint electromagnetic fluctuations that characterizes SED. As we shall try to show in what follows, these effects *are not negligible* but, on the contrary, including them we can describe semiquantitatively some important aspects of Compton's scattering.

We shall initiate our analysis by describing, with a few details, the interaction between a plane monocromatic wave (frequency ω) and a free charged particle. It is possible to find exact solutions, neglecting radiation reaction, for the equations of motion even in the relativistic case in which the magnetic force is not negligible. In Landau and Lifshitz book[36], for instance, we find a sophisticated solution to the problem. Here we only give a brief exposition of the results.

Let us consider that we have a plane wave with circular polarization which is propagating in the direction of the z axis. The electric field can be written as $\mathbf{E} = E_0(\hat{i}\cos[\omega(t-z/c)]+\hat{j}\sin[\omega(t-z/c)])$. The stationary solution is such that the coordinate r, which gives the position of the particle with charge e and rest mass m, is given by the simple expression

$$\mathbf{r} = -\frac{ecE_0}{a\omega^2}(\vec{i}\cos\omega t + \vec{j}\sin\omega t) \tag{3.1}$$

with $a^2 = (mc)^2 + (eE_0/\omega)^2$.

The conclusion is that the particle will undergo a circular motion (with the same frequency ω) in the xy plane, that is, perpendicular to the direction in which the wave is propagating. We can also verify that the particle does not recoil since, initially, it was assumed to be at rest in the origin of the coordinate system.

We are considering the case in which the wave has circular polarization only to simplify the calculation and there is no loss of generality. If the polarization is linear, for instance, the periodic motion is more complicated but there is no systematic recoil[36].

We want to analyse the situation in which the wave frequency is high . Such condition imply that we are assuming that $eE_0/mc\omega \ll 1$ for beams of X or γ–rays produced int the laboratory. This assumption ensures that the oscillatory motion will be non relativistic since $|\dot{\mathbf{r}}| = eE_0/m\omega \ll c$ as we can see from (3.1).

We can now calculate the radiation scattering cross section. The radiation intensity emitted into a solid angle $d\Omega$ around some direction characterized by the unit vector $\hat{n}$ will be

$$dI = \frac{e^2}{4\pi c^3}(\ddot{\mathbf{r}} \times \hat{n})^2\, d\Omega \quad . \tag{3.2}$$

If we use the solution (3.1), take the time average in (3.2) and divide by the modulus of the Poyting vector of the incident beam, we get

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Thomson}} = \left\{\frac{1+\cos^2\theta}{2}\right\}\left[\frac{e^2}{mc^2}\right]^2 \tag{3.3}$$

which is the Thomson cross section. This is symmetric in $\theta=0$ and $\theta=\pi$ where θ is, as before, the angle between the direction of observation and the direction of the incident wave. In doing the calculation we take $a^2 \simeq m^2c^2$ that is, $eE_0 \ll m\omega c$ which is the condition assumed above. The radiation emitted has the same frequency as the incident one. All the results of this relativistic calculation are in contradiction with the experimental facts discussed in the previous section. Our argument will be that the above calculation is *incomplete*, that is, we have not considered all the existing forces.

Let us see what happens if we take into account the radiation reaction force which is generated by the action of the self fields on the charged particle.

This difficult problem has no exact solution in the relativistic case but it is possible to use some iterative procedure as was pointed out before by Hagenbush[37], for instance. Here, however, we will use a non relativistic approximation and after we shall do an adaptation of the result to relativistic motion in the same way as is done by Landau and Lifshitz[38].

The radiation reaction force can be written approximately as $\mathbf{F}_r = 2e^2\ddot{\mathbf{v}}/3c^3$ in the reference frame in which the velocity is low. Therefore, if the particle is under the action of the electric (E) and magnetic (B) fields of wave, the equation of motion will be

$$m\dot{\mathbf{v}} = e\left[\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right] + \frac{2}{3}\frac{e^2}{c^3}\ddot{\mathbf{v}} \quad . \tag{3.4}$$

If we recognize that the radiation reaction force is small as compared with the others, we can write

$$\ddot{\mathbf{v}} \simeq \frac{e}{m}\dot{\mathbf{E}} + \frac{e}{mc}(\dot{\mathbf{v}} \times \mathbf{B}) \tag{3.5}$$

as an equation valid in the reference frame in which the particle is instantaneously at rest. In this frame we also have $\dot{v} \simeq eE/m$ and the radiation reaction force can be written as[38]

$$F_r = \frac{2}{3}\frac{e^3}{c^3}\dot{E} + \frac{2e^4}{3m^2c^4}(E \times B) \quad . \tag{3.6}$$

It is clear that the second term above is the part of the radiation reaction force which is in the direction $\hat{k}$ of the incoming wave. The first term, which is perpendicular to the incident direction, is oscillating in time and gives no contribution on the average. Therefore the time average of the modulus of the radiation reaction force can be written as

$$<F_r> = \frac{\sigma<E^2_{inc}>}{4\pi} \tag{3.7}$$

where E_{inc} is the electric field associated to the incident wave and $\sigma = \frac{8\pi}{3}\left[\frac{e^2}{mc^2}\right]^2$ is the Thomson cross section, that is, (3.3) integrated over all directions.

The force $<F_r>$ is in the direction of the incident beam, but in general is very small (except for very intense beams) and therefore generates negligible recoil[37,38]. Then the oscillatory motion characterized by (3.1) will remain with the frequency ω. This has a fundamental importance for the effects of zero–point electromagnetic fluctuations in our discussion concerning the Compton effect.

According to previous experience of many authors[3,9] working with SED we know that if we have an oscillating system (like an harmonic oscillator for instance) a *resonance*, between the system and zero–point radiation, with the same frequency, can often occur because all frequencies (and phases) are present in the zero–point electromagnetic fluctuations. Thus the same phenomenon is expected to happen to the charge which is oscillating due to the fields of the incident wave.

If we assume that the average energy density, from the background radiation (with the same frequency as the incident wave), and the volume V of the charged particle are such that

$$\frac{V<E^2_{zero\text{-}point}>}{4\pi} = \hbar\omega \tag{3.8}$$

than the energy $\hbar\omega$ can be absorbed by the particle as was suggested in the introduction. Our proposition is that this can contribute to $<F_r>$ derived above (see (3.7)) if the incident wave from the beam is in phase with the same wave (that is, same wave vector, same polarization) from the zero point electromagnetic field. If the frequency is high enough (a γ–ray for instance) $\hbar\omega$ can be as large as the rest energy mc^2 of the particle.

