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EINSTEIN-EHRENFEST'S RADIATION THEORY AND COMPTON-DEBYE'S KINETICS

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ABSTRACT

Einstein and Ehrenfest's radiation theory is modified in order to introduce the effects of the random zero–point fields, characteristic of classical stochastic electrodynamics. As a result, the Compton and Debye's kinematic relations are obtained within the realm of a completely undulatory theory, that is, without having to consider the corpuscular character of the photon.

I. INTRODUCTION

The concept of light quanta, introduced by Einstein[1] in 1905, was one of the most important concepts of the old quantum theory. Einstein's ideas about the quantum character of the radiation were difficult to reconcile with the classical undulatory theory of electrodynamics and therefore they were met with some resistence from most physicists[2]. Yet, those ideas were determinant in the development of quantum mechanics. Now, in the classical paper[3] of 1917, Einstein tried to show the necessity of introducing the concept of "photon" (actually this term represents at least four distinct models[2]) in the description of the processes of absorption and emission of radiation by atoms. However, even though the main purpose of Einstein in that classical paper was to demonstrate that emission and absorption of radiation by atoms ought to be directional, that work is nowadays famous for containing one of the first probabilistic treatments within quantum theory, and for introducing the notion of *spontaneous* and *stimulated* emissions. Due to the simplicity of the theory, part of the ideas contained in it became quite popular and are well known by physics undergraduate students. Obviously, the textbooks on modern physics try to harmonize Einstein's model with quantum mechanics but they quite often overlook some other results contained in Einstein's original work[3], e.g., the derivation of Planck's radiation law and the fact that Bohr's condition, instead of being taken as an assumption, comes out as a result[4]. Even less known are the efforts made by Einstein and Ehrenfest[4,5] in 1923, to improve the results of Einstein's work of 1917. This improvement consisted of reinforcing the concept of light quanta and extending the model for the case of free particles. Coincidently, in the same year of 1923, Compton and Debye explained, independently[6], the mysterious features of X– and γ–ray scattering, based on the light quanta model for radiation. Debye mentioned his indebtedness to Einstein's work. It should be noted, however, that in Compton's work there is no reference to the Einstein's 1917 paper.

Our aim in the present work is to further explore Einstein's and Einstein–Ehrenfest's works within the realm of a completely classical theory, the so called stochastic electrodynamics[7–11], or simply SED. According to the SED the whole universe is filled with a zero–point (that is non–thermal) and random electromagnetic radiation characterized by a Lorentz invariant spectral distribution $\rho_0(\omega) = \hbar\omega^3/2\pi^2c^3$, with ω the radiation frequency. In this theory the Planck constant $2\pi\hbar$ appears naturally and is simply the spectrum scale factor. The use of different boundary conditions on Maxwell's equations provide new and interesting informations about the behaviour of the microscopic world[7–11].

By assuming that the spontaneous emission is in fact stimulated by the zero–point radiation, we will show that, in a system of particles immersed in both thermal and zero–point fields, the exchange of energy and momentum must obey some relations in order for the equilibrium to be maintained. Curiously enough, these relations are exactly the same as those characterizing the conservation of momentum and energy in the Compton effect. Without having to use the corpuscular character of the photon, we show that the above system may exchange finite quantities of electromagnetic energy, namely $\hbar\omega$, having an associated electromagnetic momentum $\hbar\vec{k}$, where $\vec{k}$ is the wave vector. The energy and momentum are added to or removed from the *classical electromagnetic waves* according to the conservation laws predicted by Maxwell theory.

II. CLASSICAL STOCHASTIC ELECTRODYNAMICS AND THE EINSTEIN–EHRENFEST MODEL

The relationship between Planck's theory of blackbody radiation and Bohr's theory of line spectra was not very clear in the early days of the quantum theory. Only in 1916 Einstein conceived an elegant and illuminating approach to this problem; he derived both Planck's radiation formula and Bohr's condition from general (probabilistic) hypotheses concerning the interaction between radiation and matter. He proved that the consistency of his model (for describing emission and absorption of radiation by molecules) depended on the validity of the hypothesis that the electromagnetic radiation is made up of "quanta". By light quanta he meant *directional* radiation or needle radiation (Nadelstrahlung).

Einstein first considered that the matter molecules (actually polarizable objects, as we shall see later) were immersed in thermal radiation characterized by the spectral density $\rho_T(\omega)$. Then he made the following simple hypotheses:

1 – The molecules have only discrete energy states E_2 and E_1 $(E_2 > E_1)$.

2 – The Boltzmann distribution is valid for the molecules in these states.

3 – Wien's law is valid for the spectral distribution at temperature T, that is:

$$\rho_T(\omega) = \omega^3 F(\omega/T) , \tag{1}$$

where F is an arbitrary function.

Only the first hypothesis is a quantum one, due to the discrete character of the energy states. The other two are completely classical, based on thermodynamics and electromagnetism.

With these simple hypotheses Einstein deduced (in the first part of the 1917 paper[3]) two important results in a simple way. He showed that the spectral distribution $\rho_T(\omega)$ must be given by

$$\rho_T(\omega) = \frac{\hbar}{\pi^2c^3}\left[\frac{\omega^3}{\exp(\frac{\hbar\omega}{kT}) - 1}\right] , \tag{2}$$

which is Planck's formula for cavity radiation at temperature T. He also concluded that the Bohr's rule

$$E_2 - E_1 = \hbar\omega \tag{3}$$

follows from the above three hypotheses.

