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SPONTANEOUS REASONING OF GRADUATE  
STUDENTS

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## SPONTANEOUS REASONING OF GRADUATE STUDENTS

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### Summary

Investigations into student's conceptions have led to the conclusion that alternative ways of reasoning coexist with those forms learnt in science classes at school. This paper presents a study of the persistence of such conceptions among physics graduate students. Results show that for this population the conceptions are the same as those which have been found amongst secondary course students. The topics which were selected were Special Relativity Theory and the Classical Theory of Collisions. Within these contents it was possible to find some fundamental and common characteristics of the way of thinking.

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### INTRODUCTION

The large number of investigations referring to alternative conceptions, as well as the diversity of situations in which they have been detected, leaves little doubt that children of secondary and early University students possess an articulated knowledge which is different from that taught at schools and which is resistant to change.

Are University courses, with their length and extensive coverage, able to address this situations? Furthermore is the practice of scientific research by graduate students able to instill precise scientific knowledge? Some interesting hints to answer to these questions can be found in reports on the training of primary and secondary school science teachers (Perez, 1985) where alternative conceptions still play an important role in the explanation of their behaviour.

This paper reports on the results of two investigations, in Special Relativity Theory and Classical Theory of Collisions (Villani 1987 and 1988), which were carried out with over a hundred graduate students, with a BS degree in Physics, who were already involved in research towards their MSc or PhD degrees. The results will show that students present a strong tendency to interpret phenomena according to more general conceptions and systems which are very similar to what has been called spontaneous reasoning.

### METHODOLOGY OF RESEARCH

We selected two topics for which spontaneous reasoning was already known from previous investigations: speed of light (Relativity Theory) and collisions (Classical Newtonian Theory). The corresponding references are the model of Saltiel (Saltiel, 1980) for spontaneous conceptions in kinematics and change of reference frame and the models of Mariani (Mariani, 1987) for the transfer of motion in collisions.

In each investigation we used two problems: one with an immediate solution and

another which could be solved only after an articulated reasoning although without the need for any calculations. The problems demanded objective qualitative answers (such as: larger than, smaller than, equal to) with a corresponding justification. We expected that the more complex the problems would have answers in which both the formal and the spontaneous knowledge would be present. The question sets which were posed in interviews are shown in the Appendix.

In the investigation on the velocity of light, 65 students participated with 24 interviews on the first problem and 60 on the second (with partial overlap); 75% of these students were in the MSc course and 25% in the PhD course. In the transfer of motion investigation 85 students participated with 56 interviews on the simple collision problem and 59 interviews on the multiple collision problem: 50% were from the MSc course and 50% from the PhD course.

### SPONTANEOUS IDEAS ABOUT THE SPEED OF LIGHT

One problem deals with two light rays emitted by an antenna and two trains running away in opposite direction (Villani, 1987). The questions on the first problem refer to the distances between photons and trains as seen by different observers.

The second problem refers to the time spent by a light ray for going forward and backward between an antenna A and a mirror M which are tied to a relativistic rocket as seen by different observers. The questions are in Appendix A.

At first, 50% to 90% of the answers seemed adequate according to the theory of Relativity, but the analysis of justifications used by students reduced largely such percentages, since some of the predictions were accompanied by reasoning which was incomplete or mixed with spontaneous notions.

In the first problem, the general idea that underlay many of the justifications was that the invariance of the speed of light meant that a uniform displacement of light took

place in both directions. The speed of the trains in opposite directions explained why the photons observed from the trains were not at the same distance. It was as if there was only one speed for light and for the trains, with only one distance traversed by them, independent of the velocity of the observers. For this reason the ambiguous text not specifying the framework to which the data were referred did not cause any problem.

In the second problem the mixture between relativistic and spontaneous notions appeared evident: the distance traversed by light did not depend on the observer, the time dilation happened only when the object was "really" being displaced, the formal symmetries were superposed to an absolute space-time vision, the observations were qualified as "real" or "apparent" depending on its conditions of rest or motion. Examples may be found in Villani (1987) and in the comparative section.

### SPONTANEOUS IDEAS ABOUT ELASTIC COLLISIONS

This investigation aimed to find out the way students think about the transmission of motion in a collision (Villani, 1988).