The above discussion is very much idealized, because in fact a beam of γ–rays from any experimental device is not a plane monochromatic wave with circular polarization. In reality we have short pulses almost monochromatic, that is, in fact we have a wave packet with a more complicated polarization.

In this more realistic situation we believe that it is possible to calculate the *probability* (Q) to obtain in the zero–point field the same configuration as in the wave packet from the γ–ray beam. The exact value of Q must depend on the specific form of the wave–packet representing the γ–ray signal. This kind of calculation could be performed by using a method similar to that proposed by Marshall and Santos within the realm of what they call *Stochastic Optics*[18]. This theory is essentially SED of visible light. The goal of these authors is the same as ours, that is, to see if the classical zero–point fluctuations of the electromagnetic fields can generate effects similar to the corpuscular theory of light. In other words we are looking for evidence for "a reaffirmation of the wave nature of light".

Since our paper is semiquantitative, we prefer to leave for future research a more realistic calculation (with a wave packet) and, instead we maintain the simplicity by assuming that the incident beam is a monochormatic plane wave. We also assume that there is some probability Q $(0 < Q < 1)$ that characterizes the possibility of resonance with the wave with the same frequency in the zero–point background. Two waves with close frequencies and wave vectors can also present constructive interference as we can see from figure 3.

With these simplified ideas in mind we can generalize (3.7) by the inclusion of the zero–point field. Then the average value of the radiation reaction force is

$$<F_r> = \frac{\sigma}{4\pi}\left\langle(E_{inc} + E_{zero\text{-}point})^2\right\rangle = \frac{\sigma}{4\pi} <E^2> \tag{3.9}$$

where $\mathbf{E}$ is the total field. We expect E_{inc} to be comparable with $E_{zero\text{-}point}$ in order to obtain an appreciable constructive interference between these two fields[18]. In this case $<F_r> = 4\sigma\hbar\omega/V$ in order of magnitude.

We must remark that the above expression is valid in the instantaneous rest frame of the charged particle. In order to calculate the recoil velocity $v = c\beta\hat{k}$ in the laboratory frame we shall use the procedure explained very clearly in the text of Landau and Lifshitz[39]. We give here only the final result namely

$$\frac{3\sigma}{mc}\frac{<E^2>}{4\pi}\, t = \left[\frac{2-\beta}{1-\beta}\right]\left[\frac{1+\beta}{1-\beta}\right]^{1/2} - 2 \tag{3.10}$$

where $v = c\beta$ is the recoil velocity attained after the time t. In writting (3.10) we have assumed that the charge was at rest at $t=0$.

In what follows we shall assume that the charge has a radius r such that the equality $r = e^2/mc^2$ is approximately valid (this is the "classical" electron radius). The detailed motion of an extended charge is complicated[40] but here we shall ignore such details.

Returning to (3.10) and considering that the average energy $V<E^2>/4\pi$ is of order $\hbar\omega$, we see that we can have a relativistic recoil even for very short times, that is for t such $t \simeq 10^{-23}$ sec, if the frequency is high $(\omega \simeq mc^2/\hbar)$. This is simply the time the electromagnetic signal requires to cross the particle dimensions. If the charged particle is recoiling with velocity $v = c\beta$ in the fields of a transverse electromagnetic wave then it will suffer an acceleration $\dot{v} = c(\dot{\beta}_{\|}, \dot{\beta}_{\perp})$. Here $\dot{\beta}_{\perp}$ and $\dot{\beta}_{\|}$ are the components perpendicular and parallel to the incident direction. The expressions for $\dot{\beta}_{\perp}$ and $\dot{\beta}_{\|}$ can be obtained using classical electromagnetism and are given below. According to Landau and Lifshitz (ref. 36, pg.71) the transverse acceleration is (see also (3.21) below):

$$c\,\dot{\beta}_{\perp} = \frac{e}{m}\left[\frac{1-\beta}{\gamma}\right] E$$

where E is the total electric field and $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor. The parallel acceleration is[39]

$$c\,\dot{\beta}_{\|} = \frac{\sigma}{m}\frac{E^2}{4\pi}\left[\frac{1-\beta}{1+\beta}\right]\gamma^{-3} .$$

According to a well known[41] formula, obtained by Liénard in 1898, the total radiated power P_{rad} is

$$P_{rad} = \frac{2}{3}\frac{e^2\gamma^4}{c}\dot{\beta}_{\perp}^2\left[1 + \gamma^2\left[\frac{\dot{\beta}_{\|}}{\dot{\beta}_{\perp}}\right]^2\right] .$$

Using $\dot{\beta}_{\perp}$ and $\dot{\beta}_{\|}$ given above one can easily show that

$$P_{rad} = \sigma c\frac{E^2}{4\pi}\left[\frac{1-\beta}{1+\beta}\right]\left[1 + \alpha_0\left[\frac{1-\beta}{1+\beta}\right]\right] \tag{3.11}$$

where α_0 is defined as

$$\alpha_0 \equiv \frac{4}{3}\,\frac{V}{mc^2}\,\frac{E^2}{4\pi} \qquad (3.11a)$$

and the volume $V = \frac{4\pi}{3} r^3$ is defined in terms of the "classical" electron radius r we have mentioned above.

We can extract interesting information from (3.11) if we assume that total power incident upon the charge is such that

$$P_{inc} = \sigma c \frac{E^2}{4\pi} = P_{rad}(\beta) \quad . \qquad (3.12)$$

The above assumption deserves a few more comments. Firstly we are considering, as before, that the particle has a charge which is distributed over a sphere of radius r. Secondly, the equality (3.12) is an approximate form of energy conservation law for the scattering process. In other words, if $P_{inc} = P_{rad}(\beta)$ is fullfiled by some value of β (which appears in (3.11)) then $v = c\beta$ is the *stationary* recoil velocity of the charge. We have used the word stationary because if $P_{inc} = P_{rad}$ there is no incident energy to increase the velocity of the recoiling charge. Let us use the notation

$$\beta = \frac{\alpha}{1+\alpha} \qquad (3.13)$$

introduced by Compton. A simple calculation will show that (3.12) is equivalent to

$$2\alpha(1+2\alpha) = \alpha_0 \quad . \qquad (3.12a$$

This is an equation for the parameter α which always has a solution for each α_0 defined in (3.11a). This means that the stationary recoil velocity $v = c\alpha/(1+\alpha)$ is always attained (in a very short time t as we have concluded from (3.10)). From (3.12a) one can see that

$$\alpha \le \frac{\alpha_0}{2} = \frac{2}{3}\,\frac{V}{mc^2}\,\frac{E^2}{4\pi} \qquad (3.12b)$$

where the equality is valid if $\alpha^2 \ll 1$.

