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DOUBLE BETA DECAY, NEUTRINO PHYSICS, NUCLEAR STRUCTURE AND ISOSPIN AND SPIN-ISOSPIN SYMMETRIES

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ABSTRACT

Prominent features of the double beta decay processes are reviewed. Emphasis is placed on the neutrino masses and the quasiparticle random phase approximation (QRPA). The suppression mechanism for the $\beta\beta$-decay transition rates, proposed by Vogel and Zirnbauer, is found to be closely related to the restoration of SU(4) symmetry. It is suggested that the extreme sensitivity of the $\beta\beta$-decay amplitude on the proton-neutron coupling is a consequence of the explicit violation of the SU(4) symmetry and therefore an artifact of the model. A prescription is given for fixing this interaction strength within the QRPA itself, which in this way acquires predicting power on both single and double β-decay lifetimes.

Invited talk at the International Workshop on Hadronic Matter, Olinda/Recife, Brazil, August 14-18, 1989.

1. INTRODUCTION

Both the theory of the nuclear double beta ($\beta\beta$) decay and its experimental history have been extensively discussed in the literature [Pri 81, Hax 84, Doi 85, Ver 86, Cal 86, Bil 87, Lev 87, Mut 88, Ros 88, Avi 88]. Thus, these topics will be omitted whenever not indispensable for the clearness and the self-consistency of the present talk.

The $\beta\beta$ decay is one of the rarest processes in nature, with a half-life of the order of ~ 10^{20} years or longer. It became observable only when the single β decay is forbidden energetically or strongly suppressed due to a large change of spin. It is the pairing force which makes even-even nuclei more bound relative to the neighboring odd-odd nuclei. As an example the $\beta\beta$ decay from ^{128}Te to ^{128}Xe is shown in Fig. 1.

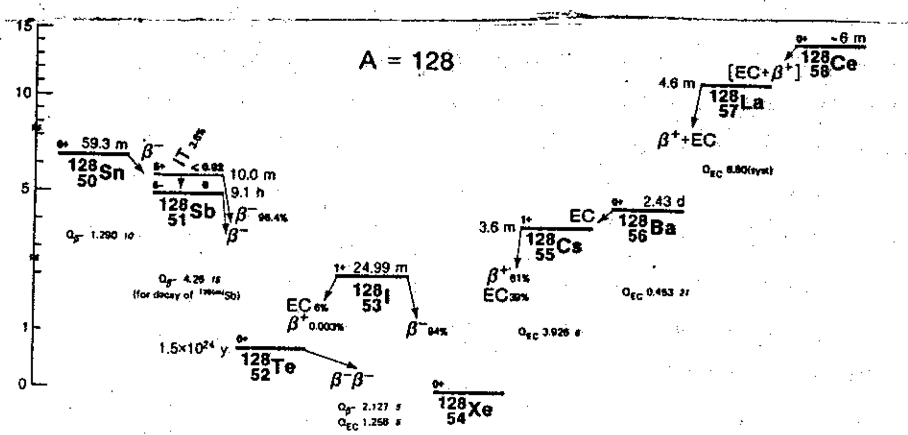


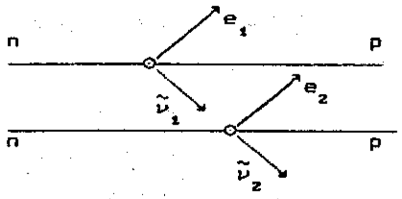
Fig.1 Mass spectrum of isobars with mass number A=128 [Led 78]. Decay modes of ground states are given. The $\beta\beta$-decay is the only possible mode for ^{128}Te.

These second order processes can take place through two different decay modes which are intimately related to the neutrino–antineutrino distinction:

1. Lepton conserving process ($\beta\beta_{2\nu}$). The $\beta\beta$-decay can be considered as a two step process

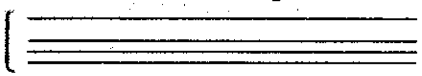
$$n \to p + e^- + \tilde{\nu}_1$$

$$n \to p + e^- + \tilde{\nu}_2$$



$$2n \to 2p + 2e^- + 2\tilde{\nu}$$

$$\text{or: } (A,Z) \to (A,Z+2) + 2e^- + 2\tilde{\nu},$$

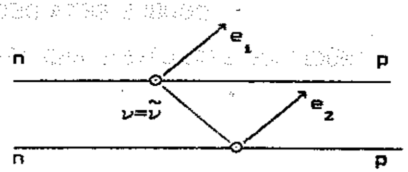
A-2 

and is described within the standard electroweak model by second order perturbation of the V–A weak Hamiltonian H_w, independently of whether neutrinos are Dirac ($\nu\neq\tilde{\nu}$) or Majorana ($\nu=\tilde{\nu}$) particles, massive or massless. However, when ν and $\tilde{\nu}$ are different particles the emission of two electrons must be accompanied by two antineutrinos.

2. Lepton number violating process ($\beta\beta_{0\nu}$). For the case in which there exists only one type of electron neutrinos the double β decay can take place according to the scheme:

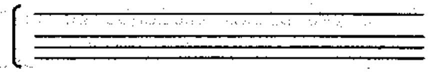
$$n \to p + e^- + \tilde{\nu}$$

$$\nu + n \to p + e^- \quad (\text{if } \nu=\tilde{\nu})$$



$$2n \to 2p + 2e^- + 2\tilde{\nu}$$

$$\text{or: } (A,Z) \to (A,Z+2) + 2e^-.$$

A-2 

Here it is assumed that a virtual neutrino is emitted in the first step and absorbed in the second. The net result is that only two electrons are emitted and therefore the lepton number is not conserved. It should be emphasized that when $m_\nu=0$, i. e., when the helicity is a good quantum number, and the lepton weak current is of pure V–A type, the helicity projection operators

$$L=(1-\gamma_5)/2$$

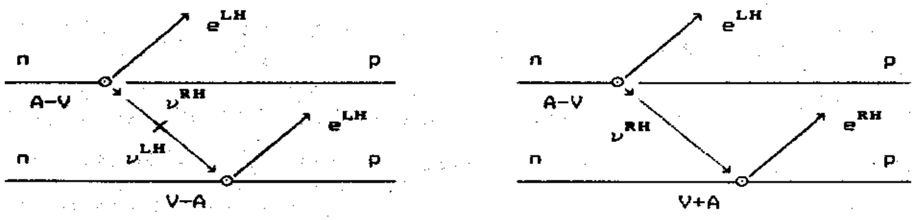
$$R=(1+\gamma_5)/2, \qquad (1.1)$$

ensure that the emitted antineutrino and the absorbed neutrino are, respectively, left-handed (LH) and right-handed (RH); that is: $n \to p + e^- + \tilde{\nu}^{RH}$, and $\nu^{LH} + n \to p + e^-$. Consequently, even when the neutrino is a Majorana particle ($\nu=\tilde{\nu}$), the $\beta\beta_{0\nu}$ decay will not take place unless at least one of the following conditions is fulfilled:

a) neutrinos are massive (the m_ν part), or

b) the right-handed (V+A) current coexists with the left-handed (V–A) current (the η part).

The diagrams in Fig. 2 illustrate these two possible mechanisms for the $\beta\beta_{0\nu}$ decay.



a) $m_\nu \neq 0$ b) $\eta \neq 0$

Fig.2 Diagrams of two possible mechanisms for $\beta\beta_{o\nu}$ decay. At each vertex the handedness of the weak current, V−A (LH) or V+A (RH) is indicated.

In order to give a more quantitative support to the above mentioned statements we adopt here the following effective weak interaction Hamiltonian

$$H_w = \frac{G}{\sqrt{2}}\, j\cdot J \ + \ h.\,c. \tag{1.2}$$

where G is the weak coupling constant,

$$j^\mu = \bar{e}\gamma^\mu\,[(1-\gamma_5) + \eta(1+\gamma_5)]\nu \tag{1.3}$$

is the lepton current, and

$$J_\mu = \bar{\Psi}_p[g_V\gamma_\mu - g_A\gamma_\mu\gamma_5]\Psi_n \tag{1.4}$$

is the left-handed (V−A) hadronic current. The transition amplitude for a $\beta\beta_{o\nu}$ decay reads

$$M_{o\nu} = \sum_{m,\nu}\left\{\frac{\langle f;e_1^-,e_2^-|H_w|m;e_1^-,\nu\rangle\langle m;e_1^-,\nu|H_w|i\rangle}{E_m - E_i + E_1 + E_\nu} - (1\leftrightarrow 2)\right\}, \tag{1.5}$$

where i,m,f, and e_1^-,e_2^- denote, respectively, the initial, intermediate, and final nuclear states and the final electron states; ν stands for the intermediate neutrino state. The lepton part of the amplitude $M_{o\nu}$ is written as [Doi 85]:

$$M_{o\nu}(\text{lepton}) = \bar{e}(x)\gamma_\sigma L\nu(x)\bar{e}(y)\gamma_\rho(L+\eta R)\nu(y)$$

$$= i\int \frac{d^4q}{(2\pi)^4}\,\frac{e^{-iq(x-y)}}{q^2 - m_\nu^2}\,\bar{e}(x)\gamma_\sigma(m_\nu L + \eta q_\mu\gamma^\mu R)\gamma_\rho e^c(y), \tag{1.6}$$

where the line connecting $\nu(x)$ and $\nu(y)$ means the contraction, which is allowed if the neutrino is a Majorana particle; the symbol C stands for the charge conjugation (see bellow), and the following relations have been employed $L(q^\mu\gamma_\mu + m_\nu)L = m_\nu L$ and $L(q^\mu\gamma_\mu + m_\nu)R = m_\nu R$. From the relation (1.6) it is now clearly seen that the $\beta\beta_{o\nu}$ decay will take place only when $\nu=\tilde{\nu}$ and when one or both of the conditions a) and b) are satisfied. The eq.(1.6) shows that for a massive neutrino, the transition amplitude is proportional to the factor

$$m_\nu/(q^2 - m_\nu^2).$$

Thus, for small neutrino mass the amplitude becomes proportional to the neutrino mass. On the other hand, for large neutrino mass the amplitude becomes proportional to $1/m_\nu$ i. e. it is again suppressed. The helicity mismatch between the main components of the left-handed interaction is also avoided by an admixture η of the lepton non-conserving right-handed interaction (Fig.2.b).