The simple collision problem deals with several collision situations between two moving balls and two fixed targets. In the multiple collision problem the situations are similar but the targets are two stationary balls. Our interest was to find out the role of reciprocal interactions between balls and the influence of the symmetries between masses in the collision analysis.

The nature of the questions is phenomenological (prediction of range and direction of motion) and technical (involving energies and impulses transferred during a collision). See Appendix B.

In the simple problem the objective answers showed that about 1/3 of students predicted the behaviour of the incident balls and targets in a way that was compatible with newtonian mechanics applied to central elastic collisions (neglecting rotations and friction).

In the replies to questions about energy transfer only 1/6 used systematically criteria which are compatible with newtonian mechanics; most of the students preferred to relate energy transfer only to the mass of the incident ball or to the target velocity (previously predicted).

Finally in replies to questions on impulse we had a reduction of newtonian scheme to 5% of the total, the large majority of answers presenting the incident mass as the only factor responsible for the impulse given to the target.

In the case of a simple collision (two colliding bodies) we found two tendencies of interpretation, for explaining the appearance of the target motion, which can be characterized as:

**Transmission process** — according to this interpretation the incident ball passes its energy/momentum to the target. The transmission is complete when the incident ball and the target are not very different; in this case the incident ball stops and the target acquires all the incident ball energy. The target speed will be higher or lower depending on the mass of the ball so that the smaller mass ball will have the higher speed. The transmission is partial in the case of a substantial difference between the masses of the incident ball and the target; the incident ball "is not capable to pass" all its energy to the target, it continues its motion either forwards when the target is small, or backwards when the target is large.

In this version, the physical elements involved are purely scalar and the "motion passes from one body to the other" in a conservation frame work.

**Production Process** — according to this interpretation since the incident ball acquired motion rolling down the inclined plane, it processes "impact force" which of course depend on its mass. On colliding with the still target the ball imparts a forward impulse to it and the speed will be higher the smaller is the target mass. After the collision the incident ball motion will depend totally on the target ball and on the resistance opposed by it. If the target is smaller it will not succeed in stopping the incident ball. If the target ball is equal to the incident ball the latter will be completely stopped. If the

target is larger the incident ball will be stopped or even "pushed" back in a scheme of reaction which resembles the principle of action and reaction.

In this second model the basic idea is that of an active force effected by the incident ball and a resistance or reaction by the target in an opposite sense. The only difference from the newtonian scheme is that action and reaction are not connected by a symmetry principle: the action depends on the incident ball and will be stronger the larger its mass, the same happening with the reaction.

In the multiple collision (record problem) the students attention is concentrated on the intermediate ball and their explanation always refers to this point. An analysis of the students answers shows that they can be classified into three different categories depending on the behaviour of the intermediate ball, which includes the ideas connected to transmission and/or production:

**Independent collision** — two interactions are considered in which the intermediate ball is first the target and then the incident ball.

**Global collision** — this is a simple collision where the target is made up by the two still balls.

**Coupled collision** — the intermediate ball appears as a mere transmitter between the incident ball and the target.

In the first two cases collisions are regulated by the same mechanism as in the simple collision. In the third case the transmission is effected either with the intermediate ball at rest or moving. When the transmission is at rest, the incident ball stops or returns and the last ball stops or comes out with a certain velocity. When the transmission is in motion, the intermediate ball has been "dragged" by the incident ball and gets rid of that only after the collision.

The ideas about energy and impulse, especially those that support the proportionality between energy loss of the incident ball and increase of target mass and sustain the difference between the energy lost by the incident ball and energy transmitted, are incompatible with the classical elastic collision. They suggest the spontaneous view of

a collision with some inelastic component, in which the incident ball speed after the collision will be smaller, the larger is the target. On the other hand, the idea that the transmission of energy and momentum depends directly on the mass of the incident ball or inversely on the target mass suggests the interpretation of energy and impulse as "movement" that is accepted more easily the lighter is the target or the stronger is the "force exerted" by the incident ball. This idea has been found also in the analysis of simple collision. Examples may be found in Villani (1988) and in the next section.

In general the answers of the students always give an account of three factors that would be present in a collision: the characteristics of the target, the way energy and momentum are transferred or generated and the elasticity of the collision.