This (more simple) inequality is good enough for our purposes. We can easily see that the average value $\langle\alpha\rangle$ depends on the average energy density $\langle E^2\rangle/4\pi$ and, according to (3.8) and the arguments for obtaining the order of magnitude of (3.9), we expect that

$$\frac{\langle\alpha_0\rangle}{2} \simeq \frac{2}{3}\,\frac{\hbar\omega}{mc^2} \quad , \qquad (3.14a)$$

if $E_{inc} \ll E_{zero\text{-}point}$ and

$$\frac{\langle\alpha_0\rangle}{2} \simeq \frac{8}{3}\,\frac{\hbar\omega}{mc^2} \quad , \qquad (3.14b)$$

if $E_{inc} = E_{zero\text{-}point}$. From these relations we conclude that $\hbar\omega/mc^2$ is a good estimative for the order of magnitude of α.

At this point it is important to recognize that α is a fluctuating quantity. In each event (characterized by a wave packet emitted by the source) α assume different values due to the random intervention of the zero–point fields. The probability distribution for α could be calculated by using the method proposed recently by Marshall and Santos (see for instance the expression (9) which appears in the Foundations of Physics paper (ref. 18)). As these authors pointed out this probability depends on the details of the wave packet coming from the source. We shall not go into such details here. For the moment we only need to know the average value of α.

Now we are going to make some approximations that have the virtue that with them the following calculation will be much more simple. We shall assume that the radiation pulse is very long in time (as compared with $t = r/c$), that is, the plane wave has an infinite duration for calculation purposes. We also assume that the particle enters into the stationary regimen (see (3.10) and (3.12)) immediately after the pulse arrives, and remains with constant velocity $v = c\beta = c\alpha/(1+\alpha)$ during all the time. Those simplistic hypothesis are idealizations which will be more discussed below.

Let us firstly see what happens with the scattered radiation if we take into account the Doppler effect. It is interesting to remember now what we said before, that is, that Compton himself used the Doppler effect in his hybrid (quantum and classical) paper in 1923[11].

The particle is moving with velocity $c\beta$ in the wave propagation direction. Due to the Doppler effect the wavelength in the proper frame will be[42]

$$\lambda_1 = \lambda\left[\frac{1+\beta}{1-\beta}\right]^{1/2} \tag{3.15}$$

where λ is the wavelength in the laboratory frame. In the proper frame the radiation emitted will have a wavelength λ_1. To an observer in the laboratory, the radiation will suffer another Doppler effect, and the wavelength observed will be such that

$$\lambda' = \lambda_1\left[\frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}\right] \tag{3.16}$$

where θ is the angle between the primary beam and the direction of observation. The above result together with (3.15) can be written as

$$\Delta\lambda = \lambda' - \lambda = \lambda\left[\frac{\beta}{1-\beta}\right](1-\cos\theta) \tag{3.17}$$

If we use now Compton's notation for β, which is given by (3.13), we get

$$\frac{\Delta\lambda}{\lambda} = \alpha(1-\cos\theta) \tag{3.18}$$

where $\alpha \simeq \hbar\omega/mc^2$ in order of magnitude.

If $\alpha \equiv \hbar\omega/mc^2$ the above result coincides with that obtained by Compton (formula (2.2)) through the relativistic kinematic relations postulated by him in his corpuscular theory of light. We are going to use $\hbar\omega/mc^2$ for α in what follows. The reason for this is that we are able to derive Compton's kinematics by using the Einstein–Ehrenfest (1923) model for cavity radiation. The model is adapted to SED and is to be considered classical in the opinion of the present authors. However the calculations are non relativistic as in the original papers by Einstein and Ehrenfest. The presentation and the discussion of all these calculations are left to the appendix.

Now we want to calculate the radiation scattering cross section, utilizing only Classical Electrodynamics, as was done by Woo[27] in 1925. We use the same assumptions, that is, the particle is moving in a straight line with relativistic velocity $v = c\beta\hat{k}$ and $\beta = \alpha/(1+\alpha)$.

If the particle has an acceleration $\dot{v}$, the radiation electric field at long distances R is given by[43]

$$E_{rad} = \frac{e}{e^2R}\,\frac{\hat{n}\times\left[\left(\hat{n}-\frac{v}{c}\right)\times\dot{v}\right]}{\left[1-\frac{\hat{n}\cdot v}{c}\right]^3} \quad ; \quad \hat{n}\equiv\frac{R}{R} \tag{3.19}$$

and the instantaneous radiation emitted in the solid angle $d\Omega$ around $\hat{n}$ is $dI = E_{rad}^2R^2d\Omega/4\pi$ or

$$\left[\frac{e^2}{\pi c^3}\right]^{-1}\frac{dI}{d\Omega} = \frac{\dot{v}^2}{\left[1-\frac{\hat{n}\cdot v}{c}\right]^4} + \frac{2(\hat{n}\cdot\dot{v})(\dot{v}\cdot v)}{c\left[1-\frac{\hat{n}\cdot v}{c}\right]^5} - \frac{(\hat{n}\cdot\dot{v})^2}{\left[1-\frac{\hat{n}\cdot v}{c}\right]^6}(1-\beta^2) \tag{3.20}$$

The incident wave has an electric field such that $\mathbf{E} = E_0\,\hat{i}\cos\omega(t - z/c) + E_0\,\hat{j}\sin\omega(t - z/c)$ and therefore the acceleration will be approximately transverse as we are going to see in a while. The exact (relativistic) expression for the acceleration is such that[44]

$$\dot{\mathbf{v}} = \frac{e}{m}\sqrt{1-\beta^2}\left[\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B} - \frac{\mathbf{v}(\mathbf{v}\cdot\mathbf{E})}{c^2}\right] \quad . \tag{3.21}$$

