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B.V. Carlson
Divisão de Física Teórica, Instituto de Estudos Avançados
Centro Técnico Aeroespacial,
12200 São José dos Campos, S.P., Brazil

M.S. Hussein
Instituto de Física, Universidade de São Paulo

R.C. Mastroleo
Divisão de Física Teórica, Instituto de Estudos Avançados
Centro Técnico Aeroespacial,
12200 São José dos Campos, S.P., Brazil

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B.V. Carlson*, **M.S. Hussein**** and **R.C. Mastroleo***

*Divisão de Física Teórica, Instituto de Estudos Avançados
Centro Técnico Aeroespacial
12200 São José dos Campos, S.P., Brazil

**Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, S.P., Brazil

ABSTRACT

A model for the yield of projectile–like fragments in intermediate and high energy heavy–ion reactions is developed within the Hussein–McVoy formalism. The primary yield of the spectator particle b is found to have the simple form $P_b = \frac{\pi}{\sigma^2} \sum_{j=0}^{\infty} (1-\hat{T}_j^b(\sigma))\hat{T}_j^x(\sigma)$ where σ is the width of the Gaussian wave function describing intrinsic motion of the projectile a, $x = a-b$ is the participant particle and the $\hat{T}$,s are Fermi motion–modified transmission coefficients. Reasonable account of the projectile–like fragment production, considered as a secondary yield, in the reaction $^{16}O + ^{208}Pb$ at 20 MeV/A and 2 GeV/A is obtained when further geometrical restriction on P_b is imposed and b is allowed to decay statistically.

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Fragment emission in heavy ion reactions at intermediate and higher energies is a common occurrence[1]. Several models have been devised to calculate the cross–section. These range from statistical[2], microscopic Monte Carlo[3] and others. A fully quantum mechanical treatment based on general reaction theory concepts is still lacking. Recently[4),5)], we have developed a general reaction theory of nuclear fragmentation and applied it successfully to several reactions involving a partial fusion of the projectile (incomplete fusion, IF). In these calculations, the inclusive spectator particle angular and energy distributions is usually presented. It is the purpose of this paper to present a theory of the yield of spectator particles in these reactions.

To be specific we consider the following process

$$a + A \equiv (b+x) + A \longrightarrow b + \sum_{\substack{\text{all}\\ \text{states}}} (x+A) \quad . \qquad (1)$$

In the spectator–DWBA treatment of Hussein and McVoy [4], the yield of b is given by

$$P_b = \int \frac{d^2\sigma}{d\Omega_b\, dE_b}\, d\Omega_b\, dE_b \quad , \qquad (2)$$

with

$$\frac{d^2\sigma}{d\Omega_b\, dE_b} = \frac{2}{\hbar v_a} \rho(E_b) <\hat{\psi}_x^{(+)}| W_{xA} |\hat{\psi}_x^{(+)}> \qquad (3)$$

where $\rho(E_b)$ is the density of states of b and is equal to $\frac{\mu_b k_b}{(2\pi)^3\, \hbar^2}$, v_a is the velocity of the projectile and $\hat{\psi}_x^{(+)}$ is given by

$$|\hat{\psi}_x^{(+)}> = (\chi_b^{(-)}| \phi_a\, \chi_a^{(+)}> \quad . \qquad (4)$$

In (4) the χ,s refer to distorted waves, and ϕ_a is the intrinsic wave function of a . The symbol $\langle\,|\,\rangle$ implies that the b coordinates are integrated over. Finally W_{xA} is the imaginary part of the x–A optical potential. Note that Eq. (3) describes the inclusive inelastic break–up. The elastic break–up piece of $\frac{d^2\sigma}{d\Omega_b\, dE_b}$ is known to contribute less than 10% at moderate and higher energies [6].