While the main emphasis here will be given to the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decay modes, a third type of $\beta\beta$ could be possible if a massless Goldstone boson, called the majoron, would existed. The majoron comes from a spontaneously broken global symmetry [Chi 80, Gel 81, Geo 81] and give rise to the lepton non conserving process

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + \chi^M ,$$

which will be designated as $\beta\beta_{0\nu,M}$.

While there is at present no indisputable evidence for neutrinoless $\beta\beta$ decay, the importance of this process for particle and nuclear physics, mainly due to its relationship with the neutrino mass, justifies the effort invested by many groups in this field. There are many open questions in neutrino physics, such as: Are neutrinos massive? If $m_\nu \neq 0$ are neutrinos Dirac or Majorana particles? Why does the nature favor only left handed currents (η=0)? Do neutrino oscillations take place? Does the majoron exist?, etc. The possibility that neutrinos have non-zero masses goes beyond the standard model and is utterly related with the grand unified theories (GUT), as well as with the astrophysics (stellar cooling and stellar evolution, the propagation of neutrinos in the interior of stars such as the sun or a supernova, etc.) and the cosmology (the question of dark matter and the formation of the universe, etc.).

On the other hand, since the $\beta\beta_{0\nu}$ decay rate depends on one or more unknown parameters (neutrino mass, RH coupling constant η, or majoron coupling) only the $\beta\beta_{2\nu}$ process is in principle calculable, and hence a comparison between experiment and theory

for the latter decay provides a measure of the confidence one may have in extracting those unknown parameters from $\beta\beta_{0\nu}$ lifetime measurements.

Until quite recently the observation of $\beta\beta$ decay has relied upon the so -called geochemical methods by measuring the abundance of daughter nuclei from $\beta\beta$ decays of elements in geological materials. In 1987 Elliott, Hahn and Moe [Ell 87] have observed for the first time $\beta\beta_{2\nu}$ decay of ^{82}Se into ^{82}Kr by direct counting method. This experiment is very important as it is consistent with the geochemical measurements on ^{82}Se and therefore it lends considerable credibility to the other geochemical determinations of the $\beta\beta$ life times (in particular that of ^{128}Te and ^{130}Te). The next exciting goal would be the laboratory test of $\beta\beta_{0\nu}$ decay and Rosen [Ros 88] has ended his recent article by saying: " If seeing the first double beta decay events in the laboratory is like hearing the first cuckoo of spring, then seeing the first no-neutrino decay will be the crowning glory of summer."

Until recently theoretical calculations of the $\beta\beta_{2\nu}$ decay rates were systematically larger than the corresponding experimental values; the discrepancy was particularly pronounced in the ^{128,130}Te isotopes. Vogel and Zirnbauer [Vog 86] have recently made important progress in this regard. They have applied the quasiparticle random phase approximation (QRPA) and shown that the ground state correlations (GSC), induced by the proton-neutron (PN) residual interaction, play an essential role in reducing the $\beta\beta_{2\nu}$ decay probability. In observing this suppression mechanism a zero-range interaction was used and the consequences of the SU(4) symmetry on the $\mathcal{M}_{2\nu}$ amplitude were discussed in the context of a schematic model. Subsequent studies [Civ 87, Eng 88, Eng 88a, Mut 88, Mut 88a], most of them performed with realistic effective interactions, lead essentially to the same conclusion: when evaluated within the QRPA, irrespective of the force employed, the predicted life-times are very sensitive to GSC within the particle-particle (PP) channel. As an example, in Fig. 3 are shown the calculated half-lives for ^{128}Te and ^{130}Te [Civ 87]. Is is seen that a minute variation of the parameter g_{pp}, which renormalizes the PN interaction in the PP-channel, gives rise to enormous variation in the half-lives. The same effect appears in the description of ordinary β^+-decay processes [Eng 88a, Mut 88a] and so far no satisfactory interpretation of this phenomenon has been put forward.

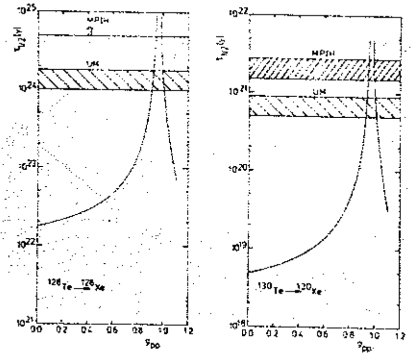


Fig.3 The calculated $\beta\beta_{2\nu}$ decay half-lives as a function of g_{pp} [Civ 87]. Experimental results are indicated either by hatched regions or lines with arrows and are taken from [Kir 86] and [Man 86].

The aim of the present talk is to point out that the extreme sensitivity of the Gamow-Teller (GT) $\beta\beta_{2\nu}$ amplitude to model parameters is closely related to the self-consistency between the residual interaction and the average mean field, as well as to the supermultiplet structure in spin and isospin space. Moreover, it is suggested that physically meaningful nuclear parameters, within the Vogel and Zirnbauer model (VZM) [Vog 86], are those which lead to the maximal restoration of the Wigner SU(4) symmetry. As a frame of reference, the results for the Fermi (F) $\beta\beta_{2\nu}$ amplitude and the isospin invariance, will be used [Hir 89].

On the other hand, as the nuclear physicists are, in general, less familiar with the issue of neutrino masses, we also address the question of several acceptable ways to modify the standard theory as to generate neutrino masses. This aspect of $\beta\beta$ decay will discussed rather extensively following mainly a recent article of Peccei [Pec 89].

2.NEUTRINO PHYSICS INVOLVED IN THE DOUBLE BETA DECAY

We adopt the following conventions with respect to the charge-conjugation (C)

$$\psi^c = C\gamma^0\psi^* = i\gamma^2\psi \ , \quad \bar{\psi} = \psi^T C \ , \tag{2.1}$$

where ψ^T denotes the transpose of ψ and $\bar{\psi} = \psi^+\gamma^0$. In the Weyl representation the γ matrices read

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \gamma^k = \begin{pmatrix} 0 & -\sigma^k \\ \sigma_k & 0 \end{pmatrix} \quad , \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; . \qquad (2.2)$$

We also use the notation

$$\psi_L = L\psi \; , \psi_R = R\psi \quad ; \; \psi_L^c \equiv (\psi_L)^c = R\psi^c = (\psi^c)_R \; . \qquad (2.3)$$

2.1 Neutrino mass terms of Dirac and Majorana type

The Dirac-type mass connects the left (L) and right (R) components of the neutrino field ν,

$$\mathcal{L}_D = - m_D \bar{\nu}\nu = - m_D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \; ,$$

$$= - m_D (\bar{\nu}_L \nu_R + \bar{\nu}^c_R \nu^c_L) \; , \qquad (2.4)$$

and the mass eigenstate is

$$\nu = \nu_L + \nu_R \; . \qquad (2.5)$$

A Majorana-type mass connects the L and R components of conjugate fields. In the notation of (2.3), we can have [Che 88]

$$\mathcal{L}^L_M = - \frac{1}{2} m^L_M \bar{n} n = - \frac{1}{2} m^L_M (\bar{\nu}^c_L \nu_L + \bar{\nu}_L \nu^c_L) \qquad (2.6a)$$

$$\mathcal{L}^R_M = - \frac{1}{2} m^L_M \bar{\mathcal{N}} \mathcal{N} = - \frac{1}{2} m^R_M (\bar{\nu}^c_R \nu_R + \bar{\nu}_R \nu^c_R) \; . \qquad (2.6b)$$

The mass eigenstates are then self-conjugate fields

$$n = \nu_L + \nu^c_L \; ; \; n^c = n$$
$$\mathcal{N} = \nu_R + \nu^c_R \; ; \; \mathcal{N}^c = \mathcal{N} \; . \qquad (2.7)$$

The Dirac mass m_D results from the coupling of the independent LH and RH fields, while Majorana masses m^L_M and m^R_M arise from the coupling of fields with their charge-conjugate fields. Only the first kind of mass term is available for the charged leptons, since Majorana masses would violate the charge conservation. The mass coupling for neutrinos and electrons is illustrated in Fig.4 [Mut 88a].

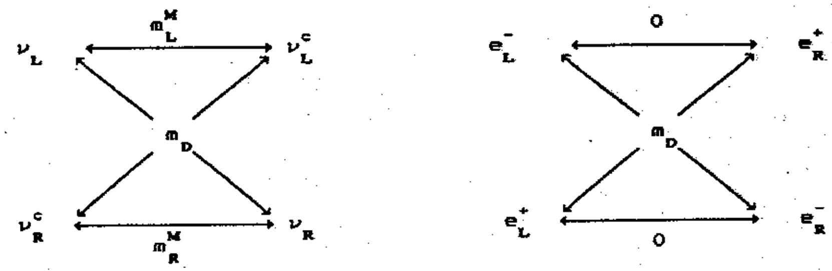


Fig.4 Coupling schemes for neutrinos and electrons.