Since the charge is in approximately uniform motion we have $\mathbf{v}\cdot\mathbf{E} = 0$ because $\mathbf{v} \simeq c\beta\hat{k}$. In this way we get

$$\dot{\mathbf{v}} = \frac{e}{m}\sqrt{1-\beta^2}\,(1-\beta)\,\mathbf{E} \quad . \tag{3.22}$$

Now we can introduce this simple expression for $\dot{\mathbf{v}}$ in (3.20) and take the time average. Here however we must remember that the expression (3.19) for the electric field at distance R must be taken in the retarded time $t' = t - R/c$. Therefore, in performing the time average, the time increment dt must be replaced by dt' and this introduces a factor $dt/dt' = 1 - \hat{n}\cdot\mathbf{v}/c$ in (3.20). The integration is trivial. The cross section is obtained by dividing the result by the modulus of the Poynting vector from the incident beam. The result is[27,45]

$$(1-\beta^2)^{-1}(1-\beta)^{-2}\left[\frac{e^2}{mc^2}\right]^{-2}\left[\frac{d\sigma}{d\Omega}\right]_{\text{Woo}} = \frac{1}{(1-\beta\cos\theta)^3} - \frac{\sin^2\theta(1-\beta^2)}{2(1-\beta\cos\theta)^5} \tag{3.23}$$

Substituting $\beta = \alpha/(1+\alpha)$ we obtain

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Woo}} = \frac{1}{2}\left[\frac{e^2}{mc^2}\right]^2(1+2\alpha)\left[\frac{1+\cos^2\theta + 2\alpha(1+\alpha)(1-\cos\theta)^2}{[1+\alpha(1-\cos\theta)]^5}\right] \tag{3.24}$$

where $\alpha \equiv \hbar\omega/mc^2$.

In the limit $\hbar\omega \ll mc^2$ the above expression reduces to the Thomson result (3.3) as expected. Both calculations, by Compton[11] and by Woo[27] lead to the expression (3.24) for the radiation scattering cross section. At this point however Compton made a correction which improved the agreement with the experimental data. In order to do this he based his reasoning on the corpuscular properties of the "photon". In other words the scattering of a "photon" in the forward direction ($\theta = 0$) is not accompanied by the recoil of the electron. In this case Compton said that it is reasonable that the cross section should be the same as in classical Thomson's theory, that is:

$$\left[\frac{d\sigma}{d\Omega}(\theta{=}0)\right]_{\text{Thomson}} = \left[\frac{e^2}{mc^2}\right]^2 \quad . \tag{3.25}$$

However in the expression (3.24) there is an additional factor $1+2\alpha$ even for $\theta = 0$. Compton[11] simply discarded this factor in order to get Thomson's result (3.25) to the scattering in the forward direction.

Within our analysis based on SED we cannot compare the differential cross section (3.24) directly with the experiments. The reason is that instead of (3.24) one must expect a result somewhat different, that is

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{SED}} \simeq Q\,N_c\left[\frac{d\sigma}{d\Omega}\right]_{\text{Woo}} \tag{3.26}$$

where Q and N_c are corrective factors to be discussed below.

Q is the probability to find a resonance (constructive interference) between the incoming wave–packet (almost monochromatic) and the background waves with the same frequencies. As we have mentioned above, this probability is difficult[18] to calculate because it depends on the details of the incoming wave–packet. Here we have simply assumed that the wave–packet is a plane monochromatic wave.

N_c is another corrective factor also necessary in (3.25) because, according to our assumptions, we believe that $(d\sigma/d\Omega)_{Woo}$ gives a overestimate of the scattering cross section in all directions. In order to understand this better let us remember one of the simplistic assumptions made before. We have asssumed that the particle is travelling with a *constant* recoil velocity $v = c\beta = c\alpha/(1+\alpha)$ in the field of a plane wave with infinite duration. This hypothesis can, of course, generate unphysical results like the factor $1+2\alpha = (1+\beta)/(1-\beta)$, which appears in $(d\sigma/d\Omega)_{Woo}$ as can be seen from (3.24). This factor produces a divergence in the cross section when $\beta \to 1$. There is no reason to expect such a behaviour with a real wave–packet falling upon an electron in the laboratory frame.

Since we have not the intention to calculate Q and N_c from fundamental principles, we leave this problem for a future more detailed analysis. We simply assume that Q and N_c are independent of the scattering angle θ and that

$$\left[\frac{d\sigma}{d\Omega}(\theta{=}0)\right]_{SED} = \left[\frac{d\sigma}{d\Omega}(\theta{=}0)\right]_{Thomson} = \left[\frac{e^2}{mc^2}\right]^2 \tag{3.27}$$

since, according to (3.18), $\Delta\lambda = 0$ *only* for $\theta = 0$. Therefore it is reasonable to assume that our qualitative calculation within classical SED should be in agreement with the classical Thomson calculations in this angle ($\theta = 0$) because in his theory $\Delta\lambda = 0$ (here our argument resembles Compton's one given above just before (3.25)).

According to these considerations and taking into account (3.27), (3.25) and (3.24) we have $QN_c = (1+2\alpha)^{-1} = (1-\beta)/(1+\beta)$ and then we get

$$\left[\frac{d\sigma}{d\Omega}\right]_{SED} = \left[\frac{d\sigma}{d\Omega}\right]_{Compton} = \frac{1}{2}\left[\frac{e^2}{mc^2}\right]^2 \left[\frac{1 + \cos^2\theta + 2\alpha(1+\alpha)(1-\cos\theta)^2}{[1 + \alpha(1-\cos\theta)]^5}\right] \tag{3.28}$$

for the differential scattering cross section.

This is the same expression as the one obtained by Compton in 1923. He has compared the theoretical calculation with the experimental results and found a behavior very close to the observations. This comparison is shown in figure 2 in which the dotted curve is the Thomson cross section (3.3) as a function of the scattering angle θ. The continuous curve represents the cross section we have calculated (expression (3.28) for $\alpha = 1.1$ which corresponds to a wavelength $\lambda = 0.022$ Å. The experimental points are the results measured by Compton.

We want to stress again that our simplified calculations claim only to give *indications* about the possibility of an approach to Compton effect within the realm of a classical theory like SED. A fully quantitative calculation (within SED) will require a higher level approach, sophisticated enough to characterize a new work. However we hope that our analysis should encourage other researchers to concentrate their efforts in this promising direction.