However, if it is not well known the fact that this 1917 work presents a derivation of both expressions, even less known is the fact that both derivations were based on *classical grounds*, as we shall see in what follows. As already mentioned, only the first hypothesis (related to the discreteness of the energy states) has a quantum character. Curiously enough, in a later work by Einstein and Ehrenfest[5], it is shown that this hypothesis is unnecessary. In this practically unknown article they generalize the previous result in such a way that the molecules are allowed to occupy a *continuous range of energies*[4]. However they have also assumed that the events of emission an absorption are *statistically independent*, i.e., there is *no interference* among the elementary processes. Then, according to Einstein and Ehrenfest, this no interference assumption is plausible if the electromagnetic radiation is made up of something (light quantum) which is emitted or absorbed *instantaneously*. This is the only "quantum" hypothesis of their model[4].

In the second part of Einstein's 1917 work, he studied the momentum exchange between the molecules and the thermal radiation in which they are immersed. This analysis was considered by Einstein himself the *most important* part of the work, because it suggested the unavoidability of a "quantum" theory of the radiation[11]. The reasoning of Einstein was based on the fact that the spontaneous emission, a process which he assumed that was not stimulated by radiation beams, is not necessarily a *directional* process, according to the *classical radiation theory*. Thus he proved that *all* the processes (induced and spontaneous) *must* be directional, otherwise the thermodynamical equilibrium reached by the molecules would be incompatible with the spectral distribution given by (2).

The detailed calculations of Einstein[3] (1917) and Einstein–Ehrenfest[5] (1923) will be presented in what follows. However we introduce a different interpretation, that is, we shall assume that "spontaneous" emission is induced by the zero–point electromagnetic radiation.

II.a. Energy Exchange

The reasons for an excited atom to radiate have remained somewhat mysterious even after the phenomenological introduction of the concept of spontaneous emission in 1917. Einstein himself always considered this concept very unsatisfactory. Only in 1927, with the efforts of Dirac[12] it was possible to understand spontaneous emission from first principles[11]. According to Dirac, spontaneous emission should be related to the so–called *radiation reaction*, a force caused by the self fields of a radiating charge[13]. However this is not the only interpretation possible, and in 1948, Welton[14] pointed out that spontaneous emission may be considered a forced (induced) emission due to the quantum zero–point electromagnetic field[15]. Also in a work by Park and Epstein[16], Welton's idea is applied directly to the Einstein's 1917 model in such a way that the molecules interact with both thermal and zero–point radiation fields. In that modified model there was no "spontaneous emission", only emission (and absorption) *induced* by these fields. Here we also introduce this modifications into the Einstein model as we shall see more explicitly below.

The notions of *radiation reaction* and *zero–point fluctuating electromagnetic field* appear in a consistent way within *classical electrodynamics*. If it is postulated the existence of *classical* electromagnetic (fluctuating) fields, persisting even at zero temperature, classical electrodynamics is provided with a new boundary condition. In this way it can be predicted some of the "quantum behaviour" of the microscopic matter *entirely on classical grounds*. Some examples may be found in the reviews by de la Peña[9] and by Boyer[7] where are discussed the microscopic properties of the harmonic oscillator, the blackbody radiation, the diamagnetic behaviour of free and harmonically bound charges, the paramagnetic behaviour of a rigid magnetic dipole[17,18], the Casimir forces between macroscopic objects and other phenomena. This new version of classical electrodynamics is often called classical stochastic electrodynamics or simply stochastic electrodynamics (SED).

The fundamental hypothesis of SED is that the zero–point fluctuations of the electromagnetic field (which pervade all space) are *real* and *classical*, with a spectral distribution (energy density per frequency) given by

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3} . \tag{4}$$

Here $\hbar$ is simply a multiplicative constant which is identified with $h/2\pi$. Planck's constant h is therefore introduced within classical SED without any quantization procedure. The constant gives the intensity of the zero–point fields, whose spectral distribution has the remarkable property of being the *only* one which is *Lorentz invariant*[19,20].

A convenient way to describe the zero–point fields and to study their interaction with matter is to consider them as *random fields*, which can be written formally as a superposition of plane waves:

$$\vec{E}(\vec{r},t) = \sum_{\lambda=1}^{2} \int d^3\vec{k}\ \hat{\epsilon}(\vec{k},\lambda)\, g(\omega) \cos[\omega_k t - \vec{k}\cdot\vec{r} + \eta(\vec{k},\lambda)] . \tag{5}$$

The polarization vectors $\hat{\epsilon}(\vec{k},\lambda)$ obey:

$$\hat{\epsilon}(\vec{k},\lambda)\cdot\hat{\epsilon}(\vec{k},\lambda') = \delta_{\lambda\lambda'} \quad , \quad \vec{k}\cdot\hat{\epsilon}(\vec{k},\lambda) = 0 \ ,$$
$$\sum_{\lambda=1}^{2} \epsilon_i(\vec{k},\lambda)\, \epsilon_j(\vec{k},\lambda) = \delta_{ij} - \frac{k_i k_j}{k^2} \ , \tag{6}$$

and $\eta(\vec{k},\lambda)$ are statistically independent random phases[7,11], uniformly distributed in the interval $(0,2\pi)$, that contain all the statistical character of the fields. The function $g(\omega)$ is

the amplitude, and is related to the spectral density $\rho(\omega)$ as $g^2(\omega) = c^3\omega^2\rho(\omega)$. Obviously, in such an approach, the relevant quantities will be the *averages* [performed over the random phases $\eta(\vec{k},\lambda)$] of the physical entities.

The detailed study of the interaction of the random fields with matter is not an easy task, mainly when one deals with nonlinear systems, e.g., the hydrogen atom[8,9]. Thus we will try to overcome this difficulty by joining the simplicity of Einstein's model (polarizable particles in equilibrium with radiation) with the hypothesis of SED, as we shall see in what follows.

We shall assume, as mentioned above, that not only the thermal, but also the zero point radiation fields [with spectral distribution given by (4)] stimulate *emission* and *absorption* of radiation in a polarizable particle possessing internal harmonic oscillations. For the sake of simplicity we shall initially study, like Einstein, only transitions between the energies E_2 and E_1 (with $E_2 > E_1$). Later on we shall consider the continuum case, based on Einstein–Ehrenfest's work[4,5].