The inverted relations of (2.7) are:

$$\nu_L = Ln \; ; \; \nu_R = R\mathcal{N} \; ; \; \nu^c_L = Rn \; ; \; \nu^c_R = L\mathcal{N} \; . \qquad (2.8)$$

and when γ_5 matrix is applied to ν, n and $\mathcal{N}$ fields one gets

$$\gamma_5 \begin{bmatrix} \nu \\ n \\ \mathcal{N} \end{bmatrix} = \begin{bmatrix} \nu' \\ n' \\ \mathcal{N}' \end{bmatrix} = \begin{bmatrix} -\nu_L + \nu_R \\ -\nu_L + \nu_L^c \\ \nu_R - \nu_R^c \end{bmatrix} \qquad (2.9)$$

Clearly this changes the sign of m_D, m_M^L and m_M^R in eqs. (2.4) and (2.6).

When both Dirac and Majorana mass terms are simultaneously present we have:

$$\mathcal{L}_{DM} = - m_D \bar{\nu}_L \nu_R - \frac{1}{2} (m_M^L \bar{\nu}_L^c \nu_L + m_M^R \bar{\nu}_R^c \nu_R) + \text{h.c.}$$

$$= - \frac{1}{2} [m_D (\bar{n}\mathcal{N} + \bar{\mathcal{N}} n) + m_M^L \bar{n} n + m_M^R \bar{\mathcal{N}}\mathcal{N}]$$

$$= - \frac{1}{2} (\bar{n}\ \bar{\mathcal{N}})\ M \begin{pmatrix} n \\ \mathcal{N} \end{pmatrix}; \quad M = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix} . \qquad (2.10)$$

After diagonalizing the mass matrix **M** we get

$$m_1, m_2 = - \{ (m_M^L + m_M^R) \pm [(m_M^L - m_M^M)^2 + 4m_D^2]^{1/2} \} , \qquad (2.11)$$

corresponding to the Majorana mass eigenstates ($N_i = N_i^c$)

$$N_1 = n \cos\theta - \mathcal{N} \sin\theta ,$$
$$N_2 = n \sin\theta + \mathcal{N} \cos\theta , \qquad (2.12)$$

with

$$\tan 2\theta = \frac{2\ m_D}{m_M^L - m_M^L} . \qquad (2.13)$$

The eq. (2.10) can now be rewritten as

$$\mathcal{L}_{DM} = - \frac{1}{2}\ m_1\ (\bar{N}_1 N_1) - \frac{1}{2}\ m_2\ (\bar{N}_2 N_2) . \qquad (2.14)$$

2.2. Mass generation options for neutrino in the SU(2)×U(1) models.

Although neutrinos play an essential role in the SU(2)×U(1) model of electroweak interaction–the standard model– there is no compelling reason to introduce neutrino masses [Gla 61, Wei 67, Sal 68]. As it is well known, in this model the chiral lepton fields ν_L and ν_R have different SU(2) properties, with ν_L being part of a SU(2) doublet and ν_R being a singlet:

$$\begin{bmatrix} \nu \\ e \end{bmatrix}_L \sim (2,-1), \quad e_R \sim (1,-2), \quad \nu_R \sim (1,0) . \qquad (2.15)$$

The first entry in parentheses on the right-hand sides of eq. (2.15) are the dimensions of the SU(2) representations and the second entries are U(1) hypercharge $Y = 2(Q-T_3)$. Within the standard model also enters into the play the Higgs doublet $\Phi = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix} \sim (2,-1)$, whose vacuum expectation value is responsible for breaking down of $SU(2)\times U(1) \to U(1)_{em}$.

The electroweak interaction acts only on the left-handed neutrino ν_L, while the right-handed neutrino ν_R has no U(1) interaction and therefore it is sterile under SU(2)×U(1). Strictly speaking there is no right-handed neutrino in the standard model and we cannot tell (up to now) if ν_R exists at all.

Thus, as $\bar{\nu}_L \sim (2,+1)$ and $\nu_R \sim (1,0)$ the Dirac-type mass term is not allowed by SU(2)×U(1). However it can be generated by a renormalizable Yukawa coupling with the Higgs doublet Φ, i. e.,

$$\mathcal{L}_d = -\frac{\Gamma^{\nu}}{\sqrt{2}} (\bar{\nu},\bar{e})_L \, \Phi \, \nu_R + \text{h. c.} \; . \tag{2.16}$$

As Φ acquires a non zero vacuum expectation value, $\langle\Phi\rangle$, with $\langle\phi^o\rangle = v_d/\sqrt{2}$, the Dirac mass for neutrino [Pec 89]

$$m_d = \Gamma^{\nu} v_d \; ; \; v_d \cong 250 \text{ GeV} \; , \tag{2.17}$$

is generated as shown schematically in Fig.5. This mass term is lepton number conserving since $\mathcal{L}_d$ is invariant under

$$\begin{bmatrix} \nu \\ e \end{bmatrix}_L \rightarrow \begin{bmatrix} \nu \\ e \end{bmatrix}'_L = e^{i\alpha} \begin{bmatrix} \nu \\ e \end{bmatrix}_L$$
$$\nu_R \rightarrow \nu'_R = e^{i\alpha} \nu_R \; . \tag{2.18}$$

It should be mentioned that since $v_d \cong 250$ GeV , to get neutrino masses in the eV range requires $\Gamma^{\nu} \sim 10^{-10} - 10^{-11}$.

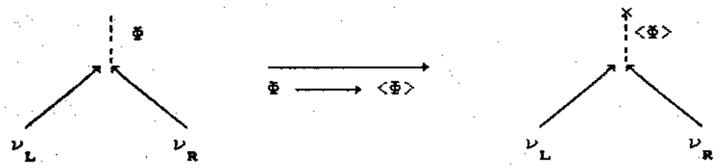


Fig.5. Dirac mass generation for neutrinos.

Being ν_R a singlet the Majorana mass m^R_M is allowed by SU(2)×U(1), and at variance with m_D, which is related to the scale of the SU(2)×U(1) breaking, m^R_M is a scale independent quantity. It is clear that the presence of m^R_M breaks the total lepton number symmetry since the combinations $\bar{\nu}^c_R \nu_R$ and $\bar{\nu}_R \nu^c_R$ have ℓ=2 and ℓ=-2, respectively.

Although m^R_M can be just a bare mass term it can also arise from the vacuum expectation value of a singlet S(2)×U(1) Higgs field σ. Here one introduces a lepton conserving coupling

$$\mathcal{L}^{\nu}_s = - \frac{h}{\sqrt{2}} \left(\bar{\nu}^c_R \sigma \nu_R + \bar{\nu}_R \sigma^{+} \nu_R{}^c \right) \tag{2.19}$$

This coupling does not violate the lepton number ℓ=2, i. e., it is invariant under a lepton number transformation

$$\nu_R \rightarrow \nu'_R = e^{i\alpha} \nu_R$$
$$\sigma \rightarrow \sigma' = e^{i\alpha} \sigma \; . \tag{2.20}$$

When this symmetry is spontaneously broken the Higgs field σ is written as

$$\sigma = \frac{1}{\sqrt{2}} (v_s + \rho_s + i\chi^M_s) , \tag{2.21}$$

where $v_s = \sqrt{2}\langle\sigma\rangle \neq 0$ is the vacuum expectation value, while the fields ρ_s and χ^M_s are massive and massless (Goldstone) fields, respectively.

Therefore two masses are possible for ν_R:

$$m_M^R = \begin{cases} m_M^R & \text{explicit mass} \\ hv_s & \text{spontaneous mass .} \end{cases} \quad (2.22)$$

and $\mathcal{L}_s$ can can be expressed in terms of the field $\mathcal{N}$ as

$$\mathcal{L}_s^\nu = -\frac{h}{2}\,[\bar{\mathcal{N}}\mathcal{N}\,(v_s + \rho_s) + i\bar{\mathcal{N}}\gamma_5\mathcal{N}\,\chi_s^M\;] \;. \quad (2.23)$$

The Majorana mass m_M^L violates SU(2)xU(1) and consequently it cannot be an explicit mass term. A possibility to generate m^L is through a Yukawa coupling of a Higgs triplet field

$$\vec{\tau}\cdot\vec{\Delta} = \begin{bmatrix} \Delta^0 & \Delta^-/\sqrt{2} \\ \Delta^-/\sqrt{2} & \Delta^{--} \end{bmatrix} , \quad (2.24)$$

to the lepton doublet $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$:

$$\mathcal{L}_t = -\frac{g}{\sqrt{2}}\left[\,\overline{(\nu,e)}_L^c\;\vec{\tau}\cdot\vec{\Delta}\;\begin{pmatrix} \nu \\ e \end{pmatrix}_L\;\right] + \text{h. c.}, \quad (2.25)$$

which is invariant under lepton number transformation

$$\begin{bmatrix} \nu \\ e \end{bmatrix}_L \to \begin{bmatrix} \nu \\ e \end{bmatrix}_L' = e^{i\alpha}\begin{bmatrix} \nu \\ e \end{bmatrix}_L$$

$$\vec{\Delta} \to \vec{\Delta}' = e^{-2i\alpha}\,\vec{\Delta}. \quad (2.26)$$

if $\vec{\Delta}$ carries ℓ=-2.