IV. SUMMARY OF CONCLUSIONS

Despite the simplicity of our calculations and the approximations introduced, we are able to justify the electron recoil without the corpuscular concept of a "photon". With our estimate of the average recoil velocity of the electrons it was possible to calculate the wavelength displacement and the radiation scattering cross section as a function of the scattering angle θ.

As far as the recoiling electrons are concerned there is an appreciable difference between our calculations and the experimental facts where a *distribution* of recoiling electrons is observed.

Based on the corpuscular radiation theory Compton and Hubbard (1924) were able to calculate the differential cross section for the recoiling electrons[46]. According to the corpuscular conception, each "photon" is scattered by an electron, and this fixes in a unique way the scattering angle between them. Therefore it was not difficult to obtain an expression for the distribution of the recoiling electrons by using the differential cross section (3.28) for the scattered radiation.

In our calculation, however, we have limited ourselves to the calculation of the average recoil velocity $v = c\alpha/(1+\alpha)$ in the direction of the incident beam. But we believe that it is clear in our picture that we have not taken into account all the possible effects of the zero–point electromagnetic fluctuations. One important fact that we have not considered is that the electrons are executing some kind of Brownian motion, due to the action of the random electromagnetic fields, before the action of the incident pulse of γ or X–rays. This, of course, introduces transversal fluctuations and the recoil velocity is not simply $\mathbf{v} = c\beta\hat{\mathbf{k}}$ but a distribution around the direction $\hat{\mathbf{k}}$ of the incident beam[42]. The conclusion is that there is an important difference between SED and the quantum interpretation as far as the recoiling electrons are concerned. In our interpretation, the electron *emission* is also influenced by the zero–point radiation. This is illustrated in figure 3. In the usual quantum (corpuscular) interpretation only the primary beam, made by "photons", is responsible for this fact.

A related important point which deserves further analysis is concerned with the energy balance in our interpretation of the Compton effect. We have concluded that the background and incident radiations, combined with the radiation reaction force, are able to give a high kinetic energy to the particle in such a way that it has a relativistic recoil. According to the quantum theory, however, the energy comes only from the primary beam. In this conventional description, very well accepted, quantum objects ("photons") with dual nature (particles and waves), are in interaction with other quanta (electrons) in such a way that the energy conservation is restricted to the system "photon"–electron, *without any mention to the quantum zero–point electromagnetic fluctuations.* Similar questions appear within the realm of Stochastic Optics[17,18].

It is quite important to stress the similarity between our approach to the Compton effect and other analyses in which the concept of the "photon" is not necessary[47] to the understanding of some important questions, as the photoelectric effect and the stimulated emission, for instance. These are semiclassical approaches (very successful in quantum optics) in which the electromagnetic radiation is considered classical but the matter has quantum behavior, since the electrons are assumed to obey Schrödinger's equation. It is this wave equation that introduces the fluctuating (quantum) character which must be invoked in order to explain the transference of energy quantum from the classical (continuous) wave to the matter. This is very well explained in a paper by Scully and Sargent III[47]. The deterministic electromagnetic fields act as a perturbation allowing the transition between the quantum states of the system (an atom for instance). We observe in this treatment the recovery of Planck's view concerning the interaction between radiation and matter. We must also stress, however, that, in order to get an accurate quantum description of some phenomena like the Lamb shift and the anomalous electron magnetic moment, *it is necessary to include the zero–point fluctuations.*

Finally, it is also important to remember that the qualitative connections of our paper with the work by Marshall and Santos[17,18], within the realm of Stochastic Optics, are more or less obvious since the goal is the same, that is, to identify *pseudo–corpuscular* properties of light by invoking the role of zero–point electromagnetic fluctuations. Future research on these subjects are quite desirable because up to the moment we only have semiquantitative, model dependent, calculations to compare with the experimental measurements. However, in our opinion, the *qualitative* features of the Compton effect have been clearly identified within the realm of classical SED.

APPENDIX – A MODEL FOR EQUILIBRIUM BETWEEN RADIATION AND MATTER WITHIN STOCHASTIC ELECTRODYNAMICS

The purpose of this appendix is to discuss other ideas connected with the concept of the "photon". These ideas are invoked in order to clarify the kinematics of the Compton effect. We believe that some of the most interesting attempts in this subject are the Einstein[4,14] (1917) and Einstein–Ehrenfest[15] (1923) works concerning the equilibrium between radiation and matter. Therefore, for the reader convenience, we decided to review (briefly) part of these papers and also to discuss how these ideas could be interpreted within SED. A similar review of this and other works by Einstein can be found in a paper by Jimenez, de la Peña and Brody[48].

a) *The original Einstein's model*

The name of Albert Einstein is directly connected to the first attempts to establish a quantum theory for the electromagnetic radiation. It is well known that Einstein, attempting to give an explanation to the photoelectric effect introduced, in 1905, the energy quanta of the electromagnetic field which, later on, were called "photons". Subsequently he tried to extend his ideas to a wide class of phenomena (involving the absorption and emission of radiation by atoms and molecules), and therefore presented in 1917 a paper with many interesting results. Traces of this work are familiar to the students of modern physics under the name of "the coefficients A and B". Unfortunately, however, the main ideas contained in the paper remain almost unknown. A very good discussion of the most important ideas contained in the Einstein–Ehrenfest work can be found in the review paper by Lewis[16]. We address the interested reader to this work.

In what follows we are going to do two things at the same time, that is, to give a brief review of the Einstein–Ehrenfest papers and also to adapt their phenomenological model to SED.

In order to understand the emission and absorption of electromagnetic radiation by atoms immersed in thermal radiation characterized by the spectral density $\rho_T(\omega)$, Einstein (1917) started from the following hypothesis[16]:

1. The atoms have discrete energy states.
2. The Boltzmann distribution is valid for the atoms in these states.
3. Wien's law is valid for the spectral distribution at temperature T, that is, $\rho_T(\omega) = \omega^3 F(\omega/T)$ where F is an arbitrary function.

The first hypothesis was named by Einstein as the quantum assumption due to discrete character of the energy states. The other two are completely classical assumptions, based on thermodynamics and electromagnetism.

With these assumptions Einstein was able to derive that $\rho_T(\omega)$ must be given by the Planck formula

$$\rho_T(\omega) = \frac{\hbar}{\pi^2 c^3}\left[\frac{\omega^3}{\exp(\hbar\omega/kT) - 1}\right] \tag{A.1}$$

if we have equilibrium between radiation and matter.