If such a system absorbs energy and undergoes a transition from the state with energy E_1 to the state with energy E_2, then, according to this modified Einstein model, the transition probability, per unit time, will be:

$$\frac{dW_{12}}{dt} = A_{12}\,\rho_0(\omega) + B_{12}\,\rho_T(\omega) \ , \tag{7}$$

where A_{12} and B_{12} are constants, i.e., temperature and frequency independent.

We would like to stress that the above expression can be justified on classical grounds, because it is well known[21] that a harmonic oscillator (frequency ω) absorbs energy from the radiation field at a rate which is proportional to the spectral density at the same frequency. Both terms in (7) represent a *classical transition probability*, because they are connected with the spectral densities $\rho_0(\omega)$ and $\rho_T(\omega)$ of the fluctuating classical electromagnetic fields.

In analogy with (7) we are going to write the transition probability, per unit time, from the state E_2 to the state E_1 as:

$$\frac{dW_{21}}{dt} = A_{21}\,\rho_0(\omega) + B_{21}\,\rho_T(\omega) \; . \qquad (8)$$

Here the term $A_{21}\,\rho_0(\omega)$ is replacing the term corresponding to spontaneous emission in Einstein and Ehrenfest's original calculation. This means that we are assuming that the *spontaneous* emission is in fact *induced* by the zero–point radiation[14–16].

The second initial assumption by Einstein (Boltzmann statistics for the particles) will be maintained, that is, if we have $n(E_1)$ particles in the state E_1 and $n(E_2)$ particles in the state E_2 the relation

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2-E_1)/kT} \qquad (9)$$

is valid on the average.

If we assume the detailed balance condition, as in the original papers by Einstein and Ehrenfest, then we have that

$$n(E_2)\frac{dW_{21}}{dt} = n(E_1)\frac{dW_{12}}{dt} \qquad (10)$$

is a sufficient condition for the particles and radiation to be in equilibrium.

Considering the expression above in the high temperature limit $(T \to \infty)$, when $n(E_2) \simeq n(E_1)$ and also $\rho_T(\omega) \gg \rho_0(\omega)$, we conclude that $B_{21} = B_{12} \equiv B$. If we also consider the low temperature limit $(T \to 0)$, when $n(E_2) \ll n(E_1)$ and $\rho_T(\omega) \ll \rho_0(\omega)$, we

conclude that $A_{12} = 0$ and $A_{21} \equiv A \neq 0$. We note here that $A_{12} = 0$ appears naturally, i.e., the zero–point radiation does not stimulate absorptions in the equilibrium situation[15].

It is easy to show from (10) that:

$$\rho_T(\omega) = \frac{\frac{A}{B}\,\rho_0(\omega)}{e^{(E_2-E_1)/kT} - 1} \; , \qquad (11)$$

and Wien's law (the third hypothesis made by Einstein) demands that $E_2 - E_1 = \hbar\omega$, where $\hbar$ is a universal constant.

The value of the constant A/B can be fixed by using the Rayleigh–Jeans law for the blackbody radiation spectral distribution $[\rho_{RJ}(\omega)]$. This law is valid for low frequencies $(\hbar\omega \ll kT)$ and must coincide with (11) in this limit, when $\rho_T(\omega) \approx \frac{A}{B}\,\rho_0(\omega)kT/\hbar\omega$. Since $\rho_{RJ}(\omega) = kT\omega^2/\pi^2c^3$, we must have $A = 2B$. The constant $2\pi\hbar$ which appears in (4) can be identified again with Planck's constant (see also (3)) and, as we have already mentioned, it is strictly connected with the intensity of the zero–point radiation fields. With this we conclude that Einstein's derivation of Planck's formula is compatible with the existence of classical zero–point electromagnetic fluctuations, and also with the concept that "spontaneous" emission can be interpreted as being *induced* by the zero–point radiation.

The relation $A = 2B$ could be interpreted as the zero–point electromagnetic fluctuations being *twice* more effective than the thermal ones in inducing the emission of radiation. In the realm of classical SED, however, both the zero–point fluctuations and the so–called radiation reaction force play equally important roles. Therefore we prefer to interpret the result $A = 2B$ in the way that was suggested by Milloni[22] and França[15], i.e., that the self–fields of the charge also induce emissions at the same rate that the zero–point fields with spectral density $\rho_0(\omega) \propto \omega^3$.

Now, based on the work of Einstein and Ehrenfest[5] we are going to remove the quantum hypothesis of discrete energy levels for the particles[4].

Let us assume that one particle suffers N *absorptions* (in the frequencies $\omega_1,\omega_2,\ldots,\omega_N$) and N' *emissions* (in the frequencies $\omega'_1,\omega'_2,\ldots,\omega'_{N'}$) in such a way that the particle goes from an initial state with energy E_I to a final state with energy E_F (where E_I and E_F are arbitrary). In the simple diagram shown in the figure we can have an intuitive feeling of the Einstein–Ehrenfest's proposition.

– FIGURE –

It is necessary a generalization of expresssions (7) and (8), in order to have a mathematical description of the processes indicated above. We then write (similarly to Einstein and Ehrenfest[3,4]) the following expression for the transition probability (per unit time) dW_{IF}/dt, from the state with energy E_I to the state with energy E_F:

$$\frac{dW_{IF}}{dt} = \overbrace{\prod_{i=1}^{N}[B\rho_T(\omega_i)]}^{\text{absorption}}\ \overbrace{\prod_{j=1}^{N'}[A\rho_0(\omega'_j)+B\rho_T(\omega'_j)]}^{\text{emission}} \quad . \tag{12}$$

For the inverse process we have:

$$\frac{dW_{FI}}{dt} = \overbrace{\prod_{i=1}^{N}[A\rho_0(\omega_i)+B\rho_T(\omega_i)]}^{\text{emission}}\ \overbrace{\prod_{j=1}^{N'}[B\rho_T(\omega'_j)]}^{\text{absorption}} \quad . \tag{13}$$

It should be remarked that the above expressions are valid only if the elementary processes (emission and absorption) are *statistically independent*[4].