Employing the Glauber approximation for the distorted waves allows writing the matrix element $\langle\hat{\psi}_x^{(+)}|W_{xA}|\hat{\psi}_x^{(+)}\rangle$ as a function of the momentum transfer $\vec{q}_b = \vec{k}_b - \vec{k}_b'$, where $\vec{k}_b$ ($\vec{k}_b'$) is the wave number of b in the incident (final) channel. Thus transforming the energy and angle integral in Eq. (1) into a momentum transfer integral allows the reduction to the following transparent form of P_b

$$P_b = \frac{2}{v_a}\frac{E_x}{\hbar k_x}\sqrt{\frac{\pi}{\sigma^2}}\int d\vec{b}_b |S_b(b_b)|^2 \int d\vec{b}_x |\phi_a(|\vec{b}_b - \vec{b}_x|)|^2 \left[1 - |S_x(b_x)|^2\right] \qquad (5)$$

where b refers to the impact parameter, and S(b) the elastic element of the S–matrix. In obtaining (5) a normalised Gaussian is used for $\phi_a(\vec{r}_b - \vec{r}_x)$, which when cylindrical coordinates are used $\vec{r} = z\hat{z} + b\hat{b}$, can be expressed as

$$\phi_a(|\vec{r}_b - \vec{r}_x|) = \left[\frac{\sigma^3}{\pi\sqrt{\pi}}\right]^{1/2} \exp\left[-\frac{\sigma^2}{2}(z_b - z_x)^2\right] \exp\left[-\frac{\sigma^2}{2}(\vec{b}_b - \vec{b}_x)^2\right] \quad . \qquad (6)$$

The z_b and z_x integrals in the original formulas Eqs. (1) and (2) has been already performed in (5).

A further reduction of Eq. (5) can be made by integrating over the polar angles of $\vec{b}_b$ and $\vec{b}_x$, which results in the following simple formula, after writing $\frac{E_x}{\hbar k_x} = v_x = v_a$,

$$P_b = \frac{\sigma^2}{\pi}\int_0^\infty b_b db_b |S_b(b_b)|^2 e^{-\sigma^2 b_b^2} \int_0^\infty b_x db_x \left[1 - |S_x(b_x)|^2\right] e^{-\sigma^2 b_x^2} \left[4\pi^2 J_0(-2i\sigma^2 b_b b_x)\right] \qquad (7)$$

where J_0 is the cylinder Bessel function. The factor $4\pi^2 J_0(-2i\sigma^2 b_b b_x)$ was obtained from the relation $\int_0^{2\pi} d\phi \int_0^{2\pi} d\beta\, e^{2\sigma^2 b_x b_x \cos(\beta-\phi)} = 4\pi^2 J_0(-2i\sigma^2 b_x b_b)$ [7]. To proceed further we use the following expansion of J_0 [7]

$$J_0(z) = \sum_{j=0}^{\infty} (-)^j \frac{z^{2j}}{(2)^{2j}(j!)^2} \quad . \qquad (8)$$

With (8), Eq. (7) can be finally written in the following simple form

$$P_b = \frac{\pi}{\sigma^2}\sum_{j=0}^{\infty}\left[1 - \hat{T}_j^b(\sigma)\right]\hat{T}_j^x(\sigma) \quad , \qquad (9)$$

where we have introduced the symbol $\hat{T}$ to represent the following dimensionless integral

$$\hat{T}_j^i(\sigma) = \frac{(\sigma^2)^{j+1}}{j!}\int_0^\infty b_i^{2j} db_i^2\, e^{-\sigma^2 b_i^2}\, T_i(b_i) \quad , \qquad (10)$$

where $T_i(b_i)$ is the transmission coefficient $1 - |S_i(b_i)|^2$ and i being either b or x . It is important to note that $\hat{T}$, Eq. (10) becomes unity in the limit $|S|=1$.