However in 1923 Einstein and Ehrenfest discarded the first (quantum) hypothesis by allowing the atoms to occupy a continuous set of energy levels[16]. This fact has changed a lot our appreciation of the Einstein–Ehrenfest work because now the derivation of $\rho_T(\omega)$ seems to be entirely classical.

b) *The Einstein–Ehrenfest model within SED*

We are going to discuss this point a little more but with one additional assumption, that is, there are also the electromagnetic zero–point fluctuations of SED and they are characterized by a spectral distribution $\rho_0(\omega)$ which is given by

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3}. \tag{A.2}$$

If we admit this, then, it is quite natural to assume that this zero–point radiation is able to stimulate emissions and absorptions in a polarizable particle with harmonic internal oscillations. For simplicity we will study initially, as well as Einstein, only transitions between energies E_2 and E_1 (with $E_2 > E_1$). Later on we shall consider the continuous case. Let us assume that the system absorbs energy, with frequency ω, from the fluctuating electromagnetic fields and suffers a transition from the state with energy E_1 to the state with energy E_2. Then, according to Einstein phenomenological model, the transition probability dW_{12}/dt will be given by

$$\frac{dW_{12}}{dt} = A_{12}\,\rho_0(\omega) + B_{12}\,\rho_T(\omega) \tag{A.3}$$

where A_{12} and B_{12} are constants independent of the frequency and temperature.

Here we want to make some remarks. The first one is that (A.3) can be considered as a classical transition *probability* because both terms on the right hand side are connected with the spectral densities $\rho_0(\omega)$ and $\rho_T(\omega)$ of the *fluctuating* electromagnetic field. The second one is that the phenomenological expression above can be justified, on classical grounds, because it is well known that a harmonic oscillator with frequency ω absorbs energy from the background radiation at a rate proportional to the spectral density at the same frequency(1,5). And finally we have introduced the term $A_{12}\,\rho_0(\omega)$ which correspond to absorption from the zero point field.

Another important remark is that when the atom absorbs energy, from a wave with frequency ω and wave vector $\mathbf{k}$, it is also absorbing momentum (in the direction $\mathbf{k}$) from the background radiation. Therefore it is reasonable to assume that all the absorption processes induced by ρ_0 or ρ_T are *directional*.

In an analogous manner we are going to write the transition probability from the state E_2 to state E_1 as

$$\frac{dW_{21}}{dt} = A_{21}\,\rho_0(\omega) + B_{21}\,\rho_T(\omega) \quad . \tag{A.4}$$

Here $A_{21}\,\rho_0(\omega)$ is replacing the term corresponding to spontaneous emission in Einstein and Ehrenfest's original calculation. This means that we are assuming that the *spontaneous* emission is in fact *induced* by the zero–point radiation. This hypothesis was put forward many years ago by Welton(49) (1948) and discussed more recently by Milonni(50).

The second initial assumption by Einstein (Boltzmann statistics for the particles) will be maintained, that is, if we have $n(E_1)$ particles in the state E_1 and $n(E_2)$ in the state E_2 the relation

$$\frac{n(E_2)}{n(E_1)} = \exp\left[(E_1 - E_2)/kT\right] \tag{A.5}$$

is valid on the average.

As in the original Einstein work we will assume that the equilibrium is reached through the detailed balance conditions:

$$n(E_2)\,\frac{dW_{21}}{dt} = n(E_1)\,\frac{dW_{12}}{dt} \quad . \tag{A.6}$$

Analysing this expression in the limits $T \to \infty$ [when $n(E_2) \simeq n(E_1)$ and $\rho_T \gg \rho_0$] and $T \to 0$ [when $n(E_2) \ll n(E_1)$ and $\rho_T \ll \rho_0$] we find, respectively, the relation $B_{12} = B_{21} = B$, $A_{12} = 0$ and $A_{21} = A \neq 0$. The fact that $A_{12} = 0$ means that the zero–point radiation does not stimulate absorptions in the equilibrium situation(51). This is expected in SED because in this theory we admit that the zero–point background is also responsible for the stability of the ground state of the atoms(3,52).

It is easy to show from (A.6) that

$$\rho_T(\omega) = \frac{\frac{A}{B}\ \rho_0(\omega)}{\exp[(E_2-E_1)/kT]\ -\ 1} \tag{A.7}$$

and the Wien's law (the third classical hypothesis by Einstein) demands that $E_2-E_1 = \hbar\omega$ where $\hbar$ is a universal constant.

The value of the constant A/B can be fixed by using the Rayleigh–Jeans ($\rho_{RJ}(\omega)$) expression for the blackbody radiation. This law is valid for low frequencies ($\hbar\omega \ll kT$) and must coincide with (A.7) in this limit. In this way, because $\rho_{RJ} = kT\omega^2/\pi c^3$ and $\rho_T(\omega) = A\rho_0(\omega)kT/B\hbar\omega$, we must have A=2B. The constant $2\pi\hbar$ which appears in (A.2) can be identified again with Planck's constant. With this we verify that the Einstein's derivation of Planck's formula is compatible with the existence of zero–point electromagnetic fluctuations.

The relation A=2B deserves a few comments. At first sight this means that the zero–point electromagnetic fluctuations are twice more effective than the thermal electromagnetic fluctuations in order to induce the emission of radiation. We are inclined to understand this result (A=2B) in the same way as was suggested by Milloni[50] and by França and Marshall[51] in recent papers. There they invoked the radiation reaction force contribution to the emission processes (A.4). In other words, the self fields of the charge induces emission as well as the zero–point spectral density $\rho_0(\omega) \sim \omega^3$. The dependence on the third power of the frequency is connected to the fact that for an harmonic oscillator (frequency ω) the Larmor formula for the emitted power P_L is such that $P_L \sim \ddot{x}^2 \sim \omega^4 x^2$. If the harmonic oscillator is immersed in the zero–point radiation we have $\langle x^2\rangle = \hbar/2m\omega$ and therefore $\langle P_L\rangle \sim \omega^4\langle x^2\rangle \sim \omega^3 \sim \rho_0(\omega)$. In summary $\rho_0(\omega)$ has two channels to contribute to (A.4).

In what follows we are going to remove, based on the work of Einstein and Ehrenfest[15,16], the hypothesis of discrete energy levels for the particles.