This point deserves some comments. Expression (12), for instance, means that we have N absorptions and N' emissions occurring in a small time interval. All these events being statistically independent. The usual justification for this fact is that radiation is made up of quanta (photons), so that absorption and emission are "naturally" considered as the result of random collisions between light corpuscles and molecules[4]. Nevertheless we

cannot say that (12) and (13) are in contradiction with SED as we know that Maxwell's equations are linear, and the waves associated to the frequencies ω_i hit the molecule randomly. Of course this point requires further analysis as we have not reached a definitive conclusion so far.

Returning to (12) and (13), we expect that under the *influence* of thermal and zero–point radiations, the particles are *induced* to exchange energy and momentum with the radiation field. This field is represented by a superposition of plane waves with all frequencies. Therefore it is reasonable to expect that each absorption (in a frequency $\omega_i = c|\vec{k}_i|$) is accompanied by a transfer of momentum (from the wave to the particle) which has the direction of the corresponding wave vector $\vec{k}_i$ as is expected from Maxwell theory. If we consider *induced emission* as the reverse of *induced absorption*, it is natural to assume that also this process involves the superposition of plane waves, each one with a definite direction for the momentum. Then, it is reasonable to accept that the energy removed from (or added to) the radiation inside the cavity will be converted into translational kinetic energy added to (or removed from) the particle. All these considerations are consistent with the Einstein–Ehrenfest model and with SED.

If we take into account these observations, the final energy, E_F, and the initial energy, E_I, of the particle are expected to be related by:

$$E_I + \sum_{i=1}^{N}\Phi(\omega_i) = E_F + \sum_{j=1}^{N'}\Phi'(\omega'_j) \quad , \tag{14}$$

where $\Phi(\omega)$ and $\Phi'(\omega')$ are unknown positive quantities to be fixed below. The first summation in (14) represents the energy extracted from the radiation field after N absorptions, and the second one the energy added to the radiation field after N' emissions.

From now on our discussion departs from the original one by Einstein and Ehrenfest, since our aim is not to derive again Planck's expression for $\rho_T(\omega)$. This formula has been derived many times within the classical realm of SED[11,23]. We shall assume now that $\rho_0(\omega)$ and $\rho_T(\omega)$ are well known and we shall obtain the unknown quantities $\Phi(\omega)$ and $\Phi'(\omega')$.

As before, we shall assume that Boltzmann distribution (9) is valid for the molecules with energies E_I and E_F.

The detailed balance condition (10) is again introduced as a sufficient condition for the equilibrium between matter and radiation.

From (10), taking into account (12), (13) and (14) together with (9) we can show that:

$$\prod_{i=1}^{N} \left[\frac{B\rho_T(\omega_i)\; e^{\Phi(\omega_i)/kT}}{A\rho_0(\omega_i) + B\rho_T(\omega_i)} \right] = \prod_{j=1}^{N'} \left[\frac{B\rho_T(\omega'_j)\; e^{\Phi'(\omega'_j)/kT}}{A\rho_0(\omega'_j) + B\rho_T(\omega'_j)} \right] . \qquad (15)$$

The above expression must be valid for any N and N' and also for arbitrary sets of ω_i and ω'_j. This means that each term inside the square brakets must be equal to 1. Since $A/B = 2$ and $\rho_0(\omega)$ and $\rho_T(\omega)$ are well known (from previous, and different, analyses based on SED), the only unknown quantities are $\Phi(\omega)$ and $\Phi'(\omega')$. From these considerations, it is easy to show that

$$\Phi(\omega) = \hbar\omega = \Phi'(\omega) . \qquad (16)$$

If we use these results and write (14) for $N = N' = 1$, we get

$$E_I + \hbar\omega = E_F + \hbar\omega' \qquad (17)$$

as a relation to be valid on the average.

We have concluded that the energy $\hbar\omega$ must be extracted from (or added to) the

classical radiation beams in order to maintain the thermodynamical equilibrium. It does not mean that we have considered the radiation as necessarily being made up of *corpuscules*. The relation (17) is a direct consequence of the hypothesis that it is possible to count the number of statistically independent elementary events of emission and absorption of electromagnetic waves.

II.b. Momentum Exchange

In the second part of the 1917 paper, which Einstein himself considered the most important part, it is presented a detailed study of the momentum exchange between the molecules and the cavity radiation. The goal of his analysis was to show the necessity of all elementary processes of absorption and emission of radiation being *directional*. This conclusion is obvious if we adopt the corpuscular model of the light quanta. However, we want to avoid this hypothesis here. We maintain the same line of thought of Einstein's 1917 work.

Since the method used in this part is more sophisticated than the one presented in the first part, nonrelativistic approximations will be explicitly introduced (as in the original Einstein presentation[3,11]) in order to greatly simplify the calculations. We shall also show that the Einstein's approach becomes much more clear if we recognize that "random spontaneous emission" may be considered as *induced emission by the random zero–point radiation* of SED (as it is well known, Einstein himself considered very strange the concept of random spontaneous emission).

Let us consider that the polarizable particle is in movement inside a recipient at temperature T, so that we can divide the radiation influence in two parts: the first one will be identified with a fluctuating interaction and the second one will have a dissipative character, in analogy with the phenomenological theory of the brownian motion. If initially the particle has a linear momentum with projection Mv in the x direction, then after a short time interval τ, it will have the momentum[3,11]

$$Mv' = Mv + \Delta - Rv\tau \quad . \tag{18}$$

Here Δ corresponds to the fluctuating part of the momentum transferred to the particle by the random action of the thermal and zero–point radiation fields. The term $-Rv\tau$ is just the dissipative part (we shall see that it only acts when $T \neq 0$) that slows down the particle. The calculation of the dissipative force $-Rv$ will be presented first.