Eq. (9) is an important result of this paper. It expresses the projectile–like primary fragment yield in a conventional total reaction cross section form, with the width σ^2 playing the role of the squared wave number, and the label j playing the role of angular momentum. It is easy to verify that Eq. (9) reduces to the known limiting cases. To see this we first note the integral $\hat{T}^i_j(\sigma)$ as a function of j looks very much like $T(b) \equiv 1-|S(b)|^2$ vs. b. Therefore $(1-\hat{T}^b_j(\sigma))\hat{T}^x_j(\sigma)$ should be a localized (window) in j. The value of j at which the "peaking" occurs can be estimated as follows. The function $b^{2j} e^{-\sigma^2 b^2}$ peaks at $b^2 = \frac{j}{\sigma^2}$ and the b integral in $\hat{T}$ counts this peak as long as $b < b_g = \sqrt{\frac{E-E_B}{E}}\, R$, where E_B is the height and R the position of the Coulomb barrier characterizing $T(b)$. Thus the critical value of j which defines the boundary of the function $\hat{T}$ is

$$j_c = \sigma^2 \frac{E - E_B}{E} R^2 \quad . \tag{11}$$

The function $1-\hat{T}$ behaves in exactly the opposite way to $\hat{T}$ and thus $(1-\hat{T}^b_j)\hat{T}^x_j$ has a window shape in j. The detailed form of this window depends upon the physical parameters that specify the functions $\hat{T}^b_j(\sigma)$ and $\hat{T}^x_j(\sigma)$.

An interesting case which received several considerations in the past is the Serber model[8], $S_b=1$. This limit corresponds to setting $1-\hat{T}^b_j(\sigma) = 1$ in our formula Eq. (10). Approximating $\hat{T}^x_j(\sigma)$ by a sharp cutoff from $\Theta(j_c-j)$, we obtain immediately for the yield

$$P_b^{\text{Serber}} \simeq \frac{\pi}{\sigma^2} \sigma^2 \frac{E - E_B}{E} R_x^2$$

$$= \pi R \frac{E_x - E_B}{E_x} = \sigma^x_{\text{Rea}} \tag{12}$$

which is just the total reaction cross section of the participant particle (x).

The more realistic expression for P_b, Eq. (9), gives, in the sharp cutoff limit, the following sum rule

$$P_b \simeq \frac{\pi}{\sigma^2}(j^x_c - j^b_c) \simeq \pi\left[\left[1 - \frac{E^x_B}{E_x}\right] R_x^2 - \left[1 - \frac{E^b_B}{E_b}\right] R_b^2\right] = \sigma^x_{\text{Rea}} - \sigma^b_{\text{Rea}} \tag{13}$$

which can be orders of magnitude smaller than P_b^{Serber}. Of course the energies E_x and E_b correspond to the incident channel, $E_x = \frac{m_x}{m_a} E_a$ and $E_b = \frac{m_b}{m_a} E_a$, and in the above equation $\sigma^x_{\text{Rea}} > \sigma^b_{\text{Rea}}$. Clearly when x is the light particle and b, the heavy one, the above approximation is not valid and one has to calculate Eq. (9) exactly.

In the calculation of P_b described below we used for the T,s the Karol soft–sphere model[9], and for σ the Goldhaber model[10].

Confronting calculations of the primary yield with experimental data, [11,12], we found the calculation to predict, for a fixed final charge Z_b, a yield which increases as the mass, A, decreases. The data, on the contrary, usually peak at the most bound isotope similar to the yields normally observed after statistical emission.

This suggests that the spectator does not play a completely inert role in the reaction but that its excitation and subsequent decay must be taken into account.

This can be done by extending our expression for the yield to one of the following form,

$$Y_b = \sum_{b'} \int d\epsilon\, P_{b'}(\epsilon) \frac{\Gamma(b'\epsilon \to b)}{\Gamma(b'\epsilon)} \tag{14}$$

where $\Gamma(b'\epsilon \to b)$ is the partial width for primary fragment b' at excitation energy to decay to b, $\Gamma(b',\epsilon)$ is the total decay width, $P_{b'}(\epsilon)$ is the primary yield of b' and Y_b is the observed secondary yield. Note that although we have taken into account the possible dependence on the excitation energy of the primary yield $P_b(\epsilon)$, we have neglected any dependence on the angular momentum.