They have assumed that one particle suffers N *absorptions*, in the frequencies $\omega_1,\omega_2,\dots\,\omega_N$, and M *emissions*, in frequencies $\omega_1',\omega_2',\dots\,\omega_M'$, in such a way that the particle goes from an initial state with energy E_I to a final state with energy E_F (E_I and E_F arbitrary). From the diagram depicted in figure 4 we can have an intuitive feeling of the Einstein–Ehrenfest proposition. In order to have a mathematical description to the processes indicated above, a generalization is necessary for the expressions (A.3) and (A.4). Therefore Einstein and Ehrenfest wrote for the transition probability dW_{IF}/dt, representing the change from the state with energy E_I to the sate with energy E_F, the following expression[16]

$$\frac{d\,W_{IF}}{d\,t} = \prod_{i=1}^{N}\left[B\,\rho_T(\omega_i)\right]\prod_{j=1}^{M}\left[A\,\rho_0(\omega_j') + B\,\rho_T(\omega_j')\right] \tag{A.8}$$

To the inverse process we have

$$\frac{d\,W_{FI}}{d\,t} = \prod_{i=1}^{N}\left[A\,\rho_0(\omega_i) + B\,\rho_T(\omega_i)\right]\prod_{j=1}^{M}\left[B\,\rho_T(\omega_j')\right]\ . \tag{A.9}$$

It is important to mention at this point that the above expressions are valid only if the "elementary" processes (emission and absorption) are statistically independent. This means that the processes of emission and absorption occur in *very short times* such that there is no interference between them[16].

By the other hand, we expect that under the *influence* of thermal and zero–point radiation, the particles are *induced* to add and subtract energy and momentum to the radiation field. This field is represented by a superposition of plane waves with all frequencies. For this reason it is reasonable to expect that each absorption (in a frequency $\omega_i = c|\mathbf{k}_i|$) is accompained by a transference of momentum (from the wave to the particle) which is *directed* according to the corresponding wave vector $\mathbf{k}_i$. If we consider *induced emission* as the reverse of *induced absorption* then it it natural to assume that also these processes involve the emission of plane waves each one with a *definite direction for the momentum.* With these considerations it is simple to accept that the energy removed or added to the radiation inside the cavity will be converted in translational kinetic added or removed from the particle. All these considerations are consistent with the Einstein–Ehrenfest model and with SED.

Taking into account these observations the final energy E_F and the initial energy E_I of a particle are expected to be related by

$$E_I + \sum_{i=1}^{N} \phi(\omega_i) = E_F + \sum_{j=1}^{M} \phi'(\omega'_j) \tag{A.10}$$

where $\phi(\omega)$ and $\phi'(\omega')$ are positive unknown quantities to be fixed below. The first sum in (A.10) represents the energy extracted from the radiation field after N absorptions, and the second sum is the energy added to the radiation field after M emissions.

From now on our discussion departs from the original one by Einstein and Ehrenfest. This happens because our intention is not to derive again Planck's formula for $\rho_T(\omega)$. This formula has been derived many times in the classical context of SED(1). Then we shall assume that $\rho_0(\omega)$ and $\rho_T(\omega)$ are well known and we change our goal, that is, we want to obtain the unknown quantities $\phi(\omega)$ and $\phi'(\omega')$.

The procedure is the same as before, that is, the Boltzmann distribution is assumed and we have

$$\frac{n(E_I)}{n(E_F)} = \exp\left[(E_F - E_I)/kT\right] . \tag{A.11}$$

Also the detailed balance condition

$$n(E_I)\frac{d\,W_{IF}}{d\,t} = n(E_F)\frac{d\,W_{FI}}{d\,t} \tag{A.12}$$

is assumed in order to keep the equilibrium between radiation and matter.

Introducing (A.8), (A.9), (A.10) and (A.11) into (A.12) we get

$$\prod_{i=1}^{N}\left[\frac{B\rho_T(\omega_i)\,\exp[\phi(\omega_i)/kT]}{A\;\rho_0(\omega_i) + B\;\rho_T(\omega_i)}\right] = \prod_{j=1}^{M}\left[\frac{B\rho_T(\omega'_j)\,\exp[\phi'(\omega'_j)/kT]}{A\;\rho_0(\omega'_j) + B\;\rho_T(\omega'_j)}\right] . \tag{A.13}$$

This expression must be valid for any N and M and also for arbitrary sets of ω_i and ω'_j. This means that each term in the square brackets above must be equal to 1. Since we know that $A/B = 2$ and that $\rho_0(\omega)$ and $\rho_T(\omega)$ are well known, from previous (different) analyses based on SED, the only unknown quantities are $\phi(\omega)$ and $\phi'(\omega')$. It is simple to show from these considerations that $\phi(\omega) = \hbar\omega$ and $\phi'(\omega') = \hbar\omega'$.

If we use these results and write (A.10) for $N = M = 1$ we get

$$E_I + \hbar\omega = E_F + \hbar\omega' \tag{A.14}$$

as a relation to be validy on the average.

This is a very suggestive result as far as the Compton's kinematic are concerned.

Einstein 1917 paper has another very interesting part which is a detailed analysis of the momentum exchange between radiation and matter. The calculation is non relativistic and very well explained in the review paper by Milloni(4). It is also possible, introducing the same hypothesis discussed above, to adapt this part of Einstein work to SED. This was done in an unpublished work by one of the authors of the present paper(53). Here we only give the result of the analysis. The conclusion was that, as is intuitively suggested by

(A.14), the absorption of energy in a frequency $\omega = c|\mathbf{k}|$ and emission in the frequency $\omega' = c|\mathbf{k}'|$ is accompained by a change in the momentum of the particle from $\mathbf{p}_I$ to $\mathbf{p}_F$. The relation between these quantities is

$$\mathbf{p}_I + \hbar\mathbf{k} = \mathbf{p}_F + \hbar\mathbf{k}' \quad . \tag{A.15}$$

The results (A.14) and (A.15) are exactly the well known Compton's kinematic relations obtained here in the non–relativistic context of classical SED.

It is also clear from (A.14) and (A.15) that apparently we have recovered the corpuscular (quantum) properties of the "photon". This is somewhat surprising because we were using only classical assumptions and SED, which is a classical theory despite the presence of $\hbar$.