Since in the Einstein model the hypotheses concerning emission and absorption were formulated in a coordinate system in which the particle is at rest, we shall investigate how does the radiation looks in such an inertial reference system. In the laboratory reference system (attached to the recipient where the particles are), the electromagnetic radiation is isotropic, that is, the energy density between ω and $\omega + d\omega$ within the solid angle $d\Omega$, around some (arbitrary) direction of propagation, can be written as:

$$\frac{d\Omega}{4\pi}\rho(\omega)\, d\omega \quad . \tag{19}$$

In our description the spectral density $\rho(\omega)$ above also includes the effects of the zero–point radiation fields, and must be written as:

$$\rho(\omega) = \rho_0(\omega) + \rho_T(\omega) \quad , \tag{20}$$

where $\rho_T(\omega)$ is given by the well known Planck's formula, and $\rho_0(\omega)$ is the zero–point spectral density given by (4).

However, for a particle that is moving with constant velocity v along the x axis (in the laboratory system), the density of energy is not isotropic in the coordinate system attached to it and therefore it will be denoted by:

$$\frac{d\Omega'}{4\pi}\rho'(\omega',\theta')d\omega' \quad . \tag{21}$$

Here θ' is the angle beween the x axis and the wave vector $\vec{k}'$ associated to a wave with frequency $\omega' = c|\vec{k}'|$.

In the original paper by Einstein[3] (see also ref. 11) it is presented the detailed calculation of the Lorentz tranformation for the spectral density. In the case $v/c \ll 1$ the result is:

$$\rho'(\omega',\theta') = \left[\rho(\omega') + \frac{v}{c}\,\frac{\partial\rho}{\partial\omega'}(\omega')\,\omega'\cos\theta'\right]\left[1 - \frac{3v}{c}\cos\theta'\right] \quad , \tag{22}$$

where $\rho(\omega')$ is the spectral density in the laboratory reference system. A dissipative force will arise due to the anisotropy of $\rho'(\omega',\theta')$.

For the sake of simplicity we shall consider that the particle will suffer transitions between two states of energies E_1 and E_2, with $E_2 > E_1$. This is not a quantum assumption, because E_2 can be arbitrarily close to E_1, as we have seen before. According to the previous considerations, a beam of radiation (thermal and zero–point) associated with the solid angle $d\Omega'$ induces[3,11]

$$N_2 = n(E_2)\left[B\rho'_T(\omega',\theta') + A\rho'_0(\omega')\right]\frac{d\Omega'}{4\pi} \tag{23}$$

emission processes per unit time. The second term does not appear in the original paper by Einstein. We shall explain why it does not contribute to the dissipative force $-Rv$.

In an analogous way we must have

$$N_1 = n(E_1)\, B\rho'_T(\omega',\theta')\,\frac{d\Omega'}{4\pi} \tag{24}$$

absorption processes per unit time.

As before, $n(E_1)$ is the number of particles with energy E_1, $n(E_2)$ the number of particles with energy E_2, and it is valid the classical Boltzmann relation (9). The coefficients A and B are the same as in the expressions (12) and (13), with $A/B = 2$.

At this point there is a fundamental difference between Einstein's approach and ours, based on SED. In Einstein's view, the fact that the absorptions and emissions ***induced*** by radiation beams are ***directional*** , is classically plausible. The fact that ***spontaneous emission*** must also be ***directional*** lead him to the hypothesis of radiation quanta. Einstein's dissatisfaction with this assumption is very clear[3]: "The weakness of the theory lies, on the one hand, in the fact that it does not bring any nearer the connection with the wave theory and, on the other hand, in the fact that it leaves moment and direction of the elementary (spontaneous) processes to *chance*". For us, however, ***all*** the emission and absorption processes are induced (by thermal and zero–point fields), and we therefore expect that in all of them will occur a ***directional*** transfer of linear momentum to the particle. Then, in writing (23) and (24) above, we have already assumed that each process involves a frequency, for instance ω', and a wave vector $\vec{k}'$ well defined. Such a wave carries momentum in the $\vec{k}'$ direction, and this will be the direction of the exchanged momentum. We intend to calculate its modulus, that we shall denote by $Q(\omega')$, because we expect it to be a function of the frequency ω'.

Since all the processes are directional the fraction of linear momentum (direction x) added to the particle per unit of time will be:

$$d\dot{p} = (N_1 - N_2)\, Q(\omega') \cos\theta' \quad . \tag{25}$$

The total variation of momentum, considering all the propagation directions, is

$$\int d\dot{p} = \frac{dp}{dt} = -n(E_2)\, A\, Q(\omega')\, \rho_0(\omega') \int \frac{d\Omega'}{4\pi} \cos\theta' + Q(\omega')\, B\left[n(E_1) - n(E_2)\right] \times$$
$$\times \int \frac{d\Omega'}{4\pi} \cos\theta' \left[1 - \frac{3v}{c}\cos\theta'\right] \left[\rho_T(\omega') + \frac{v}{c}\,\frac{\partial \rho_T(\omega')}{\partial(\omega')}\,\omega' \cos\theta'\right] , \tag{26}$$

where we have used the fact that $\rho_0'(\omega') = \rho_0(\omega')$ because the zero–point radiation has a Lorentz invariant pattern[19].

The first integral, which contains the contribution of the emission induced by the zero–point radiation ("spontaneous" emission), is zero. This happens because the emission is ***directional*** and $\rho_0(\omega')$ is ***isotropic*** in any inertial frame. Therefore only the thermal radiation contributes to (26). Of course this point is not explained in this way in the original paper by Einstein[3,11].