The analysis of the answers explanations by graduate students reveals features that are similar to the ones found with secondary students (Mariani 1987); however in the simple collision some of the more primitive ideas such as that of "dragging" and inelasticity do not appear and also the "production" model is nearly always associated to some type of scalar conservation of energy or momentum.

The more primitive ideas appear only in the multiple collision without any evident reason in the problem context which might explain this fact, unless a higher complexity in its analysis.

## COMPARATIVE RESULTS

A comparative analysis of the two investigations reveals some interesting and important similarities. The populations were similar and even contained some overlap, the methodology of research was the same and the type of analysis was the same. The contents were different, at least from the scientific point of view, but the elaboration of these contents by the students appears with common features.

First of all it is evident an *inadequate use of the physical intuition*. When the

immediate application of formulas becomes complex the percentage of replies compatible with newtonian or relativistic standards decreases significantly. In the case of the velocity of light, the comparison between measurements of the same light path made by several observers seemed more difficult than the comparison between different measurements of different light path (forward and backward) made by the same observer. In questions Q5 and Q6 of the second problem there is a fall from 90% to 50% in previsions compatible with Relativity Theory. In the case of elastic collisions, the number of replies which were compatible with a newtonian formulation decreases on passing from direct to rather technical prospects and from simple to multiple collisions. Question Q2 in problem 1 was answered "correctly" in all situations by more than 40% of the students, question Q3 only by 15%. The analogous questions Q2 and Q3 in problem 2 were answered with newtonian previsions respectively by 25% and 5% of the students.

This can be roughly taken as rather inadequate learning. However the inadequacy can provide meaning to the distance between the students physical intuition and the scientific formulation. Or putting it in another way it seems reasonable to interpret these difficulties in the more technical prospects or in multiple collision as a disagreement between the formal scientific bases and a spontaneous model which favors other more significant variables.

Depending on the complexity of the problem, students may react in two ways: either they start directly from known formulas using them as references and evaluation criteria in simple or already solved situations; or in other more complex situations the reference standard is personal and can up to a certain extent be incompatible with the scientific formulation.

These standard solutions may in our opinion have two interpretations: according to the first one the student does not know the scientific formulation in an articulated manner and that is why he does not use it as reading frame work for complex situations; according to the second interpretation the learned scientific knowledge has not been incorporated to the spontaneous personal knowledge with the building up of a bridge between the

abstraction of theory and the complexity of real situations in which prevail qualitative and personal criteria.

A second feature is the presence of common spontaneous standards of replies similar to the ones found in populations with lower degree of schooling. The model of reasoning in kinematics found by Saltiel (1988) allow an interpretation of replies given to questions on the speed of light. In fact the model is so characterized:

a) Speed, traversed distance and trajectory of a moving object are independent of a frame of reference;

b) Motion is explained in a causal way through the association of a "motor" supplying the real observable speed of the moving object and in a descriptive way by relating it to an absolute space. In this scheme, there is only one true speed and one true traversed space; any differences in measurements made by moving observers are apparent quantities caused by optical illusion.

Answers to question Q2 and Q3 of the first problem such as:

"Train  $T_B$  is farther from D and train  $T_D$  is closer to D because, beside the path of D, there is also the path of the trains."

"Train  $T_B$  is closer to B and train  $T_D$  is closer to D, because the speed of light is invariant"

may be attributed to an idea of "absolute" things. It's the same for questions Q3 and Q4 in the second problem:

"The forwards times are the same because the distances are the same."

"The backwards times are the same, because the speed of light does not depend on the frame of reference."

Finally, we should mention the presence, often explicitly but some times in a subtle form, of the qualification "real" and "apparent" in relation to time and distance.

"Apparently, light spends a longer time forwards for the station than for the rocket; but, in reality, the total time is the same."

The type of answers in collisions, found by Mariani (1987) with secondary students,

are similar to the ones found by us.

In simple collision problem we found the answers for the different situations S:

(S.2) "Incident balls M1 and M2 stop when collide with targets M3 and S1 because they transmit all their movements."

(S.5) "All incident balls loose their movements because the targets absorb all their forces."