By keeping only the pure neutrino part of $\mathcal{L}_t$ we get the Lagrangian

$$\mathcal{L}_t^\nu = -\frac{g}{\sqrt{2}}\,[\;\bar{\nu}_L^c\Delta^0\nu_L + \bar{\nu}_L(\Delta^0)^+\nu_L^c\;] + \text{h. c.}, \quad (2.27)$$

which is totally similar to $\mathcal{L}_s^\nu$, given by eq. (2.19), and after the symmetry (2.26) is spontaneously broken, the Δ^0 takes the form:

$$\Delta^0 = \frac{1}{\sqrt{2}}\,(v_t + \rho_t + i\chi_t^M\,). \quad (2.28)$$

The quantity $v_t = \sqrt{2}\langle\Delta^0\rangle$ is rather well bounded experimentally [Pec 89] and one gets

$$m_M^L = g\;v_t, \quad v_t \le 17\ \text{Gev}\;, \quad (2.29)$$

while $\mathcal{L}_t^\nu$ can be put in the form in the same form as $\mathcal{L}_s^\nu$, i. e.,

$$\mathcal{L}_t^\nu = -\frac{g}{2}\,[\bar{n}n\,(v_t + \rho_t) - i\bar{n}\gamma_5 n\,\chi_t^M\,]. \quad (2.30)$$

2.3 Why is neutrino mass small? The see-saw mechanism.

As explained in the previous section neutrino masses, whether coming from bare mass terms or Yukawa coupling (through Higgs mechanism), are arbitrary parameters. That is, they are not calculable and have to be obtained from experimental data.

The neutrino masses, if nonzero, must be small when compared with all other mass scales and the see-saw mechanism [Yan 79, Gel 79, Moh 80, Doi 85], which will be discussed now, provides a theoretical way to understand their smallness.

In the same way as the Dirac mass m_D is induced when the S(2)xU(1) symmetry is broken it can be argued that the right-handed Majorana mass m_M^R, which explicitly violates the lepton number conservation and is scale independent, arises from the symmetry breaking of grand unified theories (GUT) of the strong and electroweak interactions. This demands two vastly different mass-energy scales for m_M^R and m_D ($m_M^R \gg m_D$) corresponding to two different stages of symmetry breaking, i.e.,

$$GUT \underset{m_M^R}{\rightarrow} S(2)\times U(1) \underset{m_D}{\rightarrow} U(1)_{em} .$$

On the other hand from eqs. (2.17) and (2.29) it seems reasonable to assume that $m_D \geq m_M^L$ and thus a natural hierarchy to imagine for the mass matrix (2.10) is that

$$m_M^R = M \gg m_D \gg m_M^L \cong 0. \tag{2.31}$$

Therefore

$$M \cong \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} . \tag{2.32}$$

and its eigenvalues are, approximately ($\theta \cong - m_D/M$), $m_1 \cong - m_D^2/M$ and $m_2 \cong M$. The sign of m_1 is irrelevant as it can be changed through the chiral rotation $\gamma_5 N_1 \rightarrow N_1$ ($\theta \rightarrow -\theta$) as in eq. (2.9). Thus we end with the eigenvalues and the eigenvectors of M of the form

$$m_1 \cong \frac{m_D^2}{M} \; ; \quad N_1 \cong n - \frac{m_D}{M} \mathcal{N} ,$$
$$m_2 \cong M \; ; \quad N_2 \cong \frac{m_D}{M} n + \mathcal{N} , \tag{2.33}$$

and from the inverse relations

$$n = \sum_{j=1}^{2} U_j N_j \cong N_1 + \frac{m_D}{M} N_2 ,$$
$$\mathcal{N} = \sum_{j=1}^{2} V_j N_j \cong N_2 - \frac{m_D}{M} N_1 , \tag{2.34}$$

and the eq. (2.8) we see now that

$$\nu_L \cong N_{1L} + \frac{m_D}{M} N_{2L} ,$$
$$\nu_R \cong N_{2R} - \frac{m_D}{M} N_{1R} \tag{2.35}$$

Therefore ν_L is mostly N_1 and ν_R is dominantly N_2 and the see-saw mechanism allows a light left-handed neutrino to exists, which for all practical purposes, agrees with the massless neutrino of the standard model.

2.4. The process with a majoron emission $\beta\beta_{0\nu,M}$.

As mentioned before it is possible that lepton number is an exact global symmetry which is, however, spontaneously broken, with the creation of a zero mass Goldstone boson called majoron. There are two possible types of majoron bosons χ_s^M and χ_t^M, depending on whether ν_R or ν_L acquire a Majorana mass, and are related, respectively, with the SU(2) singlet field σ and triplet field $\vec{\Delta}$. The first one (CMP-majoron) was introduced by Chikashige, Mohapatra and Peccei [Chi 80] and the second one (GR-majoron) by Gelmini and Roncadelli [Gel 81].

As easily seen from eq. (2.23), the coupling of the CMP-majoron to neutrinos is given by:

$$\mathcal{L}^{\nu}_{s,M} = - i \frac{h}{2} \bar{\mathcal{N}} \gamma_5 \mathcal{N} \chi^{M}_{s} , \qquad (2.36)$$

and by making use of eqs.(2.34) these majoron-neutrino interaction terms can be expressed by means of physical fields N_1 and N_2.

In the case of GR-model, however, no RH neutrino is present and the only neutrino mass which enters into the play is m^{L}_{M}. Thus, the substitution $n \rightarrow N_1$ should be done in eq.(2.30) and the majoron-neutrino coupling reads:

$$\mathcal{L}^{\nu}_{t,M} = i \frac{g}{2} \bar{N}_1 \gamma_5 N_1 \chi^{M}_{t} . \qquad (2.37)$$

We will discuss only the GR-majoron which has a few more experimental consequences [Pec 89]. The $\beta\beta$ decay accompanied by the emission of a majoron ($\beta\beta_{o\nu,M}$) is represented schematically in Fig.6.

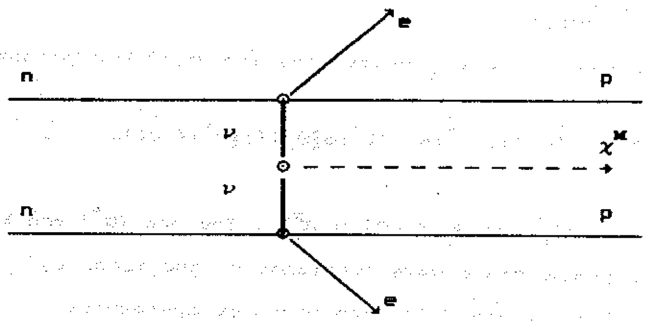


Fig.6. The $\beta\beta_{o\nu}$ decay mode with majoron emission.

The one family treatment of neutrinos presented in this section can easily be generalized to a system of n LH neutrinos ν_{iL} and n RH neutrinos ν_{iR} (i = e, μ, ...), corresponding to n families. Consequently we will have the same amount of neutrino fields n_i and $\mathcal{N}_i$, each one of them expressed as a superposition of 2n Majorana neutrinos with masses m_j. In particular, the electron neutrinos take now the form:

$$n_e = \sum_{j=1}^{2n} U_{ej} N_j \quad ; \quad \mathcal{N}_e = \sum_{j=1}^{2n} V_{ej} N_j \quad . \qquad (2.38)$$

At the same time the weak current (1.3) reads now

$$j^{\mu} = \bar{e}\gamma^{\mu} [(1-\gamma_5) n_e + \eta (1+\gamma_5) \mathcal{N}_e], \qquad (2.39)$$

and the Langrangian (2.37) is generalized as

$$\mathcal{L}^{\nu}_{t,M} = \frac{i}{2} \sum_{j,k=1}^{n} \overset{*}{N}_j \gamma_5 N_k \chi^{M}_{t} . \qquad (2.40)$$

3.NUCLEAR STRUCTURE INVOLVED IN DOUBLE BETA DECAY.

3.1 Double beta decay life times.

Having established the intimate connection between no-neutrino $\beta\beta$-decay and neutrino physics we now turn to the problem of extracting information about the neutrino from the data on the decay. The detailed evaluation of the $\beta\beta$ decay rates can be found in numerous articles [Hax 84, Doi 85, Ver 86, Mut 88a, Doi 88]. Therefore in this work only the final result for the $0^+ \to 0^+$ transitions will be presented. Moreover, the attention will be limited to the weak Hamiltonian

$$H_W = \frac{G}{\sqrt{2}} \bar{e}\gamma^{\mu} (1-\gamma_5) \nu_e J_{\mu} + \frac{1}{2} \sum_{j,k=1}^{n} g_{jk} N_j \gamma_5 N_k \chi_L^M , \qquad (3.1)$$

where J_{μ} is the hadronic current given by (1.4).

Concerning the $0^+ \to 2^+$ transitions, the contributions coming from the $\beta\beta_{0\nu}$ and $\beta\beta_{0\nu,M}$ modes are negligible [Doi 85, Doi 88a], and their measurements mainly provide information on the RH weak current, i. e., on the parameter η. On the other hand, it is also worth noting that, the nuclear structure of the first excited 2^+ states (due to the quadrupole degrees of freedom, i.e., shape and pairing vibrations) is much more complex than that of the corresponding 0^+ states. As a consequence, the uncertainties in the estimates of the nuclear matrix elements involved in the $0^+ \to 2^+$ processes could be fairly larger than in the case of the $0^+ \to 0^+$ decays.

For the Hamiltonian (3.1) the inverse half-lives can be cast in the form

$$[\, T_{1/2} (0^+ \to 0^+) \,]^{-1} = \mathcal{F}_P \mathcal{F}_N \mathcal{F}_K \qquad (3.2)$$

where $\mathcal{F}_P$ and $\mathcal{F}_N$ contain information on particle and nuclear physics, respectively, while $\mathcal{F}_K$ is a kinematical factor which

depends on the corresponding phase space.

The values of $\mathcal{F}_P$ are:

$$\mathcal{F}_P = \begin{cases} 1 & ; \text{ for } \beta\beta_{2\nu} \\ [\langle m_\nu \rangle / m_e]^2 & ; \text{ for } \beta\beta_{0\nu} \\ |\langle g_M \rangle|^2 & ; \text{ for } \beta\beta_{0\nu,M} \, , \end{cases} \qquad (3.3)$$

where

$$\langle m_\nu \rangle = \sum_{j=1}^{n} m_j U_{ej}^2 \quad ; \quad \langle g_M \rangle = \sum_{j,k=1}^{n} g_{jk} U_{ej} U_{ek} \, . \qquad (3.4)$$

The sum extends only over light neutrino mass eigenstates ($m_j < 20\, m_e$).