As a first approximation, we have taken the energy dependence of the primary yield to have the form

$$P_{b'}(\epsilon) = P_{b'}\omega_{b'}(\epsilon)/N_{b'} \tag{15}$$

where $\omega_{b'}(\epsilon)$ is the density of states obtained by removing in all possible manners Z_x protons and A_x–Z_x neutrons from the projectile. The total number of states, $N_{b'}$, is given by

$$N_{b'} = \int d\epsilon \, \omega_{b'}(\epsilon) \tag{16}$$

so that $P_{b'}$ is just the total primary yield,

$$P_{b'} = \int d\epsilon \, P_{b'}(\epsilon) \tag{17}$$

In calculating $W_{b'}(\epsilon)$ we have not renormalized the single particle energies to the final fragment ones but have simply used their values in the projectile.

The yield calculation was then repeated, using the total primary yield given above and statistical branching ratios obtained within the Weisskopf–Ewing multiple emission model. Although the isotopic distributions showed clear improvement, the calculation continued to systematically overestimate the yield for lower masses and underestimate it for larger ones. As the remaining systematic differences appeared to be primarily a function of the fragment mass, we suspected that it could be a geometrical effect of the form assumed for the fragment (b + x) decomposition in the primary yield calculation. In fact the limiting case, Eq.(13) already points to the need for a more careful analysis of the geometrical content of our P_b. Namely complete absorption of x by the target should be just as likely to occur whether x is smaller or larger than b.

We can trace this problem to the simple form assumed for the intrinsic projectile wave function in Eqs. (5) and (6), where it was taken to consist of an inert x and b in a

simple s state of relative motion. This would seem a reasonable approximation in the case of elastic breakup, but it is in no way implicit in the general expression for the x wave function given in Eq. (4) nor is it consistent with one assumption of the excitation of b. Indeed to be consistent with the latter, the x wave function would have to be a sum involving a multiplicity of excitations and relative motions.

Instead of implementing the above modifications in Eqs. (4) and (5), we have opted for the almost equivalent abrasion model. To get a feel for what to expect from this model, we have begun by performing calculations based on its simplest geometrical form. We take the total primary yield for mass A_b to be

$$\frac{d\sigma}{dA_b} = 2\pi \, b(A_b) \frac{db}{dA_b} \tag{18}$$

where the impact parameter, $b(A_b)$, and its derivative are determined by the abrasion equation

$$\frac{A_b}{A_p} = \frac{V_{p\Lambda\text{-}t}}{V_p} \approx \frac{1}{2R_p^3}\left[R_p^3 - R_t^3 + \frac{3}{8}\frac{(R_t^2 - R_p^2)^2}{b} + \frac{3}{4}(R_p^2 + R_t^2)b - \frac{b^3}{8}\right] \tag{19}$$

which relates the mass A_b to the fraction of the projectile volume which does not interact with the target, $V_{p\Lambda\text{-}t}$. To determine the isotopic primary yield, we assume the distribution of neutrons and protons in the fragment b to be combinatorial, yielding

$$A_b = \frac{d\sigma}{dA_b} \cdot p\,(Z_b \cdot N_b | A_b) \tag{20}$$

where

$$p\,(Z_b \cdot N_b | A_b) = \begin{bmatrix} Z_p \\ Z_b \end{bmatrix} \begin{bmatrix} N_p \\ N_b \end{bmatrix} / \begin{bmatrix} A_p \\ A_b \end{bmatrix} \tag{21}$$

Repeating the calculation using this expression for the primary yield as well as the densities of excited states and Weisskopf–Ewing branching ratios discussed earlier, we now obtain very good agreement with the experimental data. The results obtained for $^{16}O + ^{208}Pb$ at 20 MeV/A and 2 Gev/A are shown in Figures 1 and 2, respectively.