In the author's opinion, the discrete character was introduced with the Einstein–Ehrenfest assumption that it is possible to *count* the number N of absorptions and the number M of emissions, all statistically independent. With this assumption it was possible to write down (A.8) and (A.9). The discrete sum (A.10) is also a consequence of the *counting* hypothesis and, of course, the relation $E_I + \hbar\omega = E_F + \hbar\omega'$.

This "corpuscular" behaviour appearing in SED does not embarass us since we are able to identify where this hypothesis was introduced, at least in the Einstein–Ehrenfest model. In fact, we expect that such a *pseudo corpuscular* behaviour can appear many times in SED[17,18].

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FIGURE CAPTIONS

FIG. 1

Incident radiation (frequency ν_0), emitted radiation (angle θ) and recoiling electron (angle φ).

FIG. 2

Scattering cross section. The continuous curve corresponds to formula (3.28), the dotted curve is Thomson's formula, and the black dots are the experimental values.

FIG. 3

Schematic picture of the constructive interference between the wave from the source (electric field $\vec{E}_s$) and the zero–point wave (electric field $\vec{E}_0$). We also show the recoil velocity $\vec{V}$ of the charged particle.

FIG. 4

Schematic picture representing the processes of N absorptions ion the frequencies $\omega_1, \omega_2, \ldots \omega_N$ and M emissions in the frequencies $\omega_1', \omega_2', \ldots \omega_M'$. E_F and E_I are respectively the (arbitrary) final and initial particle energies.

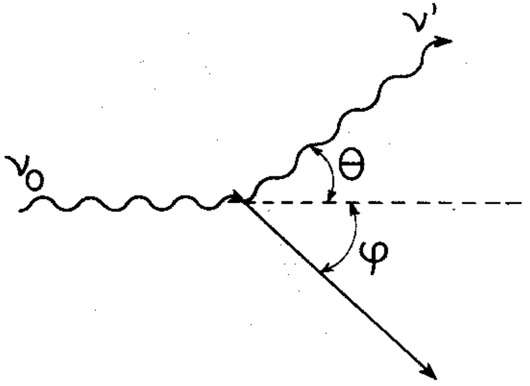

FIG. 1

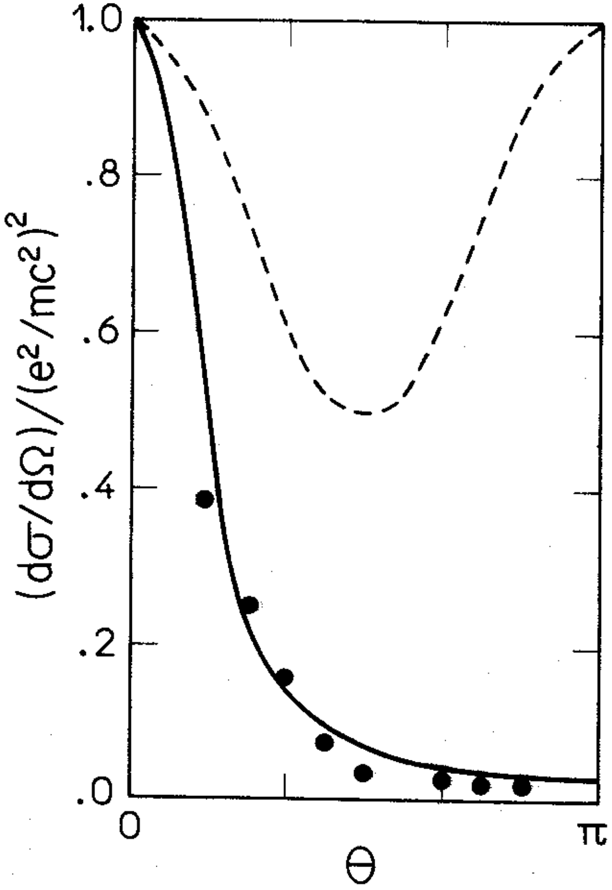

FIG. 2

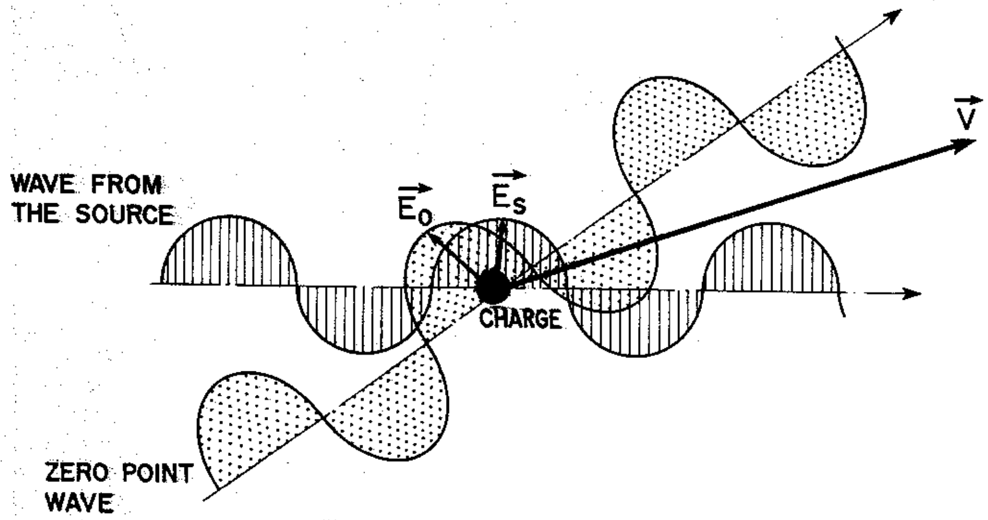
WAVE FROM
THE SOURCE
$\vec{E}_0$
$\vec{E}_s$
$\vec{v}$
CHARGE
ZERO POINT
WAVE

FIG. 3

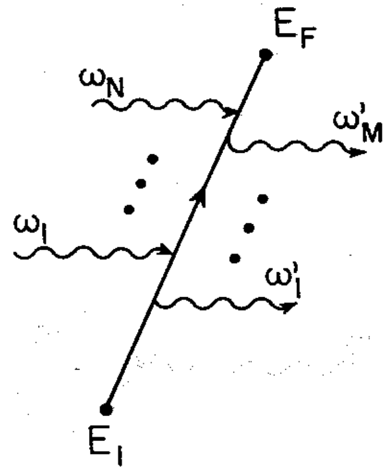
E_F
ω_N
ω'_M
ω_1
ω'_1
E_I

FIG. 4