The nuclear matrix element $\mathcal{F}_N$ is of the form:

$$\mathcal{F}_N = | \mathcal{M}^{GT} - (g_A / g_V)^2 \mathcal{M}^F |^2 \qquad (3.5)$$

where $\mathcal{M}^{GT}$ and $\mathcal{M}^F$ are, respectively, the Gamow-Teller and Fermi transition amplitudes.

Concerning the $\beta\beta_{2\nu}$ decay they are usually expressed as:

$$\mathcal{M}_{2\nu}(I) = \sum_{\alpha} \langle 0_f^+ \| \mathcal{O}_+(I) \| I^+;\alpha \rangle \langle I^+;\alpha \| \mathcal{O}_+(I) \| 0_i^+ \rangle / \, D(I\alpha) \quad , \qquad (3.6)$$

where $\mathcal{M}_{2\nu}(I=1) \equiv \mathcal{M}_{2\nu}^{GT}$ and $\mathcal{M}_{2\nu}(I=0) \equiv \mathcal{M}_{2\nu}^{F}$. The bra $\langle 0_f^+|$ and ket $|0_i^+\rangle$ stand for the ground-state wave functions of the final and initial nuclei, respectively, and $\mathcal{O}_{\pm}(I)$ are one-body operators:

$$\mathcal{O}_{\pm}(I=0) \equiv \mathcal{O}_{\pm}^{F} = \sum_{i=1}^{A} t_{\pm}(i) \; ; \; \mathcal{O}_{\pm}(I=1) \equiv \mathcal{O}_{\pm}^{GT} = \sum_{i=1}^{A} \sigma(i) t_{\pm}(i), \qquad (3.7)$$

with $\langle p|t_+|n\rangle=1$. The energy denominator $D(I\alpha)$, expressed in natural units ($\hbar=c=m_e=1$), reads:

$$D(I\alpha) = E_{I\alpha} - E_i + T_o/2 + 1 , \qquad (3.8)$$

where T_o is the Q-value of the decay. Finally, the sum in eq. (3.6) extends over a complete set of intermediate nuclear states $|I^+;\alpha\rangle$.

The $\beta\beta_{o\nu}$ and $\beta\beta_{o\nu,M}$ decay modes possess the same matrix element $\mathcal{F}_N$, with

$$\mathcal{M}^{GT}_{o\nu} \equiv \mathcal{M}^{GT}_{o\nu,M} = \langle 0_f^+| \sum_{l\neq k}^{A} H(|\vec{r}_k-\vec{r}_l|,D)\vec{\sigma}(k)\cdot\vec{\sigma}(l)t_+(k)t_+(l)|0_i^+\rangle \qquad (3.9a)$$

$$\mathcal{M}^{F}_{o\nu} \equiv \mathcal{M}^{F}_{o\nu,M} = \langle 0_f^+| \sum_{l\neq k}^{A} H(|\vec{r}_k-\vec{r}_l|,D)t_+(k)t_+(l)|0_i^+\rangle . \qquad (3.9b)$$

The function $H(r,D)$ is "the neutrino potential", being D a "typical" excitation energy $D(I\alpha)$ of the intermediate nucleus, and for light neutrinos, with masses $m_i \ll D \approx 40$, it can be approximated as [Doi 85, Tom 86, Eng 88, Doi 88]:

$$H(r,D) = (R/r)\phi(x) , \qquad (3.10)$$

where R is the nuclear radius, $x=Dr$ (with both R and r given in natural units: $1fm=2.58\times10^{-3}$n.u.) and

$$\phi(x)= \frac{2}{\pi} [\sin(x)ci(x)-\cos(x)si(x)] \cong e^{-1.5x} . \qquad (3.11)$$

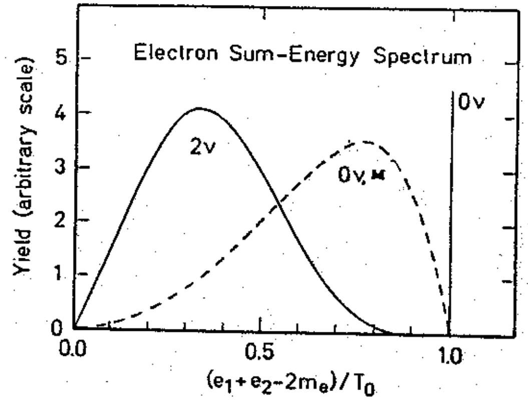


Fig.7 Sum energy spectrum of the two electrons emitted in various double beta decay processes.

The explicit results for the kinematical factors $\mathcal{F}_K$ are given by Doi et al.[Doi 85, Doi 88]. Owing to the three body phase space , the $\beta\beta_{o\nu}$ decay is kinematically favored over the other two processes. Indeed, as for this process the final state contains only two electrons and the residual nucleus, the electron sum energy spectra is a δ-function at the value of the allowed energy release $Q_{\beta\beta} = e_1+ e_2 - 2m_e$. Contrarily, the $\beta\beta_{2\nu}$ and $\beta\beta_{o\nu,M}$ decays, lead, respectively, to five-body and four-body final states and, as a consequence, both exhibit continuous sum energy spectra. Evidently, these spectra should have different shapes and this difference in kinematics can be used, at least in principle, to distinguish among these two decay modes. This is illustrated in Fig.7, which shows that the peak in the sum energy spectrum for the majoron emission is clearly at a higher value than that for the ordinary $\beta\beta$-decay. Nevertheless the former process may become an important background in the electron spectrum in $\beta\beta$ decay counter experiments.

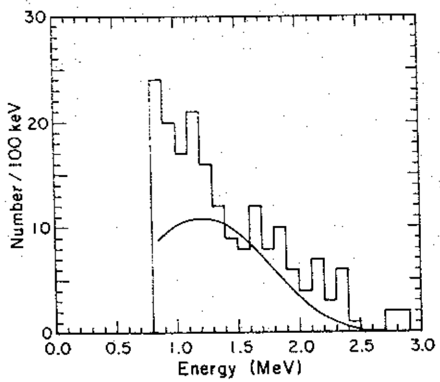


Fig.8 The observed sum energy spectrum of two-electron events in ^{82}Se [Ell 87]. Thresholds of 150 keV and 800 keV were imposed, respectively, on the single electron energy and on the sum energy of the events.

The sum energy spectrum for the two electrons, measured for the first time by the group at the University of California by studying the decay of ^{82}Se [Ell 87], is shown in Fig.8. The reported data represent 7960 h of run live time and corresponds to 36 $\beta\beta$-decay evens. The experimental histogram is in agreement in shape with expectations for the $\beta\beta_{2\nu}$ process normalized to 1.1×10^{20} yr. This result is consistent with the geochemical measurements of Kirsten [Kir 86] and Manuel [Man 86] and the cosmochemical measurement of Marti and Murty [Mar 85]. Thus, it boosts confidence in geochemical and cosmochemical measurements of $\beta\beta$-decay lifetimes

for other elements. Just from the shape of the spectrum in Fig.8, it can be inferred that the $\beta\beta_{o\nu,M}$ decay has a considerable weaker rate than the $\beta\beta_{o\nu}$ decay, and this fact was used to set the bound $|\langle g_M\rangle|\leq1.9\times10^{-4}$ on the majoron coupling [Doi 88a].

3.2 Nuclear structure models used in the evaluation of the double beta decays.

From the previous section it is clear that reliable evaluations of the nuclear matrix elements $\mathcal{F}_N$ are essential prerequisite for extracting information on the neutrino physics (contained in the quantity $\mathcal{F}_P$) from experimental data on the $\beta\beta$-decay lifetimes.

At the time being there are mainly two nuclear models which are intensively used in the literature for the calculation of the $\beta\beta$-decay processes; namely the shell model (SM) and the quasiparticle random-phase-approximation (QRPA). Application of the conventional SM technique is limited to the decay $^{48}Ca\to^{48}Ti$ [Hax 84, Zam 82, Sko 83, Tsu 84, Wu 85]. For heavier $\beta\beta$ emitters the configuration space turns out to be too large to be handled. For instance, in the case of ^{76}Ge, with four protons and fourteen neutrons distributed among valence orbits $p_{3/2}$, $p_{1/2}$, $f_{5/2}$ and $g_{9/2}$, the model space has 210.777 configurations 0^+ [Mut 88a]. Owing to this reason many people [Hax 81, 82a, Sin 88] have recourse to the weak-coupling approximation (WCA), which drastically truncates the number of configurations. Within the WCA the total Hamiltonian is conveniently expressed as [Sin 88]

$$H = H_p + H_n + H_{pn} , \qquad (3.12)$$

where H_p and H_n describe the effective Hamiltonian in proton and neutron space, respectively, while H_{pn} denotes the effective interaction between protons and neutrons. After solving the eigenvalue equations for protons and neutron systems separately, from each system a relatively small number of proton states J^p and neutron states J^n with lowest energies are taken. With these states the proton-neutron coupled basis vectors $|J^p,J^n;0^+\rangle$ are build up. Finally, the wave functions of the initial and final states are obtained from the diagonalization of H_{pn} in this new basis. The number of $|J^p,J^n;0^+\rangle$ configurations, although still large in same calculations, is always strongly reduced whit regards to the number of original multiparticle states. For instance, by employing this procedure, in a recent calculation [Sin 88] of the decay $^{76}Ge \rightarrow ^{76}Se$ the largest matrix occurs for the ^{76}Se and has dimension 2792.