The second integral is easily calculated. Retaining only terms of the order of v/c, we obtain

$$\frac{dp}{dt} = -\frac{Q(\omega)}{c}\left[n(E_1) - n(E_2)\right] B\left[\rho_T(\omega) - \frac{\omega}{3}\,\frac{\partial \rho_T(\omega)}{\partial \omega}\right] v \equiv -R(\omega)\, v \quad , \tag{27}$$

where we have simplified the notation replacing ω' by ω (since ω' is arbitrary). This expression (27) represents the dissipative force applied to the particle [the last term in (18)].

Next we shall analyse the contribution of the random electromagnetic fields to the fluctuating part Δ of the particle's momentum (see (18)) in order to establish the relation between $<\Delta^2>$ and $Q(\omega)$.

Let us assume that in a short time interval τ we have a total number S of emission and absorption processes, associated to the frequency ω. Each process transfers a linear momentum λ_i (x component). Then, the total contribution (in this small time interval τ) to the fluctuating part of the momentum will be:

$$\Delta = \sum_{i=1}^{S} \lambda_i \quad . \tag{28}$$

Since we are considering the λ_i as random variables, we have that $<\lambda_i> = 0$, and consequently $<\Delta> = 0$ on the average. However $<\Delta^2> \neq 0$, so that:

$$<\Delta^2> = \sum_{i=1}^{S} <\lambda_i^2> \quad . \tag{29}$$

We have already said that each elementary process is directional and involves a momentum transfer $\lambda_i = Q(\omega)\cos(\theta_i)$, where θ_i is the angle between the x axis and the wave vector of the emitted (or absorbed) radiation. Taking the average over all directions θ_i we obtain:

$$<\lambda_i^2> = \frac{Q^2(\omega)}{4\pi} 2\pi \int_0^{\pi} d\theta_i \,\mathrm{sen}\,\theta_i \cos^2\theta_i = \frac{1}{3} Q^2(\omega) \quad . \tag{30}$$

Thus, expression (29) is simplified to:

$$<\Delta^2> = \sum_{i=1}^{S} <\lambda_i^2> = \frac{1}{3} S\, Q^2(\omega) \quad . \tag{31}$$

The next step is just to express S, the total number of absorption and emission processes that occur in the time interval τ, as a function of known quantities. We know that $A\rho_0(\omega)+B\rho_T(\omega)$ is the rate of stimulated emission processes, and $B\rho_T(\omega)$ the rate of absorption processes. Thus, the total number S of emission and absorption processes during τ will be:

$$S = n(E_2)\, A\rho_0(\omega)\, \tau + \Big[n(E_2) + n(E_1)\Big]\, B\rho_T(\omega)\, \tau \quad . \tag{32}$$

Using this result in (31), we obtain:

$$\begin{aligned} <\Delta^2> &= \frac{1}{3} Q^2(\omega) \Big\{n(E_2)\, A\rho_0(\omega) + \Big[n(E_2) + n(E_1)\Big]\, B\rho_T(\omega)\Big\}\, \tau = \\ &= \frac{2}{3} Q^2(\omega)\, n(E_1)\, B\rho_T(\omega)\, \tau \quad , \end{aligned} \tag{33}$$

where we have again assumed the detailed balance condition.

The only point in which the above formula differs from a similar one obtained by Einstein in 1917 is related to the (up to now) unknown factor $Q(\omega)$.

Since we are considering that the equilibrium situation is maintained through the interaction of the radiation and matter, we can use the energy equipartition principle, and write:

$$\frac{1}{2} M<v'^2> = \frac{1}{2} M<v^2> = \frac{1}{2} kT \quad , \tag{34}$$

according to our assumption of Boltzmann distribution for the particles.

Now we are in position to calculate $Q(\omega)$, since we know $R(\omega)$, $<\Delta^2>$ and $<v^2>$. Squaring equation (18) and taking the ensemble average we get

$$<\Delta^2> = 2\, kT\, R(\omega)\, \tau \quad . \tag{35}$$

In establishing this (fluctuation–dissipation) relation we considered, as Einstein, that $<v\Delta> = <v><\Delta> = 0$ and also that $R^2\tau^2<v^2>$ is negligible as compared to $2MR<v^2>\tau$ in the limit of small time internal τ and large mass M.

Substituting (33) and (27) into (35) and also considering that (11) is valid (with $A/B = 2$) we get an equation for $Q(\omega)$. From this equation we can show that

$$Q(\omega) = \frac{2kT}{c}\left[3-\frac{\omega}{\rho_T(\omega)}\frac{\partial\rho_T(\omega)}{\partial\omega}\right]\left[\frac{\rho_0(\omega)}{\rho_T(\omega)+2\rho_0(\omega)}\right] . \quad (36)$$

This apparently complicated expression becomes quite simple if we substitute the formulas (4) for $\rho_0(\omega)$ and (2) for $\rho_T(\omega)$. The final expression for $Q(\omega)$ is simply:

$$Q(\omega) = \frac{\hbar\omega}{c} = \hbar|\vec{k}| , \quad (37)$$

as expected on physical grounds.

Let us summarize the above results. Consider processes which involves N absorptions (of electromagnetic waves) in the frequencies ω_i and N' emissions in the frequencies ω_j'. If the vector $\vec{p}_F(E_F)$ is the particle final momentum (energy) and $\vec{p}_I(E_I)$ is the initial momentum (energy), the relationship between the momenta, energy and frequencies must be such that

$$\vec{p}_I + \sum_{i=1}^{N}\hbar\vec{k}_i = \vec{p}_F + \sum_{j=1}^{N'}\hbar\vec{k}_j' , \quad (38)$$

$$E_I + \sum_{i=1}^{N}\hbar\omega_i = E_F + \sum_{j=1}^{N'}\hbar\omega_j' , \quad (39)$$

in order to maintain the equilibrium between matter and cavity radiation. In other words, we have combined the *classical hypotheses* contained within Einstein and Ehrenfest model with SED, and we have obtained the value $\hbar\omega$ for the energy exchanged in each process of emission (or absorption) of electromagnetic *waves* with frequency ω. Each process is also accompanied by a momentum transfer $Q(\omega) = \hbar\omega/c$ which has the direction of the wave vector $\vec{k}$. It should be mentioned that the above relations are commonly associated with the energy–momentum conservation in scattering processes in which the electromagnetic

radiation is considered as a swarm of particles. In the usual quantum language, $\hbar\omega/c$ is just the momentum of a *particle of light* (*photon*) with frequency ω.