In conclusion, we have demonstrated in this paper that the projectile fragment yield in heavy ion reactions can be described by a fragmentation/excitation process followed by statistical emission. Satisfactory agreement with the experimental data was obtained at both intermediate and relativistic energies. Our results clearly point out the need of a more general fragmentation theory which goes beyond the spectator assumption by including the possibility of fragment excitation.

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FIGURE CAPTIONS

Figure 1. Calculated secondary yields open circles in the reaction $^{16}O + ^{208}Pb$ at $E_{lab.} = 20$ MeV/A of, a) Lithium, Berilium and Boron, and b) Carbon and Nitrogen. The data of Ref. 11 are shown as full circles.

Figure 2. Same as Figure 1 at $E_{lab.} = 20$ GeV/A also shown is the yield of Oxygen. The data points (full circles) from reference 12.

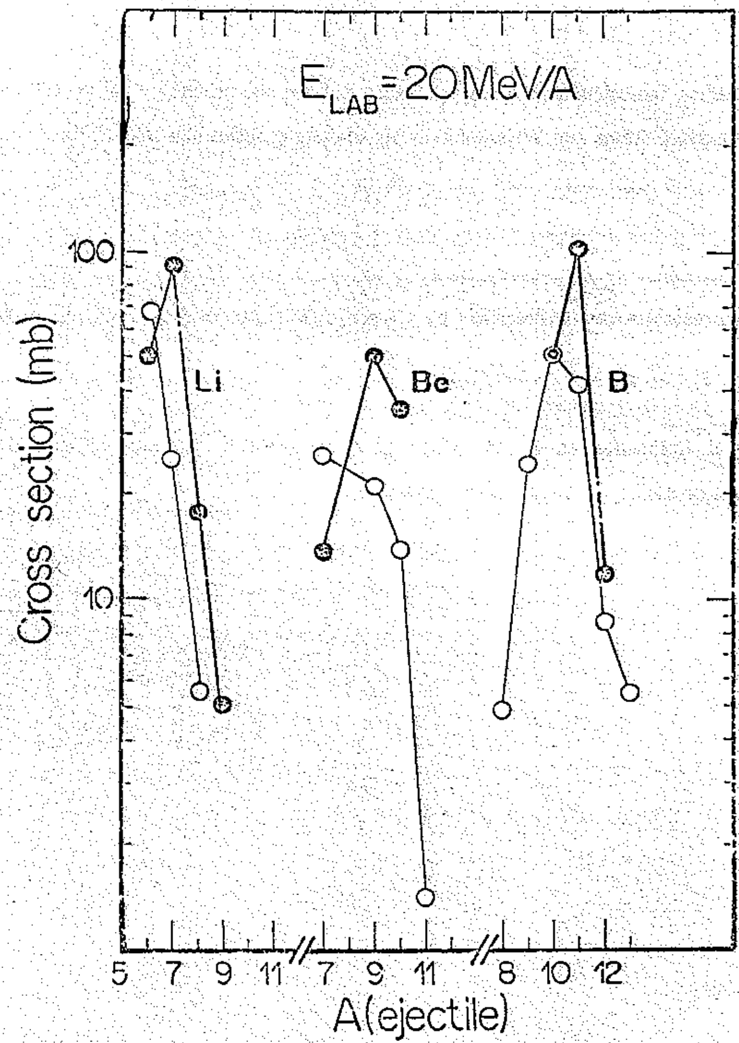


Fig. 1a

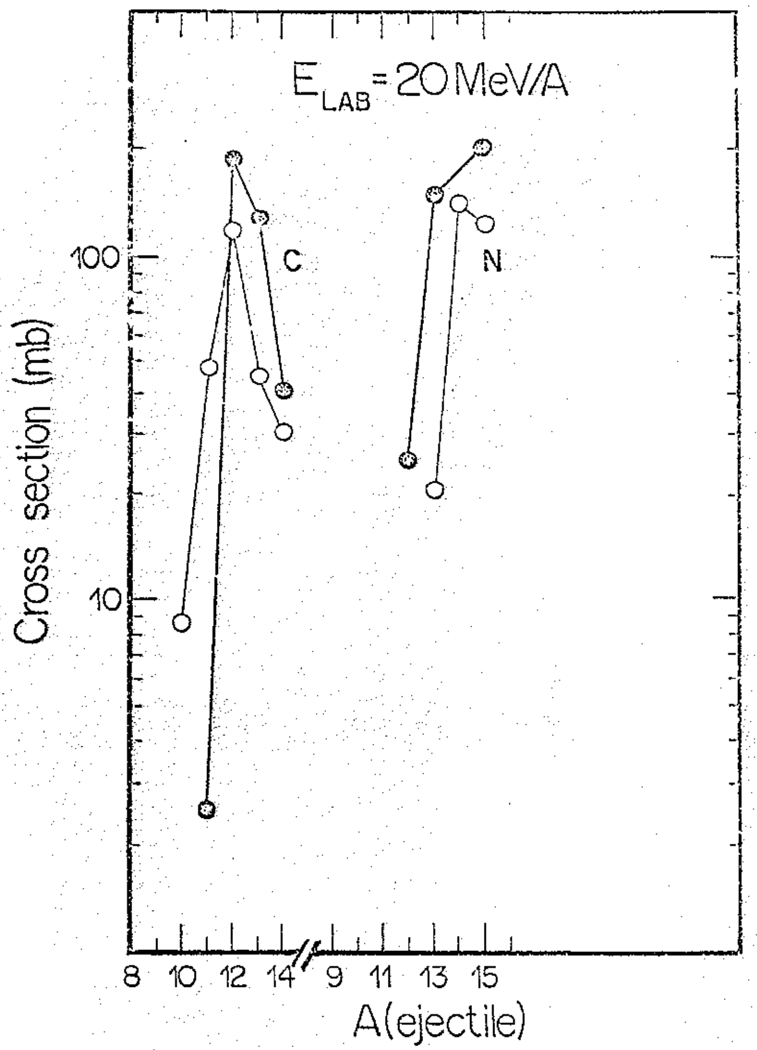

E_LAB = 20 MeV/A
Cross section (mb)
100
10
C
N
8
10
12
14
9
11
13
15
A(ejectile)