Seem to be very similar to our students' replies:

(S.1) "Incident ball M2 stops when it collides with L1, because of the conservation of energy."

(S.5) "Incident balls S1 and M1 transfer all their momentum in the collision with, respectively L1 and M2."

Also the replies:

(S.4) "The speed of S1 is greater than that of M2, because the force of L1 is larger than the one of M1."

(S.5) "S1 knocks against L1 and goes backwards because of the target's reaction; also M1 against M2"

may be confronted with

(S.4) "The impulse of the target S1 is greater than the one of M2, because the force of L1 is larger than the one of M1."

(S.5) "Incident balls S1 and M1 return for the reactions of targets L1 and M2."

On the other side, the secondary students' few answers:

(S.1) "After the collision M2 and L1 go together, because L2, which is heavy, is dragged by M2."

(S.3) "Target L2, after the collision, remains stationary, because incident ball S1 is too small to move it".

These answers may sound similar to the multiple collision ones:

(S.1) "The incident ball L1 goes together with the intermediated ball L2 and stops together...".

"When the intermediate ball is larger than the incident one, it remains stationary and functions like a barrier so that the external target doesn't start" (S. 2, 4 and 5).

The assumption that students possess a weak conceptual structure because they do not show individual coherence (Clough 1986) seems to be in contrast with the frequent return to spontaneous ideas after a long schooling. However this contradiction may be solved displacing the element of resistance to conceptual change from an explicit conceptual network to a way of observing physical phenomena: while the scientific thought is highly coherent and general, the spontaneous reasoning seems to be strongly local and context dependent, even in using almost the same basic ideas.

*Thirdly the technical language hides spontaneous ideas.*

In the case of the speed of light is out standing the coupling of the relativistic notions of contraction and dilation of space-time in a context of absolute movement or the acceptance of different relativistic measurements with a clear interpretation of "apparent" effects.

In the case of collision is frequent the use of the ideas of action and reaction in a context of absolute causality or the interpretation of "elastic" collision as a total transfer of energy or the "conservation of momentum" as scalar conservation. This tendency appears also with secondary course and early university students (at least in the case of collision) and becomes stronger with the increase of physics schooling. If on one side this may reveal a progressive consciousness of scientific knowledge power, it can also mean that there is a mismatching between the formal language and its physical content.

The inclusion of formal language in the spontaneous mode of explanation may be seen as a smaller change condition by which two learning processes that are different both in their nature and in their genesis become compatible: the first is meaningful through the natural and living growth, while the second is formal through official teaching.

The fourth point is that there is *a kind of regression to more primitive models in the case of more complex phenomena.*

In the case of collisions the meaningful appearance of the idea of "dragging" of a ball

by another or the use of a partial elasticity only in multiple collisions.

In the case of the speed of light, the complete neglecting of the change of observer or the explicit identification of an unique movement only in the more complex questions about time.

In this way (i.e. like a regression) the results quoted in Viennot (1989) and concerning the sequential reasoning may be interpreted. In fact only 10% of last years university students analyses an electrical circuit as a sequence of events without reciprocal feedback, when the circuit contains only ohmic resistences; but when condensators appear in the circuit, the number of students using such reasoning increases (more than 37%).

The occurrence of regression casts a little more light on the question of spontaneous conceptions introducing another factor which is responsible for them: the complexity of the physical situation besides the age of students and the content.

The presence of this factor suggests a coexistence of two sets of articulated ideas which are qualitatively different. In this division the scientific framework has a field of application which slowly becomes larger and leaves the still unconquered areas to the spontaneous system. Such an equilibrium is variable with time and does not include the consciousness of the limits and the possibilities of spontaneous reasoning; therefore there is no tendency to conceptual change involving also the reinterpretation of spontaneous conceptions within the scientific conceptions.

#### GENERAL REASONING WAYS

In the common features described above it's possible to found vestiges of more deep and essential alternative reasonings roughly independent of the context and of the Physics content.