In the calculations presented above we have introduced some simplifying assumptions that are the same contained in the original work by Einstein. We must take some care, however, if we want to keep these assumptions compatible with the hypotheses of classical stochastic electrodynamics.

Consider, for instance, the application of the energy equipartition principle to the translational motion of free particles. This principle is utilized when, in the calculation of the momentum exchange, we attribute to the translational energy the value $\frac{1}{2}\langle Mv^2\rangle = \frac{1}{2}kT$ as being the average of the kinetic energy acquired by the particle immersed in the Planck radiation with temperature T. This approximation, however, implies that the zero point fluctuations, important to maintain the equilibrium between radiation and matter particles, are supposed to give very little contribution to kinetic (translational) energy. As we have verified, the emissions and absorptions change the kinetic energy of the particle from E_1 to E_2, with $E_2-E_1=\hbar\omega$, in the ideal case in which only one process with frequency ω is involved. We have also calculated the random part of the momentum exchange between radiation and matter, and we have found $\langle\Delta^2\rangle$, that is, the mean square average value acquired after S absorptions and emissions (in a short time interval τ). We have obtained [see (33)]:

$$\frac{\langle\Delta^2\rangle}{Q^2(\omega)} = \frac{\langle\Delta^2\rangle}{\hbar^2\omega^2/c^2} = \frac{2}{3}n(E_1)\,B\rho_T(\omega)\,\tau . \quad (40)$$

In the limit $T\to 0$, $\rho_T(\omega)\to 0$ and therefore $\langle\Delta^2\rangle/\frac{\hbar^2\omega^2}{c^2}\to 0$. If we want the full incorporation of the zero–point fluctuations, we must modify (40). However we do not want to lose the simplicity of Einstein's model and so we decided to maintain expression (40) and the energy equipartition hypothesis. We shall now specify the domain of validity of the approximations used. For this we shall recall an earlier result obtained by Einstein and Hopf[23,24] in 1910. In this work they have calculated the mean square average momentum of a polarizable particle under the influence of random radiation. The calculation is performed within the realm of classical electrodynamics, and the result is:

$$<\Delta^2> \;=\; \frac{8\pi^4 e^2 c}{15m}\,\frac{\rho^2(\omega,T)}{\omega^2}\,\tau \;, \tag{41}$$

where e and m are the charge and the mass of the oscillating part (frequency ω) of the polarizable particle. This is a general result which does not depend on the particular form of the spectral distribution $\rho(\omega,T)$. The only hypothesis is that the particles are immersed in *random electromagnetic fields*. According to our view, based on stochastic electrodynamics, matter is *always* immersed in the zero–point radiation fields and therefore expression (41) must include them. It is usual to consider the thermal and the zero–point radiation to be statistically independent, and so the momentum fluctuations can be written as[23]

$$<\Delta^2> \;=\; <\Delta^2>_{\text{thermal}} + <\Delta^2>_0 \;=\; <\Delta^2>_{\text{thermal}} + \frac{8\pi^4 e^2 c}{15m}\,\frac{\rho_0^2(\omega)\tau}{\omega^2} \;. \tag{42}$$

In the limit $T \to 0$, the thermal part will vanish, and so expression (41) will become:

$$<\Delta^2>_0 \;=\; \frac{8\pi^4 e^2 c}{15m}\,\frac{\tau}{\omega^2}\left[\frac{\hbar\omega^3}{2\pi^2 c^3}\right]^2 \;. \tag{43}$$

Equation (43) can be rewritten to obtain a ratio between energies, i.e.:

$$\frac{<\Delta^2>_0/2M}{\hbar\omega} \;=\; \frac{\omega\tau}{15}\left[\frac{e^2}{\hbar c}\right]\left[\frac{\hbar\omega}{mc^2}\right]\left[\frac{\hbar\omega}{Mc^2}\right] \;, \tag{44}$$

where $<\Delta^2>_0/2M$ is the average kinetic (translational) energy of the particle (total mass M) due to the influence of the zero–point radiation only (temperature $T = 0$).

Now it is possible to estimate each factor in the r.h.s. of (44). We know that $e^2/(15\hbar c) \simeq 5 \times 10^{-4}$ is a small factor. Besides this, we have stressed above (see the approximations used for obtaining (35)) that the time interval τ is small. Therefore it is reasonable to assume that $\tau\omega \sim O(1)$ for not too high frequencies. However, the vanishing observed in (40) when $T \to 0$ is guaranteed in (44) only when $mc^2 \gg \hbar\omega$ (obviously this also implies that $Mc^2 \gg \hbar\omega$). In other words, at zero temperature the average kinetic (translational) energy of the particle is much smaller than the characteristic exchanged energy $\hbar\omega$. Our analysis, then, is valid in the limit of massive particles, in which is also valid the nonrelativistic approximation that we have used during the calculations. Different arguments by Boyer[20] and Jimenez[23], based on the interaction of the particles with the recipient walls also lead to the same conclusion.