In the QRPA, which is discussed in the next section, the nuclear interaction is represented in a more schematic way than within the SM but it deals with by far smaller configuration space. This allows for a more detailed analysis of the degrees of freedom relevant for a particular process.

3.3 Quasiparticle random phase approximation (QRPA).

As in the case of the WCA the Hamiltonian considered here is that given by eq.(3.12). Each of the quantities H_p and H_n, in second quantized form, is given by

$$H_t = \sum_{\tau} (e_t - \lambda_t)\, c^+_\tau c_\tau + \frac{1}{4} \sum_{\tau's} \langle \tau_1\tau_2|V|\tau_3\tau_4\rangle_{\mathscr{A}}\, c^+_{\tau_1} c^+_{\tau_2} c_{\tau_4} c_{\tau_3} , \qquad (3.13)$$

where the subscripts $t(\tau)$ stand for $p(\pi)$ or $n(\nu)$, with $\tau \equiv t, m_t$ and all the notation has the standard meaning: e_t is the single particle energy (s.p.e.), c^+_τ (c_τ) are the single particle creation (annihilation) operators, the label $\mathscr{A}$ denotes the matrix elements with respect to antisymmetric states, etc.. The Hamiltonian (3.13) is diagonalized through quasiparticle transformations [Row 70, Sol 76, Rin 80]:

$$a^+_\tau = u_t c^+_\tau - v_t c_{\bar{\tau}} ; \quad c_{\bar{\tau}} = (-)^{\tau + m_t}\, c_{t,-m_t} ; \qquad (3.14)$$

and reads

$$H_t = \sum_{\tau} \varepsilon_t\, a^+_\tau a_\tau , \qquad (3.15)$$

where ε_t are the quasiparticle energies:

$$\varepsilon_t = (e_t - \lambda_t)(u_t^2 - v_t^2) + 2\Delta_t u_t v_t = \Delta_t / 2u_t v_t , \qquad (3.16)$$

with λ_t and Δ_t being, respectively, chemical potentials and the energy gaps. The BCS ground state is represented as:

$$|0^+\rangle = |0_p\rangle|0_n\rangle \; ; \; |0_t\rangle = \prod_{\tau} (u_t + v_t\, a^+_\tau a^+_{\bar{\tau}})\,|\rangle , \qquad (3.17)$$

where $|\rangle$ represents the particle vacuum.

The second-quantized version of H_{pn} is given by

$$H_{pn} = \sum_{\pi\pi'\nu\nu'} \langle\pi\nu|V|\pi'\nu'\rangle_{\mathcal{A}} : c^+_\pi c^+_\nu c_{\nu'} c_{\pi'} : \quad , \tag{3.18}$$

where the symbol : : denotes normal product of fermion operators. After performing the quasiparticle transformations (3.14) the proton- neutron residual interaction can be put in the form:

$$H_{pn} = H_{22} + H_{04} + H_{40} \quad , \tag{3.19}$$

with

$$H_{22} = \sum_{\pi\pi'\nu\nu'} [\langle\pi\nu|V|\pi'\nu'\rangle_{\mathcal{A}} (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) - \langle\pi\bar\nu'|V|\pi'\bar\nu\rangle_{\mathcal{A}} (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})] a^+_\pi a^+_\nu a_{\nu'} a_{\pi'} , \tag{3.20}$$

$$H_{40} = H^+_{04} = \sum_{\pi\pi'\nu\nu'} \langle\pi\nu|V|\pi'\nu'\rangle_{\mathcal{A}} \; u_p u_n v_{p'} v_{n'} \; a^+_\pi a^+_\nu a^+_{\bar\nu'} a^+_{\bar\pi'} \quad . \tag{3.21}$$

To solve the QRPA equations [Row 75]

$$\hat I^{-1}\langle 0|[O(\bar I;\alpha) ,H, O^+ (I;\beta)]^0|0\rangle$$
$$= \hat I^{-1}\langle 0|[O(\bar I;\alpha) \; , O^+(I;\beta)]^0|0\rangle = \omega_{I\alpha}\delta_{\alpha,\beta} \quad , \tag{3.22}$$

where $\hat I \equiv (2I+1)^{1/2}$, the excitation operators $O^+(I;\alpha)$ are approximated by an expansion

$$O^+(I;\alpha) = \sum_\alpha \{ X(pnI;\alpha)[a^+_p a^+_n]^I - Y(pnI;\alpha)[a_n a_p]^{\bar I} \} \; ; \tag{3.23}$$

and one gets

$$(\varepsilon_p+\varepsilon_n - \omega_{I\alpha})X(pnI;\alpha) = - \sum_{p'n'} [A(pn,p'n';I)X(p'n'I;\alpha) + B(pn,p'n';I)Y(p'n'I;\alpha)] \quad ,$$
$$(\varepsilon_p+\varepsilon_n + \omega_{I\alpha})Y(pnI;\alpha) = - \sum_{p'n'} [A(pn,p'n';I)Y(p'n'I;\alpha) + B(pn,p'n';I)X(p'n'I;\alpha)] \quad , \tag{3.24}$$

with submatrices

$$A(pn,p'n';I) = (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})F(pn,p'n';I) + (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'})G(pn,p'n';I) \quad ,$$
$$B(pn,p'n';I) = (v_p u_n u_{p'} v_{n'} + u_p v_n v_{p'} u_{n'})F(pn,p'n';I) - (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \; G(pn,p'n';I) \quad , \tag{3.25}$$

where F and G are respectively the particle-hole (PH) and particle-particle (PP) matrix elements as defined in [Fuj 65].

The excitation energies $E_{I\alpha}$ in the odd-odd (A,Z+1) and (A,Z-1) nuclei are related to the QRPA energy $\omega_{I\alpha}$ as:

$$E_{I\alpha} - E_i = \begin{cases} \omega_{I\alpha} + \lambda_p - \lambda_n \; ; \; (A,Z+1) \\ \omega_{I\alpha} + \lambda_n - \lambda_p \; ; \; (A,Z-1) \; , \end{cases} \tag{3.26}$$

where E_i is the ground state energy of the parent nucleus. The corresponding transition matrix elements are expressed by means of the forward and backward going amplitudes X and Y in the form

$$\langle I^+;\alpha \| \mathcal{O}_+(I) \| 0^+ \rangle = \sum_{pn} \langle p \| \mathcal{O}_+(I) \| n \rangle [u_p v_n X(pnI;\alpha) + v_p u_n Y(pnI;\alpha)]$$

$$\langle I^+;\alpha \| \mathcal{O}_-(I) \| 0^+ \rangle = \sum_{pn} \langle n \| \mathcal{O}_-(I) \| p \rangle [v_p u_n X(pnI;\alpha) + u_p v_n Y(pnI;\alpha)]. \qquad (3.27)$$

The transition strengths defined as

$$S_\pm(I) = \sum_\alpha s_\pm(I;\alpha); \quad \begin{cases} s_+(I;\alpha) = |\langle I^+;\alpha \| \mathcal{O}_+(I) \| 0^+ \rangle|^2 \\ s_-(I;\alpha) = |\langle I^+;\alpha \| \mathcal{O}_-(I) \| 0^+ \rangle|^2 , \end{cases} \qquad (3.28)$$

fulfill the sum rule:

$$S_+(I) - S_-(I) = 2T_o(2I+1) , \qquad (3.29)$$

where $T_o=(N-Z)/2$ is the ground-state isospin.

3.4 Reconstruction of Isospin and Spin-isospin Symmetries

It is well known that, even when the isospin-breaking Coulomb force is not included in the hamiltonian H, the whole structure of isospin invariance may be demolished in an approximate treatment of the eigenstates of H. In other words, the isospin-invariance -breaking arises from the approximation that is introduced and not from the interaction. As a matter of fact, Engelbrecht and Lemmer [Eng 70] have pointed out that, in nuclei with $T_o > 0$, the isospin invariance is explicitly broken by the Hartree-Fock (HF) field as well as by the Tamm-Dancoff

approximation (TDA). They have also shown that the isospin-conserving description is recovered if the PN-correlations, generated within the random phase approximation (RPA), are included in the HF ground state in a self-consistent way. Lee [Lee 71] has explored this notion in more detail by introducing into the picture the Coulomb interaction, which leads to a dynamic breaking of isospin symmetry. His conclusion was that the isospin impurities in the ground state are also greatly reduced by the PN-correlations.

While the RPA correlations for F transitions are closely related to the isospin symmetry, the quantitative features of correlations for GT transitions depend on the more detailed properties of H [Boh 75]. The resulting nuclear states would form supermultiplets and the locations of F and GT resonances would coincide, if the nuclear forces were independent not only of the the isospin but also of the spin coordinates. Owing to the strong spin-orbit interaction in the HF field, the SU(4) symmetry is badly broken in medium and heavy nuclei and the jj-coupling scheme is established. The PN-correlations, which are responsible for building up the GT resonance, may be viewed, however, as a trend away from the jj-coupling towards the LS-coupling and the SU(4) symmetry. As shown in Fig. 9, this is reflected by the experimental energy differences between the GT and F resonances [Nak 82]

$$E_{GT} - E_F = (26A^{-1/3} - 37T_oA^{-1}) \text{ MeV}, \qquad (3.30)$$

which decreases as the mass number increases. The first and second terms arise, respectively, from the spin-orbit splitting and the PN-correlations and have a tendency to cancel each other; for example in ^{208}Pb, $E_{GT} \cong E_F$.