One of these ways is a tendency in interpreting the fundamental notions of motion and action *as an absolute one*. This fact leads to take velocities, traversed distances and

trajectories — characteristics of motion — as quantities independent of a reference frame and to interpret in an absolute limited way the invariance of the velocity of light or the relativistic contraction of distances. Kinetic energy and momentum are also considered absolute quantities according to the idea that "scalar" conservation means the crossing of motion from an object to another leaving the correspondent vacuum in the first. For action this tendency is expressed in the proposition that the action of a body over another depends only on the agent's characteristic, and is strongly connected to the previous one, since an absolute motion must have an absolute cause. Such idea can "assimilate" the newtonian law of action and reaction by associating the action to the mass of the incoming ball and the reaction to the characteristics of the target even in the most radical cases in which the target is considered as entirely passive and not producing any reaction. This kind of reasoning is expressed by the answer:

"In the collision between two incident balls M1 and M2 and respectively the targets L1 and M3, M3 has more velocity and energy than L1; in fact M1 colliding with L1, which is heavier, will be pushed back with *more force* than the one with which M1 pushes forward L1. The targets will receive the same impulse, because incident balls are the same, but their energies will not be the same; in fact M1 in going back will not leave his energy to L1."

The tendency to consider absolute quantities and unidirectional causal actions seems to have the same characteristics of the "sequential reasoning". For both is essential to neglect relative effects and feed-backs (Closset, 1983; Shipstone, 1984).

Another way of reasoning, that we call here *monoconceptual*, consists in a drastic simplification of the problem to find a unique meaningful variable. In collisions, most of the answers to the questions about impulse, refer to the mass of the incident ball, neglecting that of the target. The answer above and an other obtained in situation 2 for multiple collision are examples of this.

"The incident ball S1 loses more energy than S2, because it stroke the intermediate L which is larger than M."

This reasoning way may also be associated to the answers where a composed target is considered as made up of two balls maintained together.

In the problem about time in the context of relativity theory two factors would be simultaneously considered: the rocket displacement and distance contraction. However only one appears in the answers.

"The time spent by the light to travel the distance A-M is larger for the rocket than for the station because of the contraction of distances. The same occurs for the return M-A" (for Q3 and Q4).

"The distance travelled by the light going from A to M is greater for the station than for the rocket because the mirror is displaced".

Evidently the effect of displacement is more important, but mention about contraction of distance does not appear and this would contribute in an opposite sense.

The third reasoning way that we call *direct reasoning* consists in the application of a known formula without interpreting and reconstructing the results with physics meaning. In collisions the analysis of energy have considered frequently only the characteristics of the target (mass and speed). In these situations the students had correctly analysed the movements of incident balls, but they don't refer to them to solve the ambiguities.

"We don't know which target has more energy because L1 has more mass but smaller speed and the energy is  $mv^2$ " (in situation 1).

"Perhaps target S has more energy than M, because its mass is smaller but its speed is larger and speed is squared" (in situation 2).

In the problem about time in the context of relativity theory we have also some examples in answers to Q3 and Q4.

"The time for the forward motion is smaller for the rocket; that for backward motion is smaller for the station. For the total time I don't know".

"The time for the forward motion is  $L/c$  for the rocket but for the station I don't



know. Distance increases for the mirror's motion and decreases for the contraction effect".

In the first example the idea that the shorter time for going and return of the light is referred to the framework of the rocket does not appear. In the second one, the increase of distance must be classically considered and the contraction only relativistically.

This reasoning way was also found by Lawson (1987) in research about work-kinetic energy.

Finally there is a widely used reasoning way in situations where opposite effects are present: *compensation reasoning*.

In this case instead of considering the problem of knowing which is the dominant factor, students anticipate a (total or partial) compensation of the effects. In simple collisions, question Q3 about the target energy has a typical answer.

"The targets L1 and M1 will have the same energy, because the mass of L1 is larger, but the speed of M1 is larger and there is a compensation of the two effects; it's the same for the targets S1 and M1..." (in situations 1 and 2).

In multiple collisions this reasoning is extended for other questions.

"The lack of energy of the incident balls M1 and M2 is the same, because the intermediate ball L1 is smaller than M3, but the external ball S1 is smaller than M4 and the two effects are compensated; it's analogous for the incident balls S" (for situations 4 and 5).

For the problems about the speed of light this reasoning appears in questions Q3 and Q4 of the problem about time.

"On coming backwards, light spends a longer time for the rocket, to compensate the invariance of total time."