III. DISCUSSION

In 1923 both Compton[25] and Debye[26] independently wrote down the relativistic kinematic relations which follow from (37) and (38) when we assume that $N' = N = 1$, $\vec{p}_I = 0$ and $E_F^2 = M^2c^4 + c^2\vec{p}_F^{\,2}$. They used those kinematical relations in order to give a theoretical explanation to the experimental facts observed in the scattering of X and γ–rays by electrons. In the words of Compton "the quanta of radiation are received from definite directions and are scattered in definite directions". Debye mentioned his indebtedness to Einstein's work on needle radiation (nadelstrahlung). Compton, however, did not mention Einstein at all.

With the above assumptions Compton and Debye derived the well known expression

$$\Delta\lambda = \frac{h}{Mc^2}(1-\cos\theta) \quad , \tag{45}$$

for the difference between the wavelengths of final and initial light quanta (here θ is the scattering angle). This formula, obtained in 1923, and the explanation for the photoelectric effect, given by Einstein in 1905, are considered nowadays as very clear evidences that light quanta are also corpuscles. We want to recall that neither Einstein, nor Compton nor Debye have made such an assertion in their original papers. They have used the much more weak assumption of "needle radiation" (nadelstrahlung), that is, the elementary processes are ***directional***.

The evidences that it is not necessary to assume that photons are particles in order to explain the photoelectric effect (and also the Compton effect as we shall see in a while) start to appear a few years after the pioneering work by Einstein (1905).

In 1914, Richardson[27,28] explained the photoemission (without using the light quantum) as analogous to the evaporation from a liquid surface. Perhaps this fact and many other manifestations, against the light quantum , coming from leading physicists[28] (like Planck, Sommerfeld, Lorentz, Millikan, Bohr and Van Laue among others), lead Einstein to say in 1917: "For the rest of my life I will think what are light quanta". It is also interesting to note that Einstein published approximately twenty works on this theme within the period from 1905 to 1916.

In 1927, Wentzel[29] obtained a satisfactory description of the photoelectric effect based on the assumption that the electron was described by the Schrödinger equation and the radiation was classical electromagnetic fields. In the same year Schrödinger[30] published a paper (almost unknown) explaining the Compton effect (44) in a way quite similar to Wentzel's treatment of the photoelectric effect. According to Schrödinger the electrons are governed by a relativistic wave equation (Klein–Gordon type) and the electromagnetic radiation were classical Maxwellian waves. Later on, with the proposition

of a covariant equation for the electron by Dirac, Klein and Nishina (1929) obtained the famous expression for the cross section describing the scattering of radiation by electrons. It is interesting to note that the analysis by Klein and Nishina[31] was made without an explicit quantization of the electromagnetic field. Only Tamm (1930) performed the calculation within the realm of quantum electrodynamics (QED) for the first time[32]. More recently, however, some authors[2] have pointed out that QED is a *field* theory containing no hypothesis about the corpuscular nature of photons. Many other authors[33–37] have discussed the photoelectric effect and the Compton effect without using the concept of the photon as a corpuscle.

Here, revisiting Einstein's 1917 and Einstein and Ehrenfest's 1923 papers on the new light of SED, we have concluded that it is possible to obtain the kinematic relations (38) and (39) by using only classical electrodynamics with classical zero–point radiation. Apparently there was no need to introduce the photon as a particle–like entity. In other words, our analysis suggests that we can consider the photon as an almost monochromatic *signal* (frequency ω) *of electromagnetic waves* with average energy $\hbar\omega$. This is interesting because we know that Compton himself was able to derive the wavelenght–shift (45) by using an undulatory argument based on the *classical Doppler effect*[36]. This reasoning also helped him in the calculation (which is based on classical electromagnetism) of the X–ray scattering cross section as we can see in Compton's original paper. This fact was soon stressed in a work by Woo[38] in 1925. More recently we have discussed this point again but within the realm of SED[39].

It is gratifying to see that more than seventy years after Einstein's work one still can find new features in it, as we have perceived in combining it with classical stochastic electrodynamics. We could see the reach of Einstein's 1917 theory by appreciating its connections with Compton scattering, a phenomenon that constitutes itself a landmark in the study of the interaction of radiation with matter. It is also quite interesting to identify the qualitative connections of our discussion presented here with the work by Marshall and

Santos[40] within the realm of a new branch of SED, namely, stochastic optics. In this new theory, which is an attempt to give a local realistic interpretation of quantum optics, the zero–point electromagnetic radiation is also responsible for some pseudo–corpuscular properties of visible light. Therefore it is possible, according to Marshall and Santos, to give a satisfactory classical explanation, (that is local and realist) to recent experimental results of optical tests of Bell's inequalities[41].

It is the opinion of the present authors that the concept of light as a corpuscle must be re–examined. We believe that Albert Einstein, in his reluctance to completely accept quantum theory, was trying to convey this to us during many decades. We now quote from A. Pais[42] a statement made by Einstein to Otto Stern: "I have thought a hundred times as much about quantum problems as I have about general relativity theory". In 1951 Einstein pronounced the dramatic words: "All these 50 years of conscious brooding have brought me no near to the answer to the question: what are light quanta? Nowadays every rascal thinks he knows it, but he is mistaken". According to A. Pais, Einstein kept thinking about the quantum theory till the end of his life. In his last autobiographical notes the final sentences deal with the quantum theory and its relation with classical electromagnetism. "It appears dubious whether a (classical) field theory can account for the atomic structure of matter and radiation as well as of quantum phenomena. Most physicists will reply with a convinced 'No', since they believe that the quantum problem has been solved in principle by other means. However that may be, Lessing's conforting words stay with us: the aspiration to truth is more precious than its assured possession".

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FIGURE CAPTION

Schematic picture representing the process of N absorptions in the frequencies $\omega_1, \omega_2, \ldots, \omega_N$ and N' *emissions* in the frequencies $\omega'_1, \omega'_2, \ldots, \omega'_{N'}$. E_F and E_I are respectively the (arbitrary) final and the initial particle energies.