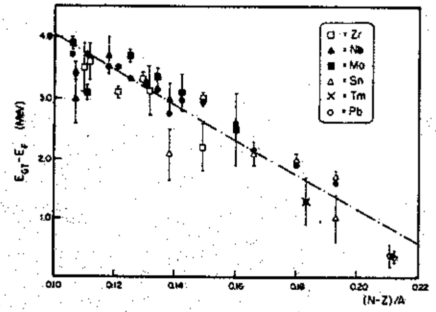


Fig.9 Plot of $E_{GT} - E_F$ versus (N-Z)/A. When the experimental results overlap (in the case of $^{90,92}Zr$ and ^{208}Pb) we displace them slightly with respect to the correct value of (N-Z)/A for the sake of clarity. The values calculated by means of eq.(3.30) are indicated by full circles.

In the limit of exact isospin and Wigner SU(4) symmetries all of the $S_+(I)$ strength is concentrated in the resonant (collective) state $|I^+;res\rangle$, there is no β^+ strength and the $\beta\beta_{2\nu}$ decay is forbidden , i.e.,

$$s_+(I;res) \equiv S_+(I) \; ; \; S_-(I) \equiv 0 \; ; \; \mathcal{M}_{2\nu}(I) \equiv 0. \qquad (3.31)$$

While for the F transitions the relations (3.31) should be exact results when the Coulomb force is excluded, they only may be considered as first approximations for the GT processes. Within the BCS approximation and the quasiparticle Tamm-Dancoff approximation (QTDA) the above mentioned symmetries are always explicitly broken and therefore the conditions (3.31) are never fulfilled.

Contrarily, within the QRPA these limits can be achieved if [Hal 67, Eng 70, Lee 71]

$$X_{pn}(I,res) \sim u_p v_n \langle p\|\mathcal{O}_+(I)\|n\rangle \; , \; Y_{pn}(I,res) \sim -v_p u_n \langle p\|\mathcal{O}_+(I)\|n\rangle . \quad (3.32)$$

After substituting (3.32) into (3.24) the following set of equations can be derived for the isobaric analog state (IAS)

$$\begin{aligned} \varepsilon_p + \varepsilon_n - \omega^F_{res} &= -U^F_{j_p=j_n} + \frac{u_n}{v_n}\Delta^F_n + \frac{v_p}{u_p}\Delta^F_p \; , \\ \varepsilon_p + \varepsilon_n + \omega^F_{res} &= U^F_{j_p=j_n} + \frac{v_n}{u_n}\Delta^F_n + \frac{u_p}{v_p}\Delta^F_p \; , \end{aligned} \qquad (3.33)$$

where

$$U^F_{j_p=j_n} = \sum_{j_{p'}=j_{n'}} \hat{j}_{p'} \hat{j}_p^{-1} (v^2_{n'} - v^2_{p'}) F(pn,p'n';0) \; , \qquad (3.34)$$

and

$$\Delta^F_t = -\frac{1}{2} \sum_{t'} \hat{j}_{t'} \hat{j}_t^{-1} u_{t'} v_{t'} G(tt,t't';0) \; . \qquad (3.35)$$

From (3.16), where

$$\Delta_t = -\frac{1}{2} \sum_{t'} \hat{j}_{t'} \hat{j}_t^{-1} u_{t'} v_{t'} G^{pair}(tt,t't';0) \; , \qquad (3.36)$$

and (3.26) one immediately sees that

$$\Delta^F_t = \Delta_t \quad ; \quad E^F_{res} - E_i = e_p - e_n + U^F_{j_p=j_n} \; . \qquad (3.37)$$

Thus, only within a self-consistent QRPA calculation the conditions (3.32) are fulfilled for the F transitions, or equivalently the explicit breaking of isospin symmetry induced by the QTDA is circumvented. In other words : i) the same residual interaction should be used in solving the gap equations, both for protons and neutrons and for the PN-particle-particle channel, i.e., $G^{pair} = G$ and ii) the symmetry energy contained in the proton s.p.e. ($e_p = e_n + \Delta_C - U_{j_p=j_p}$; where Δ_C is the Coulomb displacement energy) should be equal to the symmetry energy $U^F_{j_p=j_n}$. It is important to stress that, even when these conditions are satisfied, there is no reconstruction of isospin symmetry within the QTDA.

3.5 A few numerical results.

For the further discussion of $\beta\beta$ amplitudes we borrow now the δ-interaction [Ike 64, Fuj 65, Nak 82]

$$V = -C\,(v_s P_s + v_t P_t)\,\delta(r)\ ;\ C \equiv 4\pi \text{ MeV fm}^3, \qquad (3.39)$$

with different strength constants v_s and v_t for the PH, PP and pairing channels and calculate the ^{128}Te nucleus. In order to determine the appropriate single-particle spectra and the interaction strengths v_s^{pair} we follow the prescription proposed by Conci et al. [Con 84], which consists in utilizing the experimental data together with a Wood-Saxon plus BCS calculation. This procedure yields v_s^{pair} = 24 and 31 for neutrons and protons, respectively. The coupling strengths in the PH-channel are taken from [Cas 87], namely $v_s^{PH} = 55$ and $v_t^{PH} = 92$, while v_s^{PP} and v_t^{PP} are treated as free parameters. Two different calculations are described:

<u>Calculation I(CI):</u> The experimental values of neutron s.p.e. are used and the proton s.p.e. are adjusted according to $e_p = e_n + \Delta_C - U^F_{j_p=j_n}$; v_s^{pair} is fixed at the value of 28 for both neutrons and protons.

<u>Calculation II(CII):</u> The s.p.e. as well as the strengths v_s^{pair} were taken from the experimental data as explained previously in the text.

Usually two separate QRPA calculations are performed (one for the initial nucleus and one for the final nucleus) and some kind of average is carried out for the resulting matrix elements [Vog 85, Eng 88, Civ 87, Eng 88a, Mut 88, Mut 88a]. This effect is, however, only of minor importance regarding the objective pursued in the present work. Thus, we will only solve the QRPA for the initial nucleus (^{128}Te), and the following approximation will be done in eq. (3.6):

$$\langle 0_f^+ \| \mathcal{O}_+(I) \| I^+;\alpha \rangle \cong \langle I^+;\alpha \| \mathcal{O}_-(I) \| 0_i^+ \rangle \ . \qquad (3.40)$$

The calculated values of S_- and $\mathcal{M}_{2\nu}$, with the QRPA , are displayed in Fig. 10. The most relevant issue, which becomes self-evident at a first glance, is the great similarity between $\mathcal{M}^F_{2\nu}$ and $\mathcal{M}^{GT}_{2\nu}$. That is, both amplitudes are very sensitive to the GSC within the PP-channel and pass through zero in the vicinity of the minima of S_-. Below use will be made of this similarity in order to draw conclusions about the physical value of υ_t^{PP}, after separating the explicit breaking of the SU(4) symmetry from the dynamic one.

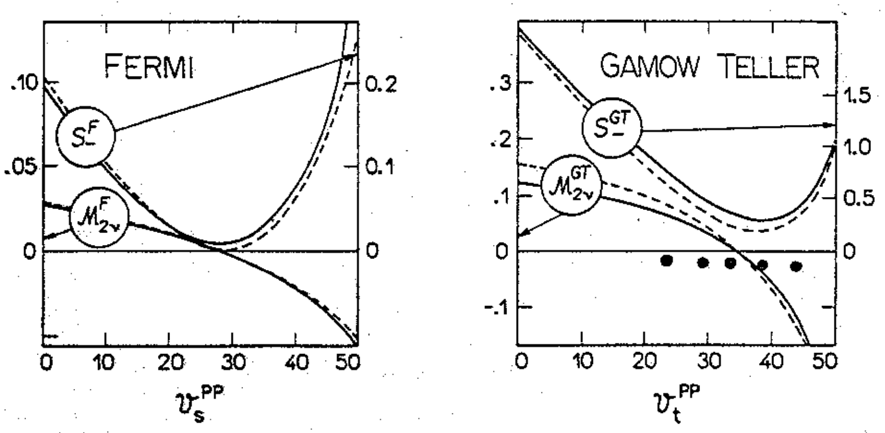


Fig. 10 The matrix elements $\mathcal{M}_{2\nu}$ and the transition strengths S_- for ^{128}Te. The results from CI and CII are indicated, respectively, by dashed and solid curves. The values of $\mathcal{M}_{2\nu}^{GT}$ obtained at the minimum of S_-^{GT} with the s.p.e. from CI and with υ_s^{pair} = 15, 20, 25, 30 and 35 are shown by dark circles.

As expected, when the condition of self-consistency (3.37) is satisfied (or equivalently when the isospin symmetry is strictly conserved), $\mathcal{M}_{2\nu}^F \equiv 0$ and $S_-^F \equiv 0$. In varying υ_s^{PP} when υ_s^{pair} is constant, a fictitious degree of freedom is introduced; thus, the variations of $\mathcal{M}_{2\nu}^F$ and S_-^F within CI, exhibited in Fig. 10, just only put in evidence the lack of self-consistency between the residual interaction and the mean field. It is clear that even after introducing isospin impurities, i.e., when the condition (3.37) is not valid anymore, the value of υ_s^{PP} should not be varied

independently of the value of υ_s^{pair}. Moreover, from CII it seems reasonable to state that the self-consistency between υ_s^{PP} and υ_s^{pair} is now achieved at the minimal value of S_-^F, which represents the amount of dynamic violation of isospin symmetry and for which $\upsilon_s^{PP} \cong \upsilon_s^{pair}$ and $\mathcal{M}_{2\nu}^F \cong 0$.