"For the rocket, light spends a shorter time forward because of the contraction of distances and a longer time backwards because of dilatation of distances in opposite direction to the motion. But the two effects compensate."

"For the rocket, light spends a longer time forwards and shorter backwards. But the total is the same as for the station."

Some original ideas express the persistence of this way of reasoning:

"Time passes more slowly in the rocket, but the distance traversed is shorter; this compensates for the fact that time passes faster for the station for which the distance is longer."

This way of reasoning appears also in the work of Lawson (1987) when the linear momentum of the pucks are confronted and, for the students, the differences between masses compensate the ones between velocities.

## FINAL REMARKS

All these occurrences are indications that the spontaneous notions remain in a latent state in graduate students minds, ready to reappear when the use of formal systems is less immediate and when students do not feel sure. The number of students which have been interviewed, their curricula and their familiarity with research do not leave any doubt that is not an accident. The similarity between such ideas and those presented by secondary course or early university students, the attempt to articulate in a "quasi" coherent statement and, especially, the strategy of adapting the new acquired notions to the former conceptual framework, besides being an evident sign of the deep roots of this type of conceptual construction, is also an evident proof of the inadequate formation in Physics.

Furthermore this inadequacy bears a specific characterization: it is not the lack of the mathematical instruments, since many of the students were working with theoretical Physics and nearly all had gone through rather difficult graduate courses (where basic training involves the solution of problems which are often complicated from the formal point of view). The origin of difficulties seems to be two fold: one specifically related to the physical content and the other generally related to the ways of reasoning. Teaching should provide some basic ideas which might articulate easily rough models, more compatible with the taught theories rather than spontaneous ideas.

The teaching of Special Relativity Theory starts by supposing that students master completely Galilean Relativity. The students probably master the transform formulas, but not their spirit and especially its incompatibility with the notion of absolute reference frame and absolute motion independent from the observers.

The lack of this Galilean-relativistic intuition appeared also in the tests on collisions; very few students used a change of reference to analyse the collision between balls to find the similarity between some of the presented situations; furthermore the very notion of action and reaction, each associated to one ball was completely fit in a context of absolute value, with the attribution of forces depending of the mass of each ball.

Similarly the teaching of elastic collisions starts by assuming that students master conservation of energy and momentum. However they misunderstand the two concepts and especially tend to dissociate conservation and reaction, hence the difficulty in working with the symmetry of this principle and in admitting the essentially vectorial character of the conservation of momentum.

These reasoning ways are constructed by individuals along their life but an important part of responsibility is in school teaching. The frequent use of compensation arguments independently of their exactitude; a rapid selection of more important variables, specially when they reduce to an only variable, releasing other possibilities without discussing the importance of the effects; the sistematic use of linear examples releasing interactive feedbacks. These situations seem to favour these reasoning ways (Viennot 1989) instead of correcting them and show that the conceptual change subject needs a big effort and a long time to be investigated and overcome.

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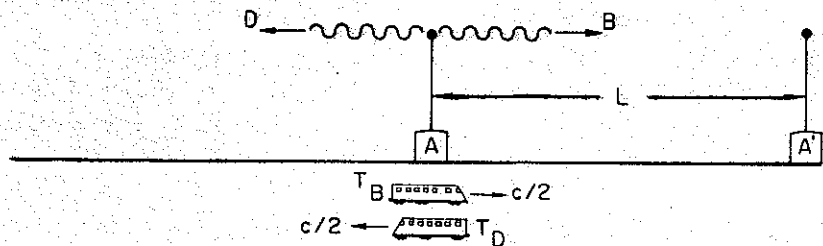
## APPENDIX A — PROBLEMS ABOUT SPEED OF LIGHT

### Problem on the distance

At the moment at which the two trains  $T_B$  and  $T_D$  which travel with speed  $v = c/2$  in opposite directions, cross the antenna A, it emits two light signals B and D in opposite directions. Train  $T_B$  and the signal B have the same direction and so have  $T_D$  and D.