Fig. 1b

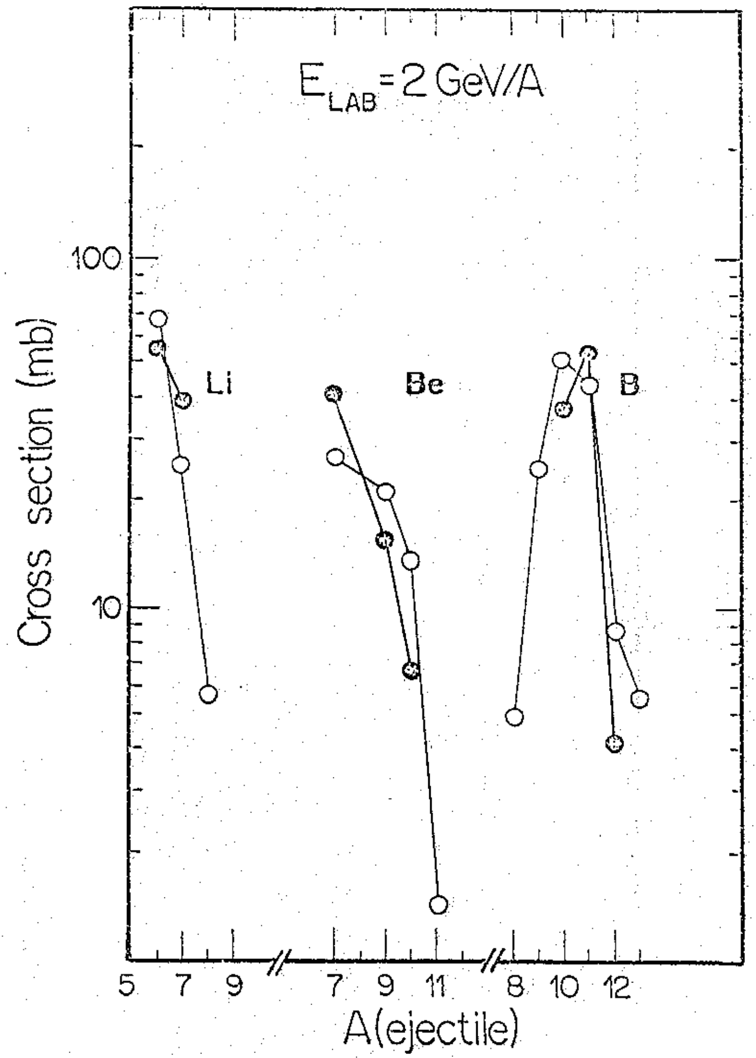
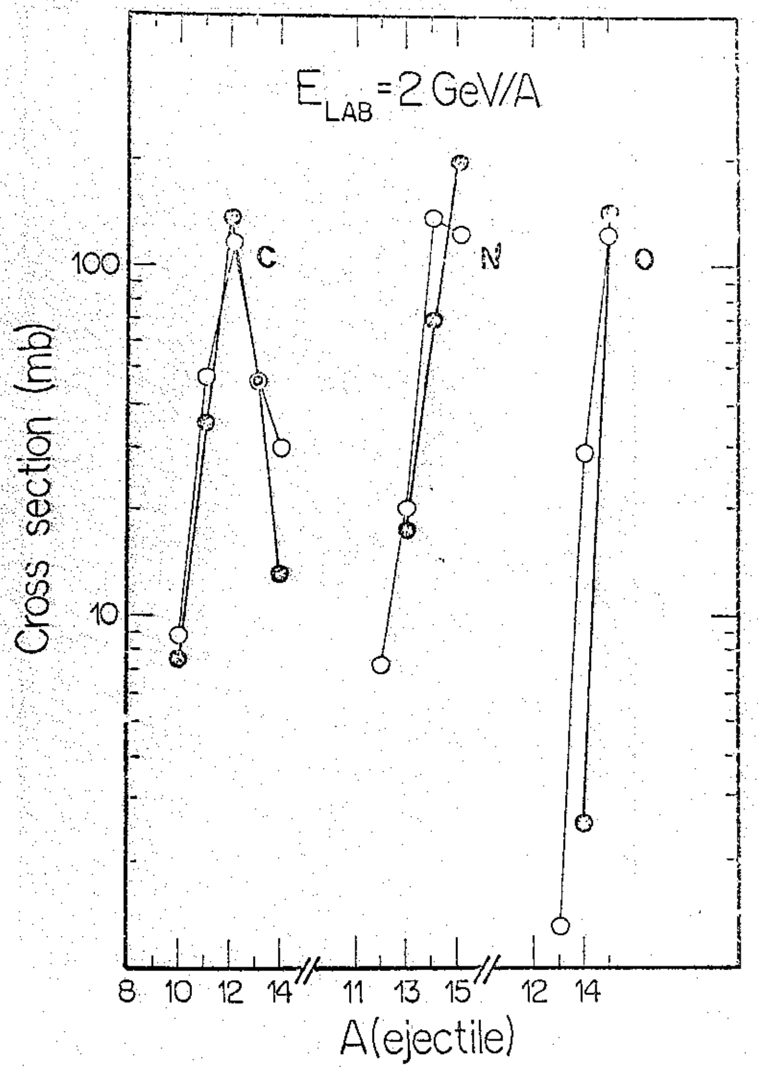

E_LAB = 2 GeV/A
Cross section (mb)
100
10
Li
Be
B
5
7
9
7
9
11
8
10
12
A(ejectile)

Fig. 2a



E_{LAB}=2 GeV/A
Cross section (mb)
100
10
C
N
O
8
10
12
14
11
13
15
12
14
A(ejectile)

Fig. 2b