The above results suggest that the physically sound value of υ_t^{PP}, for a given value of υ_s^{pair}, is that which minimizes S_-^{GT}. In other words, it might be expected that at this value of υ_t^{PP} the explicit violation of SU(4) symmetry should be totally removed and that the minimum of S_-^{GT} should measure the extent to which the SU(4) is broken by the dynamics of H. In this regard, one sees, from comparison of the lowest values of S_-^{GT} in CI and CII, that not only the spin-orbit splitting but also the isospin impurities play an important role. However, both minima of S_-^{GT} are located almost at the same value of υ_t^{PP} ($\cong$ 37) and the corresponding $\mathcal{M}_{2\nu}^{GT}$ amplitudes are very nearly equal ($\cong$ −0.03). This theoretical estimate should be compared with the experimental value $|\mathcal{M}_{2\nu}| \cong$ 0.03, obtained from $\mathcal{F}_{K,2\nu}$ = 8.54 $10^{24} y^{-1}$ [Doi 85] and the measured lifetime $T_{1/2}^{2\nu}$ = (1.4±0.4) 10^{24}y [Man 86].

The location of the minimum of S_-^{GT} does depend on υ_s^{pair} and it is moved to higher (lower) values of υ_t^{PP} when υ_s^{pair} is increased (decreased). The results for $\mathcal{M}_{2\nu}^{GT}$ obtained at the minimum of S_-^{GT} with the s.p.e. from CI and with υ_s^{pair} = 15, 20, 25, 30 and 35 are shown in Fig. 10 by dark circles. Once the above mentioned constraint between υ_t^{PP} and υ_s^{pair} is imposed, i.e., after circumvented the explicit violation of the SU(4) symmetry, the variation of $\mathcal{M}_{2\nu}^{GT}$ with respect to υ_t^{PP} is of minor importance and $\mathcal{M}_{2\nu}^{GT}$ does not pass through zero anymore. This is consistent with the recent observation of Muto and Klapdor [Mut 88a], in the sense that the vanishing of $\mathcal{M}^{GT}_{2\nu}$ is an artifact of the model.

Just for the sake of comparison let us mention a few results for the QTDA. Within CI: S^{F}_{-} = 0.55 and S^{GT}_{-} = 3.86 and $\mathcal{M}^{F}_{2\nu}$ ($\mathcal{M}^{GT}_{2\nu}$) goes from 0.062 (0.26) to 0.075 (0.40) when v^{PP}_{s} (v^{PP}_{t}) is varied from 0 to 50.

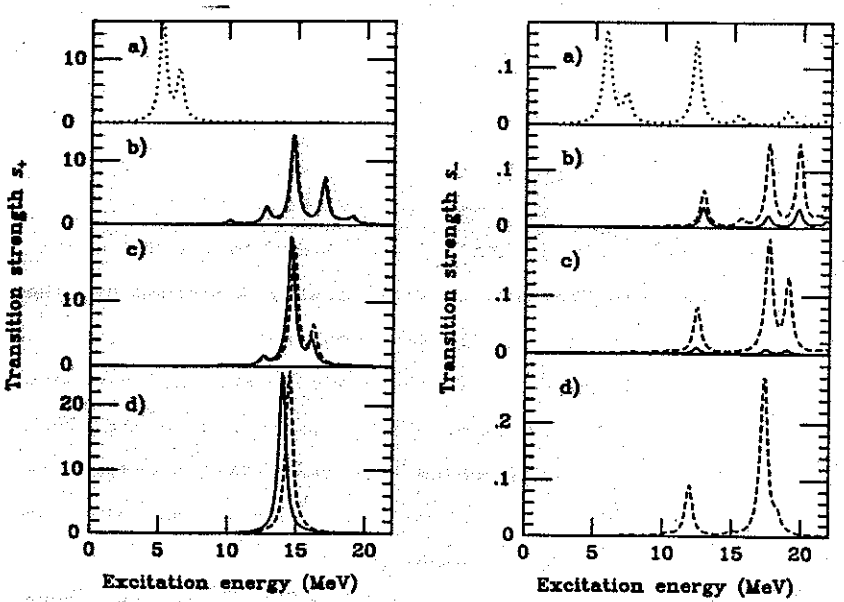


Fig. 11 Fermi strengths distributions $s_+(I;\alpha)$ and $s_-(I;\alpha)$ in ^{128}I and ^{128}Sb nuclei obtained within CII; (a) is the unperturbed strength; panels (b), (c) and (d) show the QRPA (solid curves) and QTDA (dashed curves) results for v^{PP}_{s}= 0, 15 and 28, respectively.

Finally, we discuss the consequences of the restoration of the isospin and SU(4) symmetries on the energy spectra of odd-odd nuclei ^{128}I and ^{128}Sb, which are illustrated in Figs. 11 and 12, respectively. These spectra are measured with respect of the ground state of ^{128}Te [see eq. (3.26)] and an averaging interval of 0.3 MeV was used in plotting the strengths $s_{\pm}(I;\alpha)$.

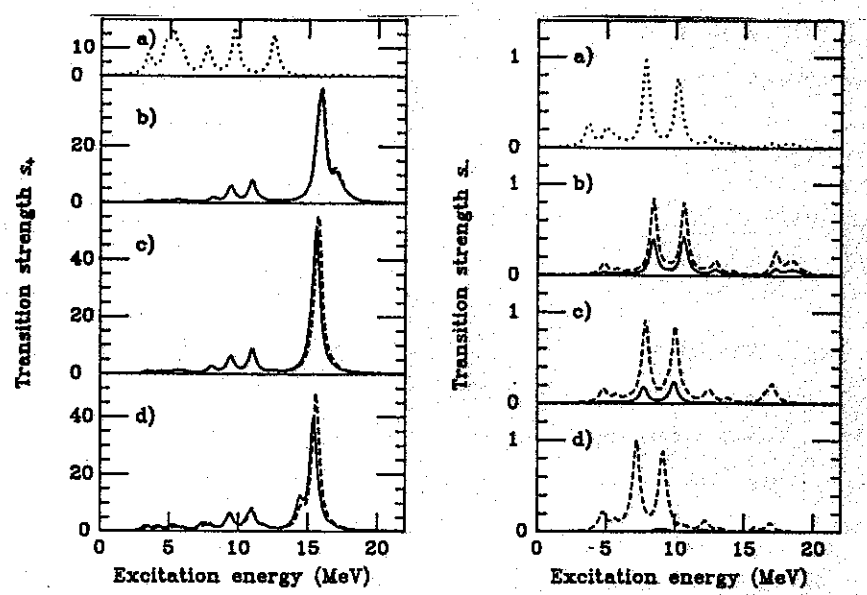


Fig. 12 Gamow-Teller strengths distributions $s_+(I;\alpha)$ and $s_-(I;\alpha)$ in ^{128}I and ^{128}Sb nuclei obtained within CI; (a) is the unperturbed strength; panels (b), (c) and (d) show the QRPA (solid curves) and QTDA (dashed curves) results for v^{PP}_{s}= 0, 18 and 37, respectively.

Most of the results, when evaluated within CI and CII turn out to be quite similar. Therefore, we only exhibit here the results CII for the F transitions (Fig. 11) and the results CI for the GT transitions (Fig. 12). Shown in panels (a) are the distributions of the unperturbed strengths. The solid and dashed curves displayed in the remaining panels represent, respectively, the QRPA and QTDA results obtained with the above mentioned values of the parameters within the PH-channel and the following PP parameters:

(b) $v_s^{PP} = v_t^{PP} = 0$, i.e, the PN interaction in the PP-channel is totally switch off;

(c) $v_s^{PP} = 15$ and $v_t^{PP} = 18$, which are intermediate values between the cases (b) and (d); and

(d) $v_s^{PP} = 28$ and $v_t^{PP} = 37$, for which the corresponding strengths S_-^F and S_-^{GT} are minima.

From the above results for the energy spectra in the odd-odd nuclei it can be immediately concluded that the GSC within the PP-channel, although of minor significance for the β^- strengths $s_+(I;\alpha)$, they are indispensable in the evaluation of the β^+ strengths $s_-(I;\alpha)$, as well as, for the restoration of isospin and SU(4) symmetries. It is worth noting, however, that only within the QRPA all the F strength is concentrated in one state. In other words, even when the condition of self-consistence (3.37) is fulfilled the isospin symmetry is not restored within the QTDA and there is some spreading of the strength. On the other hand, the SU(4) symmetry is always partially violated as signaled by the fact that the GT resonance never exhausts the sum rule, and that a

substantial part of the strength is distributed well below the resonance. This is consistent with a recent measurement [Mad 89], which shows that around 15% of the β^--GT strength in ^{128}I nucleus lies beneath the resonance bump.

Last but not least it is important to note that the PN-correlations do not built up the β^--GT resonance in the ^{128}Sb nucleus. They merely diminish the total S_-^{GT} strength, through the effect of the GSC.

4. CONCLUSIONS

We summarize our viewpoints and conclusions as follows:

(1) The extreme sensitivity of the $\mathcal{M}_{2\nu}$ amplitudes to the GSC within the PP-channel, shown in Fig. 10, is artificially generated by the explicit violation of isospin and SU(4) symmetries.

(2) The destructive interference between the forward and backward going terms in the β^+-GT amplitude, which appears within the VZM, is not an accident: the physics behind this cancellation effect is the approximate validity of eq.(3.31), or more precisely the partial restoration of the SU(4) symmetry.

(3) In order to get reliable theoretical results within the QRPA for the $\beta\beta$-decay life-times, or equivalently, to be able to extract information on the particle-physics parameter $\mathcal{F}_P$ from the experimental data, the breaking of isospin and SU(4) symmetries induced by the HF-BCS approximation should be circumvented.

(4) After fixing the PN coupling within the PP-channel, as suggested in our recent work [Hir 89] it is not necessary to resort

anymore to experimental data on β^+-strength in order to estimate this model parameter. Thus, the VZM is supplemented now with the predicting power on the life-times for both single and double β decays.

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