### Questions

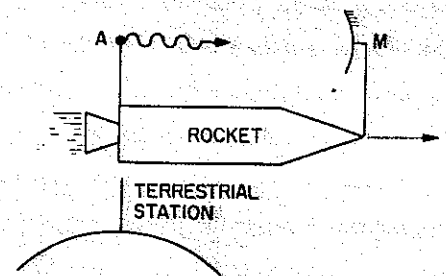
- Q1. At what distance from A will the two trains  $T_B$  and  $T_D$  and the signal D each be, when B crosses an antenna A' that is at distance L from A? Why?
- Q2. Suppose you are a passenger in  $T_B$ . For you, when B reaches antenna A, which of the two light signals will be closer to you? Why?



- Q3. Suppose you are a passenger in  $T_D$ . For you, when B reaches the antenna A, which of the two light signals will be closer to you? Why?
- Q4. Are your answers to the previous questions consistent with the invariance of the speed of light? Justify your answer.

### Problem concerning time

A rocket of length L is provided with an antenna (A) in its rear part and with a mirror (M) in the front part. When the rocket, which travels at a speed which is half the speed of light, passes over a terrestrial station, antenna A emits a light signal which, after reaching mirror (M), is reflected and returns to the antenna (A) where it is absorbed.



### Questions

- Q1. For somebody in the rocket, does the light signal spend a longer time to go from the antenna (A) to the mirror (M) or to return from the mirror (M) to the antenna (A)? Why?
- Q2. For somebody at the terrestrial station, does the light signal spend a longer time to go from the antenna (A) to the mirror (M) or to return from the mirror (M) to the antenna (A)? Why?
- Q3. Does the light signal spend a longer time for traversing the distance A-M for somebody in the rocket or for somebody in the station? Why?
- Q4. Does the light signal spend a longer time for traversing the distance M-A for somebody in the rocket or for somebody in the station? Why?
- Q5. Draw a diagram showing the light path from the antenna to the mirror and return, as seen by somebody in the rocket. Explain your drawing.
- Q6. Draw a diagram showing the light path from the antenna to the mirror and return, as seen by somebody in the terrestrial station. Explain your drawing.

## APPENDIX B — PROBLEMS ABOUT COLLISIONS

## Simple collision

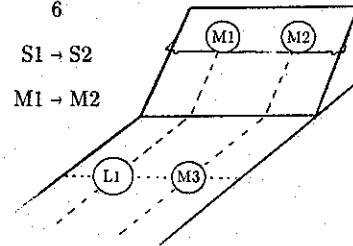
In a collision experiment there are two balls on a slope and two on the horizontal plane. The balls are all made up of the same hard material like billiard balls and may be of different sizes: small (S), medium (M) and large (L). Let us assume for simplicity that the mass of the small ball is half the mass of the medium and the mass of this is half the large ones. The balls on the slope are released from the same height and at the same time, following the broken line.

## Questions

- Q1. L1 reaches longer or shorter than M3? Why?  
 Q2. What happens with balls M1 and M2 after the shock with L1 and M3 respectively? Why?  
 Q3. After the shock L1 will have a larger or smaller energy than M3? Why?  
 Q4. M1 will impart an impulse to L1 which is larger or smaller than the impulse of M2 to M3? Why?

The same questions are put for five other situations.

1	2	3	4	5	6
M1 → L1	M1 → S1	L1 → S2	L1 → S1	S1 → L1	S1 → S2
M2 → M3	M2 → M3	S1 → L2	M1 → M2	M1 → M2	M1 → M2



## Multiple collision

In an experiment on multiple collisions there are two balls on the slope and four on the horizontal plane. The balls are the same that were used in the simple collision experiment.

## Questions

- Q1. Which of the balls S1 and M1 goes farther? Why?  
 Q2. What happens with L1, L2, L3 and L4 after the collision? Why?  
 Q3. Which of the external balls S1 and M1 has a higher energy after the collision? Why?  
 Q4. Which of the incident balls L1 and L3 loses more energy in the collision? Why?  
 Q5. Which of the incident balls L1 and L3 imparts a larger impulse to the target during the collision? Why?

The same questions are put for five situations

1	2	3	4	5
L1 → L2M1	S1 → L1M1	M1 → M2S1	M1 → L1S1	S1 → M1L2
L3 → L4S1	S2 → M2M3	L1 → M3S2	M2 → M3M4	S2 → L1M2

